Using Analogical Comparison to Reveal the Structure of the Count List

Kexin Que (kexinque@u.northwestern.edu)
Department of Psychology,
Northwestern University, Evanston, IL 60208, USA

Dedre Gentner (gentner@northwestern.edu)
Department of Psychology,
Northwestern University, Evanston, IL 60208, USA

Abstract

In many languages, number words follow a recursive morphological structure. Past research suggests that children with an extended and regular count list are earlier to notice such structure than those with a more limited and less transparent count list. Building on this line of research, we propose that children's insight into the recursive structure develops in part through comparison processes that allow them to notice morphological regularities across number words. To test this, we presented 4–5-year-old children with a visual depiction of numbers 0–49, along with comparison techniques designed to facilitate the alignment across decades. We used a pretest–training–posttest design. In pre- and post-tests, we assessed children's ability to produce the successor of numbers beyond their counting range and their understanding of numerical infinity. In the post-test, we also assessed their ability to produce the successor for novel numbers. We found that the training improved children's understanding of the successor relation between numbers, though not their understanding of infinity.

Keywords: Analogical comparison; Successor principle; Number, Number learning

Introduction

Children begin reciting number words as early as age two; however, their understanding of numbers unfolds gradually over the next three or four years (e.g., Baroody et al., 2014; Carey, 2009; Fuson, 1988; Gelman & Gallistel, 1978; Guerrero et al., 2020; Gunderson et al., 2015; Levine et al., 2010; Lyons et al., 2018; Mix, 2002, 2009; Mix et al., 2005, 2012; Schneider et al., 2020, 2021a, 2021b; Wynn, 1990, 1992). One might be misled by the fact that children can count to 10 at an early age and assume that they acquire number concepts quickly. However, children's counting range can be deceptive in reflecting their understanding of numbers at early stages. For example, a three-year-old child who can count to 10 or even higher may not know how to give 5 bananas, nor understand that adding 1 to a set of 5 bananas makes 6 (e.g., Mix, 2009; Sarnecka & Carey, 2008). Indeed, much research has documented that the process of number learning is protracted (e.g., Cheung et al., 2017; Davidson et al., 2012; Gunderson et al., 2015; Krajcsi & Fintor, 2023; Marchand & Barner, 2018; Schneider et al., 2021b; Spaepen et al., 2018).

Although learning the count list does not immediately give children an understanding of natural numbers, it is nevertheless important as a bootstrap for the development of number understanding (Carey, 2004). Recent accounts further suggest that children's discovery of the "decade+digit" morphological structure of the count list supports the development of a key principle of the natural numbers—that every number N has a successor, $N + 1$ (the *Successor Principle*) (Barner et al., 2017; Cheung et al., 2017; Chu et al., 2020, Schneider et al., 2020).

Our focus here is on how children discover the "decade+digit" structure of the count list—that is, the recursive morphological structure across decades (e.g., the parallel between 31,32,33… and 41,42,43…). Discovering this structure is a milestone for children, as it includes the knowledge that for numbers at any decade, there is a successor according to the "decade+digit" structure. This understanding is slow to build; researchers have found that three-year-olds can identify the successor (next number) only for very small numbers (Davidson et al., 2012, Schneider et al., 2021a). The scope of this understanding increases gradually over the next two to three years. By around age six, children can succeed with large numbers they are not familiar with (Cheung et al., 2017; Schneider et al., 2020; Sullivan et al., 2025). Some children also show an understanding that numbers continue indefinitely (Cheung et al., 2017; Chu et al., 2020; Sullivan et al., 2023).

Our approach borrows from other research on mathematical development that has emphasized the role of analogical comparison in children's learning of the natural numbers (e.g., Barner, 2017; Carey, 2004; Gentner, 2010; Marchant & Barner, 2018; Mix et al., 2012; Schneider et al., 2021b; Opfer & Siegler, 2007; Rittle-Johnson & Star, 2009; Sullivan & Barner, 2013, 2014). Building on these insights, we propose that children learn the "decade+digit" structure through comparison across decades. Indirect encouragement for this possibility comes from the observation that higher counters (who have more opportunity for noticing the parallels between decades) are more likely to understand the recursive structure. For example, Chu et al. (2020) found that high counters were better able to produce the successor of numbers than low counters, including those beyond their counting range (see also Cheung et al., 2017, Davidson et al., 2012, and Schneider et al., 2021b).

Another observation comes from cross-linguistic studies. Early work found that children who speak Chinese are better able to master two-digit numbers than are children who speak English (Miller & Stigler, 1987; Miller et al., 1995). This was linked to the observation that Chinese has a more transparent

In G. Boleda, I. Dautriche, M. Maldonado, C. Saldana and T. Torrent (Eds.), *Proceedings of the 48th Annual Conference of the Cognitive Science Society*

number syntax, especially for the numbers 11-19. In Chinese, these numbers are expressed as *ten one*, *ten two*, *ten three* and so on, and this regular pattern repeats throughout the count system. In contrast, the teens in English have an irregular pattern. Further research has broadened the set of language and tasks used to study this question. The results show that the transparency of recursive morphology in number words not only affects how high children can count but also their understanding of numbers (e.g., Dowker et al., 2008; Schneider et al., 2020; Srinivasan, 2024). For example, Schneider et al. (2020) compared children whose languages have fairly transparent number morphology (English, Cantonese, and Slovenian) with children whose languages have opaque number morphology (Hindi and Gujarati). They found that children with transparent language have higher counting ranges and can better produce the successor of number words (including numbers beyond their counting range) than children in the opaque language groups. A recent study by Topaloğlu (2026) further controlled for potential effects of culture and SES by comparing two populations from India: Tamil-speaking children (with transparent number words) and Hindi-speaking children. They again found that the transparency of the structure of number words is associated with an advantage in learning successors. Similar results are also found in Maheshwari et al. (2026).

Current Study

Thus, research suggests that children with an extended and regular count list are more likely to discover the "decade+digit" structure of number language than those with a limited and less transparent count list (Davidson et al., 2012; Cheung et al., 2017; Chu et al., 2020; Schneider et al., 2020; Schneider et al., 2021b). Building on this, we hypothesize that children gradually come to perceive the regular "decade+digit" structure of numbers via analogical comparison across decades. For example, when counting to 80, English-speaking children may discover that the digits from one to nine repeat at each decade beginning with twenty. If so, making the parallel structure across decades more accessible should help children grasp the structure of numbers sooner than they otherwise would. In this research, we aimed to make the parallel structure across decades more accessible to children by utilizing principles of analogical comparison.

We employed two techniques from the analogical comparison literature: (1) intended relational correspondences should be spatially aligned, and (2) the influence of potential competing correspondences should be minimized (e.g., Christie & Gentner, 2010; Matlen et al., 2020; Zheng et al., 2022). These techniques have been found to facilitate children's learning of relational concepts, including mathematical concepts (e.g., Mix et al., 2012; Opfer & Siegler, 2007; Rittle-Johnson & Star, 2009; Yuan et al., 2021).

To do this, we used a number grid, arranged so that each decade was alignable with the others (Figure 1). The idea was that by seeing, for example, 30–39 directly above 40–49, children would be able to align them and compare numbers across decades (e.g., 13, 23). We predicted this would make it easier for them to perceive the regular structure of numbers.

The grid has the advantage that the regular structure of numbers is visually available. In contrast, in young children's typical experience, numbers are recited orally, in a sequential temporal string. This format means that numbers disappear as soon as they are spoken, therefore reducing the opportunity to compare. Further, numbers ending in the same digit are interrupted by other intervening numbers—again posing difficulties in discovering the morphological analogy in number words across different decades.

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49

Figure 1: The grid used in the study.

Another feature of the grid is that it presents numbers as Arabic numerals. It has been suggested that Arabic numerals are a better source for learning the structure of numbers than number words (e.g., Schlimm, 2018; Chrisomalis, 2010; Greenberg, 1978). Arabic numerals are fully transparent, whereas verbal number systems exhibit language-specific irregularities (such as *eleven* in English). However, one potential challenge of using Arabic numerals is that young children may not be familiar with them. Therefore, our training included practice to establish correspondences between oral number words and Arabic numerals.

Other training practices were designed to highlight the regular structure of numbers, as illustrated in the grid. For example, we introduced numbers ending in zero as "decade" numbers. A large body of research suggests that providing labels helps children notice similarities (e.g., Christie & Gentner, 2014; Gentner, 2010; Loewenstein & Gentner, 2005). Using this "decade" label may help children see that decades serve as starting points, after which counting repeats in the same pattern.

In sum, this study tested the hypothesis that analogical comparisons that promote spatial alignment across decades can help children perceive the regular morphological structure of the natural numbers. We used a pretest-posttest design to assess whether our training led to gains. Our assessment tasks included the Next Number task at pre- and post-test (to assess gains in children's ability to produce the successor of a number), focusing on numbers beyond their counting range. To examine whether the training also provided generalizable insight into the structure of number language, we adapted the Novel Next Number task (Topaloğlu, 2026; Sullivan et al., 2025) for use at post-test. Finally, we adapted the Infinity Interview from Cheung et al. (2017) and Chu et al. (2020) to test the proposal that understanding recursive structure supports the discovery of infinity.

We predicted that our training would improve children's ability to produce successors for numbers beyond their counting range. We also explored whether the predicted improvement in ability to produce the next number would be associated with a generalized understanding of the "decade+digit" structure (as evidenced in the Novel Number task). Finally, we also asked whether our training would lead to an improvement in children's understanding of infinity.

Methods

The experiment was conducted over Zoom in two sessions, spaced 1–3 days apart. Each session took about 15 to 20 minutes.

Participants

We recruited 4–5-year-old children through advertisements on Instagram. The final sample consisted of 40 children (M = 56 months, SD = 5 months, range = 49–65 months; 20 females), all residing in the United States. All participants spoke English as their primary language. An additional 20 children were excluded from the final analyses based on our preregistered criteria: failure to complete both sessions (N = 11), parental intervention during the task (N = 2), a counting range greater than 80[1](N = 7).

Day 1

Pre-test

<u>Highest Count Task.</u> In this task, children were prompted to count as high as possible. The experimenter provided only motivational prompts (e.g., "21, 22, 23 and…?"). We recorded the highest count as the largest number a child could produce before making an error. The experimenter moved on to the next task when the child was unable to continue counting.

Table 1: Questions in the Infinity interview

Type	Question
Keep-going	*Do numbers keep going and going and going and never stop? [Pause] Or is there an end to numbers? What do you think?*
Gazillion	*I'm thinking about a really big number… One gazillion! It's bigger than one hundred, bigger than one billion, it's really big. Could there be a number bigger than one gazillion?*
Gazillion Construction	If passed the Gazillion question: *How could we make a number that is bigger than one gazillion? What number is bigger than one gazillion?*

<u>Infinity Interview.</u> The Infinity interview was adapted from Cheung et al. (2017) and Chu et al. (2020). It included three main question types: whether numbers go on indefinitely (Keep-going question), whether there is always a larger number (Gazillion question), and whether children know how to make a number larger than one gazillion (Gazillion construction question). For all three question types, responses of "I don't know" or no response were considered failures.

<u>Next Number Task</u>. This task was introduced as a "Number Card Game." On each trial, the child saw a card on the screen, and the experimenter explained that there was a number behind the card. As the number was revealed, the experimenter asked if the child knew the number ("What is this number? This is…"). This was to keep the child engaged. If the child could not name the number, the experimenter provided the correct number and asked the child to repeat ("This is N. Can you say N?" "Perfect!"). The experimenter then asked the next-number question: e.g. "What comes after 32?". Each child was presented with nine numbers: six critical numbers beyond their counting range and three easy one-digit numbers placed at the beginning, middle, and end of the sequence. The one-digit numbers were used to put children at ease and to remind them of the sequence one to nine. The specific critical numbers presented depended on the child's counting ability. Children whose highest count was below 30 received 32, 55, 80, 97, 104, and 151. Children whose highest count was above 30 received 80, 97, 104, 151, 1006, and 1023. We included very large numbers to test whether children generalize to levels of hundreds and thousands.

Training Part 1

<u>Count with the Grid</u>. This activity aimed to help children see the correspondence between oral and written numbers and to present numbers in a format that reflected the "decade+digit" structure of the count list. The experimenter first showed the child a blank 10×5 grid and invited them to fill it in with numbers. The grid was introduced as a "magic number grid," in which numbers appeared one by one as the child named them, thereby helping children make correspondences between number words and Arabic numerals. To demonstrate how the magic number grid worked, the experimenter began by counting 0, 1, 2, and 3 and then invited the child to continue counting. If the child was reluctant to count, the experimenter counted the remaining numbers in the row and then invited the child to count along. By the end of this activity, numbers from 0 to 49 were revealed on the grid.

<u>Fill-in-the-Blank</u>. This activity aimed to guide children to notice the regular structure within the number. On each trial, the child saw three rows of numbers (Figure 2)

[1] We preregistered this criterion because previous research (Chu et al., 2020; Schneider et al., 2020) suggests that highly competent counters (counting range > 80) show near-ceiling performance in tasks like ours.

30 numbers in a 10×3 grid) with one number missing. The task was to identify the missing number. The experimenter prompted, “Look, one number is missing. What number should we put there?” If the child did not respond, two additional prompts were given as needed: “It’s after 24. What comes after 24?” and “Let’s count from 20: 20, 21, 22, 23, 24, … Now what’s next?” The number was placed back on the grid once it was identified. If the child did not respond after the two prompts, the experimenter provided the correct answer. This practice included six trials, with the missing numbers being 6, 25, 35, 38, 48, and 148. Children answered correctly on 44% of trials without help and on 90% of trials with the experimenter’s prompts.

20	21	22	23	24		26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49

Figure 2: An example of the Fill-in-the-Blank activity.

Day 2

Training Part 2

Introducing the Decades. This activity aimed to help children recognize that the decade numbers all end in 0. Children were shown the 10 × 5 grid again, and the column of numbers ending in zero was highlighted. The experimenter named these numbers and asked children to look for commonalities among them. Regardless of children’s responses, the experimenter pointed out that all of these numbers end in zero and explained that they are called *decade numbers*. The experimenter then asked the child to repeat the word *decade*. Sixty-eight percent of children independently noticed that all the decade numbers end in zero.

Decade Counting. This activity aimed to help children recognize that the same sequence of digits repeats after each decade number. The experimenter started by highlighting the row beginning with 20 and explained that counting becomes easier after a decade number. Then, the experimenter invited the child to count along using a rhythmic tone (e.g., “After 20, we just go 21, 22, 23…”). The same procedure was repeated for the rows beginning by 30 and 40 with the experimenter providing support as needed. After children counted to 49, the number 50 was added to the next row, and children were encouraged to count on. The numbers 50-59 appeared as the child named them. Next, a row containing only the number 150 was added below the grid, and the child was asked to fill in the remaining numbers. After counting, the experimenter pointed out that counting proceeds in the same way for 50 and 150.

Fill-in-the-Blank. This followed the same procedure as in Training Part 1, but with different missing numbers: 5, 24, 34, 37, 47, and 147. Children answered correctly on 51% of trials without help and on 90% of trials when given prompts.

Post-test

Next Number Task. The questions and target numbers were identical to those used at pretest.

Infinity Interview. The questions were identical to those used at pretest.

Novel Next Number (Daxty) Task. The goal of the task was (1) to determine whether children could apply the “decade+digit” rule to infer the successor of a novel number, and (2) to determine whether children understood that adding one to the set yields the next number in the count list. We adapted the Novel Next Number task from Sullivan et al. (2025). Children were introduced to Jojo, a treasure hunter with a chest full of jewels. The experimenter first demonstrated how numbers worked in Jojo’s language (e.g., “See, Jojo says he has daxty diamonds. Look, he just found one more. Now he says he has daxty-one diamonds.”).

Children then completed four test trials. In each trial, the experimenter first showed a picture of Jojo with a chest of jewels and said, “Look! Jojo says he has *daxty-four* rubies.” After the child repeated the number, one jewel was added to the collection. The experimenter asked, “Look, he just found one more! How many rubies does he have now? (*How-many* question)” For children who did not give the correct answer, the experimenter followed up by asking, “What comes after *daxty-four*? (*What’s-next* question)”

The target numbers were *daxty-four*, *daxty-seven*, *one-hundred-daxty-two*, and *one-hundred-daxty-six*.

Coding

The videotapes of Session 1 and session 2 were coded by two trained research assistants separately.

Results

Highest Count On average, children could fluently count up to 30 (range: 8-69) without making any errors.

Effect of Training

Next Number Task This analysis focused on children’s performance on the critical numbers. Trial-level performance was analyzed using a mixed-effects logistic regression. Fixed effects included timepoint (pre vs. post), highest count, and age (in months), as well as interactions between timepoint and highest count and between timepoint and age. The highest count score and age were entered as standardized scores. Random intercepts were included for subjects and trial number.

Consistent with the prediction that training would improve children’s ability to produce successors beyond their stable counting range, the analysis revealed a significant main effect of timepoint (Figure 2; $\beta = 1.90$, $SE = 0.27$, $z = 6.98$, $p < .001$; pretest vs. posttest accuracy: 32% vs. 61%). At pretest, children on average correctly produced successors for 2 numbers beyond their counting range (M = 1.85 out of 6). After training, children correctly produced successors for 4 such numbers (M = 3.62 out of 6).

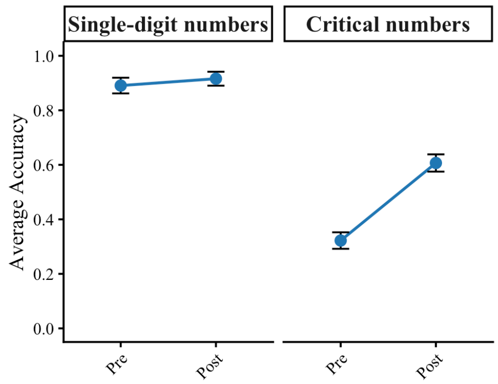

Figure 3: Pre vs. post accuracy on the Next Number task, including both single-digit numbers and critical numbers. Error bars indicate ±1 SE.

There were no significant interactions between timepoint and either highest count or age, suggesting that the training effect was similar across children with different counting abilities and ages. Highest count significantly predicted children's performance at both pretest and posttest $\beta = 0.91$, $SE = 0.35$, $z = 2.60$, $p < .01$), whereas age did not predict performance at either timepoint.

Infinity Interview We examined whether training provided children with insight into the infinite nature of numbers. Responses to each Infinity Interview question were analyzed using logistic regression models. Fixed effects included timepoint, highest count, age, as well as interactions between timepoint and highest count and between timepoint and age. Highest count and age were entered as standardized scores.

This analysis revealed no significant effect of timepoint; we saw no evidence that performance on the Infinity Interview improved from pretest to posttest (Table 2). There were also no significant interaction effects. Neither highest count score nor age significantly predicted children's performance at either timepoint.

Table 2: Percentage of children who answered the Infinity questions correctly

Type	Pre	Post
Keep-going	42.5%	40.0%
Gazillion	47.5%	52.5%
Gazillion Construction	2.5%	10.0%

Novel Next Number (Daxty) Task

For each *how-many* trial, children received a score of 1 if they gave the correct response (e.g., if the initial number was *daxty-four*, *daxty-five* would be the correct response). They received a score of 1 on the *what's-next* question if they either (a) correctly answered the *how-many* question or (b) give the correct response for the *what's-next* question.

For both questions, answering correctly provides evidence that the child can generalize the "decade+digit" understanding to novel numbers. However, answering the *how-many* question correctly requires the additional understanding that adding one item to a set corresponds to increasing by one in the count sequence. We turn first to the *what's-next* question, which we expected to be easier.

We found that children answered the *what's-next* question with a mean accuracy of 63%. In contrast, they answered the *how-many* question correctly on only 39% of trials. A paired-samples t test showed that accuracy on the *what's-next* question was significantly higher than on the *how-many* question, $t(38) = 4.46$, $p < .001$.

Next, because the *daxty* number words appeared in two structures (*daxty-n* and *one-hundred-daxty-n*), we examined whether children's performance differed across these structures. Not surprisingly, children performed better with novel two-digit numbers than with novel three-digit numbers; performance on the *daxty-n* numbers was significantly higher than on the *one-hundred-daxty-n* numbers for both the *what's-next* question (*daxty-n*: $M = .72$; *one-hundred-daxty-n*: $M = .56$, $t(38) = 2.82$, $p < .01$), and the *how-many* question (*daxty-n*: $M = .44$; *one-hundred-daxty-n*: $M = .35$, $t(38) = 2.02$, $p = .051$).

Predictors of Novel Next Number Performance

If the training provided children with generalizable insights into the decade+digit structure of number words, then children who performed better on the Next Number task at post-test should also show better performance on the Novel Next Number (Daxty) task. To examine this possibility, we fit regression models predicting children's Novel Next Number performance from their post-test Next Number performance.

As predicted, post-test Next Number performance was a significant predictor of performance on the *what's-next* question ($\beta = 0.58$, $SE = 0.10$, $t = 5.59$, $p < .001$). Interestingly, the same pattern also emerged for the *how-many* question ($\beta = 0.56$, $SE = 0.13$, $t = 4.51$, $p < .001$). These results indicate that stronger post-test Next Number performance was associated with both children's ability to produce the successor of a novel number and their understanding that adding one yields the next number in the count list.

Predictors of Infinity Interview Performance

Following the proposal that understanding the recursive structure supports the discovery of infinity (e.g., Chu et al., 2020), we fit logistic regression models predicting post-test Infinity performance from *what's-next* performance in the Novel Next Number task. Separate models were fit for each Infinity Interview question.

We found that the *what's-next* performance predicted children's understanding that there could be numbers larger than one gazillion (*what's-next*: $\beta = 0.56$, SE $= 0.23$, $z = 2.37$, $p < .05$), but not the other infinity questions.

Parallel models using post-test Next Number performance as the predictor revealed no significant effects on Infinity Interview questions.

Discussion

In this study, we asked whether intensive experience with comparisons across decades would help 4–5-year-old children grasp the structure of number language sooner than they would through typical experience. As predicted, after two sessions of comparison-based training, children were better at producing the next number for numbers beyond their stable counting range.

This improvement is noteworthy given that the total training was only about 30 minutes, whereas under typical development children often take one to two years to learn how to produce successors of numbers beyond their counting range (e.g., Chu et al., 2020; Schneider et al., 2020). The gains following our comparison training therefore lend support to our claim that noticing the parallel morphological structure across decades supports children's discovery of the "decade+digit" structure of the natural language. They also align with prior work in the comparison literature, which demonstrates that spatiotemporal juxtaposition and spatial alignment can support children's learning of relational concepts (e.g., Jee et al., 2014; Jee et al., 2022; Loewenstein & Gentner, 2001; Matlen et al., 2011; Simms et al., 2023; Yuan et al., 2017; Zheng et al., 2025).

One interesting finding was that children's performance on the Next Number task after training predicted their ability to generate successors for novel numbers (e.g., *daxty-six*). This suggests that helping children perceive the structural alignment between decades may support their development of a generalized understanding of the "decade+digit" structure. This provides encouragement for the possibility that analogical comparison is instrumental in allowing children to discover the structure of the natural numbers (Barner, 2017; Carey, 2004; Gentner, 2010; Guerrero & Park, 2023; Marchant & Barner, 2018, Schneider et al., 2021a; Sullivan & Barner, 2014; Thompson & Opfer, 2010).

In contrast to the improvement observed on the Next Number task, we found no evidence that our training improved children's understanding of infinity. We conjecture that understanding infinity through counting requires drawing out the implications of a full understanding of the recursive structure of numbers. Such an understanding likely requires grasping recursion at higher levels, such as hundreds and thousands. Indeed, although we observed improvement in the Next Number task, performance did not reach ceiling at posttest. Moreover, in the Novel Next Number task, children still showed room for improvement, and this gap was larger for one-hundred-and-daxty-n numbers than for daxty-n numbers. This suggests that children may need more experience with the count list to extend their understanding of recursion beyond the "decade+digit" structure to structures such as "hundred+decade+digit" and beyond, in order to fully appreciate the recursive nature of numbers.

Another gap in understanding we observed was between children's ability to produce successors and their understanding that adding one additional item will produce the next number (i.e., that the successor of N represents N with one additional item). This was seen in the Novel Next Number task, where many children could correctly answer when prompted with "What comes after *daxty-four*?" but could not tell how many jewel there would be if one jewel was added to *daxty-four* jewels. Similar patterns were reported by Davidson et al. (2012), who found that children perform better on the Next Number task than on a task requiring them to indicate the total quantity after one more item is added to a known set (Unit Task). Ongoing work in our lab is further investigating whether the gap exists in English numbers across different magnitudes.

The role of number language in number learning. One hypothesis that underlies our work is that understanding the structure of the linguistic count list can support children's understanding of numbers. This idea is based in part on evidence that language can provide tools of cognition that persist over time (e.g., Gentner & Christie, 2010; Gentner et al., 2013; Christie & Gentner, 2014; Pruden et al., 2011; Pyers & Senghas, 2009). For example, providing children with language for systems can improve their performance on a challenging mapping task (Loewenstein & Gentner, 2005). Here, our specific hypothesis is that learning the regular morphological structure of numbers—e.g., that after each decade number (e.g., 20), the successors will be decade+1 through decade+9 (e.g., 21-29)—will support their understanding of numbers as a generative system.

An open question is what other knowledge about the number system children can gain from learning that number language follows a recursive structure. In our study, we assessed children's improvement only in their ability to name the next number and in their understanding that numbers can go on indefinitely. However, it is also possible that, by developing a better understanding of recursive structure, children gain knowledge in other aspects of the number system. Future studies should test whether having a better grasp of recursive structure leads to improvements in understanding the compositionality of number words (e.g., fifty-one is composed of five tens and one) and the relative magnitudes of numbers—two aspects of numerical understanding that are known to be acquired later than knowledge of recursive structure (Guerrero et al., 2020; Laski et al., 2014; Mix et al., 2022; Yuan et al., 2020).

Conclusion

In conclusion, our study demonstrates that training designed to highlight the regular structure of numbers can enhance children's understanding of the structure of the count list. These findings provide support for our claim that analogical comparison plays a key role in the acquisition of the structure of the natural numbers. We suggest that investigating the developmental pathway toward a fully developed understanding of the structure of natural numbers will be valuable not only within the numerical domain, but also within relational learning more generally. Understanding how children move from local knowledge to global, abstract representations is therefore not only central to theories of number acquisition, but also informative about fundamental processes of learning and discovery.

Acknowledgements

Many thanks to Emily Espinosa and Emma Shuford for their hard work coding the videos, and to the members of the Cognition and Language Lab for their insightful feedback in lab meetings and their support throughout.

References

Barner, D. (2017). Language, procedures, and the non-perceptual origin of number word meanings. *Journal of child language*, *44*(3), 553-590.

Baroody, A. J., Lai, M. L., & Mix, K. S. (2014). The development of young children's early number and operation sense and its implications for early childhood education. In *Handbook of research on the education of young children* (pp. 205-240). Routledge.

Carey, S. (2004). Bootstrapping & the origin of concepts. *Daedalus*, *133*(1), 59-68.

Carey, S. (2009). Where our number concepts come from. *The Journal of philosophy*, *106*(4), 220.

Cheung, P., Rubenson, M., & Barner, D. (2017). To infinity and beyond: Children generalize the successor function to all possible numbers years after learning to count. *Cognitive psychology*, *92*, 22-36.

Chrisomalis, S. (2010). *Numerical notation: A comparative history*. Cambridge University Press.

Christie, S., & Gentner, D. (2010). Where hypotheses come from: Learning new relations by structural alignment. *Journal of Cognition and Development*, *11*(3), 356-373.

Christie, S., & Gentner, D. (2014). Language helps children succeed on a classic analogy task. *Cognitive science*, *38*(2), 383-397.

Chu, J., Cheung, P., Schneider, R. M., Sullivan, J., & Barner, D. (2020). Counting to Infinity: Does learning the syntax of the count list predict knowledge that numbers are infinite?. *Cognitive Science*, *44*(8), e12875.

Davidson, K., Eng, K., & Barner, D. (2012). Does learning to count involve a semantic induction?. *Cognition*, *123*(1), 162-173.

Dowker, A., Bala, S., & Lloyd, D. (2008). Linguistic influences on mathematical development: How important is the transparency of the counting system?. *Philosophical Psychology*, *21*(4), 523-538.

Fuson, K. C., & Willis, G. B. (1988). Subtracting by counting up: More evidence. *Journal for Research in Mathematics Education*, *19*(5), 402-420.

Gelman, R., & Gallistel, C. R. (1978). The Child's Understanding of Number. Cambridge, MA: Harvard University Press.

Gentner, D. (2010). Bootstrapping the mind: Analogical processes and symbol systems. *Cognitive science*, *34*(5), 752-775.

Gentner, D., & Christie, S. (2010). Mutual bootstrapping between language and analogical processing. *Language and Cognition*, *2*(2), 261-283.

Gentner, D., Özyürek, A., Gürcanli, Ö., & Goldin-Meadow, S. (2013). Spatial language facilitates spatial cognition: Evidence from children who lack language input. *Cognition*, *127*(3), 318-330.

Greenberg, J. H. (1987). *Language in the Americas*. Stanford University Press.

Guerrero, D., Hwang, J., Boutin, B., Roeper, T., & Park, J. (2020). Is thirty-two three tens and two ones? The embedded structure of cardinal numbers. *Cognition*, *203*, 104331.

Guerrero, D., & Park, J. (2023). Arithmetic thinking as the basis of children's generative number concepts. *Developmental Review*, *67*, 101062.

Gunderson, E. A., Spaepen, E., & Levine, S. C. (2015). Approximate number word knowledge before the cardinal principle. *Journal of Experimental Child Psychology*, *130*, 35-55.

Jee, B. D., Gentner, D., Uttal, D. H., Sageman, B., Forbus, K., Manduca, C. A., ... & Tikoff, B. (2014). Drawing on experience: How domain knowledge is reflected in sketches of scientific structures and processes. *Research in Science Education*, *44*, 859-883.

Jee, B. D., Matlen, B. J., Greenlaw, M., Simms, N., & Gentner, D. (2022). Spatial supports for comparison in educational science images. *Instructional Science*, *50*(6), 807-827.

Krajcsi, A., & Fintor, E. (2023). A refined description of initial symbolic number acquisition. *Cognitive Development*, *65*, 101288.

Laski, E. V., & Siegler, R. S. (2014). Learning from number board games: You learn what you encode. *Developmental psychology*, *50*(3), 853.

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge?. *Developmental psychology*, *46*(5), 1309.

Loewenstein, J., & Gentner, D. (2001). Spatial mapping in preschoolers: Close comparisons facilitate far mappings. *Journal of Cognition and Development*, *2*(2), 189-219.

Loewenstein, J., & Gentner, D. (2005). Relational language and the development of relational mapping. *Cognitive psychology*, *50*(4), 315-353.

Lyons, I. M., Bugden, S., Zheng, S., De Jesus, S., & Ansari, D. (2018). Symbolic number skills predict growth in nonsymbolic number skills in kindergarteners. *Developmental psychology*, *54*(3), 440.

Maheshwari, U., Sullivan, J., & Barner, D. (2026). Counting without end: A cross-linguistic exploration of infinity beliefs in English and Hindi learners. *Journal of Experimental Child Psychology*, *266*, 106473.

Marchand, E., & Barner, D. (2018). Analogical mapping in numerical development. In *Language and culture in mathematical cognition* (pp. 31-47). Academic Press.

Matlen, B. J., Gentner, D., & Franconeri, S. L. (2020). Spatial alignment facilitates visual comparison. *Journal of Experimental Psychology: Human Perception and Performance*, *46*(5), 443.

Matlen, B., Vosniadou, S., Jee, B., & Ptouchkina, M. (2011). Enhancing the comprehension of science text through visual analogies. In *Proceedings of the annual meeting of the cognitive science society* (Vol. 33, No. 33).

Miller, K. F., & Stigler, J. W. (1987). Counting in Chinese: Cultural variation in a basic cognitive skill. *Cognitive development*, *2*(3), 279-305.

Miller, K. F., Smith, C. M., Zhu, J., & Zhang, H. (1995). Preschool origins of cross-national differences in mathematical competence: The role of number-naming systems. *Psychological Science*, *6*(1), 56-60.

Mix, K. S. (2002). The construction of number concepts. *Cognitive Development*, *17*(3-4), 1345-1363.

Mix, K. S. (2009). How Spencer made number: First uses of the number words. *Journal of Experimental Child Psychology*, *102*(4), 427-444.

Mix, K. S., Bower, C. A., Hancock, G. R., Yuan, L., & Smith, L. B. (2022). The development of place value concepts: Approximation before principles. *Child development*, *93*(3), 778-793.

Mix, K. S., Sandhofer, C. M., & Baroody, A. J. (2005). Number words and number concepts: The interplay of verbal and nonverbal quantification in early childhood. *Advances in child development and behavior*, *33*, 305-346.

Mix, K. S., Sandhofer, C. M., Moore, J. A., & Russell, C. (2012). Acquisition of the cardinal word principle: The role of input. *Early Childhood Research Quarterly*, *27*(2), 274-283.

Opfer, J. E., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive psychology*, *55*(3), 169-195.

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter?. *Developmental science*, *14*(6), 1417-1430.

Pyers, J. E., & Senghas, A. (2009). Language promotes false-belief understanding: Evidence from learners of a new sign language. *Psychological science*, *20*(7), 805-812.

Rittle-Johnson, B., & Star, J. R. (2009). Compared with what? The effects of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology*, *101*(3), 529.

Sarnecka, B. W., & Carey, S. (2008). How counting represents number: What children must learn and when they learn it. *Cognition*, *108*(3), 662-674.

Schlimm, D. (2018). Numbers through numerals: The constitutive role of external representations. In *Naturalizing logico-mathematical knowledge* (pp. 195-217). Routledge.

Schneider, R. M., Pankonin, A., Schachner, A., & Barner, D. (2021a). Starting small: Exploring the origins of successor function knowledge. *Developmental Science*, *24*(4), e13091.

Schneider, R. M., Sullivan, J., Guo, K., & Barner, D. (2021b). What counts? Sources of knowledge in children's acquisition of the successor function. *Child Development*, *92*(4), e476-e492.

Schneider, R. M., Sullivan, J., Marušič, F., Biswas, P., Mišmaš, P., Plesničar, V., & Barner, D. (2020). Do children use language structure to discover the recursive rules of counting?. *Cognitive psychology*, *117*, 101263.

Simms, N. K., Matlen, B. J., Jee, B. D., & Gentner, D. (2023). Spatial alignment supports comparison of life science images. *Journal of Experimental Psychology: Applied*.

Spaepen, E., Gunderson, E. A., Gibson, D., Goldin-Meadow, S., & Levine, S. C. (2018). Meaning before order: Cardinal principle knowledge predicts improvement in understanding the successor principle and exact ordering. *Cognition*, *180*, 59-81.

Srinivasan, A. (2024). *Children's Learning of the Compositional Number System*. Harvard University.

Sullivan, J., Cramer-Benjamin, S., Alvarez, J., & Barner, D. (2023). Everything is infinite: Children's beliefs about endless space, time, and number. *Open Mind*, *7*, 715-731.

Sullivan, J., & Barner, D. (2013). How are number words mapped to approximate magnitudes?. *Quarterly Journal of Experimental Psychology*, *66*(2), 389-402.

Sullivan, J., & Barner, D. (2014). Inference and association in children's early numerical estimation. *Child development*, *85*(4), 1740-1755.

Sullivan, J., Alvarez, J., Cramer-Benjamin, S., Holcomb, S., Nolan, M., Morabito, A., & Barner, D. (2025). Do Preschoolers Use Rules to Represent Their Count List?. *Journal of Numerical Cognition*, *11*, e14757.

Thompson, C. A., & Opfer, J. E. (2010). How 15 hundred is like 15 cherries: Effect of progressive alignment on representational changes in numerical cognition. *Child Development*, *81*(6), 1768-1786.

Topaloğlu, S. (2026). *Counting on Language: Morphological Transparency and Children's Acquisition of Number Words and Numerical Concepts.* Harvard University.

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*(2), 155-193.

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive psychology*, *24*(2), 220-251.

Yuan, L., Prather, R. W., Mix, K. S., & Smith, L. B. (2019). Preschoolers and multi-digit numbers: A path to mathematics through the symbols themselves. *Cognition*, *189*, 89-104.

Yuan, L., Prather, R., Mix, K., & Smith, L. (2021, September). The first step to learning place value: A role for physical models?. In *Frontiers in Education* (Vol. 6, p. 683424). Frontiers Media SA.

Yuan, L., Uttal, D., & Gentner, D. (2017). Analogical processes in children's understanding of spatial representations. *Developmental psychology*, *53*(6), 1098.

Zheng, Y. S., Goldwater, M., & Gentner, D. (2025). Structural Alignment and Linguistic Contrast Help Children Learn a Key Principle of Spatial Construction. *Cognitive Science*, *49*(12), e70149.

Zheng, Y., Matlen, B., & Gentner, D. (2022). Spatial alignment facilitates visual comparison in children. *Cognitive Science*, *46*(8), e13182.