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Application of the Firefly Algorithm to Optimal Operation of Reservoirs with the Purpose of Irrigation Supply and Hydropower Production

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Abstract: Population growth and socioeconomic changes in developing countries over the past few decades have created severe stresses on the available water resources across the world, particularly in arid and semiarid regions, which are predominant in Iran. Hence, the optimal management of water resources is imperative. Reservoir operation is a challenging problem that involves complexities in terms of nonlinear functions, larger numbers of decision variables, and multiple constraints. Evolutionary or metaheuristic algorithms have become an attractive alternative to the classical methods for solving complex reservoir problems. This paper applies a metaheuristic algorithm named the firefly algorithm (FA) to reservoir operation and demonstrates the superiority of this algorithm against the genetic algorithm (GA), a commonly used optimization algorithm, using (1) five mathematical test functions, (2) the operation of a reservoir system with the purpose of irrigation supply, and (3) the operation of a reservoir system with the purpose of hydropower production. The results demonstrate the superior performance of the FA in terms of the convergence rate to global optima and of the variance of the results about global optima when compared with the results of the GA. **DOI: 10.1061/(ASCE)IR.1943-4774.0001064.** *© 2016 American Society of Civil Engineers.*

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Introduction

Among the various recent studies dealing with newly developed optimization algorithms in several fields of water resources systems analysis, such as reservoir operation (Ashofteh et al. 2013a; Ahmadi et al. 2014; Bolouri-Yazdeli et al. 2014; Ashofteh et al. 2015a), groundwater resources (Fallah-Mehdipour 2013a; Bozorg-Haddad et al. 2013), conjunctive use operation (Fallah-Mehdipour 2013a), design operation of pumped storage and hydropower systems (Bozorg-Haddad et al. 2014a), flood management (Bozorg-Haddad et al. 2015b), water project management (Orouji et al. 2014), hydrology (Ashofteh et al. 2013b), qualitative management of water resources systems (Orouji et al. 2013; Shokri et al. 2014; Bozorg-Haddad et al. 2015a), water distribution systems (Soltanjalili et al. 2013; Seifollahi-Aghmiuni et al. 2013; Beygi et al. 2014), agricultural crops (Ashofteh et al. 2014), sedimentation (Shokri et al. 2013), and algorithmic developments (Ashofteh et al. 2015b), none has focused on the application of the firefly algorithm (FA) to the optimal operation of reservoir systems with the purposes of irrigation supply and hydropower production.

Optimization methods are classified in two major groups, named classic algorithms and evolutionary or metaheuristic algorithms (EAs). Some of the classic algorithms are linear programming (LP), nonlinear programming (NLP), and dynamic programming (DP), which have been widely applied to water resources optimization problems. However, various limitations of the classic optimization methods encouraged researchers to use EAs, which do not have the typical shortcomings of classic algorithms. Some of the EAs include the genetic algorithm (GA) (Holland 1975), the simulated annealing algorithm (SA) (Kirkpatrick et al. 1983), ant colony optimization algorithm (ACO) (Dorigo 1992), the differential evolution algorithm (DE) (Storn and Price 1995), particle swarm optimization algorithm (PSO) (Kennedy and Eberhart 1995), the honeybee mating optimization algorithm (HBMO) (Bozorg-Haddad et al. 2006), the intelligent water drops algorithm (IWD) (Shah-Hosseini 2007), the imperialist competitive algorithm (ICA) (Atashpaz-Gargari and Lucas 2007), the cuckoo search algorithm (CS) (Yang and Deb 2009), and the water cycle algorithm (WCA) (Eskandar et al. 2012). The application of some of the aforementioned algorithms to reservoir operation is summarized next.

Tospornsampan et al. (2005) proposed SA for the operation of a 10-reservoir system by maximizing the total efficiency of producing hydropower energy during 12 periods of operation. Results showed the better performance of SA over GA. Jothiprakash and Shanthi (2006) used GA to develop optimal operation rules of a reservoir system in India. The objective function of this study was to minimize the sum of the annual squared differences between

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the calculated release and the designed irrigation release. Results showed the superiority of using the GA solution compared with the actual operation program. Jalali et al. (2007) introduced a multicolony ant algorithm (MCAA) to solve for the operation of a 10-reservoir system by maximizing the total efficiency of producing hydropower energy during 12 periods of operation. The MCAA performed better than the continuous differential dynamic programming (CDDP) method employed by Murray and Yakowitz (1979) to solve the same problem. However, the MCAA performance was inferior to the GA used by Wardlaw and Sharif (1999) for the same problem. In some cases, classical methods fail to solve specific optimization problems due to their complexity. Thus, EAs become an alternative option. For example, Bozorg-Haddad et al. (2008) proposed the HBMO algorithm to maximize the hydropower production in a multireservoir system. The NLP method was unable to achieve a solution to the problem under consideration. Yin and Liu (2009) employed the DE optimization algorithm to maximize hydropower production in a reservoir system. Their results were superior to those calculated by DP from the standpoint of computational efforts. More recently, Ghimire and Reddy (2013) used PSO to calculate optimal operation policies of a hydropower reservoir system by minimizing the sum of the annual squared differences between produced and target hydropower energy. Their policies increased hydropower production by 3% when compared with the energy produced by a conventional operation program.

Ashraf Vaghefi et al. (2012) introduced the ICA in the water resources field to optimize the operation of the Sefidroud Reservoir with the objective of controlling reservoir sediments. The studies of Bozorg-Haddad et al. (2014b, c) exemplify the successful application of newly developed EAs to optimal hydropower production.

The FA was introduced by Yang (2008). Yang (2009) applied the FA to solve 10 multiobjective optimization test problems whose results compared favorably with the GA and the PSO algorithm. Yang (2010) merged the levy flight (LF) approach searching with the FA and solved once more the 10 multiobjective optimization test problems by applying the proposed hybrid algorithm. The results indicated that the success rate of FA with levy flight LF algorithm was better than that of the standard FA. Yang (2011) used chaos for autotuning of the parameters of the algorithm. The results of the cited study compared favorably with those of the standard FA for the well-known problem of the welded beam. Yan et al. (2012) developed the adaptive FA (AFA) to cope with large-dimensionality optimization problems. The latter authors showed that the AFA performed better with the 10 test problems than the standard FA, DE, and PSO algorithms. Many studies have been devoted to improving the searching accuracy of the FA and have shown its better convergence rate than other algorithms. The advantage of FA from the standpoint of speed of convergence has led to its adoption in solving complex and nonlinear problems in different scientific fields. In this context, the study of Abdullah et al. (2012) is noteworthy because they introduced a new hybrid FA named hybrid evolutionary FA (HEFA) in order to improve the searching accuracy. This approach was a combination of the FA and the DE algorithm with the goal of estimating the parameters of a nonlinear and complex biological model of large dimensionality. The results showed that HEFA has an improved searching accuracy compared with the GA, the PSO algorithm, and evolutionary programming (EP).

Santos et al. (2013) calculated the amount of precipitation of a region in South America. They computed the precipitation using six different methods. In each of these methods, different effective parameters were used to calculate the precipitation. The FA was applied to find the optimal weights for the various methods. In a comprehensive review of the FA, Fister et al. (2013) concluded that the FA's solving efficiency is explained by its capacity to solve multimodal, nonlinear optimization problems. The FA is a generalization of SA, PSO, and DE and has been proven to be an efficient optimization tool in various fields of engineering.

The aim of this study is to implement the FA for solving two real reservoir operation problems with the purposes of irrigation supply and hydropower production. The FA was first employed herein to solve five mathematical test functions and its superior performance was confirmed comparing it with the GA, arguably the most commonly used algorithm in many fields of optimization. Thereafter, the performance of the FA was evaluated by solving two real reservoir operation problems with the purposes of (1) irrigation supply, and (2) hydropower production. The former case study's objective function and constraints are nonlinear and complex. The latter case study has a more complex simulation structure and a larger operational period than the first case study. One reason for evaluating the FA with hydropower optimization is the increased demand for non-greenhouse-gas-emitting technology. The results calculated with the FA were compared with the results of NLP and the GA to assess its relative efficiency and effectiveness in solving reservoir operation problems.

Methodology

This section is divided into five subsections. "Mathematical Test Functions" introduces the mathematical test functions used to validate the FA. "Reservoir Operation Model with Irrigation Supply Purpose" and "Reservoir Operation Model with Hydropower Production Purpose" present different reservoir operation models. The former is related to irrigation supply purpose and the latter is associated with hydropower production purpose. Thereafter, "Firefly Algorithm" explains the FA and its optimization process. Finally, "Penalty Functions" introduces the approach of penalty functions, which are mainly used in the constrained optimization problems.

Mathematical Test Functions

Several test functions were chosen to test the FA. These test functions are (1) sphere, introduced by De Jong (1975), which is the simplest of De Jong's functions, and (2) Ackley reported (Ackley 1987), Styblinski-Tang reported by Styblinski and Tang (1990), Rosenbrock defined by Rosenbrock (1960), and the Holder table function. Table 1 shows the details of these test functions, where $f(x_1, \ldots, x_d)$ = objective function for x_1 to x_d decision variables; di = counter for dimension; and d = total number of dimensions.

Reservoir Operation Model with Irrigation Supply Purpose

The objective function of the reservoir operation model whose purpose is irrigation supply and is expressed as follows:

$$\text{Minimize OF}_{\text{IS}} = \sum_{t=1}^{T}\left(\frac{\text{De}_t - \text{Re}_t}{\text{De}_{\max}}\right)^2 \quad (1)$$

in which OF_{IS} = objective function for reservoir problem with irrigation supply purpose; t = counter of periods; T = total number of operation periods; $\text{De}_{\max}$ = maximum downstream agricultural water demand during an operation period; De_t = downstream agricultural demand during period t; and Re_t = reservoir release during period t.

Table 1. Definition of the Selected Unconstrained and Constrained Test Functions

Test function	Formula	Search domain	Global optimum	Reason for selecting
Sphere	Minimize $f(x_1,\ldots,x_d)=\sum_{di=1}^{d}x_{di}^2$	$-5\le x_{di}\le +5$	$f(x_1,\ldots,x_d)=f(0,\ldots,0)=0$	Continuous, strongly convex, and unimodal
Ackley	Minimize $f(x_1,\ldots,x_d)=a\cdot\exp(-b\sqrt{\frac{1}{d}\sum_{di=1}^{d}x_{di}^2})$ $-\exp\left(\frac{1}{d}\sum_{di=1}^{d}\cos(cx_{di})\right)+a+\exp(1)$ $(a=20, b=0.2,\ \text{and}\ c=2\pi)$	$-5\le x_{di}\le +5$	$f(x_1,\ldots,x_d)=f(0,\ldots,0)=0$	Continuous, includes linear and exponential functions, and poses a risk for algorithms to be trapped in one of its many local minimums
Styblinski-Tang	Minimize $f(x_1,\ldots,x_d)=\frac{1}{2}\sum_{di=1}^{d}(x_{di}^4-16x_{di}^2+5x_{di})$	$-5\le x_{di}\le +5$	$f(x_1,\ldots,x_d)=f(-2.903534,\ldots,-2.903534)$ $=-39.16599d$	Continuous and nonconvex
Rosenbrock	Minimize $f(x_1,\ldots,x_d)$ $=\sum_{di=1}^{d-1}[100(x_{di+1}-x_{di}^2)^2+(x_{di}-1)^2]$	$-2\le x_{di}\le +2$	$f(x_1,\ldots,x_d)=f(1,\ldots,1)=0$	Unimodal, also referred to as the valley or banana, and even though the valley is easy to find, convergence to the minimum is difficult
Holder table	Minimize $f(x_1,x_2)=-\lvert\sin(x_1)\cos(x_2)$ $\exp(\lvert 1-\sqrt{x12+x22}/\pi\rvert)\rvert$	$-10\le x_{di}\le +10$	$f(8.05502,9.66459)=-19.2085$ $f(-8.05502,9.66459)$ $=-19.2085$ $f(8.05502,-9.66459)=-19.2085$ $f(-8.05502,-9.66459)=-19.2085$	Multimodal, defined for two dimensions, and has four global minimums

The reservoir storage equation is expressed is given by

$$S_{t+1}=S_t+Q_t-\mathrm{Loss}_t-\mathrm{Re}_t-\mathrm{Sp}_t \tag{2}$$

in which S_{t+1} = reservoir storage volume at the beginning of operation period $t+1$; S_t = reservoir storage volume at the beginning of operation period t; Q_t = monthly inflow volume to the reservoir during period t; Loss_t = loss volume of water during period t; and Sp_t = volume of spilled water during period t.

Eqs. (3) and (4) show the calculation of Loss_t and Sp_t, respectively

$$\mathrm{Loss}_t=A_t\times\mathrm{Ev}_t,\quad A_t=g[S_t] \tag{3}$$

$$\mathrm{Sp}_t=\begin{cases}0 & \text{if } S_t\le S_{\max}\\ S_{\max}-S_t & \text{if } S_t>S_{\max}\end{cases} \tag{4}$$

in which A_t = area of the reservoir lake at the beginning of operation period t and is a function of S_t; Ev_t = depth of loss during the period t; and $S_{\max}$ = maximum allowable reservoir storages during period t.

Constraints on reservoir storages and releases are respectively expressed as follows:

$$0\le\mathrm{Re}_t\le\mathrm{De}_t \tag{5}$$

$$S_{\min}\le S_t\le S_{\max} \tag{6}$$

in which $S_{\min}$ = minimum allowable reservoir storages during period t.

Reservoir Operation Model with Hydropower Production Purpose

The chief objective of hydropower reservoir operation problems is to maximize the hydropower production or minimize the hydropower deficits. Eq. (7) expresses the objective function that minimizes hydropower production deficits

$$\text{Minimize}\ \mathrm{OF_{HP}}=\frac{1}{T}\left[\sum_{t=1}^{T}\left(1-\frac{P_t}{\mathrm{PPC}}\right)^2\right] \tag{7}$$

in which $\mathrm{OF_{HP}}$ = objective function for reservoir problem with hydropower production purpose; P_t = power generated by power plant during period t; and PPC = total installed capacity of the power plant.

The storage in the reservoir, the loss of water, and the water spilled from reservoir are computed using Eqs. (2)–(4). Moreover, constraints on reservoir storages are given by Eqs. (5) and (6). Reservoir problems with hydropower production purpose have other constraints.

The power generated by the power plant during period t is computed as follows:

$$P_t=\frac{\gamma'\times\eta\times\Delta H_t\times\mathrm{DisRe}_t}{10^6\times n} \tag{8}$$

in which γ' = specific weight water; η = efficiency of the power plant; ΔH_t = difference between the average level of water surface and the tailwater at the beginning and the end of period t, which is a function of DisRe_t and is assumed constant in this study; DisRe_t = discharge of the water through power plant during period t; and n = performance coefficient of the power plant.

The computations for ΔH_t and DisRe_t are as follows:

$$\Delta H_t = \left(\frac{H_{t+1} + H_t}{2}\right) - \mathrm{TR}, \quad H_t = k[S_t] \tag{9}$$

$$\mathrm{DisRe}_t = \frac{\mathrm{Re}_t}{\mathrm{CF}_t} \tag{10}$$

in which H_t = water level at the beginning of period t; H_{t+1} = water level at the end of period t; TR = tailrace level; and CF_t = conversion factor from million cubic meters to cubic meters per second during period t, and is calculated as follows:

$$\mathrm{CF}_t = \frac{24 \times 3{,}600}{1{,}000{,}000}\,\mathrm{day}_t \tag{11}$$

in which day_t = number of days in the operation period t.

Constraints on releases and generated powers are respectively expressed as follows:

$$0 \le P_t \le \mathrm{PPC} \tag{12}$$

Firefly Algorithm

The FA is inspired by the behavior of fireflies in nature. Fireflies flash their stored energy as light in order to mate, hunt, or evade predators. Fireflies produce attractiveness by shining light. Three idealized rules are assumed in the FA as follows:

1. All fireflies are unisex so their attractiveness depends on the amount of light flashed by them regardless of their sex.
2. The attractiveness of fireflies is proportional to their brightness. Thus, for any two flashing fireflies, the firefly that flashes less will move toward the firefly that flashes more. As the distance between fireflies increases, the attractiveness and the brightness of fireflies decreases. Thus, the movement of fireflies continues in this manner until there is no brighter firefly in a group. Once this happens the fireflies move randomly.
3. The brightness of a firefly is determined by an objective function.

For simplicity, it is assumed that the attractiveness of a firefly is determined by its brightness, which in turn is associated with the encoded objective function. The attractiveness of a firefly can be expressed as follows:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \tag{13}$$

in which $\beta(r)$ = firefly's attractiveness; β_0 = attractiveness at a distance equal to $r = 0$; and γ = light absorption coefficient.

The distance r between a pair of fireflies i and j that are located at x_i and x_j positions, respectively, is computed according to Eq. (14)

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{14}$$

in which r_{ij} = Cartesian distance between a pair of fireflies i and j; $\|\|$ distance vector between a pair of fireflies i and j in space; $x_{i,k}$ = kth dimension of the spatial coordinate of the ith firefly's position; d = number of dimensions; and $x_{j,k}$ = kth dimension of the spatial coordinate of the jth firefly's position. Parameter r_{ij} defined in Eq. (14) is not limited to the Euclidean distance. In fact, any measure that can effectively characterize the quantities of interests in the optimization problems can be used as the distance depending on the type of the problem at hand (Yang 2013).

If a pair of fireflies i and j is to be considered so that firefly j is better than firefly $i$in terms of brightness, then firefly i is attracted by firefly j and will move toward the position of firefly j. As the result of this movement, firefly i would be located at a new position, which is computed by Eq. (15)

$$x\mathrm{new}_i = x_i + \beta_0 e^{-\gamma \wp_{ij}^2}(x_j - x_i) + \alpha(\mathrm{rand} - 0.5) \tag{15}$$

in which $x\mathrm{new}_i$ and x_i = new position of firefly i with less brightness and current position of firefly i with less brightness, respectively; x_j = position of firefly j with more brightness; α = a randomized parameter; and rand is a randomized value in the range (0, 1). The second and third terms are due to the attraction and randomization, respectively, according to Eq. (15).

Yang (2009) pointed out that for most implementations the value of β_0 can be considered equal to 1. Moreover, according to Yang (2009), the range of values of α is (0, 1), even though Yang (2013) remarked that it is better to use a time-dependent α_{damped} so that randomness can be decreased gradually as the iterations proceed. Recall that γ is a light absorption coefficient that takes values in the range $(0, \infty)$, in theory. However, in practice it is usually taken to be in the range (0.1, 10) (Yang 2009). The value of γ is key in determining the convergence speed and the capability of the algorithm. Thus, a sensitivity analysis of this parameter is of vital importance. Fig. 1 illustrates the FA's flowchart.

Penalty Functions

Penalty functions are introduced in constrained problems in a typical evolutionary optimization method. The combination of the penalty functions and the objective function is named fitness function. Penalty functions can be applied in two ways, collected and multiplied. In the multiplied state, the penalty is multiplied by the objective function. In the collected state the penalty is added to the objective function. The algebraic sign of multiplication or addition depends on the type of problem under consideration. In minimization or maximization problems the penalty functions are multiplied or added with a positive or negative sign, respectively.

Case Studies

Two reservoir systems are studied in the present paper. Brief details of study areas and data are presented in the following.

Aydoghmoush Reservoir with Irrigation Purpose

The rock fill Aydoghmoush Dam with a clay core is situated 23 km southwest of the city of Mianeh in the East Azarbayejan province of Iran. The dam site is located across the Aydoghmoush River in the Caspian Sea catchment. The purpose of the dam is to develop and improve irrigation with a cultivation area equal to 15×10^3 ha. The length and width of the dam crest are 297 and 12 m, respectively, while the height of the dam is 1,350 m above sea level. The maximum and minimum storage volumes of the reservoir are 145.7×10^3 and 8.9×10^3 m^3, respectively.

The average annual inflow to the Aydoghmoush Reservoir is estimated to be 228×10^3 m^3/month. A diagram of the monthly inflow volume along with the monthly projected demand volume for a 10-year period (1991–2000) are illustrated in Fig. 2(a), according to which the maximum, average, and minimum volumes of water demand are 39.57×10^6, 12.12×10^6, and 0×10^6 m^3/month, respectively. As shown in Fig. 2(a), the distributions of inflow and demand time series are not equal in most periods. In the periods for which inflow is considerable, the agricultural demand is less

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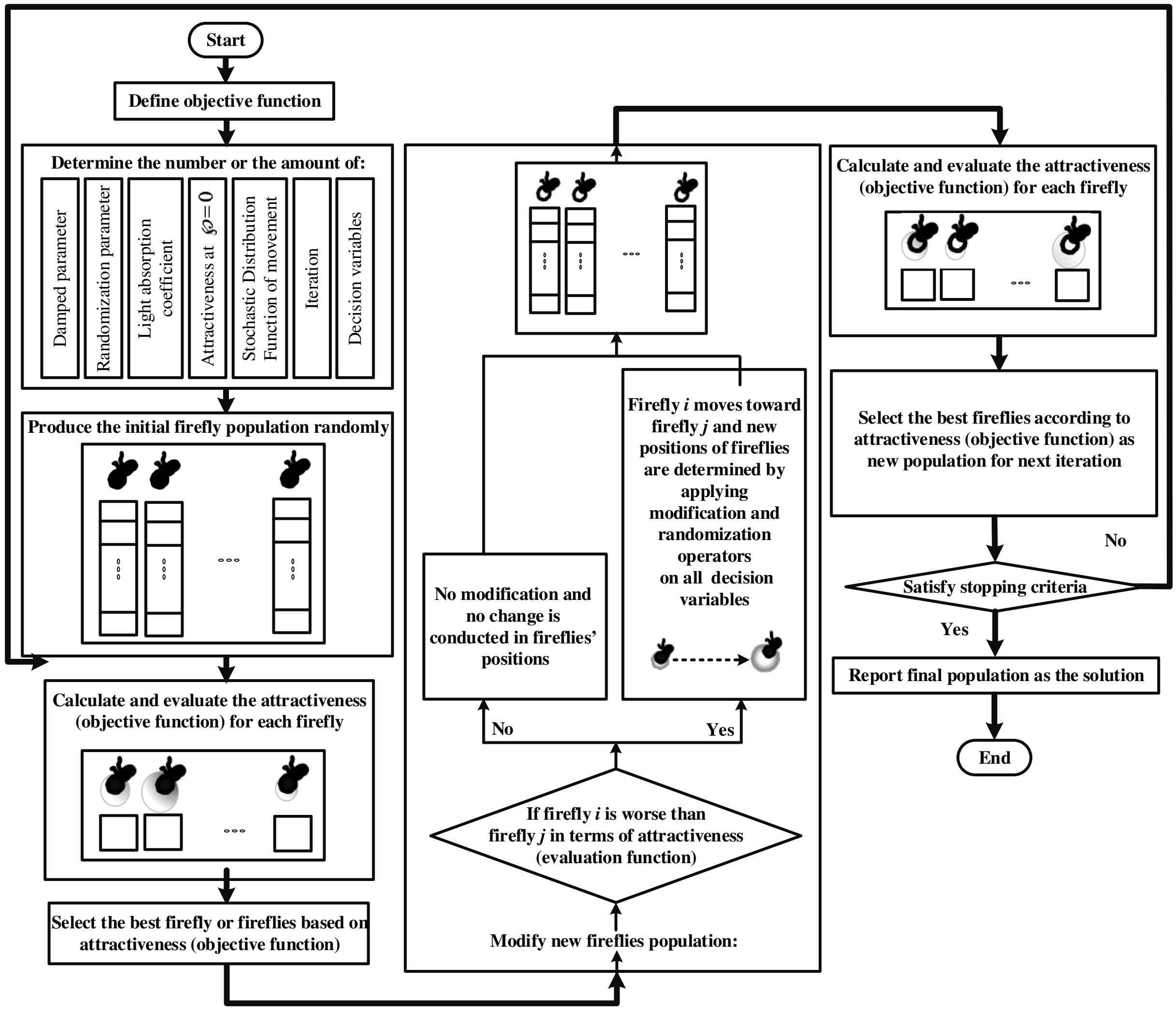

Fig. 1. Flowchart of the firefly algorithm

than inflow, and in periods with low inflow, demands exceed inflow considerably. The discrepancy between water demand and inflow is one of the reasons for constructing a reservoir. Furthermore, the values of monthly evaporation depth are shown in Table 2 for 12 months (1 year) and is repeated for other years during the operational period.

Eq. (16) is a formula relating the area and volume of the Aydoghmoush Reservoir

$$A_t = -0.0002S_t^2 + 0.0804S_t + 0.4093 \tag{16}$$

The penalty functions used in the Aydoghmoush Reservoir operation model are defined by Eqs. (17)–(19)

$$P_{1,t} = \begin{cases} 0 & \text{if} S_{t+1} > S_{\min} \\ \frac{(S_{\min}-S_{t+1})^2}{S_{\min}} & \text{Otherwise} \end{cases} \tag{17}$$

in which $P_{1,t}$ = penalty applied to the violation of the minimum storage in period t

$$P_{2,t} = \begin{cases} 0 & \text{if} \quad S_{t+1} < S_{\max} \\ \frac{(S_{t+1}-S_{\max})^2}{S_{\max}} & \text{Otherwise} \end{cases} \tag{18}$$

in which $P_{2,t}$ = penalty applied to the violation maximum storage in period t

$$P_{3,t} = \begin{cases} 0 & \text{if } \mathrm{Re}_t < \mathrm{De}_t \\ \frac{(\mathrm{Re}_t-\mathrm{De}_t)^2}{3{,}957} & \text{Otherwise} \end{cases} \tag{19}$$

in which $P_{3,t}$ = penalty applied to the violation release in period t.

The Aydoghmoush Basin was studied by Ashofteh et al. (2013a) to survey the impact of climate change on reservoir performance indices for agricultural water supply. Thus, pertinent information and data of reservoir, river, and agricultural network of Aydoghmoush was retrieved from the study by Ashofteh et al. (2013a).

Karun-4 Reservoir with Hydropower Production Purpose

The double-arch concrete Karun-4 Dam is located 180 km southwest of the city of Shahrekord in Chaharmahal and Bakhtiary province of Iran across the Karun River, which is one of the largest and longest rivers of Iran. The chief purpose of Karun-4 Dam is hydropower generation. The potential energy production of the Karun-4 Reservoir equals 2,107 MWh annually. Maximum and minimum storage volumes of the reservoir are $2{,}019 \times 10^3$ and $1{,}144.29 \times 10^3\ \mathrm{m^3/month}$, respectively. Moreover, the maximum release volume is $450 \times 10^3\ \mathrm{m^3/month}$. The power plant capacity (PPC), performance coefficient, and efficiency of Karun-4 power plant are equal to $1{,}000 \times 10^6$ W, 20%, and 88%, respectively.

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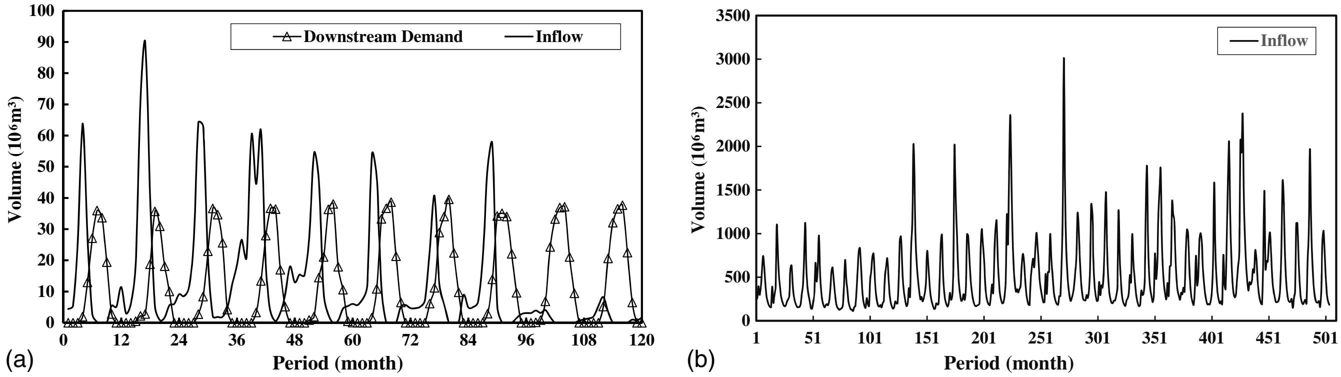

Fig. 2. Diagrams of monthly (a) inflow along with monthly projected demand volume for a 10-year period (1991–2000) of Aydoghmoush Reservoir; (b) inflow for a 42-year period (1957–1998) of Karun-4 Reservoir

Table 2. Values of Monthly Evaporation Depth in the Aydoghmoush and Karun-4 Reservoirs

	Time period											
Reservoir	1	2	3	4	5	6	7	8	9	10	11	12
Aydoghmoush (mm)	26.2	41.5	85.7	132.1	183.6	233.5	265.8	239.6	165.5	101.1	45.4	26.1
Karun-4 (mm)	158.4	77.9	55.2	49.9	64.4	80.7	131.1	165.8	238.3	253.3	259.8	208.2

The average annual inflow to the Karun-4 Reservoir is estimated to be $6{,}045 \times 10^3\ \mathrm{m}^3/\mathrm{month}$. A diagram of the monthly inflow volume along with monthly projected demand volume for a 42-year period (1957–1998) is depicted in Fig. 2(b). Furthermore, the values of monthly evaporation depth are shown in Table 2 for 12 months (1 year) and is repeated for other years during the operational period.

Eqs. (20) and (21) express the relations between (1) area and volume and (2) height and volume of the Karun-4 Reservoir, respectively

$$A_t = -3 \times 10^{-6} S_t^2 + 0.019413 S_t + 1.915948 \tag{20}$$

$$H_t = -3 \times 10^{-5} S_t^2 + 0.13810 S_t + 873.66716 \tag{21}$$

Table 3. Characteristics of the GA and FA Used with the Test Functions

Method	Characteristics	Sphere	Ackley	Styblinski-Tang	Rosenbrock	Holder table
GA	Decision variables	2	2	2	2	2
	Population	50	80	50	50	50
	Iterations	500	500	100	500	100
	Selection type	Roulette wheel	Roulette wheel	Roulette wheel	Roulette wheel	Roulette wheel
	Crossover type	One-point	One-point	One-point	One-point	One-point
	Mutation type	Uniform	Uniform	Uniform	Uniform	Uniform
	Pc	0.2	0.2	0.1	0.1	0.2
	Pm	0.02	0.03	0.02	0.02	0.02
FA	Decision variables	2	2	2	2	2
	Population	50	80	50	50	50
	Iteration	500	500	100	500	100
	Random function	Uniform	Uniform	Uniform	Uniform	Uniform
	β_0	1	1	1	1	1
	γ	5	0.01	0.01	0.01	0.1
	α	0.005	0.1	0.001	0.01	0.01
	Damped coefficient of α	0.99	0.99	0.99	0.99	0.99

Table 4. Results of Five Independent Runs of GA and FA for Mathematical Test Functions

Method	Objective function value	Sphere	Ackley	Styblinski-Tang	Rosenbrock	Holder table
GA	Global optimum	0	0	−78.33198	0	−19.2085
	Run 1	4.86×10^{-5}	0.004378	−77.4015	0.004458	−19.1968
	Run 2	6.76×10^{-4}	0.008095	−78.2894	0.004678	−19.1632
	Run 3	1.78×10^{-4}	0.014437	−78.1063	0.004104	−19.1846
	Run 4	6.60×10^{-5}	0.074197	−78.2797	0.531337	−19.1830
	Run 5	9.79×10^{-4}	0.014561	−78.1107	0.500002	−19.2076
	Minimum (best)	4.86×10^{-5}	0.004378	−78.2894	0.004104	−19.2076
	Average	3.89×10^{-4}	0.023134	−78.0375	0.208916	−19.1870
	Maximum (worst)	9.79×10^{-4}	0.074197	−77.4015	0.531337	−19.1632
	Standard deviation	4.17×10^{-4}	0.028873	0.3663	0.280246	0.016648
FA	Global optimum	0	0	−78.33198	0	−19.2085
	Run 1	5.03×10^{-14}	3.19×10^{-7}	−78.3323	1.03×10^{-13}	−19.2085
	Run 2	2.57×10^{-14}	1.71×10^{-7}	−78.2910	1.69×10^{-13}	−19.2085
	Run 3	1.61×10^{-14}	3.97×10^{-7}	−78.3323	1.63×10^{-13}	−19.2084
	Run 4	4.82×10^{-15}	7.28×10^{-7}	−78.3323	1.65×10^{-14}	−19.2085
	Run 5	5.05×10^{-14}	1.71×10^{-7}	−78.0723	3.77×10^{-14}	−19.2085
	Minimum (best)	4.82×10^{-15}	1.71×10^{-7}	−78.3323	1.65×10^{-14}	−19.2085
	Average	2.95×10^{-14}	3.57×10^{-7}	−78.2720	9.78×10^{-14}	−19.2085
	Maximum (worst)	5.05×10^{-14}	7.28×10^{-7}	−78.0723	1.69×10^{-13}	−19.2084
	Standard deviation	2.05×10^{-14}	2.29×10^{-7}	0.11308	6.99×10^{-14}	4.5×10^{-5}

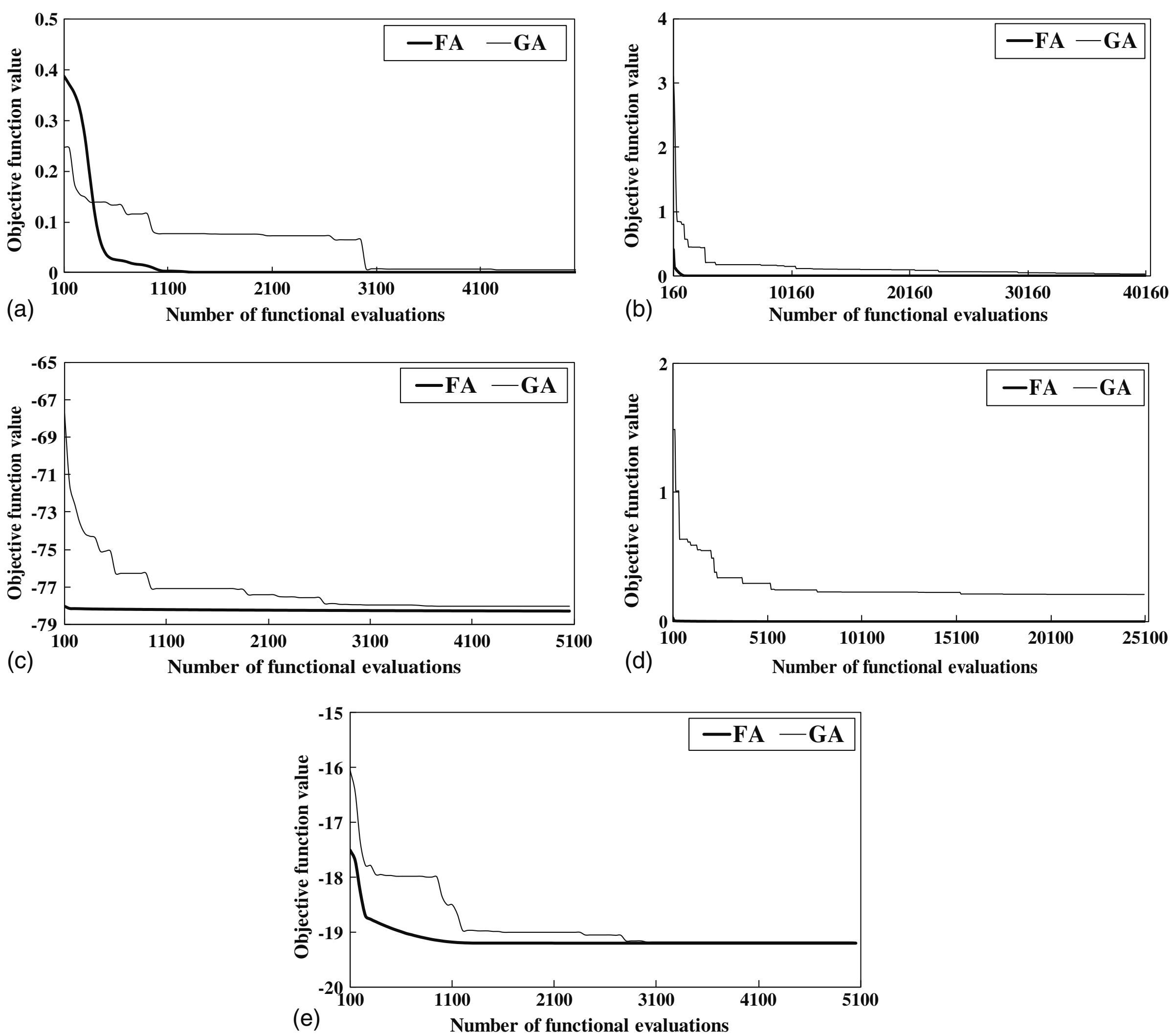

Fig. 3. Average rates of convergence over five runs obtained from GA and FA for (a) sphere; (b) Ackley; (c) Styblinski-Tang; (d) Rosenbrock; (e) Holder table

In addition, the formula relating discharge (0.0033DisRe_t) and the height of water (in relation to sea level) of the Karun-4 River (TR_t) is given by Eq. (22)

$$\text{TR}_t = 0.000016\text{DisRe}_t^2 + 0.0033\text{DisRe}_t + 843 \qquad (22)$$

The applied penalty functions in the Aydoghmoush Reservoir operation model are defined by Eqs. (23)–(25)

$$P_{4,t} = \begin{cases} 0 & \text{if } S_{t+1} > S_{\min} \\ \frac{S_{\min} - S_{t+1}}{S_{\min}} & \text{Otherwise} \end{cases} \qquad (23)$$

in which $P_{4,t}$ penalty applied to the of the minimum storage in period t

$$P_{5,t} = \begin{cases} 0 & \text{if } S_{t+1} < S_{\max} \\ \frac{S_{t+1} - S_{\max}}{S_{\max}} & \text{Otherwise} \end{cases} \qquad (24)$$

in which $P_{5,t}$ penalty applied to the violation of the maximum storage in period t

$$P_{6,t} = \begin{cases} 0 & \text{if } P_t < \text{PPC} \\ \frac{P_t - \text{PPC}}{\text{PPC}} & \text{Otherwise} \end{cases} \qquad (25)$$

in which $P_{6,t}$ = penalty applied to the deviation from the capacity of power plant in period t.

The global optima of the two reservoir operation models, Aydoghmoush and Karun-4, were obtained with NLP in the *LINGO* program. The problems were also solved with the GA and FA in the *MATLAB* software and the optimal operation rules were developed with the former three methods. The best solutions from the GA and FA strongly depend on the best settings of algorithmic parameters. Thus, a sensitivity analysis is herein performed for the parameters of the GA and the FA. The results of the sensitivity analysis are presented in the results section. The process of producing an initial population in the GA and the FA is random. Therefore, the final value of the objective function differs each time the algorithm is run. For this reason, the algorithm is run several times and the average of the calculated objective functions is reported as the solution to use for the purpose of interalgorithmic comparisons. Bozorg-Haddad et al. (2008) tested the HBMO convergence in reservoir operation with 10 runs. Guo et al. (2013) tested the convergence of the nondominated PSO (NSPSO) algorithm with 25 independent runs. Zhang et al. (2013) considered 100 runs as the number of independent runs in order to survey the convergence of the multiguide PSO (MGPSO) algorithm in a multireservoir operation problem. This study found the GA and FA parameters and the results of convergence and calculated releases, storages, and power production of the best of five runs were calculated and compared with those computed with the NLP method. All the results and conclusions are summarized next.

Results and Discussions

This section is divided into three subsections. "Mathematical Test Functions" presents the results of mathematical test functions. "Aydoghmoush Reservoir Operation" presents the results from NLP, GA, and FA for the Aydoghmoush Reservoir operation problem. "Karun-4 Reservoir Operation" reports the results of NLP, GA, and FA for the Karun-4 Reservoir operation problem.

Mathematical Test Functions

The five test functions are (1) sphere, (2) Ackley, (3) Styblinski-Tang, (4) Rosenbrock, and (5) Holder table, which were solved using the GA and the FA implemented in the *MATLAB* 12 software so that after a primary sensitivity analysis in GA, the selection, crossover function, mutation function, crossover probability (Pc), and mutation probability (Pm) parameters are listed in Table 3. The sensitivity analysis with the FA were conducted according to Yang's (2009) recommendations.

Table 5. Characteristics of the GA and FA Used with the Aydoghmoush and Karun-4 Reservoirs

Method	Characteristics	Aydoghmoush Reservoir	Karun-4 Reservoir
GA	Decision variables	120	504
	Population	10	10
	Iteration	1,000	10,000
	Selection type	Roulette wheel	Roulette wheel
	Crossover type	Two-point	Two-point
	Mutation type	Uniform	Uniform
	Pc	0.1	0.5
	Pm	0.01	0.01
FA	Decision variables	120	504
	Population	10	10
	Iteration	1,000	10,000
	Random function	Uniform	Uniform
	β_0	2	2
	γ	0.01	10
	α	1	1
	Damped coefficient of α	0.99	0.99

Table 6. Results of Five Independent Runs of GA and FA for the Aydoghmoush and Karun-4 Reservoirs

Method	Objective function value	Aydoghmoush Reservoir	Karun-4 Reservoir
GA	Global optimum (NLP solution)	3.3727	0.0045
	Run 1	6.4770	0.0094
	Run 2	6.8940	0.0096
	Run 3	7.0980	0.0089
	Run 4	6.5290	0.0099
	Run 5	6.3790	0.0096
	Minimum (best)	6.3790	0.0089
	Average	6.6754	0.0095
	Maximum (worst)	7.0980	0.0099
	Standard deviation	0.3063	0.0004
FA	Global optimum (NLP solution)	3.3727	0.0045
	Run 1	3.5581	0.0085
	Run 2	3.6166	0.0082
	Run 3	3.5365	0.0079
	Run 4	3.6898	0.0078
	Run 5	3.6427	0.0086
	Minimum (best)	3.5365	0.0078
	Average	3.6087	0.0082
	Maximum (worst)	3.6898	0.0086
	Standard deviation	0.0624	0.0003

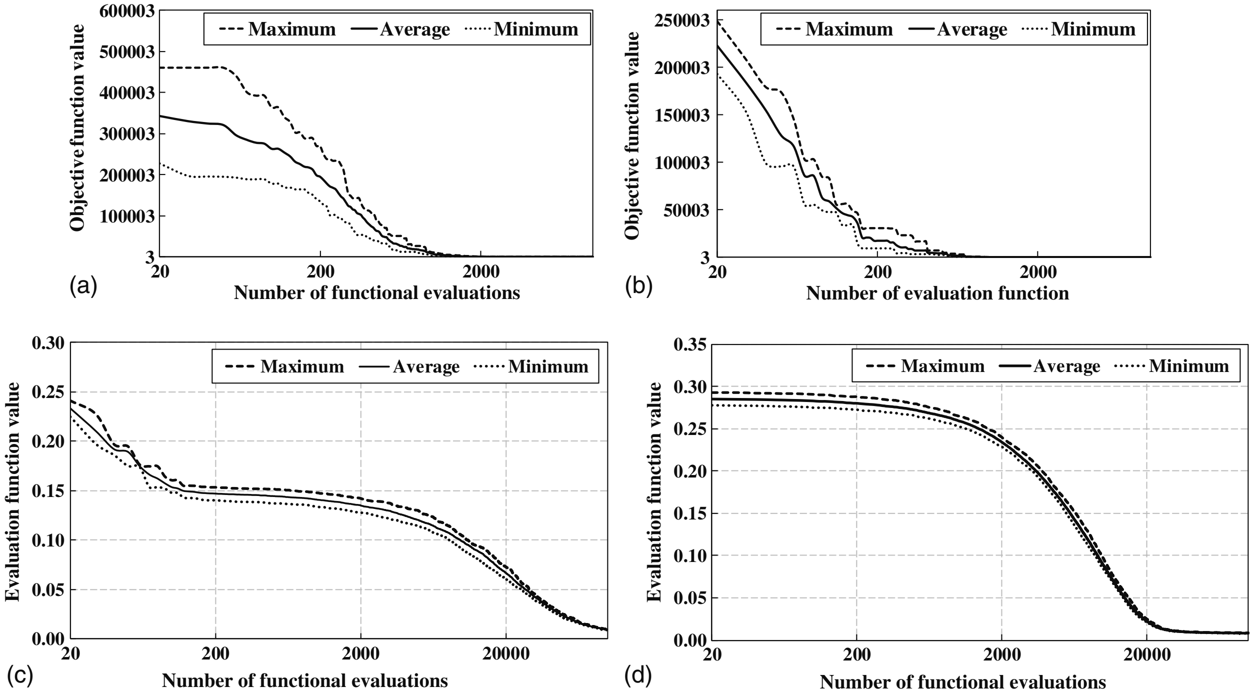

Fig. 4. Minimum (best), average, and maximum (worst) rates of convergence over five runs for reservoir of (a) Aydoghmoush obtained from GA; (b) Aydoghmoush obtained from FA; (c) Karun-4 obtained from GA; (d) Karun-4 obtained from FA

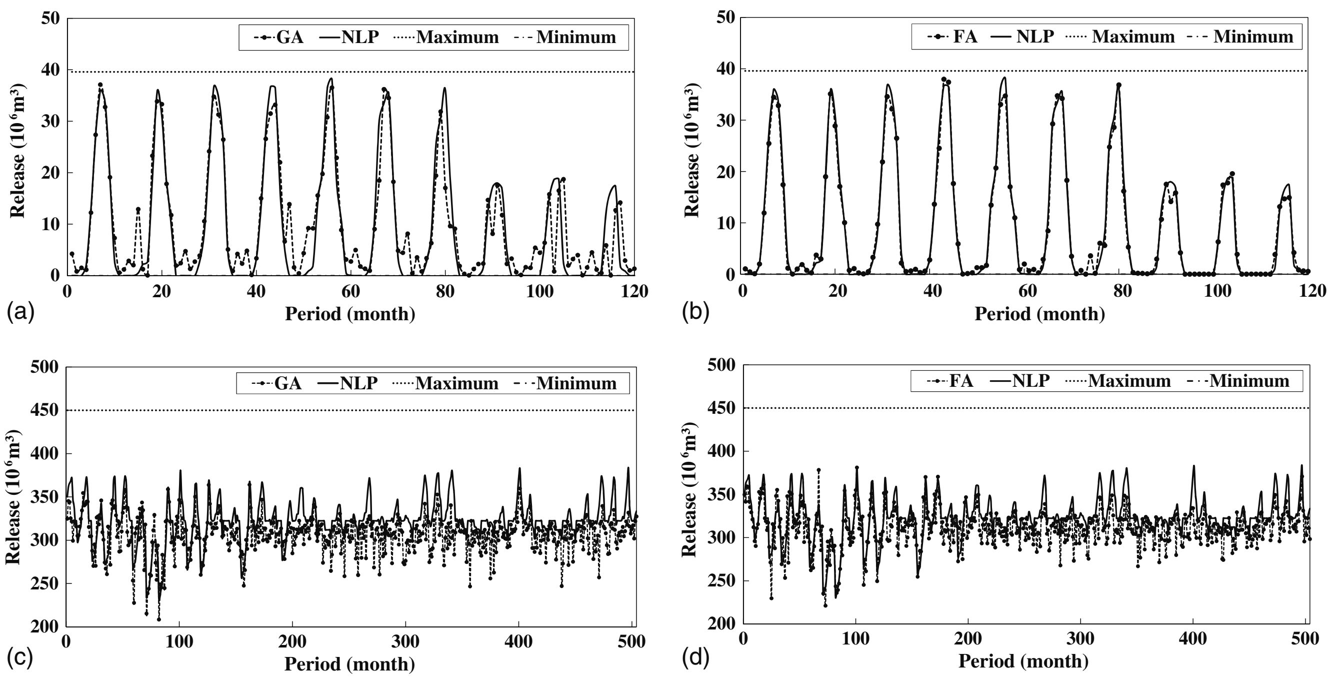

Fig. 5. Monthly reservoir releases of the best run of five runs for (a) Aydoghmoush obtained from GA; (b) Aydoghmoush obtained from FA; (c) Karun-4 obtained from GA; (d) Karun-4 obtained from FA

It is seen in Table 4 that the FA produced the objective function values with lower standard deviation over five independent runs compared with the GA. Moreover, the best (minimum) values of objective functions for all test problems obtained by the FA are closer to the global optimum, so that FA could precisely reach the global optimum value of the Holder table test function. The results of FA for sphere, Ackley, and Rosenbrock, which have global optima equal to 0, are acceptable with a very close approximation. The best calculated objective function value of the FA for the Styblinski-Tang problem differs less than 5×10^{-6} from the global optimum, whereas the best obtained objective function value of GA differs approximately 0.0005 from the global optimum

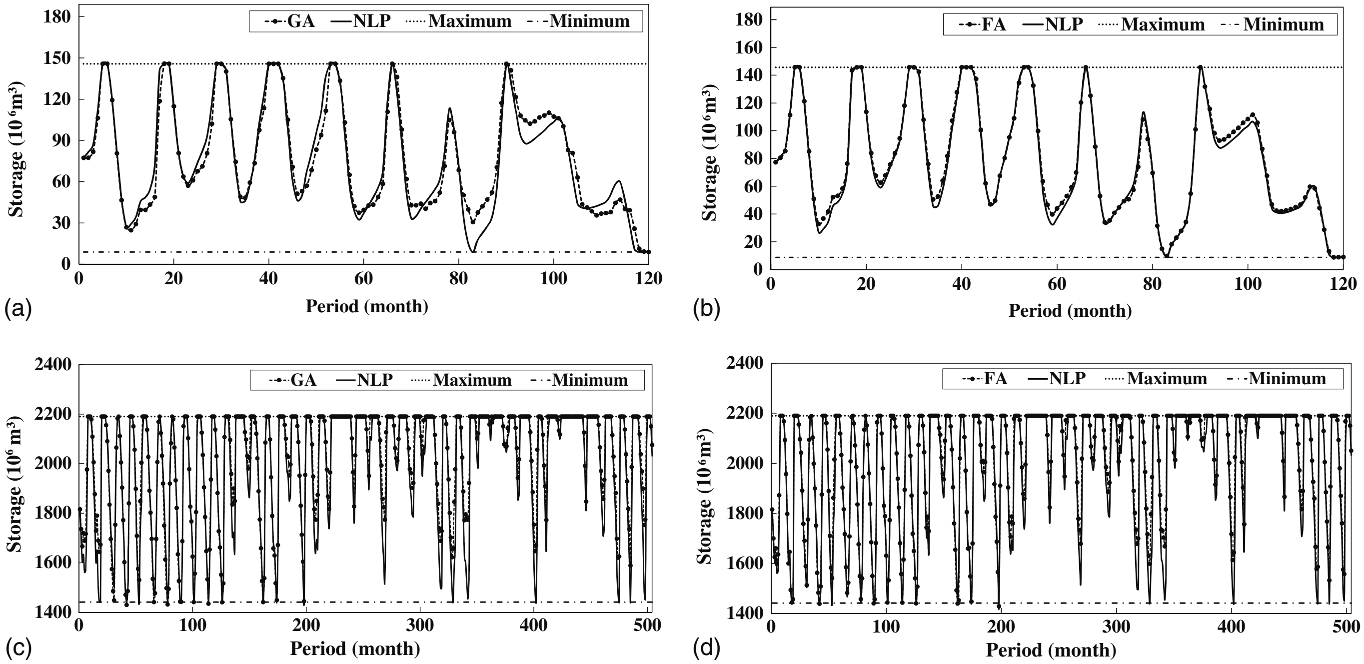

Fig. 6. Monthly reservoir storages of the best run of five runs for (a) Aydoghmoush obtained from GA; (b) Aydoghmoush obtained from FA; (c) Karun-4 obtained from GA; (d) Karun-4 obtained from FA

for the same problem. In other words, the results of the GA for Styblinski-Tang is 100 times worse compared with the result of the FA in term of closeness to the global optimum. The average rates of convergence versus the number of functional evaluations of the GA and the FA over five runs are illustrated in Fig. 3 for each of the five test functions. It is concluded from Fig. 3 that the FA converges faster than GA for all test functions and it converges closer to the global optimum than the GA.

Aydoghmoush Reservoir Operation

This subsection presents the results of NLP, GA, and FA methods applied to the Aydoghmoush Reservoir. The value of the objective function evaluated by NLP using *LINGO 14.0* optimization software equals 3.3727. Furthermore, the GA and the FA were implemented in the *MATLAB 12* software and their parameters were set equal to the values listed in Table 5 after a primary sensitivity analysis (10,010 evaluations). The results listed in Table 6 indicate that the best value of the objective function over five runs obtained with the GA and the FA are 6.3790 and 3.5365, respectively. In fact, the best value of the objective function of the FA is approximately 95% of the global optimal solution (3.3727). Moreover, it can be seen that the FA produced objective function values with lower standard deviation (0.06) over five independent runs compared with GA (0.31). This means that the variations of GA results are approximately five times larger than those of FA based on the calculated standard deviations.

Figs. 4(a and b) also illustrates the minimum (best), the average, and the maximum (worst) rates of convergence over five runs for the Aydoghmoush Reservoir obtained from the GA and the FA. In accordance with Fig. 4(a) it can be seen that the GA converged after approximately 2,000 evaluations. Fig. 4(b) shows that the FA converged after 1,000 functional evaluations. It is concluded that the FA achieved a faster convergence rate compared with the GA. Figs. 5 and 6 demonstrate monthly reservoir releases and storages of the best run of five runs for the Aydoghmoush Reservoir obtained from both algorithms versus NLP results. It is seen in Figs. 5(a and b) that the FA releases are very close to the NLP releases. Figs. 6(a and b) show that there are large differences between the GA storages and NLP storages, particularly in the last 40 periods of operation, compared with those of the FA.

Karun-4 Reservoir Operation

The value of the objective function evaluated by NLP for this problem is equal to 0.0045. The values of the GA and FA parameters are listed in Table 5 after a primary sensitivity analysis (100,010 evaluations). It is seen in Table 6 that the best values of the objective function over five runs calculated with the GA and the A for Karun-4 equal 0.0089 and 0.0078, respectively. The best value of the objective function of the FA differs approximately 73% from the global optimal solution (3.3727). The best value of the objective function of the GA differs approximately 97% from the global optimum solution. Moreover, it can be seen that the FA yielded objective function values with lower standard deviation (0.0003) over five independent runs compared with GA (0.0004). In other words, based on the standard deviation, the variations of GA results are approximately 1.3 times larger than those of FA.

Figs. 4(c and d) depict the minimum (best), the average, and the maximum (worst) rates of convergence over five runs for the Karun-4 Reservoir obtained from GA and FA. Fig. 4(c) shows that the GA converged after approximately 100,000 functional evaluations, whereas Fig. 4(d) shows the FA after 20,000 functional evaluations. Hence, a comparison of these two figures indicates that the convergence speed of the FA is much higher than that of the GA. Figs. 5 and 6 show monthly reservoir releases and storages of the best run of five runs for Karun-4 Reservoir obtained from both algorithms versus the NLP results, respectively. According to Figs. 5(c and d), the FA releases are closer to the NLP releases than those of the GA. Similarly, with regard to Figs. 6(c and d), the differences between GA storages and NLP storages are larger than those between the FA and NLP, particularly at low values.

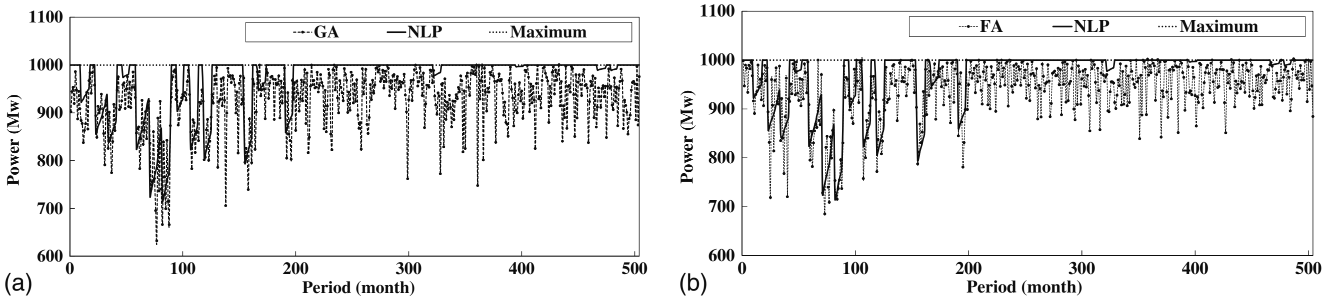

Fig. 7. Monthly reservoir power productions of the best run of five runs for Karun-4 (a) obtained from GA; (b) obtained from FA

Figs. 7(a and b) depict the computed power generation with NLP versus the GA and FA results. Based on Fig. 7 it is inferred that the deviations of the GA from NLP are larger than those of the FA. Moreover, the power generated by the FA is closer to NLP than those generated by the GA. Fig. 7(b) clearly demonstrates the lower standard deviation of the FA compared with the GA.

Concluding Remarks

This study applied the FA to solve two long-term single-reservoir operations with different purposes, namely irrigation supply and hydropower production. First, the performance of the FA was examined with five different mathematical test functions in which the FA converged more rapidly than the GA to near-global solutions. Also, FA was able to obtain the closer value of the objective function to the global solution compared with the GA. Thereafter, the results of reservoir operation systems partially revealed the high potential of FA and emphasized its capacity in solving complex constrained optimization problems. The results of the NLP method were herein considered as the global optimal solutions.

It is concluded that the FA achieved closer average value of objective function (3.6078) to NLP (3.3727) than the average value of objective function obtained from GA (6.6754) concerning the Aydoghmoush Reservoir dealing with irrigation supply. Moreover, the results of five runs of the FA exhibited lower standard deviation (0.06) than the GA (0.31), which is nearly five times worse.

The results of the Karun-4 Reservoir with the purpose of hydropower production demonstrate that the FA was capable of reaching better optimal solutions than the GA. The FA converged more rapidly than the GA and its average value of the objective function obtained (0.0082) is closer to NLP's (0.0045). The GA's average value of the objective function (0.0095) is approximately 1.2 times worse than FA compared with the NLP. This paper's results show that the FA achieved better solutions and with faster convergence rate than the GA in all the test problems.

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