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Vehicle Sorting for Platoon Formation: Impacts on Highway Entry and Throughput

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March 10, 2002

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ABSTRACT

This paper develops and evaluates strategies for organizing vehicles into platoons, with the objective of maximizing the distance that platoons stay intact. Fundamentally, this entails grouping vehicles according to their destination. We evaluate various strategies in which vehicles are sorted on entrance ramps, with respect to platoon sizes, throughput and queueing characteristics.

EXECUTIVE SUMMARY

Automated Highway Systems (AHS) are intended to increase the throughput and safety of roadways through computer control, communication and sensing. In the "platoon" concept for AHS, vehicles travel on highways in closely spaced groups. To maximize benefits, it is desirable to form platoons that are reasonably large (five or more vehicles), and it is also desirable to ensure that platoons remain intact for considerable distances. This paper develops and evaluates strategies for organizing vehicles into platoons, with the objective of maximizing the distance that platoons stay intact. Fundamentally, this entails grouping vehicles according to their destination. We evaluate various strategies in which vehicles are sorted on entrance ramps, with respect to platoon sizes, throughput and queueing characteristics.

Among four strategies investigated, a static grouping of destinations provided the largest throughput in most situations. However, with a small number of lanes (2 or 3) and the uniform trip length distribution, dynamic grouping performed better. The flexibility of dynamic grouping appears to be important when the ratio of number of lanes to number of exits is a small number. The random assignment strategy, as could be expected, produced the smallest platoon ratio and throughput in all cases. We also found that throughput is not a strictly increasing function of the number of entrance lanes in all cases.

INTRODUCTION

Automated Highway Systems (AHS) are intended to increase the throughput and safety of roadways through computer control, communication and sensing. In the "platoon" concept for AHS, vehicles travel on highways in closely spaced groups. Within a platoon, vehicles are separated by very short distances (on the order of 1m) Spacing from platoon to platoon can be considerably longer, to minimize the likelihood that platoons collide with each other. The advantages and disadvantages of platoon operation are discussed in Browand and Michaelian (2000), Shladover (1979), Tsao and Hall (1994), Tsao et al (1993), and Rajamani et al (2000).

To maximize benefits, it is desirable to form platoons that are reasonably large (five or more vehicles), and it is also desirable to ensure that platoons remain intact for considerable distances. Unfortunately, when an individual vehicle needs to exit from the highway, it may need to be separated from its platoon. The separation process can force vehicles to travel farther apart, consuming more highway capacity. It also exposes vehicles to additional safety risk. Thus, the frequency in which vehicles enter and exit platoons can affect highway performance.

As a simple illustration, suppose that a highway is homogeneous with respect to origin/destination patterns, has an average trip length of L, spacing between exits of x and platoons designed to be size N. The probability that a randomly selected vehicle will choose to leave the highway at an exit is then x/L. If platoons are formed through an independent selection process, the probability that a platoon has no exiting vehicles is $(1-x/L)^N$. Example calculations are shown in Table 1.

Table 1. Probability that Platoon Remains Intact at Exit

	Designed Platoon Size (N)									
x/L	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0.20	0.80	0.64	0.51	0.41	0.33	0.26	0.21	0.17	0.13	0.11
0.10	0.90	0.81	0.73	0.66	0.59	0.53	0.48	0.43	0.39	0.35
0.05	0.95	0.90	0.86	0.81	0.77	0.74	0.70	0.66	0.63	0.60

For example, with exits spaced 2 miles apart, an average trip length of 20 miles and a platoon size of 5, there is only a 59% chance that a platoon will remain intact between one exit and the next, thus creating considerable instability. On the other hand, if vehicles are grouped by destination, platoons would remain intact over longer distances, adding to the safety and throughput of the highway.

This paper develops and evaluates strategies for organizing vehicles into platoons, with the objective of maximizing the distance that platoons stay intact. Fundamentally, this entails grouping vehicles according to their destination. There are, however, many ways to accomplish this goal, along with significant trade-offs with respect to construction costs, queueing and throughput. Within this paper, both analytical and simulation results are provided. We limit the research to strategies for sorting vehicles at highway entrances. Future research will examine strategies for sorting vehicles on highway lanes. Our analysis is limited to a single class of vehicles, thus precluding sorting vehicles by characteristics other than destination (such as size; see Hall and Li, 1999, for instance).

A variety of authors have developed capacity estimates under platooned operation. In the interest of brevity, we simply list some of the related work: Rao et al (1993), Rao and Varaiya (1993, 1994) and Tsao et al (1993), Tsao and Hall (1994), Broucke and Varaiya (1995), Hall (1995b), Hall (1996a,b), Hall and Caliskan (1997), Hall and Li (2000), Hall et al (2001), Alvarez (1997), Ramaswamy (1995, 1997) and Tsao et al (1997),

The current paper is most similar to the entrance models developed in Hall et al (2001), Hall and Li (2000) and Hall and Li (1999) in which the entrance capacity of an AHS was evaluated via simulation. The focus here, however, is on grouping vehicles by destination in order to increase the distance that platoons can travel without splitting apart. Unlike these prior papers, we do not explicitly model the merging of vehicles on the entrance ramp with vehicles on the mainline and instead concentrate on the formation and characteristics of platoons that can be created on ramps.

System performance is evaluated on the following dimensions:

Platoon Ratio: Ratio of vehicle miles traveled to platoon miles traveled

Platoon Distance: The average distance traveled by platoons before separating.

Highway Throughput: Upper bound on highway throughput, derived from the platoon ratio, combined with inter- and intra- platoon spacing parameters.

Waiting Time: Average waiting time for platoon formation.

Policies are classified according to the following factors:

Platoon Splitting: Whether an exiting vehicle causes the entire platoon to split apart, or whether continuing vehicles can remain as a platoon until subsequent splits.

Static/Dynamic: Whether the rules for classifying vehicles into platoons are constant over time, or whether they dynamically respond to the state of the system.

The following section presents a set of policies for sorting vehicles and develops analytical models for some system characteristics. Section 3 simulates the policies with respect to a greater range of performance characteristics and situations. Section 4 provides interpretations and conclusions.

Strategies for Forming Platoons at Entrances

The focus of this section is first to define a set of platoon formation strategies, and second to develop analytical models for performance measures. The analytical models are limited to a set of special cases and approximations. For instance, in some cases an exponential trip length distribution is used and in other cases a uniform trip length distribution is used. More detailed results are presented later, based on a series of simulations, and based on different types of trip length distributions.

In this paper, we do not permit platoons to form on the highway itself (strategies in which platoons are formed on the highway are investigated in a subsequent report). Thus, once a vehicle splits from its platoon, it stays split for the remainder of its journey. Several strategies are examined for forming platoons on ramps, which are described in the subsections. All strategies assume that vehicles are grouped by lanes, and that each platoon represents an uninterrupted sequence of vehicles within an individual lane.

Destination Group (DG)

Under the DG strategy, platoons are formed at the entrance ramp on the basis of destination groups. Each entrance lane represents one group, which comprises a set of adjacent highway exits. Each exit is assigned to exactly one destination group. Vehicles enter the highway as platoons, which remain intact until a distance y upstream from the first ramp in the group. At this point, the platoon separates, and vehicles travel individually until reaching their exits. The distance *y* must be sufficient for completion of de-platooning maneuvers, and to maneuver into appropriate exit lanes.

Distance Traveled by Platoon

To optimize highway capacity, it is desirable to form destination groups that maximize distance traveled by platoon. The following terminology defines the optimization problem:

$p(i)$ = proportion of demand that is destined for ramp i

$x(i)$ = distance from entrance to ramp i

m = number of exit ramps

n = number of destination groups

b_j = index for the first ramp in destination group j

($b_1 < b_2 < \ldots < b_n$)

We assume that $x(1) < y$ and $x(b_2) > y$. The expected distance traveled by platoon can then be calculated through the following objective function.:

$$\max_{b_j} \; P = \sum_{j=2}^{n} \sum_{i=b_j}^{b_{j+1}-1} (x(b_j) - y)p(i) \qquad (1)$$

The problem posed by Eq. 1 is equivalent to finding the following longest path. Let:

a_{ij} = length of arc (i,j)

$$= \sum_{k=i}^{j-1} (x(i) - y)p(k) \;, \qquad x(i) \geq y \qquad (2a)$$

$$= 0 \qquad x(i) < y \qquad (2b)$$

$d_{i,l}$ = length of longest path from node i to node m+1, allowing for no more than l arcs

$$= \max_{m+1 \geq j > i} \{a_{ij} + d_{j,l-1}\} , \qquad l < m-i+1 , \ i \leq m \tag{2c}$$

$$d_{m+1,0} = 0 \tag{2d}$$

The solution is found through solving the backward recursion in Eqs. 2 by dynamic programming. $d_{1,l}$ defines the optimal path, and optimal destination grouping, for l destination groups. That is, any arc (i,j) in a path defines a destination group: i, i+1, ..., j-1.

A necessary condition for optimality is that P cannot be increased by switching an exit ramp from its assigned group to an adjacent group. First, consider switching the first ramp in a group to the prior group. This would have the effect of decreasing platoon distance for the switched ramp and increasing platoon distance for the remaining ramps in the group (Figure 2). The necessary condition for optimality is that:

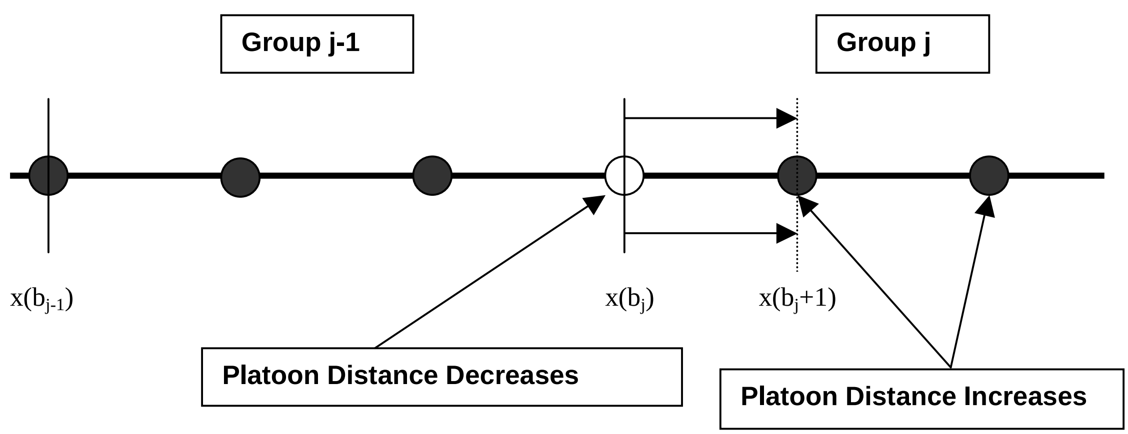

Figure 1. Effect of Switching Destination Into another Group

Decrease in Platoon Distance for First Ramp in Group >
Increase in Platoon Distance for Remaining Ramps , or

$$[x(b_j)- x(b_{j-1})]p(b_j) > [x(b_j+1) - x(b_j)] \sum_{i=b_j+1}^{b_{j+1}-1} p(i) , \qquad j = 3,4,\ldots,n \qquad (3)$$

Eq. 3 depends on a combination of four factors: (1) distance between the start of the prior destination group and the start of destination group j; (2) proportion of trips that are destined for b_j; (3) distance from ramp b_j to the next downstream ramp, and (4) proportion of trips that are destined for other ramps in group j.

A similar necessary condition can be written for switching the last ramp in a destination group into the subsequent group:

Increase in Platoon Distance for Last Ramp in Group <
Decrease in Platoon Distance for Remaining Ramps , or

$$[x(b_{j+1}-1)- x(b_j)]p(b_{j+1}-1) < [x(b_{j+1}) - x(b_{j+1}-1)] \sum_{i=b_{j+1}}^{b_{j+2}-1} p(i) , \qquad j = 2,3,\ldots,n-1 \qquad (4)$$

Because of the *y* parameter, necessary conditions must be expressed differently for ramp b_2 and b_2-1. In the interest of brevity, results are omitted.

<u>Continuous Approximation</u>

The principles for formation of destination groups are more clearly seen through a continuous approximation. Let:

$f(x)$ = probability density function for trip destinations, based on distance
$F(x)$ = probability distribution function for trip destinations, based on distance
z_j = location where destination group j begins

We define $F(z_{n+1}) = 1$, where n is again the number of destination groups. Then our objective becomes:

$$\max_{z_j} P = \sum_{j=2}^{n} (z_j - y)[F(z_{j+1}) - F(z_j)] \tag{5}$$

Assuming continuity for the distribution function, the optimal values of z_j can be derived from the following recursion, once the optimal value of z_2 is determined. The recursion is derived from the derivative of Eq. 5 with respect to z_j:

$$(z_j - z_{j-1}) = [F(z_{j+1}) - F(z_j)] / f(z_j), \qquad j = 3,4 \ldots, n \tag{6a}$$

$$(z_j - y) = [F(z_{j+1}) - F(z_j)] / f(z_j), \qquad j = 2 \tag{6b}$$

or

$$z_{j+1} = F^{-1} [F(z_j) + f(z_j)(z_j - z_{j-1})], \qquad j = 3, \ldots, n \tag{6c}$$

$$z_3 = F^{-1} [F(z_2) + f(z_2)(z_2 - y)], \tag{6d}$$

Figure 2 provides a graphical interpretation of Eq. 6c, using the exponential distribution as an example.

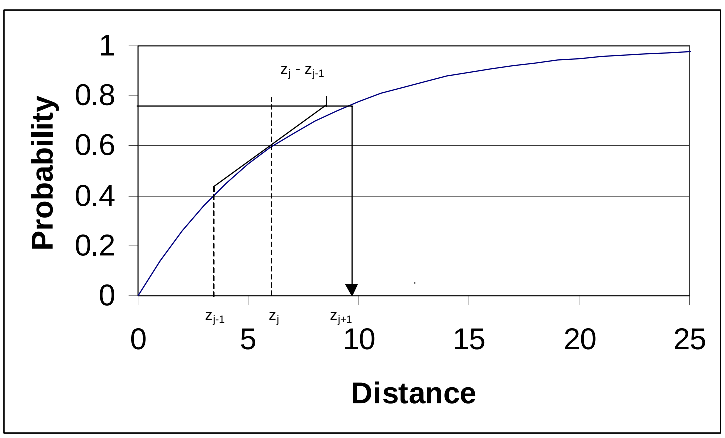

Figure 2. Graphical Interpretation of Equation 6c (continuous) for Exponential Distribution

z_n is a special case, which can be simplified to the following:

$$z_n = z_{n-1} + [1- F(z_n)] / f(z_n) , \quad n \geq 3 \tag{7}$$

Following the recursion, the entire solution can be determined through a one-dimensional search for z_2.

A Special Case: Exponential Distribution

The exponential distribution is of special interest because it captures an empirical trend seen in trip length distributions: that the likelihood of a given trip length declines as trip length increases. For this special case, Eqs. 6 and 7 can be simplified to the following:

$$(z_j - z_{j-1}) = (1/\lambda)[1- e^{-\lambda(z_{j+1} - z_j)}] \qquad j = 3, \ldots, n \tag{8a}$$
$$(z_2 - y) = (1/\lambda)[1- e^{-\lambda(z_3 - z_2)}] \tag{8b}$$

$$z_{j+1} = z_j - (1/\lambda)\ln[1 - \lambda(z_j - z_{j-1})] \qquad j = 3, \ldots, n \tag{9a}$$
$$z_3 = z_2 - (1/\lambda)\ln[(1 - \lambda(z_2 - y)] \tag{9b}$$

$$z_n = z_{n-1} + 1/\lambda \tag{10}$$

Beginning from Eq. 10, a backward recursion can be formed, resulting in the following pattern:

$$z_{n-1} = z_{n-2} + (1/\lambda)(1- 1/e) , \quad n > 4 \tag{11a}$$

$$z_{n-2} = z_{n-3} + (1/\lambda)[1- e^{-(1-1/e)}] , \quad n > 5 , \ldots \tag{11b}$$

For the special case where $y = 0$, these equations lead to the following numerical values for z_j and P (expected distance traveled by platoon). z_j is expressed as a ratio to the mean trip length ($1/\lambda$).

Table 2. $z_j/(1/\lambda)$

j	n =	2	3	4	5	6	7
2		1	.632	.469	.374	.312	.268
3		--	1.63	1.10	.842	.686	.580
4		--	---	2.10	1.48	1.16	.954
5		--	---	---	2.48	1.79	1.42
6		--	---	---	---	2.79	2.06
7		--	---	---	---	---	3.06
$P/(1/\lambda)$		.368	.531	.626	.688	.732	.765

The last row of Table 1 shows that, with two destination groups, less than 37% of the trip length is traveled by platoon; even with six destination groups less than 75% of distance is traveled in platoon. These results do not factor in splitting of platoons prior to exits (represented by the parameter *y*), which would further reduce the percentages. On the other hand, non-exponential trip length distributions should support more platooned travel, especially if a large portion of traffic shares a common destination (as is sometimes the case when major trip generators are located near highways).

It can also be observed that with $y = 0$ and the exponential distribution, the following property is satisfied for an optimal destination grouping:

$$1-F(z_2) = P^*/(1/\lambda) \qquad (12)$$

or, the probability that a vehicle enters a platoon equals the proportion of total vehicle mileage that is traveled within platoons. The validity of this relationship is easily seen for the following special case with two destination groups

$$z_2^* = 1/\lambda \tag{13a}$$

$$1\text{-}F(z_2) = e^{-\lambda(1/\lambda)} = e^{-1} \tag{13b}$$

$$P = (1/\lambda)[1\text{-}F(z_2)] = (1/\lambda)(e^{-1}) \tag{13c}$$

Because of the memoryless property of the exponential distribution, the relationship also holds for larger numbers of destination groups.

Queueing Considerations

Vehicle queues build at the entrance ramp as part of the platoon formation process. Consider two policies, a fixed time release and a fixed queue size release, and let:

τ = time gap between platoon releases

N = platoon size for fixed queue size release

For a fixed time release and Poisson arrivals, average wait in queue is simply $\tau/2$. For fixed platoon size, average wait in queue is derived from Little's formula and equals $(N-1)/(2\lambda)$, where λ is the arrival rate per lane. The average platoon size for the former is $\lambda\tau$, while the average platoon size for the latter is N. In both cases, additional queueing can occur as vehicles wait to enter gaps in the highway traffic stream.

Allowance for multiple entrance lanes can cause average platoon size to decrease, average waiting time to increase, or both. It should be noted that the policies set forth for grouping destinations do not assure equal allocations of traffic among lanes, and therefore waiting times and platoon sizes can vary among lanes. It should also be noted that creating more destination groups requires more entrance lanes, and therefore more space to accommodate queued vehicles at the entrance.

It is also possible to form platoons through a two (or more) staged sorting process, which can reduce the width of the entrance ramp (Figure 3). For instance, stage one could divide vehicles into three groups, and stage two could subsequently divide each group into three subgroups. Thus, three lanes would produce nine destination groups. Suppose that the process follows a fixed cycle (length T), divided into n_1 phases (or groups). At the start of each phase, the queued vehicles in one lane are sent to phase 2, where vehicles are sorted into n_2 sub-groups. The sub-groups are released to the highway as soon as the vehicles from stage 1 are sorted in stage 2, and the process repeats with the next lane in the cycle. The principle drawbacks of this approach are a drop in entrance throughput (due to loss time switching between phases), along with additional entrance delay as vehicles are processed through multiple stages.

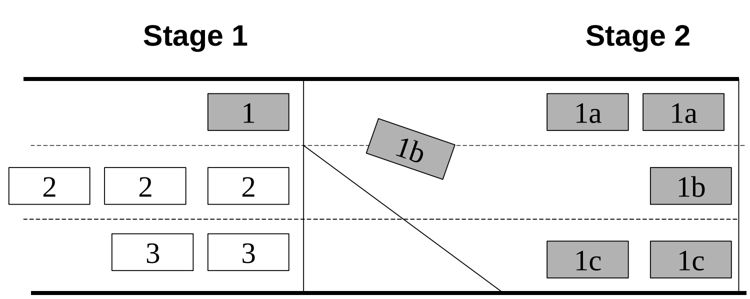

Figure 3. Multi-stage Sorting, First Grouped 1-2-3, then into subgroups a-b-c.

Dynamic Grouping (DYG)

Under the dynamic grouping strategy, destination groups are not permanently assigned to lanes. We propose the following policy:

- Platoons are constrained to have a maximum destination range of r, representing the difference in index between the closest and the furthest destinations in the group.
- An arriving vehicle is assigned to a feasible platoon (i.e., satisfying the range r), if one exists. If no feasible platoon exists, the largest waiting platoon is released, and the arriving vehicle initiates a new platoon in this lane.
- If more than one feasible platoon exists, the arriving vehicle is assigned to the platoon with the "closest boundary."

To illustrate the closest boundary concept, suppose that one group currently has destinations {1,2}, another currently has {5} and destinations are equally spaced. Also, suppose that r =2. If the newly arriving vehicle has destination 3, it is assigned to the first group, even though it would be feasible to assign it to the second. Ties are broken arbitrarily. Because a platoon can serve vehicles destined for either further, or closer, destinations than its first vehicle (or vehicles), the effective range is larger than r. This means that when platoons are small, the probability that a new arrival generates a release is smaller than would be indicated by range alone (leading to somewhat larger average platoon sizes than fixed destination groups).

EDG has the potential to create more tightly spaced groups of destinations without adding to the number of lanes. However, it is more difficult to form large platoons, unless the range and the number of lanes are large enough to cover all destinations. Thus, to make room for a new arrival, a platoon may be forced to depart prior to reaching its maximum size.

As an illustration, consider a simple case with a single lane. Further assume that r << m (thus minimizing end effects), all destinations are equally likely and independent, and that entrances are spaced at unit distance. The system can be modeled as a Markov process, where the state, r', represents the destination range among the vehicles currently in the queue. A state transition occurs when each vehicle arrives. The matrix below shows transition probabilities, which we label as p_{jk}:

Table 3. Transition Probabilities (p_{jk})

From	0	To 1	2	3	….	Departure (0)
0	p	2p	2p	2p	…	1-(2r+1)p
1	0	2p	2p	2p	…	1-(2r)p
2	0	0	3p	2p	…	1-(2r-1)p
…						

For example, if the current range is one and r = 3, any of four events is possible:

Event	Probability
New arrival falls in existing destination group, and range remains the same	2p
New arrival falls one outside existing group, and range increases by one	2p
New arrival falls two outside existing group, and range increases by two	2p
New arrival falls more than two outside existing group; Current platoon is released, new arrival forms new group, and range Returns to state 0	1-6p

Proportion of Distance Traveled by Platoon

If all platoons reach the maximum range r, and destinations are spaced at unit distance, then each vehicle will approximately travel, on average, a distance r/2+y outside of a platoon (This presumes that destinations are symmetrically distributed within the range of r). With destinations that are equally likely and equally spaced, the average trip length is (m+1)/2. Thus, the proportion (m+1-r-2y)/(m+1) will be spent traveling in platoon. In reality, a somewhat higher portion of distance will be spent in platoon, as not all platoons will reach the maximum range before being released.

Expected Platoon Size

By ignoring end effects, an upper bound on the expected platoon size can be computed from state transition probabilities. Let:

P_{ik} = probability that a platoon eventually reaches size i with range k.

$$= \sum_{j=0}^{k} P_{i-1,j} p_{jk}, \qquad i > 1,\ k \le r \tag{14}$$

$$= 1, \qquad i = 1,\ k = 0$$

$$= 0, \qquad i = 1,\ k > 0$$

Then, if platoon size is unrestricted:

$$E(\text{platoon size}) = \sum_{i=1}^{\infty} \sum_{k=0}^{r} iP_{ik} \tag{15}$$

The model (Table 4) tends toward over-estimating E(platoon size) for small n, as it does not account for end effects. For instance, if the first arrival in a platoon has destination 1, then the probability that the following arrival generates a release is 1- (r+1)/n, which is larger than assumed.

Table 4. E(platoon size) [upper bound]

n =	8	10	15	20	large n
r=2	2.07	1.76	1.43	1.30	1+(2r+1)/n
r=3	3.20	2.40	1.71	1.47	1+(2r+1)/n

The limiting equation (large n) is a first-order approximation, applicable when it is very unlikely to form platoons larger than size two.

A lower bound on the departure probability is created for equally likely destinations by assuming that each lane is always limited to accommodating exactly r destinations. (Recall that the effective range can be larger when a single vehicle is in queue). The lower bound follows:

$$E(\text{platoon size}) \geq n/[n-(r+1)] \tag{16}$$

It should be noted that the tightness of this bound increases as r+1 approaches n. When r+1 equals n, the lower bound predicts an infinite queue size, which is effectively exact (new arrivals would always fall in the existing destination group). For n = 8, the lower bound equals 1.6 for r = 2 and 2.0 for r=3. Other results follow.

Table 5. E(platoon size) [lower bound]

n =	8	10	15	20	large n
r=2	1.6	1.43	1.25	1.18	1
r=3	2.0	1.67	1.36	1.25	1

It should be apparent that when all destinations are equally likely, a single lane is unlikely to form very large platoons, unless r is quite large relative to n. However, when r is large relative to n, platoons will be unable to travel far before they need to separate. By expanding the state space, similar stochastic models can be created for multiple lanes. We have instead created simulations to represent these situations, to be presented later.

Dynamic Grouping and Platoon Splitting (DGPS)

We now consider a dynamic policy for grouping destinations that permits platoons to continue after some vehicles split off. This is accomplished by ensuring that vehicles in each platoon are sorted, front to back, in order of non-increasing destination. Thus, the same vehicle can remain as platoon leader through the platoon's lifetime, while the platoon "drops off" vehicles that have closer destinations. This also provides flexibility to group vehicles with a greater range of destinations within a single platoon, which provides flexibility in the entrance process.

Suppose there are n lanes, and let d_j be the destination index for the last vehicle in lane j. The policy is implemented through three rules, representing (1) lane assignment, (2) platoon release, and (3) platoon splitting.

<u>Lane Assignment</u>

Upon arrival, a vehicle with destination θ is assigned to the lane for which: $d_j \geq \theta$, and $d_j - \theta$ is minimized. If no lane satisfies $d_j \geq \theta$, then platoon release is invoked.

Platoon Release

A platoon is released when any of the following events occurs:

- The elapsed time since the first vehicle arrived equals the release time τ
- The number of vehicles in the platoon reaches the maximum N, or
- An arrival cannot be assigned to any current platoon, and the platoon has the smallest value of d_j

Platoon Splitting

A platoon is split when reaching a distance *y* before the destination of the last vehicle in the platoon. Vehicles with more distant destinations remain in the platoon until reaching a distance *y* before their destinations.

Suppose, without loss in generality, that lanes are numbered according to the destination indexes: $d_1 \leq d_2 \leq \ldots \leq d_n$. If $\theta > d_n$, then platoon one is released, and θ is inserted at the end of the sequence, creating a state vector of $(d_2, d_3, \ldots, d_n, \theta)$. Otherwise, θ is inserted at the end of one of the platoons. For instance, if θ is greater than d_1, but less than or equal to d_2, the state becomes: $(d_1 , \theta ,d_3, \ldots, d_n)$. If τ and N do not constrain queue length, P(release) is then defined by $P(\theta > d_n)$, and the expected platoon size is defined by $1/P(\theta > d_n)$.

Calculation of Platoon Sizes

Consider the simple case where there is a single lane, and platoon size is unbounded by τ and N. Then the last vehicle in the queue will always be the last vehicle that arrived. The probability that a new arrival causes a platoon to be released is the probability that the new arrival has a more distant destination than the last vehicle that joined the queue. If independence can be assumed:

$$P(\text{release}) = \sum_{i=1}^{m} p(i) \sum_{j=i+1}^{m} p(j) \tag{17}$$

A continuous approximation for Eq. 17 would be:

$$P(\text{release}) = \int_0^{\infty} f(x) \int_x^{\infty} f(z)dzdx = 1 - \int_0^{\infty} f(x)F(x)dx \tag{18}$$

The expression is equivalent to computing the expectation of the function F(X), where X is a random variable with density function f(x). For any continuous distribution, F(X) varies in value from 0 to 1, with mean ½. Therefore, the P(release) = ½ and expected platoon size equals two (inverse of P(release)). For discrete destinations, P(release) is somewhat less than ½, as a new vehicle has a non-zero probability of having an identical destination as the currently queued vehicle. Nevertheless, the P(release) is reasonably large and the expected platoon size would be only slightly larger than 2. Clearly, the policy is ineffective at forming large platoons when there is just one lane.

For more than two lanes we can approximate the expected platoon size by assuming that, at any time, the probability that $d_j = i$ equals p(i) for all j, and that d_j are independent among lanes. Then the probability of release is defined by:

$$P(\text{release}) = P(\max\{d_j\} < \theta) = \sum_{i=2}^{m} p(i) \left[\sum_{k=1}^{i-1} p(k) \right] \quad . \tag{19}$$

where θ is the destination for a randomly arriving vehicle. If destinations are equally likely, P(release) equals the values in Table 6:

Table 6. P(Release) for DGPS When New Vehicle Arrives

			m		
L	**5**	**10**	**15**	**20**	**∞**
1	.4	.45	.47	.48	.50
2	.24	.28	.30	.31	.33
3	.16	.20	.22	.23	.25
4	.11	.15	.17	.18	.20
5	.08	.12	.14	.14	.17

With three entrance lanes, the approximation produces expected platoon sizes in the range of four to six (inverse of P(release)). The policy is reasonably robust with respect to changing the number of destinations, and approaches the limiting value of 1/(m+1) as m becomes large (a continuous approximation result). However, the policy will not produce very large platoons (on the order of 10) unless the number of entrance lanes is quite large (five or more).

These results are premised on equally likely destinations. Demand concentrations around a limited number of destinations would improve results.

SIMULATION

A simulator was developed to evaluate platoon formation policies with respect to a greater range of performance measures, and for a greater range of scenarios. The following features were common for all policies: (1) Vehicles arrive by stationary Poisson process, (2) Platoon size is constrained not to exceed N, and (3) Vehicle waiting time was constrained not to exceed τ. In our simulation, N was set at 8 and τ was set at 180 seconds.

Performance was evaluated with respect to: Platoon Ratio, Highway Throughput, and Waiting Time. For comparison, we also evaluated a policy in which vehicles were randomly assigned to lanes. In this policy a platoon was split as soon as the first vehicle needed to exit.

The policies were evaluated for a set of scenarios, defined as follows:

Trip length was exponentially distributed, uniformly distributed or clustered. For clustered, (1) 20% of the exits accounted for 50% of demand; (2) demand was identical within each group (those with high demand and those with low demand); and (3) demand followed a repeating pattern, with four low demand exits between each pair of high demand exits.

Exit Spacing was either large (5 miles) or small (1 mile)

Number of Entrance Lanes varied from 2 to 7

Average Trip Length equaled 10 miles in all cases

Highway Length depended on the trip length distribution. For exponential, the highway was limited to 60 miles (6 x mean trip length); for uniform, the highway was limited to 20 miles (e.g., 20 exits with 1-mile spacing).

<u>Throughput Calculation</u>

An upper bound on highway throughput was calculated from spacing parameters and expected platoon sizes. We assume that different types of platoons are intermixed in lanes, and that throughput can be derived from the platoon ratio (which is averaged across all highway segments). In this model, spacing is defined by the time-separation between fronts of vehicles, which eliminates the need to parameterize vehicle sizes. Let:

a = intra-platoon time spacing, front-to-front (seconds)

b = inter-platoon time spacing, front-to-front (seconds)

Π = expected platoon size

μ_1 = vehicle flow per lane

μ_2 = platoon flow per lane = μ_1/Π

Then

$$a\,\mu_1 + (b-a)\,\mu_2 \leq 3600 \tag{20a}$$

or

$$\mu_1 \leq 3600/\,[a + (b-a)/\,\Pi] \tag{20b}$$

For expected platoon size, we use the platoon ratio (expected vehicle miles divided by expected platoon miles). Following Hall and Li (2000), we evaluated throughput for a = .26 s and b = 1.36 s.

<u>Dynamic Grouping Range (DYG)</u>

For the DYG strategy, the range was adjusted to produce the maximum throughput in each situation. This was accomplished by simulating system performance for different values of r, and selecting the best quantity. For 5 miles spacing, a range of 2 was used in all cases. For 1 mile spacing, the range varied from 2 to 7, depending on the trip length distribution and number of lanes. The optimal range increased as the number of lanes increased, and was larger for exponential trip lengths than uniformly distributed or clustered trip lengths.

Simulation Results

Figures 4 to 13 provide results for a range of cases. As a general trend, adding lanes tends to provide longer average waiting time, larger platoon ratios and larger throughput. Waiting times increase because each lane handles fewer vehicles, meaning it takes longer to form a platoon of a given size. The platoon ratio increases because platoons can serve a smaller range of destinations, and because (for some strategies) larger platoons can be formed. Throughput increases because the platoon ratio increases. However, the benefits of adding lanes diminish rapidly beyond four entrance lanes.

The maximum attainable platoon ratio is 8 in all cases, which is the maximum allowed platoon size in the simulations. The maximum is attained for DG (destination grouping) when the number of entrance lanes equals the number of exits (e.g., when exit spacing is 5 miles and there are four entrance lanes and trip length distribution is uniform).

Among the four strategies, DG provided the largest platoon ratio and throughput in most situations. However, with a small number of lanes (2 or 3) and the uniform trip length distribution, dynamic grouping (DYG) performed better. The flexibility of dynamic grouping appears to be important when the ratio of number of lanes to number of exits is a small number. The random assignment strategy, as could be expected, produced the smallest platoon ratio and throughput in all cases. It should be noted that throughput is not a strictly increasing function of the number of lanes for DGPS. The range of destinations within a platoon can be smaller with fewer lanes, meaning that platoons remain intact over longer distances.

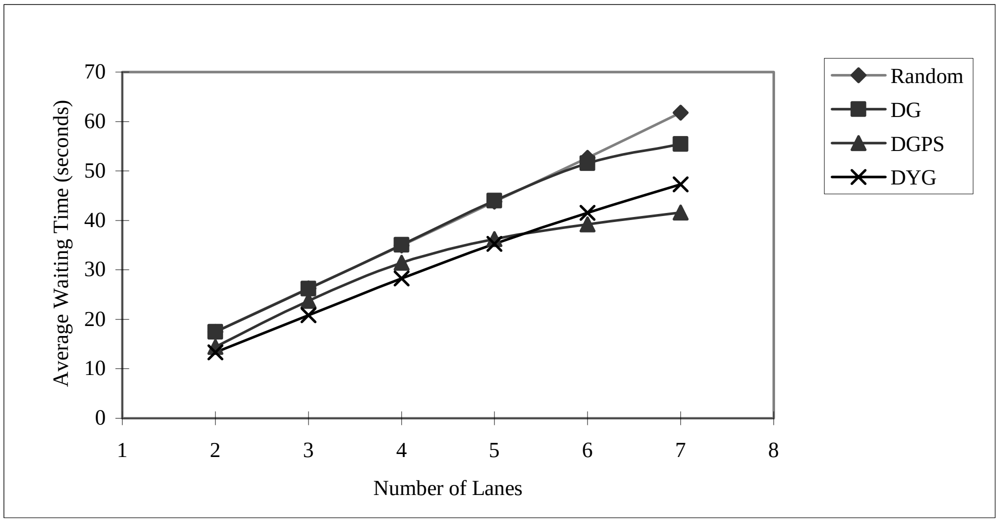

Figure 4. Average Waiting Time Versus Number of Lanes, Exponential Trip Length with 5-mile Exit Spacing

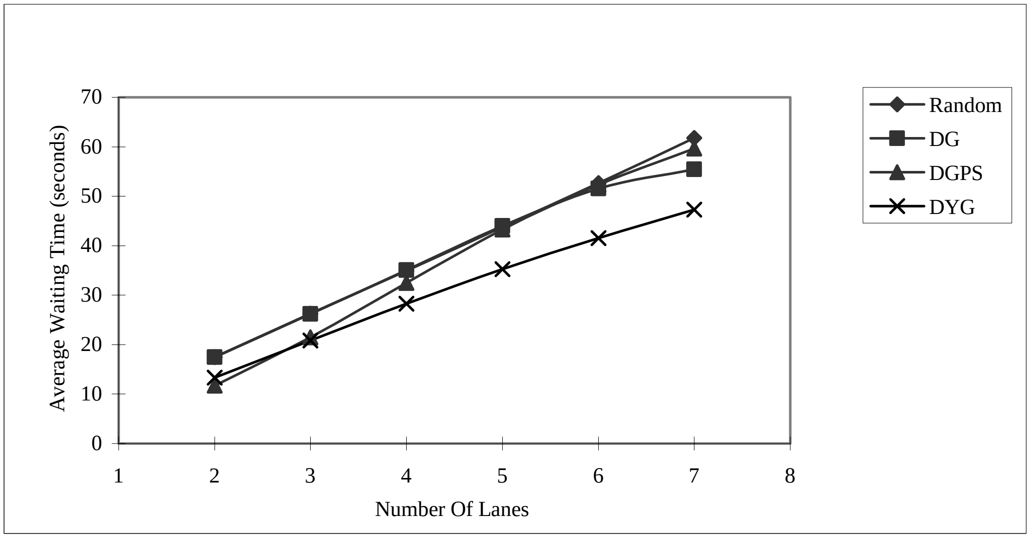

Figure 5. Average Waiting Time Versus Number of Lanes, Exponential Trip Length with 1-mile Exit Spacing

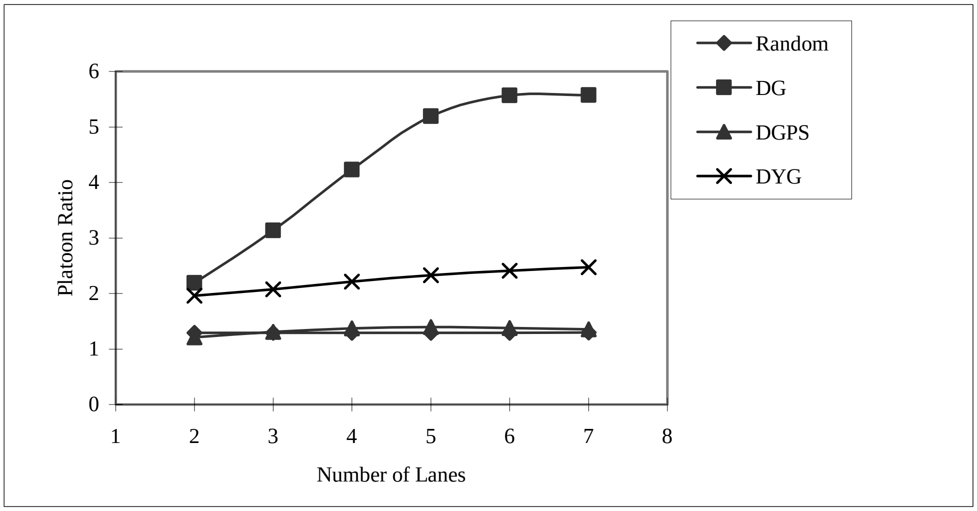

Figure 6. Platoon Ratio Versus Number of Lanes, Exponential Trip Length with 5-mile Exit Spacing

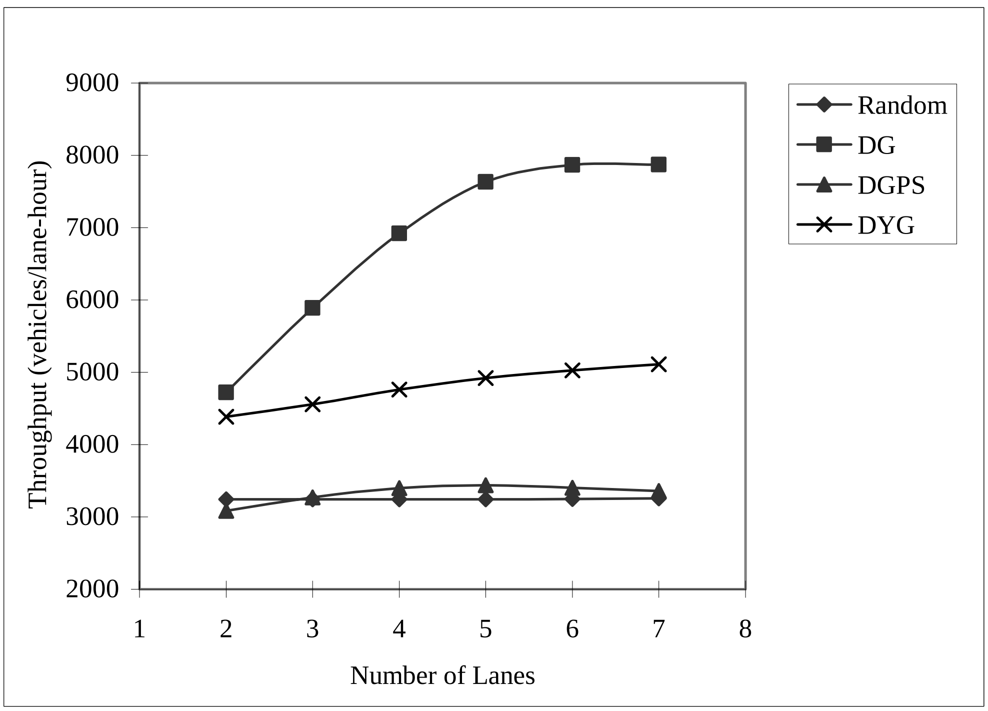

Figure 7. Highway Throughput Versus Number of Lanes, Exponential Trip Length Distribution and 5-mile Exit Spacing

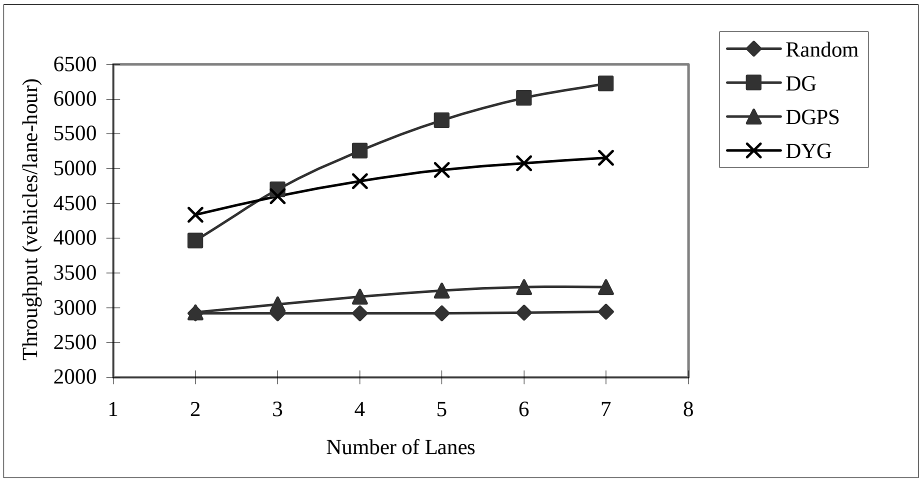

Figure 8. Highway Throughput Versus Number of Lanes, Exponential Trip Length Distribution and 1-mile Exit Spacing

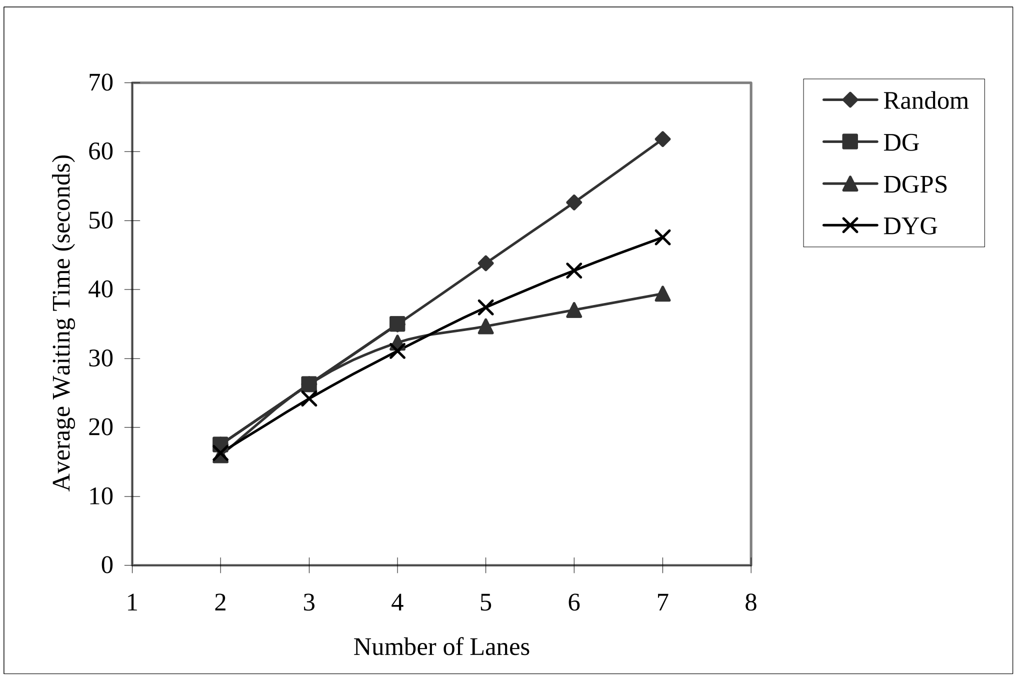

Figure 9. Average Waiting Time Versus Number of Lanes, Uniform Trip Length Distribution and 5-mile Exit Spacing

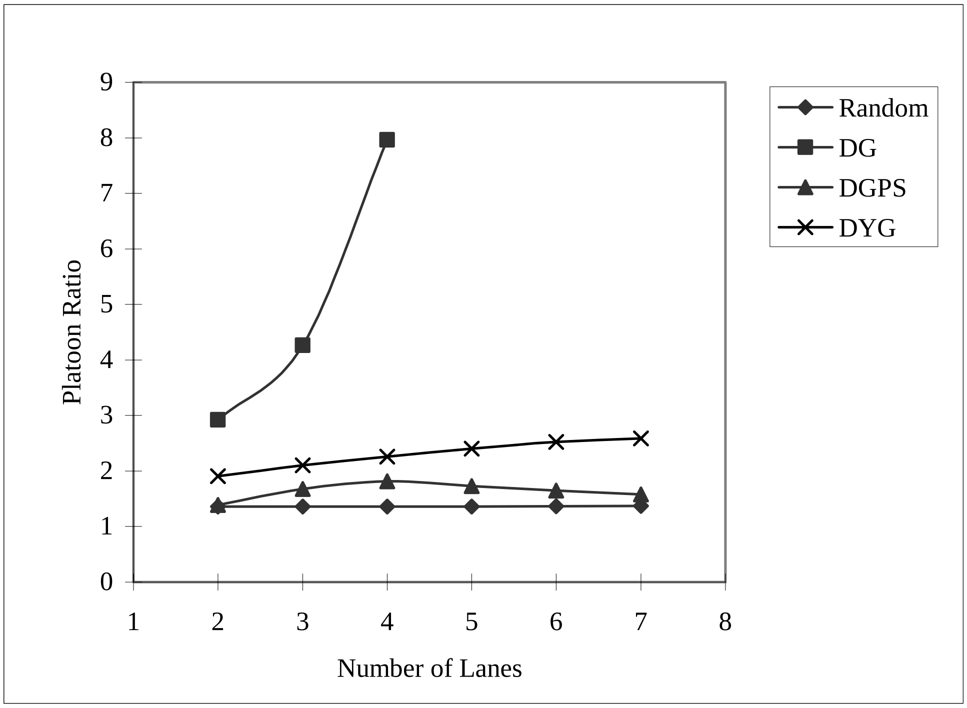

Figure 10. Platoon Ratio Versus Number of Lanes, Uniform Trip Length Distribution and 5-mile Exit Spacing

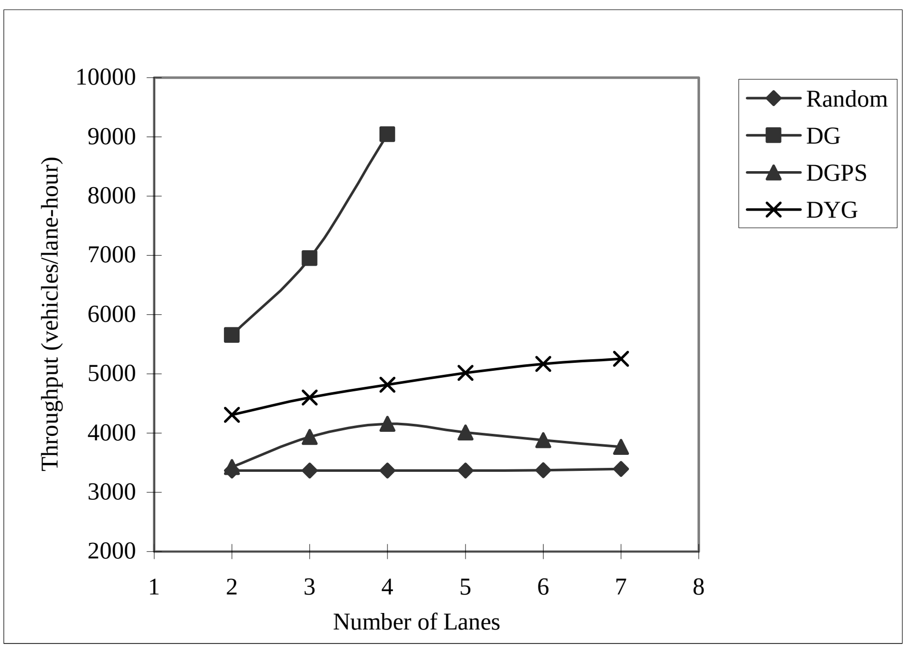

Figure 11. Throughput Versus Number of Lanes, Uniform Trip Length Distribution and 5-mile Exit Spacing

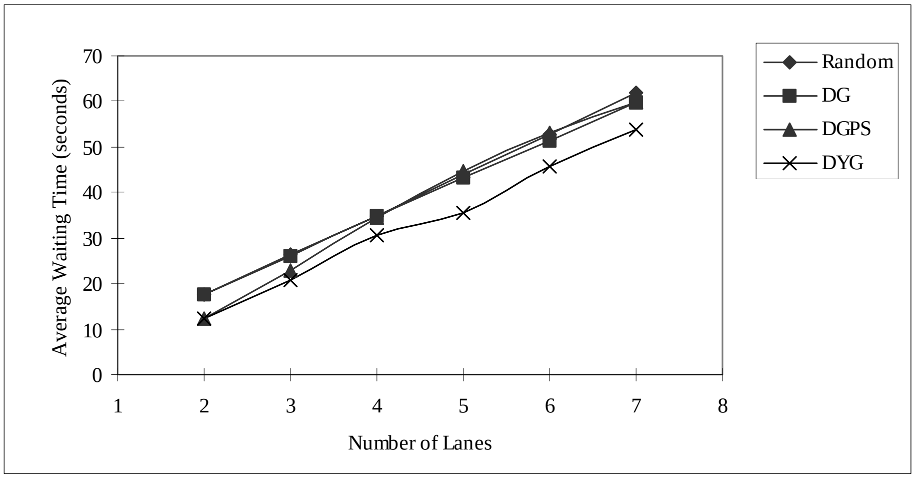

Figure 12. Average Waiting Time Versus Number of Lanes, Clustered Pattern with 1-mile Exit Spacing

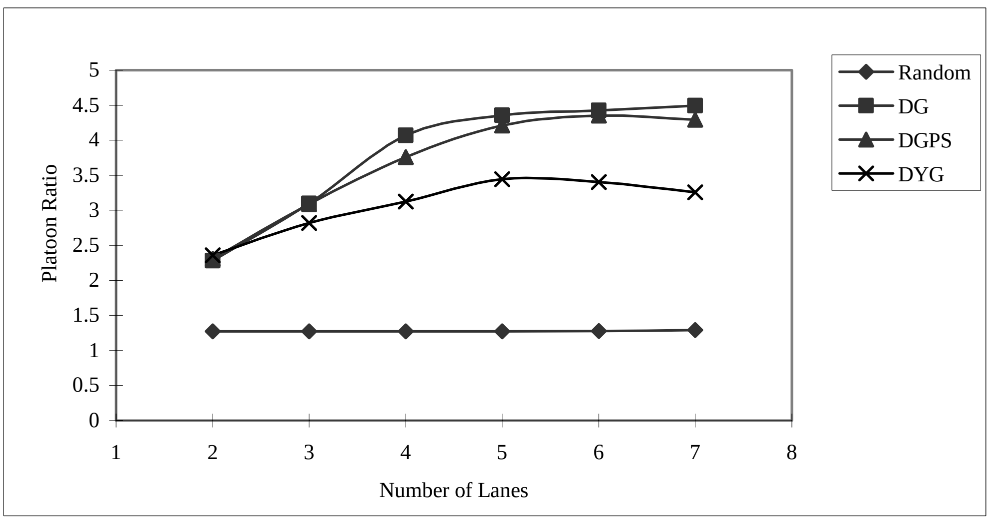

Figure 13. Platoon Ratio Versus Number of Lanes, Clustered Pattern with 1-mile Exit Spacing

Average waiting time is a nearly linear function of number of lanes in most cases. When vehicles are split into more categories, it takes longer to form a platoon of a given size. And although platoon size is also a function of number of lanes, the relationship is fairly insensitive. DYG tends to produce the smallest expected waiting time, though the range among strategies is not so great as the range for platoon ratio or throughput. However, shorter waits do not seem sufficient to compensate for lower throughput (relative to DPGS and DG).

CONCLUSIONS

To maximize highway throughput, it is desirable to create platoons that are large in size, and that remain intact over long distances. Sorting vehicles by destination at the entrance is one way to accomplish this objective. Toward this end, this paper evaluated a range of strategies, and determined how to optimize a dedicated assignment of vehicles to entrance groups. For the cases studied in this paper, dedicated assignment performed better than dynamic assignments with respect to platoon ratio and throughput. However, average waiting time at entrance was somewhat larger.

In future research, we will examine the integration of vehicle sorting at entrances with vehicle sorting on highways. In combination, the strategies will group vehicles according to exit, to facility egress from the highway.

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