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Macroeconomics, Aging and Growth

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Introduction

As low fertility and longer life, there is increasing concern about possible macroeconomic consequences. These consequences unfold on the time scale of demographic change, which is to say not from quarter-to-quarter or year-to-year, but rather decade-to-decade. Will rising old age dependency reduce consumption per capita? Will there be a deepening of capital, and if so, will rates of return drop, leading to an "asset price meltdown"? Or will capital instead flow to the younger developing countries, so that rates of return in developed countries fall less? Might the quality of more highly educated labor effectively substitute for quantity? Will an older labor force be less productive and will innovation and technological progress slow down? Longer-working elderly crowd out employment of the young? Has population aging already brought secular stagnation to the rich nations of the world, underlying the anemic recovery from the Great Recession and condemning their economies to recurring asset price bubbles? This chapter discusses these and related points.

Samuelson (1958, 1975a) was one of the first economists to focus attention on the broader issues surrounding population growth rates, old age dependency, capital deepening, and the central role of intergenerational transfers in shaping the relations among these. A landmark paper by Cutler, Poterba, Sheiner and Summers (1990) framed and analyzed the macroeconomic issues in greater detail with more demographically realistic models. Weil (1997) is an excellent Handbook chapter on the economics of aging and Weil (2008) is an excellent Palgrave article on the topic.

If individuals lived independent lives in isolation they would not care about the ages of others – the age distribution of the population would be irrelevant to them. However, human generations overlap so economic interactions among them can and do occur, both through markets and through non-market intergenerational transfers, both private and public. To quote Weil's (2008) Palgrave article, "Population aging has economic effects whenever some economic interaction (the sale of a good or service, the provision of a government benefit, and so on) brings together people whose participation is a function of their age….Old-age pensions, child rearing, and the combining of old people's capital with young people's labor are all cases where a change in the relative numbers on either side of the equation will have important effects." Similarly, Sheiner et al (2006:16) in their Primer on the macroeconomics of population aging tell us that "If generations were not linked, then, by definition, the consumption choices of one generation and the generation's size would not affect the consumption possibilities of subsequent generations. But intergenerational linkages are important. Such linkages include not only transfer programs and bequests but also, in a closed economy, connections through saving and the return to capital."

This chapter will focus on these interactions with particular emphasis on public and private intergenerational transfers and on capital. I will begin with a brief discussion of population aging. Next I consider economic dependency and the economic life cycle, followed by a consideration of how resources are reallocated from those in working ages to the young and old who consume more than they produce. I will then consider the special case of economies and populations in steady state, and develop the analytically simple but enlightening case of comparative golden rule steady states and equilibrium in the capital market. With that background we will turn to more dynamic models of non-steady state. After this we will consider a number of important special topics including the productivity of an aging labor force, innovation, and intensification of human capital, followed by a concluding discussion.

Population Aging

It is natural to think of population aging in terms of longer life, calling for adjustments in life cycle planning to accommodate the longer expected time horizon. However, in populations closed to migration, the evolution of the age distribution depends on fertility, mortality, and the initial population age distribution. Mortality decline and longer life do tend to make the population older, since on average individuals in each generation live longer. However, lower mortality also makes a population grow faster, so that younger generations are larger at birth than older ones, which tends to make the population younger. In low mortality countries, the net outcome from mortality decline is population aging, but in high mortality countries mortality decline can make populations younger, as it did for developing countries after WWII. Fertility decline, however, leads unambiguously to population aging, because it slows population growth. Life cycle adjustments to longer life can help to meet the economic challenges of population aging, but these adjustments do not address what is often the greater challenge of aging arising from low fertility and slow or negative population growth.

The process by which populations move from initially high fertility and mortality with young age distributions to low fertility and mortality with old age distributions is called the demographic transition (Lee, 2003). This transition, starting with declining mortality, can take around two centuries to unfold. Most countries in the world have started the process, and most have advanced from mortality decline to fertility decline, and are aging due to both. For each country, its own changing population age distribution poses economic problems but may also present important advantages. There are profound differences between countries and regions of the world in the stage of the transition that has been reached and the age distribution that results. These demographic differences may lead to differences in factor prices that yield at least transitory economic opportunities to be realized through international flows of trade, capital and labor.

It will be useful to look more closely at the demographic drivers, starting with mortality decline which leads to longer lives and later deaths. When mortality decline means that an actual individual lives longer, it is clear *ex post* that time has been added at the end of that individual's life. However, *ex ante* people are exposed to the risk of death at every age starting from birth, and deaths occur at all ages. Survival is a matter of probabilities. Based on these probabilities we can describe the distribution of expected person-years lived at each age for a new birth or for a synthetic cross-sectional cohort. The sum across all ages of these expected person-years lived is life expectancy at birth, either actual, for a generation, or synthetic, for a particular year or period. When mortality declines, the expected person-years lived at each age rises, and the sum of the increases is the increased years of life expectancy. These increases are not distributed evenly across age (see Lee, 1994; Eggleston and Fuchs, 2012). When mortality declines from a high level, most of the gains in person-years lived occur in childhood. When mortality declines from a low level as in the rich industrial nations today, most of the gains occur above age 65. The more mortality declines, the more the possibilities for improvement at younger ages are used up, and the more subsequent gains must occur at older ages. The reproductive age of females is bounded at 45 or so. When survival from birth to age 45 approaches unity, further mortality declines have no effect on the number of births and raise the population growth rate only through increased numbers at the older ages without affecting the steady state growth rate.

The discussion so far has been based on chronological age, age measured in calendar years. Changes in age distribution are of interest for two reasons. The first is that physiological state varies with age: children are largely incapable of providing for themselves and have poor judgment; older people have lower health, strength, energy levels, abilities to learn, flexibility, and cognitive capacities compared to prime age adults. At the same time, knowledge, skill, judgment and wisdom increase with age. Consequently productive efficiency at first rises with age and then later declines and the disutility of labor rises (Skirbekk 2008). The second reason is that chronological boundaries are often embedded in public policies, most notably at age 65 which has become an almost official threshold for becoming elderly in many countries.

However, over the past century or so the physiological state of health and vigor associated with any particular age has been improving, or put differently, age specific rates of disability and ill health at any older age have been declining. For the earlier part of the 20th century this has been shown for the United States by Robert Fogel (2004) and collaborators. For the years since the early 1980s, Freedman et al (2013) have demonstrated a similar but more rapid decline. Although this decline appears to have stopped since 2000 in the US, the health status of the elderly at any given age is much improved relative to earlier decades, as reflected in statements such as "70 is the new 60" (National Research Council, 2012, Ch.4). If we redefine "aging" based on physiological status rather than chronological age, then the extent of population aging projected from today through 2050 is much reduced in all industrial nations (Sanderson and Sherbov, 2010). Projected improvements in health and functional status offset the increase in elderly projected on the basis of chronological age. In particular, the fraction of expected adult years lived in good health out of all expected adult years remains roughly constant. We reach similar conclusions when defining old age by distance from death, that is, by remaining life expectancy, so that the threshold automatically rises as life expectancy increases. This approach is consistent with the finding that health costs depend much more on time until death than on chronological age.

Based on the improving functional status measures reported in Sanderson and Sherbov, chronological population "aging" should pose no fundamental economic problem. However, to the extent that both public institutions and popular culture have enshrined age 65 as the appropriate age for retirement, a problem is created. The greatest challenge in the coming decades may be to adjust our institutions and cultural perceptions as rapidly as populations experience chronological aging.

In any event, chronological population aging is inevitable, since with longer life fertility cannot remain above replacement level in a finite world. Countries differ greatly in their position in the transition, with some late starting populations still extremely young and others far older. However, no country is yet near to its long term level of population aging given its current birth and death rates. Based on United Nations medium projections, for example, every country in the world will experience at least a doubling of the ratio of those over 65 to those age 20-64, the old age dependency ratio (OADR). Even Japan, the oldest sizable country in the world, is projected to go from an OADR of .38 in 2010 to .76 in 2050 (United Nations, 2011).

As economies become increasingly open with globalization, global aging rather than national aging will govern capital intensity and factor prices in an economy. Differences in the timing and degree of population aging may shape international flows of capital, trade and labor (Boersch-Supan et al, 2006). While aging is currently most advanced in the rich industrial nations (other than the US), many developing nations including Brazil and China have experienced very rapid fertility decline and will age exceptionally rapidly in coming decades.

Population aging is coming. How will it affect the macroeconomy?

Population aging, dependency, and redistribution of income across generations

Because the biological capacities and needs of individuals vary with age, and also for cultural, institutional and behavioral reasons, labor income and consumption vary with age over the life cycle. Labor income depends on labor force participation rates, efficiency per hour of work, and hours worked per participant. Here it will be useful to focus on the age profile of labor income itself, drawing on estimates from the National Transfer Accounts (NTA) project (Lee and Mason, 2011; United Nations, 2013; ntaccounts.org).

In NTA the age profile of labor income is defined as the average across all individuals at a given age, whether working or not, of before-tax wages and salaries including fringe benefits, plus labor's share of self-employment income and unpaid family labor (assumed to be two thirds, with the remaining third going to assets). The age profile of consumption includes an average across all individuals at a given age of an imputed portion of household consumption expenditure for the household in which each individual lives (including health and education), plus in-kind public sector transfers (mainly health care, long term care and education) to each individual. Imputation is done in proportion to equivalent adult consumer weights that increase linearly from .5 at ages 0-4 to 1.0 at age 20, except for private expenditures on health and education which are estimated separately. These age profiles are cross-sectional estimates based on standard surveys. Longitudinal age profiles would reflect similar age patterns superimposed on a growth-induced upward trend.

Figure 1 shows the resulting age profiles of labor income and consumption. For comparison across countries, these have been standardized by dividing both labor income and consumption at each age by the average level of labor income across ages 30 to 49 for each country. The figure shows the average of these ratios for five rich industrial nations and five lower income countries. The shapes are broadly similar. Consumption greatly exceeds labor income at younger ages and older ages, and ages between earn a substantial surplus. Consumption is also funded out of asset income of various kinds we will consider later. There are important differences between the rich and poor countries. Labor income begins at younger ages in lower income countries, although high youth unemployment in some African countries pulls down the average. Labor income peaks later in the rich countries and then declines much more rapidly, in part due to the availability and incentive structures of public pensions. Beyond age 65 or so labor income is very low. In the lower income countries the decline in labor income begins earlier but is much more gradual, and labor income remains substantial even in the 80s. There is interesting regional and national variation in these patterns (Lee and Mason, 2011, Chs. 5 and 6).

Consumption per child is distinctly higher in the richer nations reflecting heavier investment in human capital, particularly education (recall that public and private expenditures on education are included in this measure of consumption, as described above). The figure also shows a very important difference at adult ages. In lower income countries, consumption is relatively flat across ages, with younger adults and the elderly consuming similar amounts. In the rich countries, however, the pattern is very different: consumption rises throughout the older ages, particularly above age 80. This pattern is not universal among the richer nations, but is particularly pronounced in the US, Sweden, Finland, Germany, the UK and Japan. At older ages it reflects the growing importance of public expenditure on health care, and above 80 on long term care as well.

Population aging increases the share of people at the older ages where consumption greatly exceeds labor income, and to varying degrees may reduce the share of people at the younger dependent ages. The net effect of changing population age distribution on overall dependency is measured by the “support ratio”. The support ratio is sometimes measured as the share of the population in the working ages, say 20-64 or 15-60, divided by the total population. Cutler et al (1990) introduced a different approach, using data for the US to calculate the base period age profile of labor income and of consumption, which were used to calculate the support ratio by weighting the changing population age distributions. This is the approach we take here, calculating the support ratio as the population-weighted sum across ages of labor income divided by the population weighted sum of consumption, using base period NTA age profiles. Let y be the average product of labor, L and C be the population weighted sums of labor income and consumption, respectively, and s be the share of output saved. Then consumption per equivalent adult consumer (EAC), an age-standardized measure of consumption per capita, is given by $c = (L/C)(1-s)y$. That is, other things equal (we will relax this assumption later), consumption per EAC varies in proportion to the support ratio, *L/C*, and proportional variations in the support ratio will correspond to proportional variations in consumption per EAC (Cutler et al, 1990). If y is trending upward with productivity growth, the support ratio still locates consumption relative to this trend.

The age profiles for some base year can be combined with population projections to form hypothetical projections of the support ratio. Of course, we expect both the consumption and labor income age profiles to change in the future, for example if the retirement age rises or public transfers to the elderly are reduced. The projected support ratio is just an analytic tool to capture the pure demographic consequences, but it is in no way a forecast; that would require more comprehensive procedures and serve a different purpose.

Figure 2 shows projections of support ratios based on NTA data and United Nations (2011) population projections from 1960 to 2060. Population aging begins earliest in Japan, around 1970, and progresses rapidly starting around 1990, reaching its deepest point around 2060. Spain’s aging starts later but is even more rapid. The US has only modest aging, with rate of decline of the support ratio only half that of Japan and Spain. In China, starting around 1970, we see a very rapid increase in the support ratio due to its rapid fertility decline, which has the initial effect of reducing child dependency and raising the share of the population in the labor force, generating what is called the “demographic dividend”. In this case, the dividend (calculated as the rate of increase in the support ratio) adds .8% annually to the growth rate of c (over 1974-2012, on average). Starting just after 2012, however, fertility decline has slowed labor force growth enough that the elderly population begins to predominate, bringing an end to the dividend phase, and initiating population aging so that between 2012 and 2050, .4% will be subtracted from the growth rate of c. India has a number of decades before its dividend phase ends and population aging begins, with the whole process slowed by the gradualness of its fertility decline. Finally, in Nigeria fertility decline has barely begun, and it will experience a modest dividend over many decades. These contrasts between the dividend phase and population aging phase are important, but they would be swamped by productivity growth rates of recent decades in China or India.

The support ratio and its changes here reflect all sectors of the economy. If we instead focus on the fiscal support ratio for the public sector (based on age profiles of taxes paid and benefits received), or even more if we focus on specific public transfer programs for the elderly such as pensions or health care, then the effects of population aging can look very much larger. It is important to realize that they refer to only a fraction of the economy. For some purposes, the fiscal support ratio may be informative because it could be viewed as reflecting external costs and benefits of fertility, while private transfers correspond to private costs of childbearing which may be taken into account when individuals choose their fertility.

The support ratio is widely used as a simple and intuitive measure of the effect of the changing population age distribution on per capita consumption, but let us take a deeper look at it. The interpretation of the support ratio given above begins with the identity Y=Ly, where L is labor and y is average output per worker. In the standard model with constant returns to scale and capital and labor the only factors of production, average output is given by y=f(k) where k is capital per worker. If we now add one worker to the population the identity Y=Ly suggests that Y will rise by y to (L+1)y. But if this worker brings only labor and no capital, then k will fall and Y will rise by the marginal product of labor, not by its average product, y. If instead we add an elderly non-worker to the population then we are led to expect that output will not rise at all and will now be shared by one more consumer causing consumption per capita to drop. But if this elder is like the average old person in the US he/she will own a substantial amount of capital accumulated during the working years, and this will raise K so that Y will rise by the marginal product of capital times the elder's capital holding, or by the elder's capital income. Continuing this line of thought we might consider that the worker would also come with some modest amount of capital in addition to pure labor, but less than k. This discussion suggests that the support ratio may overstate the advantage of having more workers and overstate the cost of having more elderly.

An alternative measure is what Lee and Mason (2013) call the "general support ratio" or GSR. It reflects the extent to which individuals and governments have actually prepared for population aging by accumulating assets in advance. It has the same denominator as the standard support ratio, that is, effective consumers. The numerator includes effective labor as in the standard support ratio, but it also includes the effective amount of asset income used to pay for consumption, that is the population weighted sum of asset income less saving at each age. This numerator at each age equals the amount of consumption that is paid for out of net transfers, both public and private. If T is the sum of population weighted net transfers and C is effective consumers, then T/C = 1-GSR, where T/C is aggregate net transfers as a share of aggregate consumption. In a closed population this will be 0 in the base period, because transfers given and received must cancel and add up to 0, and for the same reason the GSR will be unity in the base period. However, as the population age distribution changes while we hold all the relevant age profiles fixed (for consumption, labor income, asset income, and saving) T/C will move away from 0 indicating changes in the extent to which transfers would have to increase or decline to maintain balance, and the GSR will move away from unity indicating changes in the true level of dependency.

Lee and Mason (2013) use NTA data to show that in countries like Sweden, Finland and Slovenia, where older people rely virtually 100% on public transfers to fund their consumption, there is no difference between the standard and the general support ratio as the populations age. However, in countries like the US or Mexico, where older people rely heavily on asset income to fund their consumption, and rely much less on public transfers, the general support ratio declines much less than the standard ratio, by only 40% as much in the case of the US, for example. The general support ratio also indicates a later end to the first demographic dividend as countries move through the demographic transition and a larger dividend.

Reallocation Systems

We saw, not surprisingly, that the young and the old in all countries consume much more than the labor income they generate. How do they do this? What is the nature of their claims on a portion of current output? There are three basic systems that support consumption in excess of labor income for young and old: private transfers, public transfers, and asset income. Asset income is used to fund consumption to the extent that it is not saved. In NTA, asset income minus saving at each age – that is, the portion used to fund consumption-- is referred to somewhat awkwardly as “asset based reallocation”. Children’s consumption is largely provided by parental expenditures for child rearing, that is private transfers, and human capital investment (which I will consider to be part of consumption for the moment) is paid for by a mixture of public and private transfers. Consumption by the elderly in excess of their labor income is funded by a mixture of public transfers, private transfers, and asset based reallocations. The excess labor income of the prime working years goes for taxes, private transfers, and saving.

A seminal article by Cox (1987) asked whether transfers are motivated by altruism or by exchange, where elders might make inter vivos transfers or bequests in exchange for care and attention from a child, attention that has no close market substitute. (The Cox terminology is different than in this chapter, since “Transfer” as I use it here cannot involve a *quid pro quo* by definition. This difference is purely semantic. We can certainly ask whether a given intergenerational payment is made for altruistic reasons or as part of an exchange.) The Cox question is important for understanding the consequences of public transfers, among other things. For example, Barro (1974) showed that if generations are altruistically linked by intergenerational transfers from young to old or from old to young, then in the simplest case changes in private intergenerational transfers would simply offset changes in public intergenerational transfers such as a new public pension system[1]. The argument is that the altruistic intergenerational linkage, even if only between parents and children, would through recursion mean that families effectively have infinite horizons. In this case new government bonds or new tax and transfer programs do not change family wealth over the infinite horizon and therefore do not change the amount that the current generation would want to consume relative to its descendants so long as all families are at interior solutions for optimal bequests rather than at a corner. It follows that, in theory, government bonds or taxes and transfers would have no effect on consumption of different generations, and private intergenerational transfers would simply offset public transfers. The relation of public transfers to private ones is also discussed in Cigno’s chapter in this Handbook, and Arrondel and Masson (2006) review the empirical evidence on this topic.

Figure 3 illustrates these patterns for the United States. In childhood, private transfers fund most of ordinary consumption (food, housing, clothes, recreation) while public transfers largely fund education. In addition, children receive transfers of public services such as national defense, roads, police, and research that are allocated equally on a per capita basis by NTA. Prime age adults pay taxes and make private transfers to their children. They also consume increasing amounts of asset income although some of it is saved (saving is not shown in this figure). After age 65 or so transfers received from the public sector outweigh taxes paid, and public transfers contribute substantially to old age consumption. The main source of funds for old age consumption in the US, however, is assets. In NTA data saving by the elderly remains positive at all ages in most countries, but much asset income is nonetheless consumed. Older people in most countries continue to make net transfers to younger family members rather than receiving net support from them on average above age 65, although at older ages above 75, it is common for the elderly to receive net transfers from their families. East Asia and Thailand are exceptions, where net private transfers to the elderly can be quite important starting at earlier ages. Returning to the prime adult years in the US, we see that there is no age at which even young adults are saving more than the asset income they receive. From where do these assets that are generating asset income come, if not from savings? It appears that they come from end-of-life bequests that are not yet measured by NTA. Although most such bequests are surely received at later adult ages given current mortality patterns, enough may be received at younger ages to account for this outcome, an outcome which is quite common in NTA countries.

The Changing Age Pattern of Consumption

Among the many important messages in Samuelson's (1958) classic paper on the consumption loan economy was this: the life time consumption and welfare for individuals in a society with a given level of per capita consumption can vary greatly depending on how that consumption is distributed by age and generation. For example, suppose a population with two overlapping generations (age groups) is doubling each generation so that the younger age group is always twice the size of the older one, and income at each age is constant over time and derives entirely from labor. Gaps between consumption and labor income are made up by transfers. Consider two different lifetime consumption patterns, A and B. In A members of the younger generation always consume twice as much as the older generation whereas in B the reverse is true, and they consume half as much as the older generation. In A lifetime consumption (the unweighted sum) will be 20% less than in B because in A, the larger age group also has the higher per capita consumption. In a different regime of population decline and aging scenario A could instead yield higher lifetime consumption than B[2].

A social planner would maximize lifetime welfare by choosing the lifetime consumption pattern that an optimizing individual would choose if faced with an interest rate equal to the population growth rate plus the rate of productivity growth. In an aging society with slow or negative population growth this interest rate will be lower and perhaps negative, and for standard intertemporal utility functions the longitudinal age profile of consumption would also flatten, rising more slowly with age and time. In contrast, the market equilibrium rate of interest in Samuelson's economy (with three adult age groups) was strongly negative, inducing young people to borrow and consume at a high rate, while older people consumed very little. Consequently lifetime welfare was very low relative to the social planner optimum.

In the real world, age patterns of consumption are determined in part by individual decisions, but also by unpredictable changes in government pension programs and in-kind transfers of health care, long term care and education. It is also well-established in behavioral economics that individuals are not good long term planners, which is particularly relevant for retirement planning. Consumption by children is largely driven by their parents' decisions. For these reasons we should not expect age trajectories of consumption to conform closely to predictions of life cycle saving models.

In fact, age patterns of consumption have changed a great deal in recent decades in the rich nations. Figure 1 showed the difference between the cross-sectional age profiles of consumption in the lower income countries and the rich countries. In NTA, the lower income age pattern holds on average all the way through the third income quartile of countries, including countries like Taiwan and S. Korea. Figure 4 shows how the cross-sectional age profile of consumption has changed in the United States, comparing 1960, 1981, and 2011. In 1960, total consumption declined substantially after age 60. In 1981 it rises across all ages rather than declining. And by 2011 it rises very strongly across the whole range of ages, and accelerates after age 85 (this cannot be seen for 2011 because age details were truncated at 85+, but the acceleration is clear in 2007 and other earlier years). Inspection of the components of consumption suggests that much of this increase is due to increasing public provision of health care including long term care. In the United States, public coverage of health care for those 65 and over (Medicare) began in 1967, and similarly for long term care for those meeting an asset test (Medicaid). Inspection also shows that private expenditures at older ages increased greatly, with a sharp drop in 2007 at age 65 when Medicare becomes available. But total private consumption has also risen at older ages, and it seems likely that this reflects the increased generosity of public pension benefits (Social Security). The ratio of consumption at age 80 to consumption at age 20 doubled between 1960 and 2007, from .83 to 1.67.

This tilt toward older ages in the cross-sectional age distribution of consumption in the United States is opposite to what Samuelson's social planner would choose in the face of declining population growth rates and projected population aging. At the same time, we note that the health care component of consumption has special features since health and survival are preconditions for enjoying standard consumption.

Population aging and economic growth in comparative steady states

The support ratios we discussed earlier indicate the consequences of shifting levels of dependency in the economy resulting from changing population age distributions. However, that discussion implicitly focused on labor as the sole productive factor, whereas capital is also important, as reflected in the discussion of the General Support Ratio. In this and following sections we will broaden the framework to include both labor and capital in a neoclassical growth model. Even without behavioral heterogeneity by age, the same slower population growth that produces population aging also raises the capital intensity and per capita income of the economy, assuming that saving rates remain constant. If we allow heterogeneity across age and assume that life cycle saving prevails then we would expect older populations to have higher proportions of dissavers and therefore have lower aggregate saving rates. But we would also expect that the lower fertility and longer life that lead to population aging would lead individuals to save more to provide for a longer life in retirement and to save more because with lower fertility they would expect to consume more in retirement, and therefore would plan to save more when younger. The net effect on saving rates is not clear (Tobin, 1967; Lee et al, 2000; Sanchez-Romero, 2013w, and for a different view, Deaton and Paxson, 1994). However, it is clear that the elderly would hold more assets than the young, and as their share in the population rises, assets per worker and per capita in the population would also rise. Depending on whether assets are invested domestically or abroad, and whether they are held as equities or bonds, population aging could raise domestic capital intensity, labor productivity, wages and per capita incomes. The notion that an older person who pays for her consumption out of her own asset income is “dependent” on workers can also be questioned.

Samuelson (1975a,1975b) pointed out that when populations grow more rapidly, there will be less capital per worker (for a given saving rate), tending to reduce per capita output and consumption. At the same time, more rapid population growth will mean fewer old people per worker, and therefore a lower old age dependency cost, which would tend to raise per capita consumption. He suggested that consumption would be maximized at some intermediate population growth rate. (Deardorff (1976) showed that on Samuelson’s assumptions the optimum would be at plus or minus an infinite growth rate, and Samuelson agreed but showed that his result would still follow given more realistic assumptions (Samuelson, 1976). We will discuss these issues starting with capital dilution and then turning to the population age distribution.

In the standard Solow (1956) model with homogeneous population and a constant saving rate, it is well known that for a given saving rate, when the population growth rate slows then capital per worker rises, and consequently wages and per capita income rise while interest rates fall (Solow, 1956). In this model, the golden rule saving rate (which maximizes steady state per capita consumption) is lower when the population growth rate is lower, but at the same time the capital/labor ratio, wages, and per capita income are higher and interest rates are lower. Across golden rule steady states, proportional changes in per capita consumption c are given by: $d\ln c/dn = -k/c$. If the ratio of capital to consumption is 4, for example, per capita consumption would be 4% lower in a golden rule steady state with population growth rate 1% per year higher. Slower population growth permits capital deepening even with lower saving rates.

Individuals in this Solow model are homogeneous in all respects except age, although the fact that each individual is born at a particular time and therefore has a particular age hardly matters, since individuals of all ages behave identically. The average age in a stable population growing at n>0 would be 1/n, which rises as the growth rate falls (for $n \le 0$, the average age is infinite). Individuals also have finite lives with life expectancy at birth given by 1/d, where d is the proportion of the population dying each year. Now let us move beyond age heterogeneity in this trivial sense to the realistic case in which economic behavior varies with age. We will see that the important features of the Solow model discussed above are preserved once behavioral age heterogeneity is introduced. However, for simplicity we will begin by adding age-related behavior in a consumption loan model with no capital, as in Samuelson (1958).

We will assume that there is no capital or durable good. Labor is the only source of income. Let y(x) be labor income at age x, and c(x) be consumption at age x, with both assumed constant over time for simplicity. Let N(x,t) be the population age x at time t. Aggregate output in year t, Y(t), is given by total labor income, and aggregate consumption is given similarly:

101* MERGEFORMAT (.)

$$\int_0^w N(x,t)\,y(x)\,dx = Y(t)$$

$$\int_0^w N(x,t)\,c(x)\,dx = C(t)$$

Consider a demographic steady state in which every age group grows at the same rate n and the proportional age distribution of the population is fixed. Demographers refer to this as a stable population. The number of births is $B(t)=B(0)e^{nt}$. Assume that the population is closed to migration.

Let l(x) be the proportion of births that survive from birth to age x, which is assumed constant over time. In this case, the proportion of the population at age x is:

202* MERGEFORMAT (.)

$$\frac{N(x,t)}{N(t)} = \frac{e^{-nx}l(x)}{\int_0^w e^{-na}l(a)\,da} = be^{-nx}l(x)$$

where b is the crude birth rate, B(t)/N(t). Substituting from 02 into 01 and assuming that Y(t)=C(t) as must be true in an economy with no durable good and no waste:

303* MERGEFORMAT (.)

$$\int_0^w e^{-nx}l(x)\,c(x)dx = \int_0^w e^{-nx}l(x)\,y(x)dx$$

This equation describes a cross-sectional budget constraint that is sometimes called the social budget constraint. Note, however, that you could also view this as an individual life cycle budget constraint that says that the survival weighted present value of consumption at birth equals the survival weighted present value of labor income, with discounting at the rate of population growth n.

Now differentiate both sides of the social budget constraint with respect to the population growth rate n while holding survivorship, l(x), constant. Since mortality is held constant, the changes in n must reflect differences in fertility. In order for the identity C=Y to continue to hold when n changes, the age profiles of consumption, or labor income, or both, must change. For simplicity, suppose that it is consumption that changes. To represent these changes in a particularly simple way, suppose that consumption by age varies proportionately with a multiplicative factor of β which is 1.0 for the initial value of n. Let γ(x) be the baseline age schedule of consumption such that c(x) = βγ(x). Note that dc(x)/dn = (dβ/dn)γ(x).

404* MERGEFORMAT (.)

$$\frac{dY}{dn}=\frac{dC}{dn}$$

$$-\int_0^{\omega} xe^{-nx}l(x)\,y(x)dx=-\int_0^{\omega} xe^{-nx}l(x)\,c(x)dx+\int_0^{\omega} e^{-nx}l(x)\frac{\partial c(x)}{\partial n}dx$$

$$-\int_0^{\omega} xe^{-nx}l(x)\,y(x)dx=-\int_0^{\omega} xe^{-nx}l(x)\,c(x)dx+\frac{\partial \beta}{\partial n}\int_0^{\omega} e^{-nx}l(x)\gamma(x)\,dx$$

$$\frac{\int_0^{\omega} xe^{-nx}l(x)\,c(x)dx}{C}-\frac{\int_0^{\omega} xe^{-nx}l(x)\,y(x)dx}{Y}=\frac{\partial \beta}{\partial n}$$

$$A_c-A_y=\frac{\partial \beta}{\partial n}$$

On the left we have the average age of consuming in the population less the average age of producing, in the cross-section.[3] On the right we have the proportional change in consumption at each age and overall in the population cross-section, for a change in the population growth rate across steady states. This result tells us that when the population growth rate rises (and therefore the population age distribution becomes younger) consumption must adjust so that the budget balance identity C=Y continues to hold. If $A_c > A_y$ then the left side is positive and age specific consumption will rise. The reason is that people on average produce at younger ages than they consume, so having a greater proportion of younger people relaxes the social budget constraint and permits higher consumption at every age. We can drop the artificial simplifying assumption about β, and the assumption that labor supply is constant, and instead get a result that constrains the weighted sum of variations in y(x) and in c(x) over the life cycle, while permitting adjustment to take the form of an increase in the retirement age, for example, or a reduction in specifically old age consumption.

If A_c-A_y = 4 years, then an increase in the population growth rate by .01 (1% per year) would permit an increase in consumption of 4% at every age. Suppose that people work from age 20 to age 65, and then survive to age 80 when all die, and that the population growth rate is 0. Suppose they consume the same amount c at every age, and earn the same amount y at every age that they work. Then the average age of consumption is 40 and the average age of earning is (20+65)/2 = 42.5. In this example, A_c-A_y = -2.5, and higher fertility and more rapid population growth require a reduction in consumption at every age. If $A_c > A_y$ then the same logic says that age specific consumption must be reduced to balance the social budget constraint in an older population. Later we will see that many rich countries have $A_c > A_y$ and would benefit from more rapid population growth (at least so far as age distribution is concerned) while almost all non-rich countries have $A_c < A_y$ and would benefit from slower population growth.

The Golden Rule Case

We now want to analyze economies with population age distributions and with capital. There are various ways we could proceed. One approach is to analyze the way that population aging affects the decisions of a Social Planner in a Ramsey-Cass-Koopmans (Koopmans, 1967) model with a population of immortal individuals. This setup also applies to a population of dynastic altruistic household heads, who through the altruistic linkage of generations, effectively act like infinitely long-lived individuals. In this general approach, the optimal trajectories of saving, consumption, capital accumulation and economic growth are determined independent of the population age distribution, and then output each period is distributed through transfers to achieve the maximum welfare according to the criterion chosen. Thus there is a two-stage optimization in which first the trajectories of saving and capital are optimized and second consumption is allocated across ages and generations to maximize the welfare function (Calvo and Obstfeld, 1988; Samuelson, 1975b).

There are two drawbacks to using this setup to explore the consequences of population aging. First, the dynastic altruistic assumption is extreme and assumes away most of the interesting consequences of population aging. Second, in the standard setup in which the Planner optimizes the integral over time of population-weighted utility, the Planner maintains the same capital-labor ratio regardless of the population growth rate (or age structure) determined solely by the rate of (intergenerational) time preference in the welfare function (Calvo and Obstfeld, 1988; Cutler et al, 1990), so capital deepening does not occur in an aging population and the entire response takes the form of variation in the aggregate savings rate. For these reasons, this version of the Ramsey model does not seem appealing, although the interested reader can find a useful empirical application to the US in Cutler et al (1990).

If instead the Planner maximizes the integral of unweighted per capita consumption, however, then the marginal product of capital is set equal to the population growth rate plus the social discount rate. If the social discount rate is zero, then the economy converges to the "golden rule" path, which chooses the saving rate to maximize per capita consumption. In this case, with slower population growth (and consequent aging), the optimal saving rate is reduced and capital intensity rises. For me, this is a plausible and appealing outcome. However, with this per capita objective function, the Planner would seek to raise consumption by small generations and reduce it for large generations during the approach to steady state, to take advantage of the generational differences in cost of achieving higher utility, and that is unappealing.

Turning now to the second stage of the optimization, often and perhaps typically (based on my own simulations) the optimal or actual pattern of consumption by age would require massive downward intergenerational transfers of capital, which could take place either as bequests at death or alternatively as inter vivos transfers, to maintain the optimal capital intensity. Thus transfers are a key component of the age structured Ramsey model, although they are seldom discussed.

Another approach is to assume non-altruistic individuals with finite lives as in Diamond (1965). Although the assumption of zero altruism may seem extreme, there are mechanical ways to build in parental investments in offspring. However, the Diamond model, like many OLG models, has only two adult age groups, and this has been shown by d'Albis (2007) to be too limiting, since for example it imposes an exogenous limit to the ages of positive saving. This implies a monotonic negative relationship between the rate of population growth and the capital intensity and per capita income of the economy, whereas d'Albis shows that with a continuous age distribution the relationship should be non-monotonic, with a population growth rate which maximizes per capita income.

Below I will follow Arthur and McNicoll (1978) in adding a continuous population age distribution to a Solow growth model, and take the shapes of the age profiles of consumption and labor income to be as observed in various actual populations, rather than to assume that they are chosen optimally by the actors. I will assume that saving and capital intensity are consistent with golden rule, without specifying how golden rule is brought about.
The golden rule growth path is the steady state that maximizes consumption per capita, and so it assumes that every generation has the same per capita consumption (absent technological progress, purely for simplicity). When the population growth rate changes then both the saving rate and the capital labor ratio change, while the steady state assumption avoids the problem of treating small and large generations asymmetrically – but also gives no guidance outside of steady state. Despite these restrictive features, the golden rule case is useful for several reasons: it is very tractable analytically with a flexible and realistic population age distribution; it incorporates many of the core effects of population aging including the key role of intergenerational transfers; and it is neutral with respect to institutions and behavior and in that sense reveals the fundamental and general consequences of population aging.

In a comment on Samuelson (1975), Arthur and McNicoll (1978) build into the Solow model a population with continuous age distribution. They consider the effect of a small difference in the population growth rate across golden rule steady states, deriving the following elegant result:

505* MERGEFORMAT (.)

$$\frac{\partial \beta}{\partial n} = A_c - A_y - \frac{k}{c}$$

where k is the capital labor ratio, β has the same meaning as in equation 04, that is indexes the level of lifetime consumption, and c is ordinary per capita consumption, C/N. In other words, the effect of slower population growth and population aging across golden rule steady states is a simple sum of the beneficial capital deepening effect and a possibly deleterious dependency effect. Samuelson (1958, 1975) considered the costs of old age dependency but not of child dependency. The Arthur-McNicoll (1978) result takes both into account.

Based on data from the National Transfer Account project and a few other sources, average ages of consumption and labor income have been calculated for a large number of countries at different levels of income and in different regions of world. Figure 5 charts these data as arrows with the head at the average age of consumption and the tail at the average age of earning labor income. The length of the arrow is $A_c - A_y$ with an arrow pointing to the right indicating a positive difference and to the left a negative one. The countries are organized by geographic region, and the regions go from lowest per capita income at the bottom to highest at the top. Countries within regions are ranked in the same way. In the bottom panel are found two hunter-gatherer communities, and Kenya, the lone African country.

All the arrows point downward (leftward) outside of Europe and the US except for Japan, indicating that the average age of consumption is less than that of earning labor income. In these countries, higher fertility and more rapid population growth lead unambiguously to lower golden rule consumption. The Samuelson tradeoff does not occur. these countries would benefit from lower fertility. In Japan and in four European countries (Hungary, Slovenia, Austria and Germany), however, the arrows have reversed direction and point to the right, indicating that consumption occurs at an older age than labor income. In these, the Samuelson tradeoff occurs, but given how short the arrows are the capital dilution effect dominates and the overall effect of faster growth is negative—despite the perception by governments that fertility is too low.

The average ages and the direction of the arrows depend both on the ages of consumption and of labor income across the individual life cycle and on the age distribution of the population itself. In particular, because labor income is fairly symmetric around its average age, and low at both young and old ages, population aging has relatively little effect on its average age. Consumption, however, is lower in childhood, and in the rich countries tilted toward older ages. The mean age of consumption is strongly affected by population aging. Consequently, countries with lower fertility and slower population growth tend to have $A_c > A_y$. Population aging has a powerful effect, although such factors as age at retirement and publicly provided health care matter as well (Mason and Lee, 2013).

The Arthur-McNicoll equation 05 tells a straightforward story about Samuelson’s (1975) tradeoff between dependency and capital dilution in the golden rule case. And yet one might expect that the result would depend on the extent to which individuals provide for their own old age through life cycle saving versus relying on intergenerational transfers from younger family members or the public sector. The equation appears to abstract entirely from such institutional and behavioral factors, and this was also true of the original Arthur-McNicoll formulation. However, further development of the model with age distribution permits us to introduce transfers and saving/asset accumulation as alternative ways to smooth consumption relative to labor income. Above, I asserted that aggregate consumption must equal aggregate labor income, but I said nothing about how the young and the old managed to fund their consumption in excess of their labor income at those ages. Now we will look at these arrangements in more detail in a model that includes capital.

Consider the budget for an individual at age x. The difference between consumption and labor income at that age must be made up by net transfers received (transfers received minus transfers made to other ages) and by asset income received minus saving:
606* MERGEFORMAT (.)

$$c(x) - y_l(x) = \left[\tau^+(x) - \tau^-(x)\right] + \left[nk(x) - s(x)\right]$$

The first bracketed quantity on the RHS is net transfers received. It can be decomposed into public and private components. The second bracketed quantity can be decomposed into public asset income minus public savings and the private equivalent. I will refer to this bracketed quantity as “asset based reallocation”. Under the golden rule assumption with no technological progress, the return to capital is n, the population growth rate. We can interpret k here as assets rather than capital, and take it to include negative values (debt) as well as positive, and the whole bracketed quantity therefore to include public and private borrowing and lending as well as saving and dissaving.

It will be useful to define transfer wealth held by an individual at age x, T(x), as the survival-weighted present value (discounted at n) of expected transfers to be received in the future minus transfers to be made. This transfer wealth concept is well known in the context of Pay As You Go public pensions, but it can be applied to all kinds of transfers, including transfers from older to younger such as public education. We can then calculate the aggregate transfer wealth T for the whole population as the population-weighted sum across all ages of T(x). The flows of transfers at any given time t must sum to 0 across individuals in a closed economy, since every dollar of transfer given is exactly offset by the dollar of transfer received. However, the aggregate stock of transfer wealth T can be positive, negative, or zero. To see this, consider a pure Pay As You Go pension system. Every adult has paid into the system ever since reaching the age of labor force entry, so adults of every generation, young or old, expects to receive more in the future than they expect to pay in the future, and therefore at every age the pension wealth is positive. Therefore the aggregate transfer wealth T held through this pension system is positive.[4]

We can take this analysis a step farther. Willis (1988) showed that aggregate transfer wealth T in a golden rule economy is given by:
707* MERGEFORMAT (.)

$$T = \tau^+ \left[A_{\tau^+} - A_{\tau^-}\right]$$

Here τ^+ is the annual gross flow of transfers made in the population (which is identically equal to the flow of transfers received in a closed population), and the quantity in brackets is the average age of receiving transfers minus the average age of making transfers in the population. When the preponderance of transfers goes from parents to the children they are rearing, then the average age of receiving transfers will be much lower than that of making transfers, and T < 0. When the preponderance of transfers goes from working age adults to the elderly, then T>0.

It will also be useful to define life cycle wealth of an individual at age x, W(x), as the survival-weighted present value (discounted at n) of expected future consumption minus expected future labor income. This is the amount of wealth that the individual would have to hold in order to be able to consume the expected amount at each age given expected future earnings. Then aggregate life cycle wealth W is the population-weighted sum of W(x). Willis (1988) (see also Lee, 1994a and b, Bommier and Lee, 2003) showed that:
808* MERGEFORMAT (.)

$$W = c\left(A_c - A_y\right)$$

where c is ordinary per capita income.

In Figure 5, the column to the right indicates the life cycle wealth W as given by 08, which is the area of each arrow. The thickness of the arrow indicates c relative to per capita income, so the area is life cycle wealth as a multiple of per capita income.

This life cycle wealth can be held in two forms, as capital or as transfer wealth. For example, a 50 year old expects to be able to consume more in old age than she earns, by combining a public pension with retirement savings (Willis, 1988; Lee, 1994a and b; Bommier and Lee, 2003; Lee and Mason, 2011, Chapter 2).

909* MERGEFORMAT (.)

$$W = T + K$$

Debt and credit do not enter into this equation because in a closed economy they must cancel (when government debt is allocated to tax payers). Substituting from 07 and 08 into 09 and rearranging, we find that:
10010* MERGEFORMAT (.)

$$T = \left(A_c - A_y \right) - k/c$$

Referring back to 05, we have the striking result from Willis (1988) that:
11011* MERGEFORMAT (.)

$$\frac{\partial b}{\partial n} = \frac{T}{c}$$

That is, across golden rule steady states, the change in age-specific consumption is proportional to the ratio of aggregate transfer wealth to per capita consumption. T may be positive, negative, or zero, depending on context. In societies with aging populations that make heavy pension and health care transfers to the elderly, T will be positive and this result tells us that higher fertility and more rapid population growth will be beneficial. In low income societies with high fertility and young populations, T will be negative and this result tells us that lower fertility and slower growth will be beneficial.

Now consider an economy in which either there are no transfers whatsoever, or upward and downward transfers are perfectly balanced so that $A_{t+} = A_{t-}$ and T=0. Since by assumption this economy is golden rule, there must be very substantial life cycle wealth held in the form of capital in order for the elderly to be able to consume more than they earn. In this case, we see from 011 that a small difference in the population growth rate due to different fertility would have no effect at all on age-specific consumption, since the derivative is zero. The increase in the consumption needs of the elderly would be exactly offset by the increase in life cycle asset holdings arising from life cycle savings. The population growth rate for which the derivative is zero is the one that maximizes life cycle consumption (the second derivative condition is met). It follows that in golden rule economies, population aging due to low fertility is a problem only to the extent that the economy relies on asymmetric transfers to redistribute income to the elderly. When the elderly provide for their own retirements by relying on their earlier life cycle savings, population aging has no adverse effect. The obvious question, then, is to what extent do economies in fact rely on asymmetric transfer systems?

We draw on National Transfer Accounts data to assess the sign and size of T. NTA finds that average private transfers are invariably downward, from older to younger. Even in those countries in which there is substantial familial support of the elderly, as in East Asia, it is dominated by familial child rearing transfers (see Lee and Mason, 2011, Chapter 4). However, net public transfers go from old to young in some countries, but from young to old in others (Lee and Mason, 2011, Chapter 4). Figure 6 plots arrows for total transfers, that is, public and private combined, in the same format as Figure 5. The head of each arrow is at the average age of receiving transfers and the tail at the average age of making transfers in various populations. The Figure plots the arrows based on 2010 population data and then it plots the arrows that would result if the current age profiles of transfers remained unchanged while the population age distributions changed as projected to 2050 by the United Nations (2011). The current arrows almost all point to the left, indicating net transfers flowing from older to younger ages, except for a few European countries. However, the projections to 2050 indicate that more countries would shift to upward transfers, and the regional average arrows for Europe and the US and for East Asia would both point to the right, indicating upward transfers.

Taking these results at face value, they suggest that today almost all countries would benefit from having somewhat older age distributions (Lee and Mason, 2014), but by 2050 many would benefit from having younger age distributions. However, there are a number of problems with this interpretation. First, none of these countries is on a golden rule growth path; second, there is every reason to expect the age profiles of consumption, labor income and transfers themselves to change in both rich and non-rich countries; and third, none of these populations is stable now. Assessment of the consequences of ongoing population aging in the 21st century requires dynamic simulations of more complex models. Nonetheless, these simple golden rule steady state models, and the empirical estimates of transfer flows, provide some helpful insights. In particular, the macroeconomic impacts of population aging depend on: 1) the age pattern of consumption in relation to labor income over the life cycle; 2) the extent to which the gap between consumption and labor income is made up through transfers rather than by drawing on assets accumulated through earlier saving, and the extent to which these transfers are asymmetric with age; 3) population aging raises the capital intensity of the economy and affects wages, profits and interest rates accordingly.

So far we have considered only population aging that arises from low fertility and slow or negative population growth. Some part of population aging is due to declining mortality, however. The same model setup can be used to analyze the effect of mortality decline as in Lee (1994a) and Lee and Tuljapurkar (1997). In general, mortality decline raises the population growth rate and thereby has all the effects just analyzed for fertility change. However, in countries where mortality is already low, the effect on population growth rates of further decline is negligible. The important effect arises from the interaction of increased person years lived at each age in the life cycle with the age profiles of consumption and labor income. It is particularly important to note that in early years of mortality decline, starting from high mortality, most of the person years gained occur in childhood and during the working years. However, as mortality declines and life expectancy rises, the share of the person years gained above age 65 rises. By the time life expectancy reaches 70 or 75, further mortality reductions lead to person years gained that are mostly at older ages. In the US and other high income countries, 75% of the gains in life over the last 25 years have accrued to ages 65 and over and in most rich industrial nations years of labor force participation have declined as a share of life expectancy since 1980 (Eggleston and Fuchs, 2012).

International Comparisons of Support Systems for Elderly

While Figure 6 showed the general direction of total transfers, whether toward older or younger members of the population, it is also useful to see more institutional detail. National economies vary a great deal in how consumption by older people is funded. For starters, in some countries the elderly continue to work to a substantial degree while in others they earn very little labor income. In NTA data, we find that labor income at older ages is particularly low where either public or private transfers are an important source of old age support, as in Europe, Latin America, and East Asia. Where asset income is more important older people tend to continue to work, as in the US, Mexico, and parts of South and Southeast Asia.

Figure 7 focuses on how the gap between consumption and labor income in old age is funded, which makes it possible to use a triangle graph in which the shares of funding from asset income, private transfers and public transfers sum to 1.0. Each point in the figure represents one country, identified by the two letter code of the United Nations. Each vertex refers to a single funding source, as labeled. A point located at a vertex indicates that in that country, funding for the elder gap comes entirely from that source. Positions along the line connecting two vertices indicate a mixture of funding from those two sources, with zero from the third. Points located inside the triangle have funding from all three sources. Points outside the triangle to the right have negative funding from family transfers, which is to say that older people on net make transfers to younger, rather than the reverse.

Five countries are located on or near the public transfers vertex, indicating 100% funding from that source: Austria, Slovenia, Hungary and Sweden, but also Brazil (Brazil's position indicates that public transfers equal the gap between consumption and labor income, but that additional funds from asset income are used to make large private transfers to younger family members). In another group of countries, public transfers fund two thirds of the gap: Germany, Uruguay, Spain, Costa Rica, and Chile, with China and Japan close by. In nearly two thirds of the countries shown the elderly rely very heavily on public transfers. It is striking that reliance on private familial transfers is so rare. No countries are near the private transfer vertex. Four countries in East and Southeast Asia (China, S. Korea, Taiwan and Thailand) rely one sixth to one half on private transfers. Japan, interestingly, is not among them. In five countries, the elderly rely at least 50% on asset income to fund the gap: the US, Mexico, Philippines, Thailand, and Mexico. The fact that the elderly in relatively few countries rely much on asset income in their old age does not necessarily mean they do not own assets. It is possible that they own substantial assets but that they save all the asset income rather than using it to fund consumption. In this case they will leave very substantial bequests, intentionally or not, when they die.

These measures of flows do not correspond exactly to the variables in equations 09 and 011 but they nonetheless are suggestive in that regard. In general, the greater reliance of the elderly on asset income, the less the impact of population aging on consumption across the life cycle.

Population Aging and Capital Market Equilibrium with Transfers

Consider an hypothetical economy with pure life cycle saving (for consumption smoothing alone, with no altruistic motives toward children or parents, and no precautionary motive), in the extreme case in which individuals must borrow at some interest rate to fund their consumption starting at birth, and then continue to save and dissave over their lifetimes to achieve their desired consumption trajectory in the complete absence of public or private transfers, with perfect annuitization, and with zero saving for any other purpose. Producers in this economy have a demand for capital that depends on the interest rate at which funds can be borrowed. The economy reaches an equilibrium with some interest rate and some amount of capital, where the amount that individuals want to invest is just equal to the amount that producers want to borrow.

In this economy and population the capital held for life cycle smoothing purposes would be different than the golden rule amount, except by unlikely accident. Suppose that capital is less than golden rule. If the age trajectories of consumption and labor income remain the same, but we now introduce downward transfers from older to younger, for example in the realistic form of parents bearing the costs of rearing their children, then we can see from 09 that as transfer wealth declines, capital would increase. The desire to make downward intergenerational transfers, including both inter vivos transfers and end of life bequests, generates a demand for wealth beyond life cycle saving. This point was made forcefully by Kotlikoff and Summers (1981, 1988) in a series of articles suggesting that the desire to make intergenerational transfers is a more important source of demand for capital than is the desire to smooth consumption over the life cycle as in the life cycle saving hypothesis (Modigliani, 1988). There is some hypothetical level of downward transfers that would make T sufficiently negative that K would be raised to the golden rule level. In a golden rule steady state of this sort with $T<0$, population aging due to lower fertility and slower population growth would raise life time consumption (see 011 and Arthur-McNicoll, 1978).

This case is illustrated in Figure 8 (based on Willis, 1988; see also Eggertsson and Mehretra, 2014:7) which shows the discount rate on the vertical axis, including the golden rule rate where $r=n$, and shows on the horizontal axis the corresponding demand for life cycle wealth by individuals, W, and its components transfer wealth T and capital K, expressed on a per capita basis. The extent to which this demand for wealth by households exceeds transfe wealth T is the supply of investment funds for the production sector. The amount of investment funds demanded by the production sector is plotted as the heavy black line which is lower when the interest rate is high and conversely. The intersection of the demand and supply of capital curves locates the equilibrium interest rate and the equilibrium values of W, T and K. The golden rule equilibrium without transfers occurs when the demand for wealth is given by W_B, which intersects the Producers' demand for capital at interest $r=n$. In this case, $T=0$, and small changes in the population growth rate and aging in the neighborhood of n will have no effect on life cycle consumption (evaluated at the initial interest rate $r=n$). Given labor supply by age and intertemporal consumption preferences, this corresponds to a sort of double optimum for both the capital labor ratio and the desired level of capital holdings to achieve consumption smoothing.

The situation described in the paragraph above occurs when the demand for wealth curve lies to the left of W_B, here shown as W_A, which intersects the demand for capital curve at an interest rate above the golden rule rate n. However, with a sufficiently strong desire to make downward transfers to children T will be sufficiently negative so that the overall demand for wealth can lead to the golden rule outcome. In this case, population aging (a reduction in n) will raise age standardized consumption across golden rule steady states as indicated by equation 011.

Now suppose that the pure life cycle demand for capital would exceed the golden rule amount, as some have feared might be the case in aging populations. This leads to the so-called dynamically inefficient outcome with $r<n$, where the rate of return on capital is less than the rate of economic growth and less than could be earned through a program of upward transfers such as PAYGO public pensions. All transfer systems earn a rate of return of n in steady state (assuming no productivity growth). This situation is illustrated by the demand for wealth curve W_C in Figure 8, lying to the right of W_B. In this case, by introducing a system of upward transfers such as public pensions, public health care, or national debt, or private transfers to support the elderly, T can be raised to a positive value which would displace enough capital to reach the golden rule steady state with r=n. In this case, life cycle consumption could be raised by making the population younger through increasing fertility, since T>0 (see 011).

A number of these theoretical and empirical themes are drawn together in Table 1 which shows results for the golden rule case (for those economies that have capital) under particular assumptions made in the literature. The first case with both capital and transfers, covered in the first four rows of the table, has already been discussed. The next row assumes T=0. In this case, as we have seen, population aging has no effect on age adjusted consumption. But this case is entirely unrealistic, since it does not even allow for parental expenditures for rearing children, or else assumes perfect symmetry of transfers.

In the next row we have the case when there is no demand for life cycle wealth, so W=0. This would be true, for example, in the standard Solow growth model in which individuals are homogeneous other than in age. Since labor and consumption are the same at all ages, there is no demand for life cycle wealth. With W=0 we have K=-T. That is, all saving must be motivated by the desire to transfer capital to the new members of the population. For this reason a slight increase in the age of the population (reduction in n) will raise golden rule consumption with an elasticity of –k/c, as we know it does from earlier discussion.

In Samuelson (1958) there were no durable goods so no K. He initially sought a market equilibrium with no transfers, so T=0 as well. With two age groups, the first working and the second retired, each had to consume its own production. With three age groups, the first two workers, a solution with intergenerational borrowing and lending exists, but it requires a strongly negative interest rate to induce the younger workers to borrow and consume. Since debts and credits must add up to zero, in this outcome we must have $A_c=A_{yl}$. In either case, population aging through slower growth has no effect on consumption in the neighborhood of initial n.

Finally we consider the Samuelson (1958) case with transfers (a social contract or fiat money that changes value from generation to generation). In this case, the proportional effect on consumption of a small change in the growth rate and population aging is given by A_c-A_{yl} with no offset due to capital dilution.

For a given level of mortality and survival, the steady state population growth rate in a closed population is determined entirely by the level of fertility. Drawing on the foregoing discussions and on NTA data, and summarizing what has been learned, we can ask what level of fertility would maximize the present value of survival-weighted lifetime consumption discounted at the golden rule discount rate, either the population growth rate or that plus the productivity growth rate. This question can be asked in various ways. What level of fertility would maximize the support ratio? What level would maximize the fiscal support ratio? What level would maximize the present value of survival weighted consumption, assuming that a capital/output ratio of 3.0 is maintained, consistent with recent international values? And finally, what level would maximize this under golden rule assumptions? The calculations, which can be done for individual countries and for different mortality assumptions, are reported in Lee and Mason et al (2014). Results vary from country to country and region to region, reflecting differences in the age profiles of labor income and consumption, and taxes and public benefits. The results can be summarized as follows: Typically fertility levels near replacement level would maximize the overall support ratio. However, fertility well above replacement would be required to maximize the fiscal support ratio, because in so many countries governments transfer resources to the elderly. Neither of these calculations takes into account the role of capital in production, and when we do, lower fertility is beneficial, with lower costs of providing capital for new workers through current saving. For the Western nations (Europe and the US), a TFR of 3.1 would maximize the fiscal support ratio, 2.4 would maximize the support ratio, 1.9 would maximize lifetime consumption if a capital/output ratio of 3.0 is maintained, and 1.5 would maximize lifetime consumption across golden rule paths. Of these, the third case (K/Y=3) is probably most relevant, and it suggests that current fertility levels are reasonably close to a desirable level. The outcome surface is generally quite flat in the neighborhood of the maximizing values in any case.

Secular Stagnation

To this point, I have suggested that population aging leads to increased capital intensity (or at least increased assets per capita) which raises labor productivity and offsets the rising dependency costs of the elderly. But another possibility is that instead, rising capital intensity pushes the long term equilibrium real interest rate below zero.

In 2013 in a speech to the IMF Larry Summers revived long dormant fears of secular stagnation, suggesting that it lies behind economic stagnation since 1990 in Japan, the lack of recovery from the Great Recession in Europe, and the deficient recovery in the US. Fears of what Hansen later called “secular stagnation” were first raised by John Maynard Keynes (1937) in a brief article. Keynes suggested that “an increase in population increases proportionately the demand for capital” and therefore that slower or declining population growth would lead to deficient aggregate demand, an idea he attributed to Malthus. Keyens recommended “policies of increasing consumption by a more equal distribution of incomes and of forcing down the rate of interest“. Hansen (1939) developed this idea more broadly and in more detail, introducing the term “secular stagnation”. In modern discussions, secular stagnation is taken to mean that “negative real interest rates are needed to equate saving and investment with full employment” (Teulings and Baldwin, 2014:2). In fact, a measure of the global real interest rate suggests that it has declined steadily since 1985 from above 4% to around 0% in 2012-2014 (Summers, 2014:35, but also see Eichengreen, 2014) while the “natural rate of interest” for the US, that is the real rate calculated to correspond to full employment of resources, has declined from 5 or 6% in the 1960s to around 0% in 2011. Declining population growth rate is only one possible cause of secular stagnation, but it is the one emphasized by Keynes and Hansen, it is emphasized by some but not all economists writing about secular stagnation today, and it is the one I will emphasize here. A case can be made that declining population growth and population aging raise the supply of savings and reduce the demand for savings, reducing the equilibrium interest rate and thereby creating problems for monetary policy and perhaps worsening financial instability.

Most discussions of secular stagnation suggest that slower population growth rates reduce investment demand in one of two ways: by reducing expected future GDP and consumption demand (Hansen, 1939), or because slower labor force growth leads to rising capital per worker and rising wages, and therefore reduced returns to investment (Eichengreen, 2015). Hansen (1939) also suggested that slowing population growth rates and population aging would tilt the composition of consumption demand toward services, and particularly health care, that require relatively little capital, exacerbating the problem of insufficient investment demand. This is a researchable topic that has received little attention in recent discussions.

These demographic forces join with other factors such as the low capital intensity of information technologies, and their lack of linkage with other sectors to drive capital accumulation (Gordon, 2014), and a surge in saving in developing countries which now export capital to the US and Europe (Summers, 2014b, but disputed by Eichengreen, 2015) could reduce the equilibrium real interest rate below zero. Rising wage costs would further reduce the demand for capital by producers. The net result could be a negative equilibrium real interest rate.

It is generally assumed that nominal interest rates cannot fall below zero, and indeed they have never reached zero from 1800 to the present in the US (Eichengreen, 2015). In fact, in 2015 monetary policy in a number of countries targeted and achieved negative nominal interest rates (Rognlie, 2015). In any event, with inflation, real interest rates can and do fall below zero, and have been substantially so in the US since 1900 for protracted periods centered on 1920 and 1950, and more marginally so in recent years. Nonetheless, a negative equilibrium real interest rate constrains options for monetary policy, and the task when it must aim for 4% inflation rather than 2%, in order to achieve a negative real rate of interest.

Another concern, raised forcefully by Summers (2014, 2015) is that if monetary policy is able to achieve a stable inflation target and the resulting negative real interest rate needed for full employment, a consequence may be financial instability. When real interest rates are less than the growth rate of the economy, then speculators will buy goods (for example, houses, apartments, artworks, gold) that are expected to appreciate, leading to bubbles, and encouraging other risky financial practices (Summers, 2014, 2015).

While most discussions of secular stagnation ignore population age distribution and treat slowing population growth itself as the key factor, Eggertsson and Mehrotra (2014) develops an OLG model of secular stagnation in which age distribution plays a fundamental role. Their model is in the spirit of Samuelson (1958) with three adult generations, two working and one retired. It is a consumption loan economy with no capital or other durable good. When population growth slows the population grows older, and the interest rate must be lower to induce young workers to borrow from older workers. It is this mechanism that produces an equilibrium r below zero, as occurred in Samuelson's example for the competitive market equilibrium.

Secular stagnation can be diagrammed with a setup like that in Figure 8. When the population growth rate is slower or more negative, then the proportion of the population at older ages is higher, driving the per capita demand for life cycle wealth higher at every interest rate. This shifts the demand for life cycle wealth curve outward to the right, reducing the equilibrium interest rate r. Likewise, for a given level of fertility and population growth rate, longer life will raise the amount of wealth needed to provide for consumption in retirement, and will further shift the demand for wealth curve to the right, further reducing the equilibrium interest rate (as emphasized by Teulings and Baldwin, 2014b). At the same time, the demand for capital by producers at each interest rate is reduced because slower population growth reduces the future demand for consumption goods, and because slower labor force growth raises wages, both shifting the demand curve for capital inward to the left. Both shifts lead to a lower equilibrium real interest rate which may go negative.

There are various possible policy responses to raise the equilibrium real interest rate, such as increasing public investment in infrastructure. In the context of this chapter, two suggestions are of particular interest, both made by Teulings and Baldwin (2014b:18). One is to raise the retirement age, which would both reduce the demand for life cycle wealth by reducing the period of retirement, and also raise the supply of labor and thereby increase the demand for capital by producers, shifting the producers' demand for capital curve up. A second possibility is to increase the generosity of public PAYGO transfers to the elderly, for example through more generous health care benefits, which would create transfer wealth that would substitute for asset wealth in the desired total wealth holdings. This would reduce the demand for capital or other assets as a form of wealth, leading to a new positive equilibrium real interest rate.

Capital markets are global, which means that aging in foreign countries can drive down interest rates for all countries, even young ones. Conversely, the growing importance of countries like India that are still young could in principle attract foreign capital and raise interest rates even in countries with aging populations. While capital markets are global, public transfer systems are local, which means that the kind of policy response described above could in principle be effective.

Dynamic Analysis of Population Aging

Real world populations are not stable, because fertility, mortality and migration continue to change. Population aging itself is the dynamic process by which the population becomes older. Even in Japan the old age dependency ratio is projected to double between 2010 and 2050. Comparative steady state analysis can provide some insights for the effects of population aging, but these insights are limited. In my view, the most useful approach is to take the demographic change as given, using historical values and projections for the future, rather than treating fertility, mortality or migration as endogenous, because the theoretical basis is not sufficiently well established.

Once we drop the steady state assumption, there are at least four different ways to analyze the consequences of changing population age distributions. The simplest approach, often used by government statistical agencies and public pension authorities, is to assume some underlying trend in productivity growth, reflecting technical progress as well as the accumulation of physical and human capital. Then, given the projection of the population by age and labor supply, and given relative consumption and the structure of public programs, many outcomes of interest can be derived easily since virtually all feedbacks are assumed away. The annual Trustees Report of the US Social Security Administration is an example.

A second approach is to incorporate some economic feedbacks while remaining in a partial equilibrium context. For example, one can model saving by assuming life cycle saving behavior, with due allowance for the effect of children on parental decisions about consumption and saving, and some assumptions about the degree of foresight about future mortality, earnings growth, and interest rates. Assumptions about public or private transfers can be incorporated. Tobin (1967) pioneered this general approach (although he analyzed comparative steady states) developing a model which has subsequently been used in a more dynamic context by others (Lee, Mason and Miller, 2000, 2003). In a different vein, Mason and Lee (2007) develop a dynamic partial equilibrium model in which the cross-sectional age profile shapes for consumption and labor income are preserved, along with a specified fraction of net old age consumption that is funded through transfers, with the balance funded through saving and asset income, and consumption is maximized along a closed form dynamic path.

A third approach is a Computable General Equilibrium model with Overlapping Generations (CGE/OLG) (Ríos-Rull, 2001; Sanchez-Romero, 2012; Sanchez-Romero et al, 2012), where the OLG models used for such purposes are increasingly demographically realistic. For example, detailed historical and projected trajectories of rising life expectancy and falling fertility can be incorporated, and the consequences of demographic change can be examined for different assumptions about public and private systems of intergenerational transfers (Sanchez-Romero, 2012 and Sanchez-Romero et al, 2012). Different assumptions about bequests and annuitization can also be made.

A fourth approach, closest in concept to the golden rule assumption, uses a Ramsey model to analyze future trajectories of age-specific consumption, saving and capital accumulation that are optimal by some criterion, given the assumed demographic trajectories, as in a Social Planner model (Cutler et al., 1990; Calvo and Obstfeld, 1988). The optimal outcome depends on the criterion chosen.

Special Topics on Population aging, saving, and capital intensification

a. Pop aging and saving – will saving rates decline?

The literature on this topic is reviewed elsewhere in this Handbook, so it will not be discussed here. Suffice it to say that the topic is complex. For example, it is quite possible that saving rates might decline with population aging while at the same time the capital intensity of the economy rises, since with slower population and labor force growth the saving rate necessary to achieve any level of capital intensity is also reduced. For this reason it may be simpler to focus on how the demand for assets changes in the economy rather than on the saving rate itself. For example, Lee, Mason and Miller (2000 and 2003) simulate life cycle saving across the demographic transition in an open economy, following a general setup from Tobin (1967), and they find that the capital-output or capital-labor ratio rises steadily, doubling or tripling by the later stages of the transition, while the aggregate saving rate first rises strongly while the proportion of the population in the labor force rises (dividend phase), and then declines strongly as the population ages. The salience of PAYGO pensions matters as well, since a rising demand for wealth may be met by increased pension wealth offset by implicit debt rather than by increased holdings of capital.

b. The role of PAYGO transfers, public and private, implicit debt, asset accumulation, and capital intensity. Diamond (1965).

Diamond extended Samuelson's (1958) consumption loan model to include capital and a public sector. In Samuelson's model, the competitive market rate of interest was determined by the demand and supply for consumption loans. In Diamond's model, it is determined by the marginal product of capital which depends on the amount of capital per worker. In Figure 8 we see this is determined by both the private demand for capital for each interest rate and demand by producers for capital in relation to the interest rate. The equilibrium interest rate in the absence of transfers can occur either above the golden rule rate (equal to the population growth rate n), as at D in Figure 8, or it can occur below it, as at E. If the equilibrium without transfers is at D, there is less capital than in golden rule, and adding a system of upward transfers such as government debt, a public pension program, or familial support of the elderly, would substitute for capital and move the equilibrium leftward, farther from the golden rule, with less capital and higher interest rates. A system with increased downward transfers, such as greater spending on children or larger bequests, would instead move the equilibrium closer to golden rule, and with large enough transfers could reach golden rule at A. Equilibria like D are called "efficient". Alternatively, the equilibrium without transfers could occur at E where there is more capital than the golden rule amount. Such equilibria are "inefficient" because the introduction of upward transfers (e.g. government debt), which earn a rate of return equal to the population growth rate, lead to an equilibrium closer to the golden rule, and if large enough the equilibrium reaches the golden rule at C. Transfers always yield a rate of return of n (or n plus productivity growth rate) in steady state, and if this is greater than r=rate of return on capital, then it is efficient to use transfers rather than capital at the margin as a store of wealth for private individuals. Diamond (1965) shows that in the case of government debt, the discussion above holds for internal debt but not necessarily for external debt.

Consider an economy at an equilibrium like D, with less capital than the golden rule amount, and r>n. Suppose that life cycle saving is an important source of the private demand for holdings of capital, so that the demand for capital is positive at all ages, but rises until the age at retirement and then soon after declines as the older individual spends down her capital to pay for consumption. Older people hold more capital than younger in this case. As the population ages, the demand schedule for capital (like $W_A(r,\tilde{n})$ in Figure 8) will shift to the right, and the equilibrium amount of capital will rise while the rate of return falls. Numerical simulations for a number of countries are shown in Lee and Mason (2010a). To put this in very concrete terms, an older population with a pre-funded pension system or private retirement accounts will demand more assets than a younger one. In a closed economy with a balanced budget, this demand for assets will translate into a demand for capital. It is possible that deep population aging, such as is projected for many low fertility nations by 2050, could push an economy from a situation like A today (in the absence of transfers) to a situation like C in three or four decades. In the absence of old age support by public or familial transfers, population aging should lead to capital intensification.

c. Public sector transfers

In most countries and in all rich industrial nations consumption by the elderly is funded by a mixture of public and private transfers rather than solely by asset income, as shown earlier in Figure 7. However, transfers are also made downward to children. Figure 6 showed that around 2010, almost all countries have net downward transfers when public and private are combined. These countries are likely to be found somewhat to the right of A in Figure 8, with downward transfers generating a demand for capital in addition to the life cycle saving demand. However, projected population aging between now and 2050 will lead to a reversal of the net direction of transfers, assuming the current age profiles of transfers are unchanged in shape. By 2050, if there are no adjustments in the shape of the public and private transfer age profiles, net transfers will be upward, and economies would tend to have less capital than otherwise, because upward transfers substitute for life cycle savings and asset holdings. If we restrict our attention just to public transfers (see Lee and Mason, 2011, Figure 4.6) we find that among the rich industrial nations all have net upward public transfers flows except for the United States which has a relatively young population and relatively low public pension benefits.

It has been well established that public transfer programs in many rich nations will be seriously unbalanced as populations age over the next few decades unless benefits for the elderly are cut sufficiently and/or taxes to pay for these benefits are raised sufficiently (later retirement is viewed here as a reduction in benefits and an increase in taxes). If such adjustments are not made, then the public sector will dis-save and government debt will rise. Rising government debt is a form of upward transfer from future generations to current ones, and substitutes for current private saving and asset accumulation for life cycle purposes. Government borrowing will compete with efforts by producers to borrow for investment in capital and lead to lower capital intensity.

I suggested earlier that population aging would raise the capital intensity of economies. But it is possible that the reverse will happen if governments fail to make appropriate adjustments in their transfer programs for the elderly, and if growing public debt displaces private capital. For this reason, any conclusions about aging and capital intensification must be conditional on assumptions about the response of the public sector to the need for reform in pensions, health care, and long term care (National Research Council, 2012, Chapter 7).

d. Open and closed economies in an aging world.

Many of the conclusions reached above hold only in a closed economy. In an open economy that is not too large, interest rates, wage rates, and therefore capital intensity are all set on international markets. It is arguable that even the US should be viewed as open and not too large in this context. In the polar case of a small, open economy, population aging would not affect domestic capital intensity. Population aging would lead to an increase in domestic asset intensity (an increased ratio of assets to labor income) but assets would be held substantially in the form of investments in foreign capital or foreign debt. In this case, domestic population aging would raise asset holdings and asset income, and the share of asset income in national income would rise, but rates of return to capital would not fall and wages would not rise. These would continue to be set on international markets.

The bigger picture is that the whole world is aging, or will be soon, so global population aging will affect factor returns in international markets. For example, the median age of the population in rich developed nations is projected by the United Nations (2011) to rise from 39 years in 2010 to 48 years in 2050, and in the rest of the world from 27 in 2010 to 37 in 2050. The global Old Age Dependency Ratio (OADR= population ≥ 65 divided by population 20-64) is projected to rise from .134 in 2010 to .283 in 2050, more than doubling. If each country's population is weighted by its projected per capita income for each future year, then the global weighted average OADR is projected to rise from .195 in 2010 to .378 in 2050, nearly doubling (National Research Council, 2012, 8-4).

That is the big picture, but of course, the timing and pace of population aging vary from country to country and from region to region. The currently rich industrial nations experienced earlier demographic transitions and in many cases also deeper fertility declines, and their populations are older now and will be aging rapidly between now and 2050. Japan experienced an early, rapid and deep fertility decline, but other East Asian countries were not far behind including China, and they also will age rapidly between now and 2050. The same is true of some countries in Latin American, the Middle East and North Africa. Much of the developing world including India, however, is moving more slowly through the demographic transition and aging will come to it later and more slowly. These differences in population age distribution combined with differences in economic growth create important opportunities and incentives for international capital flows as simulated by BÖRSCH-SUPAN, Ludwig and Winter (2006). Williamson and

Higgins (1997), in a cross-national empirical study, found that indeed population age distribution influenced international capital flows.

For some countries, the international context will ameliorate the impact of population aging. For example, aging European economies, if closed, would experience capital deepening with falling rates of return, and rising wages. However, the rest of the world is younger and these European economies, by exporting some of their savings, can reduce the decline in rates of return and reduce the rise in wages (Boersch-Supan et al 2006). The US is considerably younger than Europe, and Krueger and Ludwig (2011) find that openness reduces the rate of return to capital and raises wages rather than the reverse, because the US is younger than most other OECD countries and through openness they effectively import population aging. To a greater degree, this will be true of many developing countries. For Europe, openness means that older people who hold assets benefit through higher income while younger people with fewer assets but more labor lose through lower wages.

While the reasoning just given seems straightforward, multi-regional OLG models with actual and projected demographic change show more subtle effects that can be counter-intuitive (BÖRSCH-SUPAN et al, 2006). The expected effects do occur initially, but after a few decades these effects wear off and are replaced by weakened or reversed effects in some cases. Using an international OLG model Fehr et al (2008) find that in developed nations as a region, even a large increase in fertility or mortality would not have much effect until 2070, due to the momentum of demographic change.

e. Will population aging cause an asset price meltdown?

Older people often rely to some degree on funded retirement plans, whether their own or through an employer's pension. The ability of such plans to deliver income for retirees depends on the rate of return on assets. This lends urgency to the question whether population aging will lead to an "asset price meltdown", a topic that has been the subject of an academic literature (Poterba, 2001 and 2005; Brooks, 2006; Kruger and Ludwig, 2007; Geanakoplos, 2004) accompanied by more strongly voiced concerns in the popular press. It is suggested that a decline in rates of return or meltdown might occur for two reasons. First, population aging is expected to lead to capital intensification with a resulting decline in the return to capital (Arnott and Chaves, 2011; Liu and Spiegel, 2011), as capital-labor ratios rise, as discussed earlier. Second, a subtler argument is that older people are more risk averse and prefer an investment portfolio with a higher proportion of bonds. As a baby boom moves into old age its members will sell off their equities in order to buy bonds, reducing the price of equities and raising the price of bonds. There could be a similar effect on houses and housing prices as older people sell their homes and move into smaller rental units.

Population aging can be foreseen decades in advance, so if participants in asset markets are forward looking, it is unlikely that slowly moving population aging would lead to a sudden decline in asset prices or to a dramatic meltdown. There have been many empirical studies of the relation between demographic change and asset price movements, reviewed in National Research Council (2012, Chapter 8). The results of these studies have been mixed, and when analysis is modeled in the international open economy context, the studies tend to find at best small effects of population aging (BÖRSCH-SUPAN et al, 2006; Ludwig and Vogel, 2011). Open international contexts dilute and counteract the effects of population aging.

It is also important to keep in mind, however, that the return on capital will be influenced by the trend in labor supply as well, which like capital will be strongly influenced by international flows, that is by labor migration. Furthermore, it is not only the quantity of labor in relation to capital that matters. It is also the amount of human capital per worker. This brings us to the topic of the next section, human capital intensification.

f. How much population aging is optimal?

From the Arthur-McNicoll (1978) result given in 05 we have the effect on consumption of a slight change in the population growth rate, where consumption is measured as the height of the consumption age profile. We have age profiles of labor income and consumption for different countries. If we assume an elasticity of output with respect to capital of 1/3 under constant returns to scale, depreciation at .05 and productivity growth at .02, we can find the population growth rate and age structure that maximizes the height of the consumption profile. In fact, this has been done in Lee, Mason et al (2014) and Weil and Hock (2007). Results vary by country. In general, however, a slightly negative steady state population growth rate is “optimal” in this sense, and the associated fertility rates are around 1.6 births per woman. This calculation is based solely on the simple neoclassical growth rate with age distribution and fixed age-shapes of consumption and labor income. Nonetheless, the result is striking. Taking the capital costs of population growth into account changes the outcome. Without including these capital costs, the maximizing growth rate would be close to 0. If we focused only on the age profiles of tax payers and recipients of public transfers, then substantially more rapid growth and higher fertility would be desirable.

One might be uncomfortable with the idea that a negative population growth rate could be sustained and be optimal. However, it is completely standard to work with steady states in which population growth and productivity growth are both greater than zero. The long run problems posed by that scenario are vastly greater than those caused by mildly negative population growth.

Population aging and human capital intensification

There are good reasons to expect that population aging will be accompanied by increased investment in the human capital of children so that the quality of the workforce will rise, and a reduction in the relative quantity of labor will be offset to some degree by an increase in its quality. Such an increase in labor quality would have several relevant effects. First, it would raise labor income and therefore tend to reduce capital intensification measured by the ratio of capital to number of workers. This would reduce any decline in the rate of return to assets as populations aged. Second, it would raise the earnings and incomes of more recent generations relative to older ones, and thereby ease the problems of pay-as-you-go (PAYGO) pension systems in which benefits are determined by wages at retirement and are not indexed to wage levels in subsequent years (the US Social Security system is of this sort, for example). In such systems growth in wages per worker is just as helpful for fiscal balance as growth in number of workers. Third, it would raise the level of per capita income.

The main cause of population aging is low fertility. In the well-known quantity-quality tradeoff theory of fertility (Becker and Lewis, 1973; Willis, 1974), couples seek to maximize their utility which depends on their own consumption and also on the number of children they have and on the average quality of those children. They face a budget constraint in which number of children is multiplied times the quality of children because this product is total spending on children. For simplicity assume that a couple first decides on the share of income they will devote to children, and then given that amount, decides how to allocate this amount between quantity and quality subject to the constraint. In this case, variations in tastes or prices or incomes across otherwise similar couples would lead to a negative relation between quantity and quality with an elasticity of approximately -1. The theory suggests that income elasticity for quality is substantially higher than for quantity, so that when incomes rise over time in a country, couples choose to have fewer children and to invest proportionately more in each child.

A version of this theory might also be applied to public education to suggest increasing spending per child in countries in which fertility is falling. Figure 9 presents National Transfer Accounts data on human capital spending per child relative to fertility. Human capital spending is defined as the sum over ages 3 to 26 of per capita spending at each age (a synthetic cohort measure), combining both public and private spending, plus a similar sum for public and private health care spending from age 0 to 17. For comparative purposes, this measure for a country is divided by that country’s average labor income across ages 30-49. Fertility is measured as the Total Fertility Rate in the five years before the base year of the survey used for the NTA estimates. The data are plotted on a log-log scale with each point corresponding to a country, distinguished by region. There is a moderately strong negative relationship with a slope of -.7. For present purposes the direction of causality does not matter, whether from fertility to human capital, from human capital to fertility, or from economic development to both. The point is that lower fertility is associated with higher investment per child. In high fertility countries about one to two years of labor income is invested in human capital per child, whereas in a country with fertility near 1.0, 5 or 6 years of labor income are invested in each child. A similar or stronger relationship is observed over time within countries (Lee and Mason, 2010b, 2011). As it happens, this relationship is largely due to variations in public education rather than in private expenditures, although within the Asia Region, it holds also for private expenditures on education. Lee and Mason (2010b) also develop an OLG model incorporating these estimates as well as estimates of returns to human capital in the literature, and conclude that the substitution of quality of workers for quantity of workers could greatly reduce the old age dependency costs of population aging.

Another reason to expect human capital investment to rise as populations age is that capital intensification reduces returns to capital and raises returns to labor and to human capital. This creates an incentive to shift investment from physical capital to human capital. Ludwig, Schelkle and Vogel (2012) and Ludwig and Vogel (2009) develop OLG models incorporating this insight which are applied to the US and to a set of countries, and they find that increased human capital investment substantially offsets the effect on wages and rates of return of population aging. Becker and Murphy (1988) provide an elegant theory of the interrelation of human capital investment, public education, public transfers to the elderly (pensions and health care), and capital accumulation.

A different approach to a similar set of issues takes fertility and longevity as exogenous, and models the investment in human capital as a matter of individual optimization over the life cycle. Heijdra and Reijnders (2012) exemplify this approach. In an OLG model with these features, they find that an increase in longevity accompanied by a postponement in the depreciation of human capital leads to an increase in both physical capital and human capital, with the human capital increase larger so that wages (per efficiency unit of labor) fall and the rate of return to capital rises, reversing the usual result, and illustrating the importance of taking human capital into account in this context. Cervellatti and Sunde (2013) show that so long as mortality declines within the working ages, it raises the incentives for investment in human capital.

Aging and Productivity Growth

There are two very different issues to consider here. One is that population aging means aging of the labor force which may reduce its efficiency, dynamism and ability to adopt new technology and its willingness to take risks. The other is that the very machinery generating technological progress may move more slowly in an aging population. We will begin by considering the aging labor force.

Age and productivity of workers

As the population ages, there will also be a shift toward older ages in the labor force even if labor supply by age of individuals remains the same. This shift will be strengthened if the age at retirement rises with longer life, as appears very likely. The projected aging of the labor force raises two basic questions. First, are older workers less productive than younger ones, and if so will this lead to slower productivity growth, at least during a transitional stage? And second, will an older population be less creative and innovative than a younger one, leading to slower technological progress and productivity growth?

It is tempting to look to the age profile of average real wages to show us how labor productivity varies with age. There are serious difficulties with this approach. First, older people who continue to work may be selected for health, strength, and vitality and therefore not be representative of older people in general or those older people who might be induced to supply more labor. Second, seniority and rules against age discrimination may influence the wages paid to older workers beyond what their productivity might warrant. Third, cross-sectional data will show effects of generation as well as age. Difficulties like these have led some analysts to study direct measures of ability by age, for example performance in athletic events or in chess (Fair, 2008; Skirbekk, 2004), or cognitive ability (Skirbekk et al, 2012). Pekkarinen and Roope (2012) use piece rates for Finnish blue collar workers and fund that hourly productivity rises up to age 40 and then remains constant. A thoughtful review of this literature is given in National Research Council (2012, Ch.6). Based on its survey of the literature and its own calculations, it concludes that “there is likely to be a negligible effect of the age composition of the labor force on aggregate productivity over the next two decades" (p.120) while cautioning that the evidence is very fragile.

Endogenous Growth and Population Aging

It is arguable that the mechanisms and consequences considered so far – support ratios, capital intensity and human capital intensity--are all just one-time adjustments that do not alter the trajectory of economic growth. Even a small effect of an aging labor force and slower population growth on the pace of technological change would come to dominate all other influences. Will an aging labor force be less innovative and slow technological progress?

For changes in long term economic growth rates we need to turn to endogenous growth models. There are many examples of these, which are briefly reviewed in Jones (2002) and Prettner (2013). In Jones' model, growth is driven by "the discovery of new ideas throughout the world" (p.221) and because new ideas come from people, perhaps engaged in formal research and development efforts, population size has an effect and therefore "long-run per capita growth is ultimately tied to world population growth"(p.224). In this view, then, the slowing population growth that causes population aging will also reduce the rate of technological progress. Jones emphasizes, however, that it is population growth in those countries that are at the technological frontier that matters, and surely these would include the rich developed nations but increasingly also developing nations like China and newly industrialized nations like Taiwan and S. Korea. This perspective suggests that as population growth rates fall and populations age, technological progress will slow. A contrary view, drawing on the relation of human capital investment to fertility levels, is that slowing population growth will go hand in hand with increasing human capital intensity, and this might prove more important than raw population numbers in generating new ideas and progress.

Many analyses in this literature abstract from age and treat population as homogeneous. Prettner (2013), in a useful paper, introduces fertility, mortality and population age distribution into models of endogenous technological progress (dependent on population size) and semi-endogenous (dependent on population growth rates, as in Jones above) that previously had homogeneous individuals. In his analysis lower fertility does impede progress but longer life favors it by reducing discount rates and encouraging investment in human capital, and it is the balance of these effects that matters. Although the two modeling approaches yield different implications of aging for economic growth, he concludes that current demographic changes featuring declines in both mortality and fertility will not necessarily slow technological progress or economic growth, and could indeed make them more rapid.

Population aging might also affect the direction of technological progress, for example by influencing its factor bias. Slower labor force growth and capital intensification could incentivize labor saving technological change, for example. Population aging also changes the composition of demand, tilting it toward health care services, and incentivizing research and development in this area. Additionally, it raises the demand for workers in elder care, including elder care in the home. In Japan we see advances in the use of specialized robots to substitute for elder care workers in the home. Because the public sector is heavily involved in provision or funding of long term care, it might play a special role in this area. Research on the way that population aging affects the direction of research and development, and technological change, is very limited.

Another set of issues arises around the political influence of the elderly on the process of innovation. Lancia and Prarolo (2012) use an OLG model with children, workers and retirees to analyze the interests of each in the costly adoption of existing new technologies. They find that as longevity increases and the population ages, even though individuals respond by investing more in human capital, the growing share of elderly tends to oppose the adoption of new technologies through the political process, and a slowdown in innovation and growth is one possible outcome.

More prosaically, it is worth noting that population aging, and more generally changing population age distributions, have a mechanical effect on measured aggregate unemployment rates and on employment rates. For example, Horn and Heap (1999) calculated age-standardized unemployment rates for the US over the period 1980 to 1997, and found that it varied relative to the standard BLS measure from .3 percent less to .5% more over this 17 year period.

Perhaps this should not be surprising, because the passage of the Baby Boom through the population age distribution was a major event. Nonetheless, it is worth noting that the young have higher unemployment rates than older workers, and so population aging will affect the standard measure and should be taken into account. Similarly, and probably more important, employment rates vary by age. For example, Fujita (2014) notes that the aggregate participation rate declined from 2000 to 2013 by more than 4 percentage points, and that an important share of this drop was due to the aging of the Baby Boom, particularly starting in 2010. Hotchkiss (2009) finds that changing population age distribution accounts for much of the variation in the aggregate participation rate since 1950, and holding age specific participation rates fixed at 2008 levels, would predict a decline of 2.4 percentage points from 2008 to 2020 – mainly reflecting the retirement of the Baby Boom generations. Changes in population age distribution can have an important impact on these aggregate measures which in turn have important roles in policy discussions, and failing to take their role into account can lead to erroneous interpretations.

Political Economy of Population Aging and the Public Sector

There are two leading questions in the political economy of population aging. One is how public spending on human capital investment will be affected. The other is how public transfers to the elderly will be affected. The first question is complicated by the quantity-quality issues surrounding fertility and human capital investment, as discussed earlier. But an additional factor is how longer lives and the growing proportion of elderly voters will affect spending on education. This is related to the second question, since increased public spending on transfers to the elderly could crowd out public investment in children.

From an empirical perspective, Gruber and Wise (2002) report how changes in the share of elderly are associated with public spending on the elderly in OECD countries from 1980 to 1995, in a setup with country and period fixed effects. They found that a 1% increase in the share of the elderly was accompanied by only a .47% increase in transfers to the elderly as a share of GDP, so that per capita transfers to the elderly were reduced by population aging, while aggregate transfers to the elderly were increased. At the same time, there was no increase in total transfers, because transfers to other age groups, including children, were reduced. In sum, as the population ages, per capita transfers to individuals at each age, including the elderly, are reduced, even though aggregate transfers to the elderly are increased. This analysis suggests that population aging leads to less human capital investment than otherwise.

Another literature focuses on theoretical aspects of the political economy of aging. Razin et al (2002) develops a two age group OLG model with different skills classes of labor and different ability levels. The voting population forms coalitions around spending on public intra- and inter-generational transfers. The elderly would like to raise transfer spending, since they are major beneficiaries, while many working age voters would rather reduce public spending on transfers because they stand to lose. They show that population aging would actually lead to reduced payroll taxes and reduced transfers unless population ages to the point where the median voter is old – a highly unlikely prospect.

Gradstein and Kaganovich (2004) consider the effect of longer life on public spending on education. They develop a two age group OLG model in which younger adults want to invest more in the human capital of the young, because they are concerned about future productivity growth which will affect rates of return earned by their savings over a retirement period that will be longer due to increased longevity. The already old and retired, however, would rather spend less on human capital investment, and with longer life there share in the population rises. They find that the interests of the younger adults will dominate, favoring increased spending on human capital, even though the cross-sectional relationship of aging and educational spending across local areas could be the opposite as is sometimes found. A much-cited paper by Becker and Murphy (1988) presents a coherent theory in which public education arises to remedy inefficiencies and institutional failures in familial investment in human capital, and public pensions arise to repay the wind-fall losses suffered by the initial generations who pay taxes for public education but received none themselves. Bommier et al (2010) explores this theory empirically in the case of the US.

Policy Issues

Population aging is inevitable in a world with rising life expectancy, because fertility must decline. Some parts of the world (Europe, N. America, East Asia, parts of Latin America) are farther through the Demographic Transition with lower or negative growth rates and early population aging. Possibly these countries can reduce the impact of population aging through increased immigration, but this is doubtful because immigrants themselves become old and in the long run tend to make receiving countries older (Goldstein, 2009). Nor are pronatalist policies likely to have a major impact on fertility.

Because net upward public transfers are pervasive, population aging will be costly and will require adjustments which could take various forms (Weil, 1997; Elmendorf and Sheiner 2000; Sheiner et al 2007). These include raising labor supply by delaying retirement or increasing labor force participation of women; raising saving rates; raising taxes; reducing government benefits. For illustrative calculations of tradeoffs among these options see Sheiner et al (2007) for the case of the US. In each case, adjustments can be made earlier or later, with corresponding differences in generational impacts, but they will have to be made. These adjustments will have to be greater in countries with greater population aging in coming decades such as Japan, Spain, or Germany, and in countries with more generous public transfers to the elderly, as in Sweden, Austria, or Brazil. In countries with less projected population aging and less generous transfers to the elderly, such as the US or Mexico, the needed adjustments will be smaller.

To get an idea of the magnitude of the fiscal consequences of population aging, assume that public benefit profiles (pensions, education, health care, long term care) keep the same shape as they have around 2010, but shift upward with projected productivity growth. Project GDP forward based on population projections and productivity growth. We can then project the percentage point increase in government spending on these programs as a share of GDP. Such projections are not intended to give a realistic picture of the future, because they assume no adjustments to program benefits and adjustments will surely occur, even if by economic collapse. Mason et al (2015:Figures 3 and 4) reports such projections of the fiscal consequences of population aging, from 2015 to 2065 based on NTA data. For the US, the projected *increase* in government spending due to population aging would be a relatively modest 8.5 percentage points of GDP. For Brazil it would be 14 percentage points. For Japan, 16 and for Germany 18 percentage points. These are very large increases, and for some countries they will come on top of already very high shares of government spending out of GDP. These increases indicate a growing dependency burden that will lead to reductions in consumption below what it would otherwise be. But a more serious concern may be the distortions to economic behavior that would result from the increased taxes needed to support such big increases.

The distortions might take many forms. Large increases in payroll taxes would distort incentives for supplying labor, perhaps leading to declining labor supply at all ages. It has been well established that the incentives built into many public pension plans have a strong effect on retirement behavior (Gruber and Wise, 1999). Higher payroll taxes would also reduce private returns to human capital and might lead young adults and their parents to invest less in their education, reducing productivity growth and perhaps reducing technical progress as well. Some have suggested that higher payroll taxes, by reducing after tax incomes, might lead to reductions in fertility which would in turn increase population aging and reduce the tax base. Paying for the increased costs of these transfers for the elderly through other kinds of taxes, such as a tax on capital, would lead to other kinds of problems and distortions, as discussed by Razin et al (2002).

Such concerns suggest that government should not simply raise taxes to cover the rising cost of benefits for the elderly. There are alternatives for policy. One is to facilitate and incentivize the postponement of retirement by older workers. This means removing mandatory retirement laws, barring employment discrimination against older people, removing early retirement incentives from public and private pension programs, and encouraging employers to offer part-time work. Longer work life is a natural response to longer and healthier life, however, it cannot be expected to solve all the economic problems of population aging, some of which arise from low fertility rather than from longer life. Another policy option is to maintain the current PAYGO program structure for a portion of benefits, but to introduce private accounts for additional costs. Alternatively, PAYGO programs may be restructured to mimic defined

contribution private accounts, as in the Notional Defined Contribution pension program introduced by Sweden and subsequently adopted by other countries. The incentive structures in such programs may avoid some of the distortions in ordinary public pension programs.
On a more optimistic note, Dolls et al (2015) use a microsimulation model including a partial equilibrium treatment of the labor market, to analyze the labor market and fiscal response to projected demographic change in 27 European countries, and to assess the impact of five year increases in the statutory retirement aged. Their microsimulation approach enables them to take into account population heterogeneity including by educational attainment, and they also incorporate both a demand side response of wage rates to declining labor supply, and the supply side response of labor to rising wage rates. They find that taking into account labor market responses eliminates more than half of the adverse fiscal consequences. When a policy response of increased retirement age (parallel to increased life expectancy) is taken into account, again with wage responses, the adverse fiscal impact of aging is completely removed, on average.

Research directions

There are many questions of interest to be explored. Here are a few.

- Could increased investment in human capital, public or private, offset the decline in the number of workers relative to elderly? Would the possibility of such substitution of quality for quantity be exhausted after a generation or two or could it be a long term solution? This would involve a mixture of empirical analysis and theoretical analysis.
- To what extent could international capital flows reduce capital intensification and declining rates of return in aging economies? How would consumption, savings, bequests and intergenerational equity be affected?
- What would be the consequences of a transition from an unfunded to a funded public pension program or system of private accounts as in Chile? How would asset holdings, saving, consumption, and bequests be affected? How would intergenerational equity be affected?
- Similarly, what would be the consequences of a transition from familial support of the elderly to a public transfer system or a public or private funded system? How would saving and asset holdings be affected? How would human capital investment be affected? How would intergenerational equity be affected?
- How does the transition to lower fertility and to very low fertility (e.g. one birth per woman) affect life time consumption, saving, asset holdings, bequests and intergenerational equity in systems of familial or public support of the elderly?
- To what extent could postponed retirement offset the increase in dependency and decline in the support ratio across the demographic transition? Would postponed retirement lead to lower saving rates?
- Would there be important advantages for lower income developing countries in instituting private retirement accounts or funded public pensions early in the transition before aging begins to appear? Might population aging then raise capital intensity and promote economic growth?

Questions of this sort could be investigated using models for simulation, optimization or other kinds of analysis. Here are some suggestions for the kinds of models that might be useful.

Fertility, mortality, and possibly health status
There are two sources of population aging: rising life expectancy and low or falling fertility. Some macroeconomic impacts of population aging depend on its source. It is important to consider both sources, and this is possible only if both are in the model. Often in OLG models mortality is omitted entirely. Except for the few countries that still have high mortality, mortality during the first 50 years of life is no longer very important, so the key is to include mortality in the transition from working life into retirement, or from early retirement to later retirement. Adult mortality rises exponentially at about 10% per year of age, doubling roughly every seven years, so including this age variation and including mortality at older ages are important. Another key question is whether improving longevity is accompanied by improving health. If so, this alters the consequences, since the problems of population aging are then more

institutional and behavioral than fundamental, in the sense that the proportion of expected life spent in need of care and unable to work may remain unchanged. Trends in health status are particularly important if the age at retirement is made endogenous. Heijdra and Reijnders (2012) demonstrate the importance of these points by analyzing several different demographic/health scenarios, each leading to a different conclusion.

My personal view is that trajectories of fertility, mortality and health should be taken as exogenous. While theories are available to relate these to individual choices, they have little predictive power and their use might obscure the workings of some better-understood mechanisms.

Intergenerational transfers

Between the public sector and household sector 55% of GDP is transferred across age groups, mostly to children or the elderly (Lee and Donehower, 2011:185). Within the household, lower fertility may reduce the total costs of transfers to children, but it also is associated with greater investments in the human capital of each child. Public sector transfers to the elderly may substitute for saving during the working years or may enable asset preservation and increased end-of-life bequests by the elderly. Intergenerational transfers play a central role in investment in human and physical capital, and for this and other reasons are centrally important for the consequences of population aging. It is not clear how best to model bequests. Sometimes they are assumed away by assuming complete annuitization of wealth, and sometimes they are made simply accidental by assuming annuitization is not available. With fewer children per decedent, and with retreat from defined benefit pensions in the US, bequests may become more important in relation to the macroeconomy.

In my view, patterns of intergenerational transfers should be taken as given rather than treated as exogenous, until more progress has been made in understanding the relevant individual behavior and political economy. But including transfers is important, and saving and investment decisions can be modeled as if individuals took the transfer context as given when planning for risk buffering and life cycle consumption smoothing.

Age at retirement

Retirement age is obviously important, and it has been shown to respond to pension incentives, policy incentives and other changes in the economy. This suggests that for some purposes it should be endogenous, and for other purposes it should be taken as a policy variable.

Global perspective

Population aging is coming all over the world, but with uneven timing. Treating an economy as closed would capture some important implications of population aging while missing the consequences of demographic change elsewhere. It would exaggerate capital intensification and falling rates of return on capital in the older rich countries, by ignoring the possibilities of investing abroad. It would understate capital intensification in a country like the US with moderate aging, but with open financial markets. Treating an aging economy as financially open would minimize the effects on its factor prices, while maximizing the changes in assets and asset income. Both perspectives are useful.

Technological progress and endogenous growth

Even a very small difference in the rate of technical progress will dominate age distribution effects in the long run. For models based on population size it is not clear that population aging would be relevant. However, for models such as Jones' in which the rate of technological progress is driven by the population growth rate, aging economies will have slower progress even if aging is not the root cause.

Conclusions

Population aging is inevitable and will affect the macroeconomies of all nations. However, because cultural values and institutional arrangements differ from place to place, the consequences of aging will likewise differ. In some countries, the elderly continue to work productively into old age, in others they withdraw early from the labor force, or perhaps are unproductive relative to younger workers. In some countries, the elderly are supported by transfers from their families, in others by transfers from the public sector. In still others, they rely more heavily on income from assets accumulated earlier in life. In some countries, the elderly consume far more than younger adults, while in others they consume about the same as younger adults. For all these reasons, the elderly depend to varying degrees on transfers from the younger population, and consequently population aging has larger or smaller macroeconomic impacts.

There are a number of widely held assertions about these impacts, but these are not always true.

a. *Population aging raises dependency and reduces support ratios.* **This assertion is true but yet may be misleading**. Elderly people are not dependent unless they rely on transfers from the working age population. Even if elderly people do little work, they may not be highly dependent on transfers. They may instead use asset income to fund their consumption. In this case dependency ratios and support ratios may be misleading. Having more elderly may mean having more asset income. At the same time, we have seen that consumption by the elderly has risen faster than at other adult ages in recent decades in many rich industrial nations. There are great differences among nations in how the consumption of the elderly is funded.

b. *Population aging will mean lower consumption than otherwise*. **Not necessarily**. Population aging is due mainly to low fertility which is accompanied by increased human capital investment per child leading to a more productive labor force. Quality substitutes for quantity in the labor force. Also, population aging may be accompanied by capital intensification that offsets some of the increased dependency. Furthermore, as life lengthens and health improves at older ages, labor supply may increase, for example through rising age at retirement. International trade and international capital flows may effectively export population aging to younger countries, reducing its consequences.

c. *Population aging leads to capital deepening, with rising wages and falling rates of return.* **Not necessarily.** This will not be true for an open economy. It will not be true if there are important public transfers to the elderly and the government runs up national debt in the face of population aging rather than balancing the budget (National Research Council, 2012). And it will not be true along an optimal growth trajectory if a Social Planner's objective function weights per capita utility by the size of the population (Cutler et al., 1990). However, it is true in the golden rule case discussed earlier, and in that case fertility below replacement, population decline and the population aging it brings, appear to be beneficial.

d. *Population aging brings an aging labor force which is less productive and less innovative.* **Not necessarily.** Simple calculations based on age-specific earnings suggest that population aging will have only a tiny effect on average labor force productivity ("negligible" according to National Research Council, 2012, Chapter 6), and a study finds that mixed teams including older workers make fewer major errors than young teams. As for creativity, innovation and technological progress, the National Research Council (2012) concluded that "While age is an important determinant of invention and innovation, it explains very little about actual performance across societies. Other factors, such as education, support institutions, economic and social rewards, and religious institutions, tend to dominate the actual distribution of scientific output." It is also true that technological advances are a global public good, and aging in a given country would have little effect on the technologies available to it for adoption.

Population aging will pose difficult problems in societies that rely heavily on public or private transfers to the elderly. In other countries where aging will be less profound, or the elderly rely more on continuing work and on asset income and less on public or private transfers, the problems will be muted. Increased investment in human capital, adjustment of taxes and benefits in the face of population aging, and facilitation of postponed retirement, will go a long way to soften the impact of population aging.

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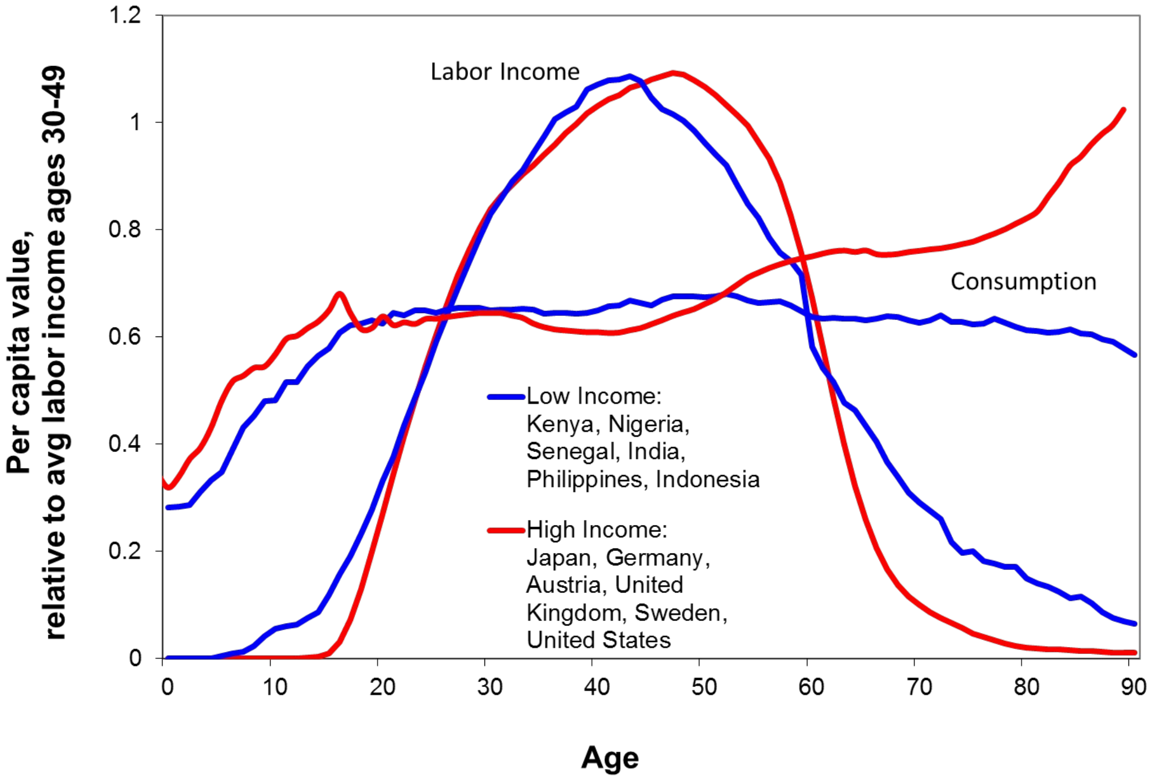
Table 1. The effect of slower population growth and population aging on life time consumption across golden rule steady states under different assumptions about presence of capital (K) and aggregate transfer wealth (T) which can be positive or negative. W is the aggregate demand for life cycle wealth. K* is the golden rule level of K.

Constraints on System for reallocating income across ages through capital or transfers	**Proportional effect on life time consumption of a small decrease in pop gr rate, population aging.**	**Other comments on this case.**
General Case: **W=T+K**	-T/c With T>0, pop aging and slower growth are costly	Since capital exists (K>0), age profiles of consumption and labor income are not equal at all ages, and transfers are pervasive, this general case is the only realistic one.
T>0	>0	Positive transfer wealth displaces some K, which is Feldstein's concern with Social Security. Slower population growth tightens the budget constraint and life time consumption decreases. Population aging is costly. However, if W greater than K* then positive transfer wealth would be necessary to reach golden rule, as per Peter Diamond.
T<0	<0	When the life cycle demand for wealth is less than K* then negative transfer wealth from a strong bequest motive (Kotlikoff and Summers) makes it possible to achieve golden rule. For example, tax revenues invested in capital which is passed on from generation to generation. Slower population growth from lower fertility and population aging is beneficial.
No transfer wealth (T=0), pure market economy	0 With T=0, small variations in n do not affect life time consumption.	T=0; W=K. Pure life cycle saving (Tobin, Modigliani). This is also Samuelson's goldenest golden rule case, described in his serendipity theorem.
No life cycle wealth (W=0), because life cycle is symmetric or age-homogeneous.	-K/c This is the pure capital dilution case of the standard Solow model. Age doesn't matter at all.	W=0 so K = -T. Demand for K is entirely bequest motive. This is standard Solow and pure Kotlikoff-Summers. W=0 implies that $A_c=A_{yl}$ which is consistent with reallocation through credit, but in Solow comes about because there is no age heterogeneity, and c(x) = $y_l(x)$ at every age. With no life cycle saving motive, all capital is held in order to transfer to the next generation.
No store of wealth; neither capital nor transfers. (K=T=0) implies W=0.	0 $A_c=A_{yl}$	With no store of life cycle wealth, W must be 0 and therefore $A_c=A_{yl}$. There can be reallocation of income over the life cycle by borrowing and lending, but it must be symmetric. This is the initial Samuelson consumption loan case with a market solution. Aggregate credit wealth must be 0, so with only two age groups each must consume exactly what it produces. With three age groups symmetric reallocation through credit is possible but requires a highly negative interest rate to induce the youngest age group to borrow.
No capital (K=0), consumption loan economy but with possible T.	Effect of a decrease in pop growth rate, d[ln(C)]/d(-n) =-T/c =-(A_c-A_y)	W=c(A_c-A_{yl}) With no capital, everything depends on the dependency ratio which is defined by the age shapes of c and y_l. This is the Samuelson case with transfers or "fiat money". The effect of reduced population growth would be costly in the case Samuelson envisioned, but could also be beneficial if a childhood stage is permitted and dominates the flow of transfers.

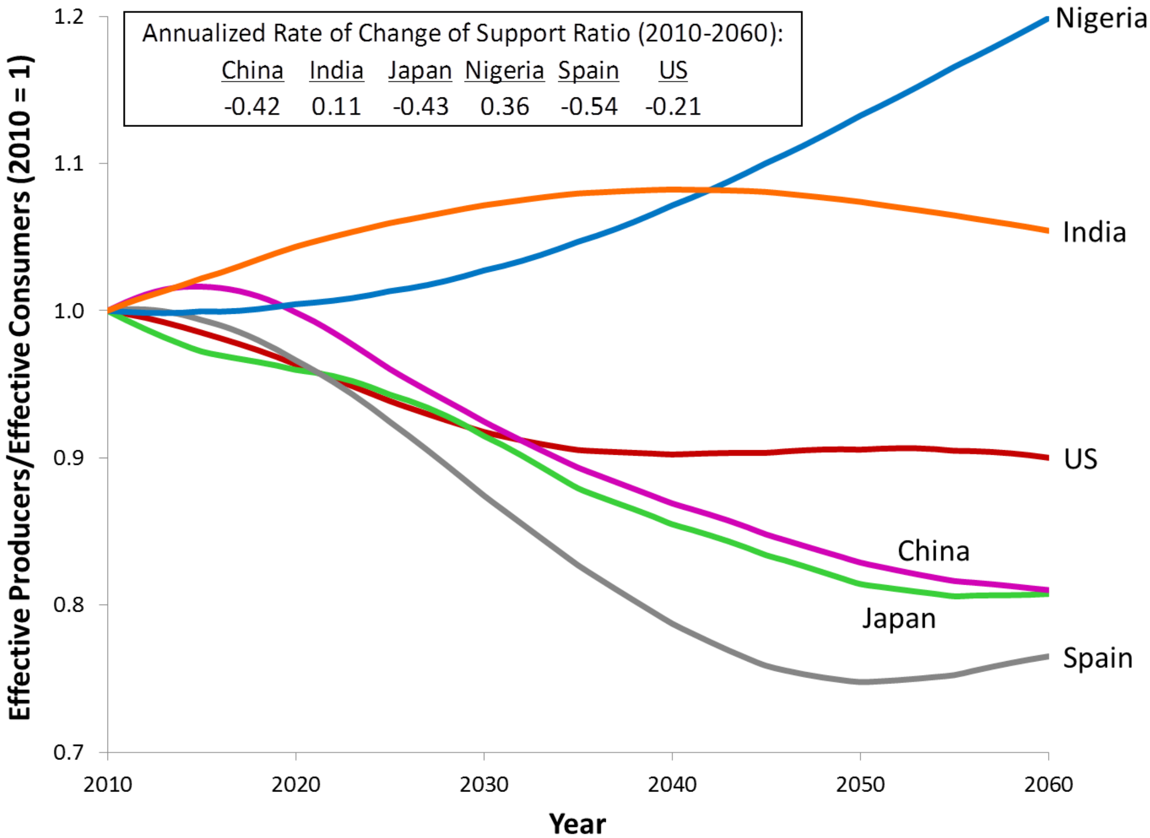
Note: See text for explanation. K is expressed per capita. T is aggregate transfer wealth per capita. At age x, T(x) is defined as the difference between the survival weighted discounted transfers expected to be received in the future, $t^{+}(x)$, minus those expected to be given in the future, $t^{-}(x)$, per capita. Aggregate transfer wealth T is the population weighted sum across all ages of T(x), divided by the total population. A_c is the average age of consuming in the stable population and A_{yl} is the average age of earning. See Willis (1988) and Bommier and Lee (2003).

Figure 1. Labor income and consumption age profiles from NTA averaged for six low income and six high income countries



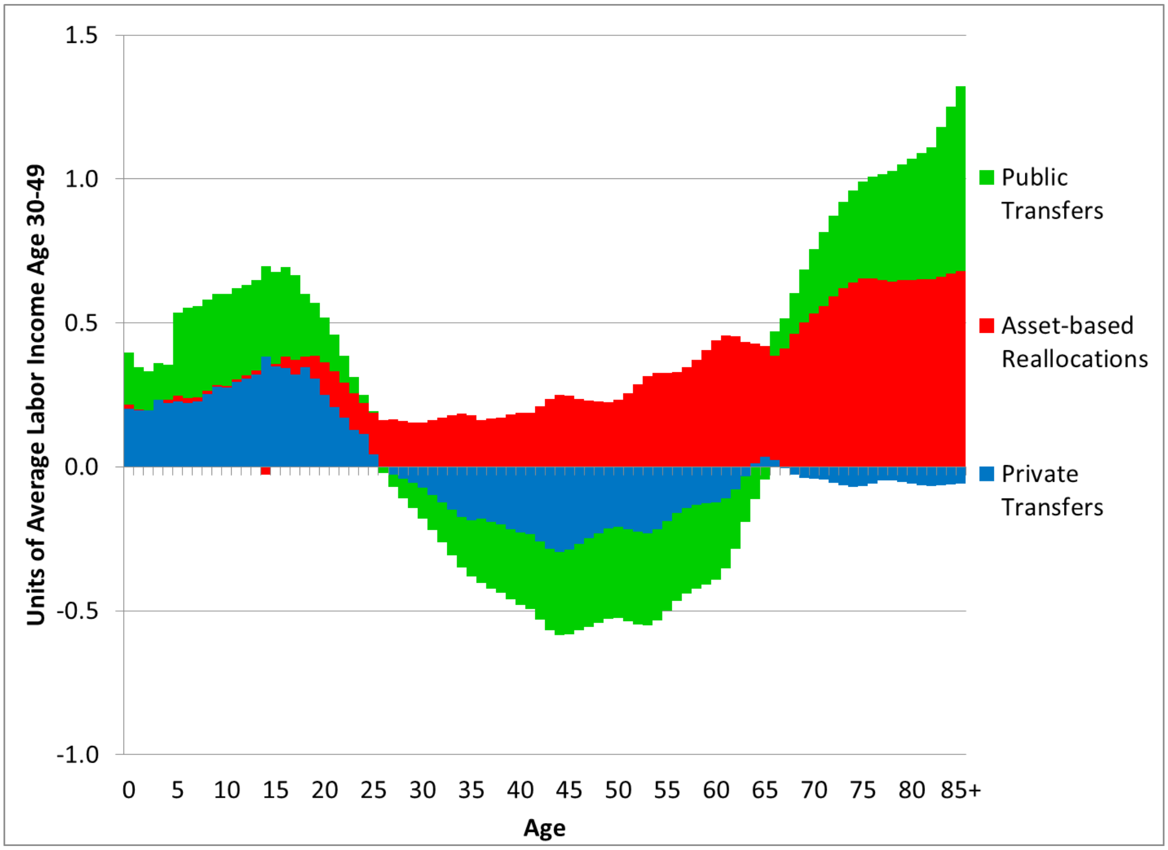
Note: For comparability, each age profile is divided by the average level of labor income in the country for ages 30-49, before averaging across countries. Labor income is an averages across males and females including zero values, based on pre-tax wages and salaries plus employer provided benefits, plus two thirds of self-employment income (the other third is allocated to assets), and includes unpaid household labor as reported in surveys, allocated to household members with reported unpaid labor. Consumption is household consumption expenditure on health and education allocated to recipients of these, plus other consumption allocated in proportion to weights that are .5 for age 0-4 and then rise linearly to 1.0 at age 20. It also includes public in-kind transfers such as public education, publicly provided health care, and publicly provided long term care, allocated to recipient.

Figure 2. Support ratios for selected countries based on UN projections and NTA age profiles, indexed to 1.0 in 2010.



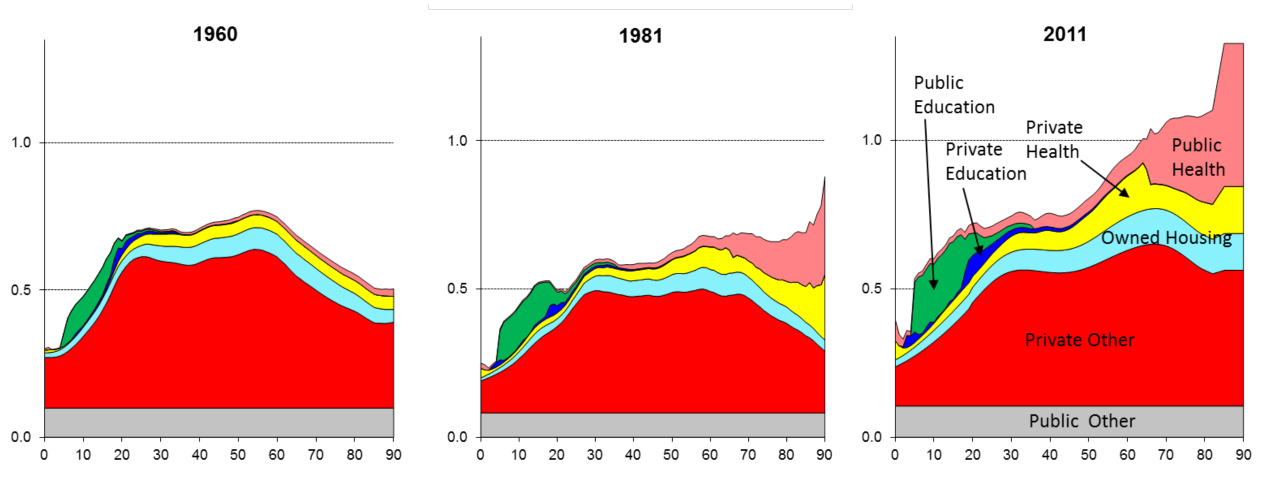
Note: Support ratios for selected countries based on United Nations (2011) projections and NTA age profiles like those shown in Figure 1 (see note to Figure 1). The support ratio is the ratio of effective labor to effective consumers, where effective labor is the sum of the population age distribution times the baseline age profile of labor income, and effective consumers are defined similarly. The support ratios are all standardized to 1.0 in 2010 to facilitate visual comparison of changes. The inset annualized rate of change is given as a percent, e.g. .11 indicates .11 percent per year increase.

Figure 3. How the gap between consumption and labor income at each is made up in the US (NTA data): Components are net public transfers received; net private transfers received; and asset income minus savings.



Note. The figure shows the cumulative sum of the three components when all are the same sign, otherwise the components of different sign must be added. The sum equals consumption minus labor income at each age as shown in Figure 1. Private transfers include interhousehold transfers as reported in surveys and intrahousehold transfers which are calculated as the difference between consumption (see note of Figure 1) and income from labor income and assets. Asset-based allocations are the difference between asset income and saving at each age.

Figure 4. How the cross-sectional consumption age profile has tilted toward older ages over the past 50 years in the US (NTA).



Note. Consumption has been divided by average labor income at ages 30-49 in each year, and the vertical axis units give this ratio. In 2011, the age detail is no longer available to make a full set of age profiles to age 90+ because survey data are truncated at 85+. The figure shows the composition of consumption in each year with components labeled in 2011. Each component sums to the corresponding item in the National Accounts when weighted by population by age in that year. Public Other is items that are not age-targeted such as national defense, roads, and medical research. These items are given the average per capita value at each age.
Source. Updated from Lee, Donehower and Miller, 2011.

Figure 5. The direction of redistribution of income across age in selected populations. The head of the arrow points to the average age of consumption and the tail is at the average age of earning labor income. The length of the arrow is $A_c - A_y$. An arrow pointing to the right indicating a positive difference with income redistributed from young to old, and to the left a negative one.

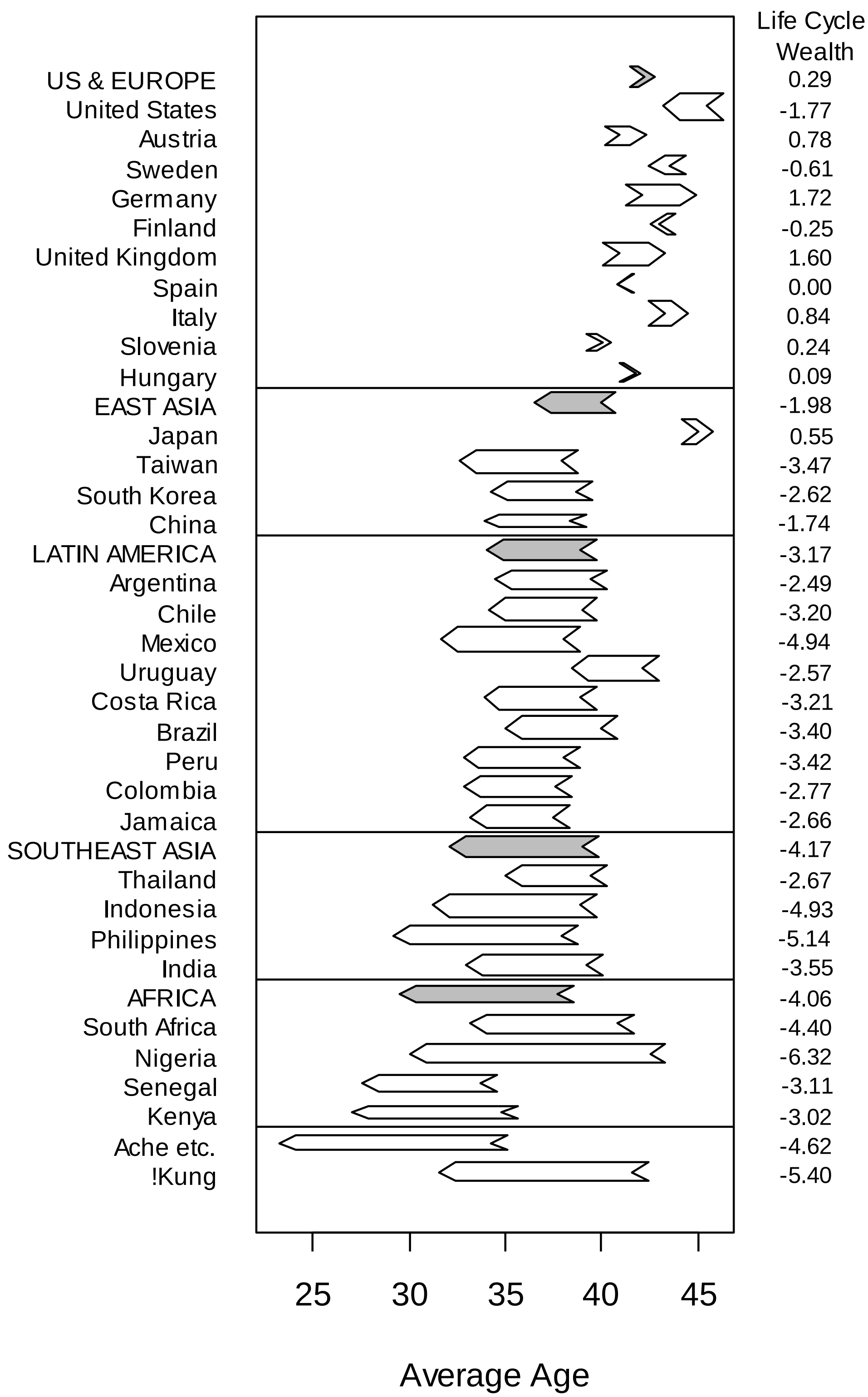


Figure 6. The direction of transfers, private plus public, across age in selected populations. The head of the arrow points to the average age of receiving a transfer and the tail is at the average age of making a transfer. The length of the arrow is $A_{t+} - A_{t-}$. An arrow pointing to the right transfers flowing upward from young to old, and to the left a downward transfer.

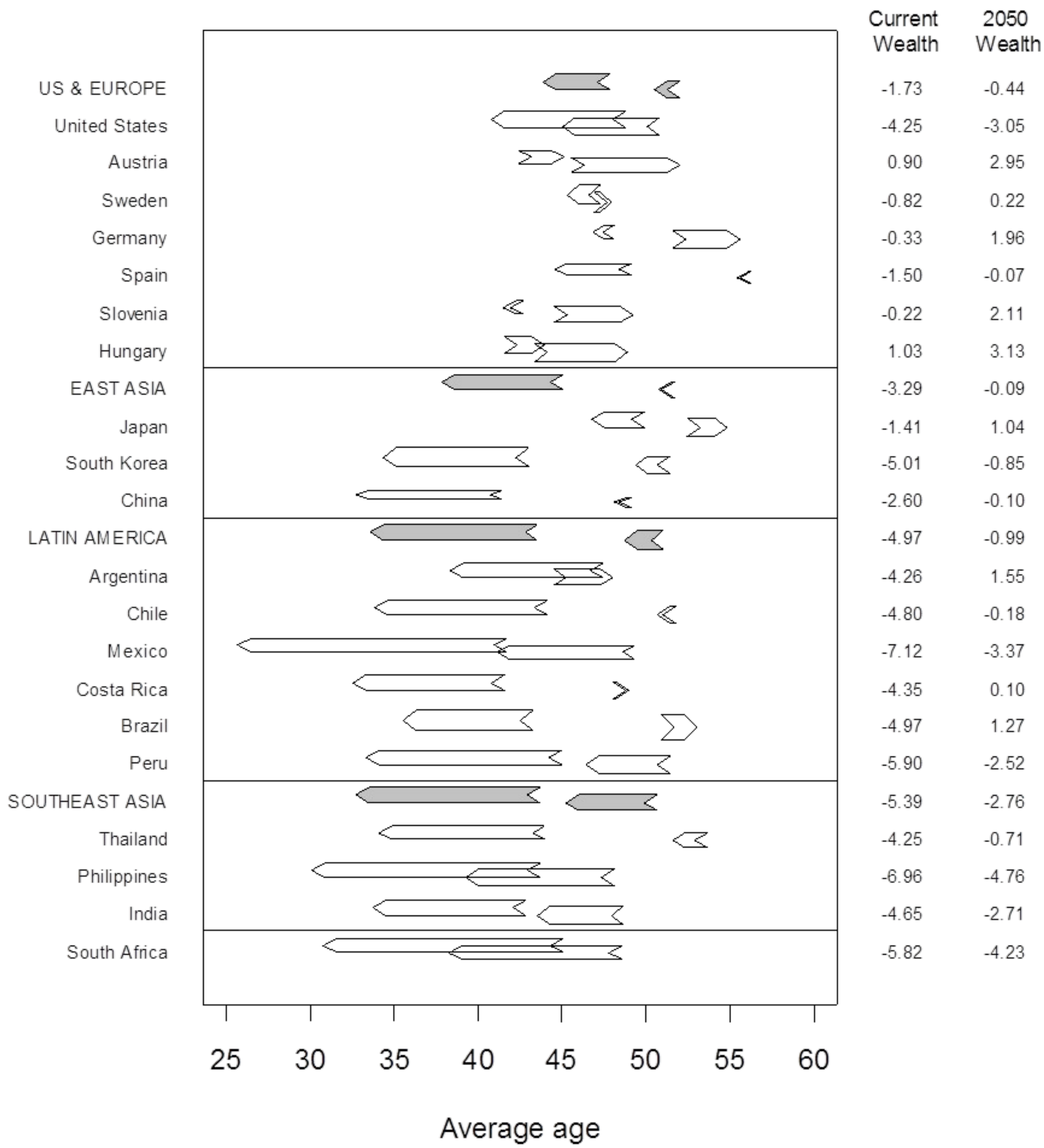


Figure 7. How the gap between consumption and labor income for people 65+ is funded in different countries, by public transfers, by private transfers, or by asset income. The shares add to 1.0.

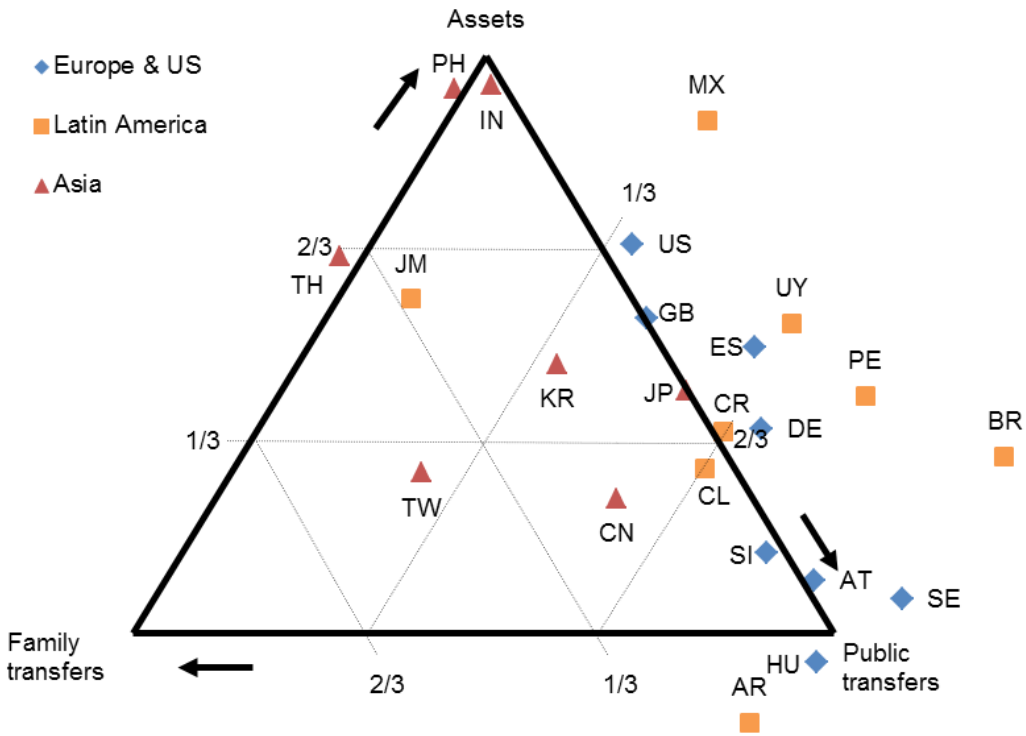


Figure 8. Equilibrium in the capital market and the roles of the aggregate demand for wealth (W_A) and aggregate transfer wealth (T), illustrated for three different demand for wealth schedules. The population growth rate n is fixed at ñ.

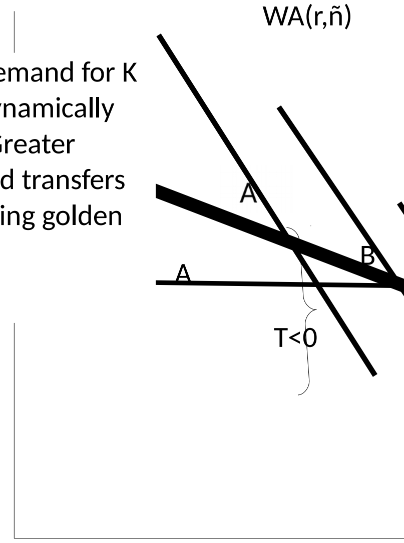
At D, pure life cycle demand for K less than golden rule. If augmented by desire to make downward transfers capital rises, reaching golden rule at B.

At D an golden rule. If augmented by desire to make downward transfers capital rises,

At B the pure life cycle demand for K is coincidentally exactly right for golden rule

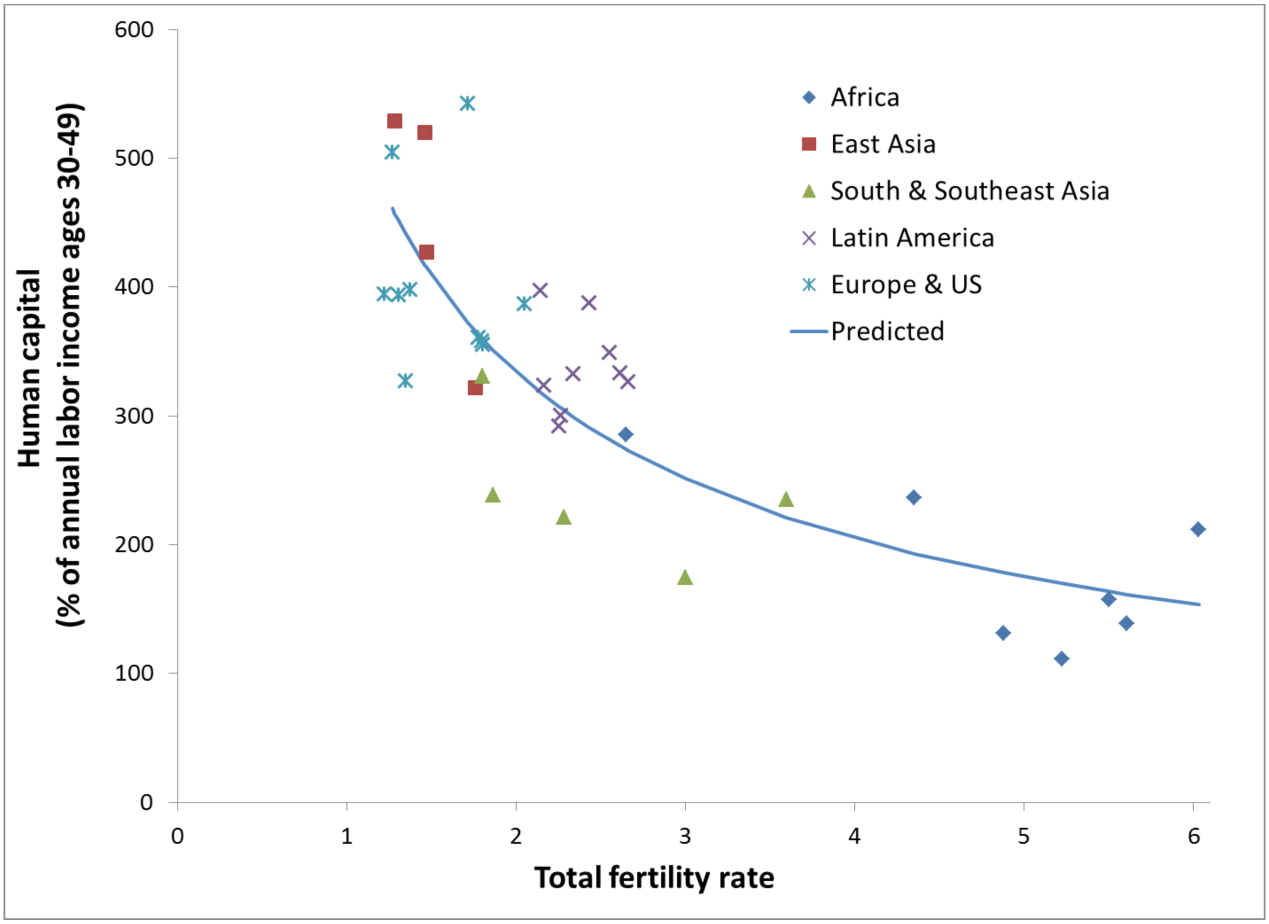
At E, pure life cycle demand for K above golden rule, dynamically inefficient with r<n. Greater efficiency with upward transfers T>0, eventually reaching golden rule at B.

E



Note: The heavy line is the demand for capital by producers, equating r to the marginal product of capital. The lifecycle demand for wealth by households (for purposes of achieving planned life cycle consumption given planned life cycle labor earnings) is shown for three different cases. In case B, when r= ñ it just happens to equal exactly the producers' demand for capital at r= ñ with transfer wealth T=0, and therefore corresponds to the golden rule case. In case A the life cycle demand for wealth when r= ñ is less than the golden rule amount, perhaps because people plan to work longer in old age or to consume less in old age. In this case, only if people make substantial transfers to the young either privately, e.g. through bequests, or publicly, e.g. through a large publicly owned capital stock to which new births automatically gain a share, can golden rule be achieved. In case B the life cycle demand for wealth at r=ñ is greater than the golden rule level, perhaps because people plan to retire very early and to consume more in old age than in youth. In this case to achieve golden rule it would be necessary to satisfy a part of the demand for life cycle wealth through upward transfers such as familial support of the elderly or a Pay As You Go public pension system. D marks a possible non-golden rule steady state equilibrium for Case A with transfer wealth less negative so that r>ñ. Other institutional arrangements leading to other values of T would generate different non-golden rule steady state equilibria.

Figure 9. Human capital investment by fertility across countries based on NTA and United Nations fertility



Note: Human capital spending is public and private spending on education and health by year of age summed from age 3 to 26 for education and 0 to 17 for health. Fertility is the TFR in the five years preceding the NTA base year.

1 Barro's result requires the assumption that parental altruism is strong enough to be operative and that markets are perfect so that receipt of an inheritance does not change lifetime production opportunities.
2 Consider a given period (subscript suppressed) which has some level of output, all of which is consumed; call the aggregate amount C. Let be the amount consumed by each young individual in Case A, and be consumed by each old person. By assumption . If N_O is the number of old and N_Y is the number of young, with , then the social budget constraint is . Solving for and , and summing, we find . The same calculation for Case B yields . It follows that lifetime consumption (undiscounted) is 20% less in Case A than B.
3 More precisely, this is the age at which the average unit of output is consumed minus the age at which the average unit of output is produced.
4 Every steady state system of transfers pays a rate of return equal to the population growth rate plus the productivity growth rate. If transfer wealth is calculated using a discount rate r>n then the pension wealth will typically be negative at some ages, but that does not happen in golden rule conditions when the discount rate equals the population growth rate plus productivity growth rate.