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Abstract

The d-d total cross section is expressed, in the high-energy approximation, in terms of nucleon-nucleon elastic scattering amplitudes and the deuteron ground state wave function. Single, double, triple, and quadruple interactions are treated. Calculations are compared with measurements.

The study of d-d collisions can yield information about n-n or n-p cross sections in much the same manner as the study of collisions between single particles and deuterons has yielded information about single-particle-neutron cross sections [1). The d-d collision is furthermore the simplest one which involves neither neutron beams nor neutron targets and yet yields some information about n-n cross sections. As such it may become a means by which the charge symmetry of nuclear forces can be tested or, if charge symmetry is assumed, a means by which the n-p cross section can be estimated. We present here the results of an analysis of d-d total cross sections at high energies by means of the Glauber high-energy approximation [2).

The methods used for the present note are similar to those used in a recent analysis [3) of the single and double interactions in collisions of <u>single particles</u>

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with deuterons. We characterize high-energy d-d collisions by single, double, triple, and quadruple interactions and the interferences between the corresponding scattering amplitudes. The d-d elastic scattering amplitude $f_{dd}(q,k)$ may be given in terms of the n-n, p-p, p-n, and n-p elastic scattering amplitudes f_{nn}, f_{pp}, f_{pn}, and f_{np} by the relation*

$$f_{dd}(q,k) = 2\{S(q/2)\ S(-q/2)\ [f_{nn}(q,k/2) + f_{pp}(q,k/2)] + S^2(q/2)\ f_{np}(q,k/2)$$

$$+ S^2(-q/2)\ f_{pn}(q,k/2)\} + \text{multiple interaction terms} \qquad (1)$$

where $\hbar q$ is the momentum transferred to the target and $\hbar k$ is the momentum of the incident deuteron in the laboratory system. $S(q)$ is the form factor for the deuteron ground state

$$S(q) = \int e^{iq\cdot r}|\phi(r)|^2\, dr \quad .$$

We note that $f_{dd}(q,k)$ is given in terms of the nucleon-nucleon (N-N') elastic scattering amplitudes for nucleons incident at one-half the deuteron laboratory momentum.

The four terms written explicitly in eq. (1) are those which would obtain in a simple impulse approximation. Since the multiple interaction amplitudes are lengthy, we shall only record here some of the useful results that can be derived from them. The d-d and N-N' total cross sections $\sigma_{dd}(k)$ and $\sigma_{NN'}(k/2)$ are given by the optical theorem:

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*In the present work the Pauli principle and the spin-dependence of nuclear forces have been neglected.

$$\sigma_{dd}(k) = (4\pi/k)\ \mathrm{Im}\ f_{dd}(0,k)$$

$$\sigma_{NN'}(k/2) = (8\pi/k)\ \mathrm{Im}\ f_{NN'}(0,k/2) \quad .$$

We define the d-d total cross section defect $\delta\sigma$ by the relation

$$\sigma_{dd}(k) = \sigma_{nn}(k/2) + \sigma_{np}(k/2) + \sigma_{pn}(k/2) + \sigma_{pp}(k/2) - \delta\sigma \quad . \tag{2}$$

We shall analyze $\delta\sigma$ in terms of contributions $\delta\sigma_2$, $\delta\sigma_3$, and $\delta\sigma_4$ which arise from double, triple, and quadruple interactions, respectively, and write

$$\delta\sigma = \delta\sigma_2 + \delta\sigma_3 + \delta\sigma_4 \quad . \tag{3}$$

The following relations are obtained:

$$\delta\sigma_2 = -8k^{-2} \int \{S(\underset{\sim}{q})\ S(-\underset{\sim}{q})\ \mathrm{Re}[f_{nn}(\underset{\sim}{q})f_{pp}(-\underset{\sim}{q})] + S^2(\underset{\sim}{q})\ \mathrm{Re}[f_{np}(\underset{\sim}{q})\ f_{pn}(-\underset{\sim}{q})]$$

$$+ \mathrm{Re}\Big([S(\underset{\sim}{q})f_{np}(\underset{\sim}{q})+S(-\underset{\sim}{q})f_{pn}(\underset{\sim}{q})][f_{nn}(-\underset{\sim}{q}) + f_{pp}(-\underset{\sim}{q})]\Big)\}d^{(2)}\ \underset{\sim}{q} \tag{4}$$

$$\delta\sigma_3 = (8/\pi k^3)\int S(\underset{\sim}{q})S(\underset{\sim}{q}')\mathrm{Im}[f_{nn}(-\underset{\sim}{q})f_{pp}(-\underset{\sim}{q}')f_{np}(\underset{\sim}{q}+\underset{\sim}{q}')+f_{nn}(\underset{\sim}{q})f_{pp}(\underset{\sim}{q}')f_{pn}(-\underset{\sim}{q}-\underset{\sim}{q}')$$

$$+ f_{np}(\underset{\sim}{q})f_{pn}(-\underset{\sim}{q}')f_{pp}(\underset{\sim}{q}'-\underset{\sim}{q})+f_{pn}(-\underset{\sim}{q})f_{np}(\underset{\sim}{q}')f_{nn}(\underset{\sim}{q}-\underset{\sim}{q}')]d^{(2)}\underset{\sim}{q}d^{(2)}\underset{\sim}{q}' \tag{5}$$

$$\delta\sigma_4 = (8/\pi^2 k^4) \int S(\underset{\approx}{q})\, S(\underset{\approx}{q}')\, \mathrm{Re}[f_{nn}(\underset{\approx}{q}-\underset{\approx}{q}'')\, f_{pp}(\underset{\approx}{q}'-\underset{\approx}{q}'')\, f_{np}(\underset{\approx}{q}'')\, f_{pn}(\underset{\approx}{q}''-\underset{\approx}{q}-\underset{\approx}{q}')]$$

$$\times\, d^{(2)}\underset{\approx}{q}\; d^{(2)}\underset{\approx}{q}'\; d^{(2)}\underset{\approx}{q}'' \quad . \tag{6}$$

In these expressions we have suppressed the argument $k/2$ in the N-N' scattering amplitudes.*

Equations (3-6) give the general expression for the cross section defect in the high-energy approximation. We shall discuss here two sets of assumptions regarding the N-N' amplitudes and the structure of the deuteron which lead to particularly transparent results for the total cross section.

The first set consists of those assumptions which when employed in the analysis of single-particle-deuteron collisions yield the widely used Glauber asymptotic shadow formula [4] $\sigma_{xd} \approx \sigma_{xn} + \sigma_{xp} - \sigma_{xn}\sigma_{xp} \langle r^{-2}\rangle/4\pi$, where x represents the incident particle and $\langle r^{-2}\rangle$ is the expectation value of the inverse square of the n-p separation in the deuteron taken with respect to the deuteron ground state. We should note that this simple asymptotic expression and the corresponding one for d-d collisions which we shall present give only very rough estimates for the cross section defects [3]. If we assume that the average n-p separation in the deuteron greatly exceeds the ranges of the N-N' interactions, that the real parts of the N-N' forward elastic scattering amplitudes may be neglected, and that $\sigma_{np} = \sigma_{pn}$, we obtain the asymptotic results

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*Using results from ref. 3, we may re-express eq. (2) as $\sigma_{dd} = 2(\sigma_{nd} + \sigma_{pd}) - (\sigma_{pp} + \sigma_{nn} + 2\sigma_{np}) - (\delta\sigma_2' + \delta\sigma_3 + \delta\sigma_4)$, where $\delta\sigma_2'$ is given by the first two terms on the right-hand side of eq. (4). The N-N' and N-d cross sections refer again to incident nucleons with laboratory momenta $\hbar k/2$.

$$\delta\sigma_2^{(A)} \approx \frac{\sigma_{np}(\sigma_{nn}+\sigma_{pp})}{2\pi} \langle r^{-2}\rangle + \frac{\sigma_{np}^2+\sigma_{pp}\sigma_{nn}}{4\pi} \langle \frac{1}{2rr'} \ln \frac{r+r'}{|r-r'|}\rangle \qquad (7)$$

$$\delta\sigma_3^{(A)} \approx - \sigma_{np}(2\sigma_{pp}\sigma_{nn}+\sigma_{np}\sigma_{nn}+\sigma_{np}\sigma_{pp}) \quad \langle r^{-2}\rangle^2/16\pi^2 \qquad (8)$$

$$\delta\sigma_4^{(A)} \approx 32\, k^{-4} \langle r^{-2}\rangle^2 \int \mathrm{Re}[f_{pp}(-\underset{\sim}{q})\, f_{nn}(-\underset{\sim}{q})\, f_{np}{}^2(\underset{\sim}{q})] \;\; d^{(2)}\underset{\sim}{q} \qquad (9)$$

where

$$\langle \frac{1}{2rr'} \ln\frac{r+r'}{|r-r'|}\rangle = \iint \frac{1}{2rr'} \ln \frac{r+r'}{|r-r'|} \,|\phi(\underset{\sim}{r})|^2|\phi(\underset{\sim}{r}')|^2 d\underset{\sim}{r}d\underset{\sim}{r}' \quad . \qquad (10)$$

Considerably different asymptotic expressions for $\delta\sigma_2$ and $\delta\sigma_3$ have been stated by Queen [5]. They are based on the assumption that only elastic (i.e., unexcited) intermediate states occur in the scattering processes, an assumption which is rather doubtful for high-energy collisions, and they are not equivalent to eqs.(7) and (8). If quadruple interactions are small at high energies, and our calculations will show that they are, then eqs. (2,3, and 7-9), together with a knowledge of σ_{dd}, σ_{pp}, σ_{np}, and the deuteron wave function can yield an order of magnitude estimate for σ_{nn}.

A second set of assumptions is that in which $f_{pp} = f_{nn}$ and $f_{np} = f_{pn}$, and in which a commonly used form for the N-N' scattering amplitudes is assumed, namely

$$f_{jp}(\underset{\sim}{q},k/2) = (i+\alpha_j)(k\sigma_{jp}/8\pi)e^{-\gamma q^2/2} \quad , \qquad j = n,p.$$

If this expression is used in eqs. (4-6) the results may be written as

$$\delta\sigma_2 = (1/4\pi) \int_0^\infty \{[1-\alpha_p^2)\sigma_{pp}^2 + (1-\alpha_n^2)\sigma_{np}^2]\ S^2(q)$$

$$+ 4(1-\alpha_p\alpha_n)\sigma_{pp}\sigma_{np}\ S(q)\}\ e^{-\gamma q^2}\ q\ dq \quad , \tag{11}$$

$$\delta\sigma_3 = -[(1-2\alpha_p\alpha_n-\alpha_n^2)\sigma_{np} + (1-2\alpha_p\alpha_n-\alpha_p^2)\sigma_{pp}]\ (\sigma_{np}\sigma_{pp}/8\pi^2)$$

$$\times \int_0^\infty S(q)e^{-\gamma q^2}\ q\ dq \int_0^\infty S(q')e^{-\gamma q'^2}\ I_0(\gamma q q')q'dq', \tag{12}$$

$$\delta\sigma_4 = (1-4\alpha_p\alpha_n-\alpha_p^2-\alpha_n^2+\alpha_p^2\alpha_n^2)(\sigma_{np}^2\sigma_{pp}^2/256\pi\gamma)\ [\int_0^\infty S(q)e^{-\gamma q^2/2}\ q\ dq]^2 \quad , \tag{13}$$

where $I_0(y) = J_0(iy)$ is the Bessel function of imaginary argument.

No complete set of measurements for σ_{np}, σ_{pp}, α_n, α_p, γ, and σ_{dd} are yet available at a given nucleon momentum. In a calculation, therefore, it may be necessary to use as input data measurements made at somewhat different momenta. We have calculated $\delta\sigma_2$, $\delta\sigma_3$, $\delta\sigma_4$, and σ_{dd} for deuterons with a momentum of approximately 2.8 GeV/c by means of eqs. (11-13). As input data we have used $\sigma_{pp} = 46.487$ mb and $\sigma_{np} = 35.8$ mb which were taken from measurements at 1.408 GeV/c [6] and 1.38 GeV/c [7], respectively. We have obtained a value of $\gamma = 0.189\ F^2$ by making a least-squares fit of the elastic p-p scattering data of McManigal et al. [8] at 1.386 GeV/c. From theoretical analyses [9] we have $\alpha_p = 0.21$ and $\alpha_n = -0.31$ at 1.408 GeV/c. The form factor $S(q)$ was obtained analytically from a representation of the deuteron ground state wave function given by Moravcsik [10]. The results of our calculations are

$\delta\sigma_2 = 21.0$ mb, $\delta\sigma_3 = -1.8$ mb, $\delta\sigma_4 = 0.2$ mb, $\delta\sigma = 19.5$ mb, and $\sigma_{dd} = 145$ mb. This may be compared with the measured d-d total cross section at 3.0 GeV/c for which an estimate of 122±7 mb has been given [11]. Despite the fact that the five nucleon-nucleon input data were obtained from five different sources, including three independent measurements, it is difficult to see what would reduce the calculated value of σ_{dd} by more than a few millibarns and it is suggested that perhaps another cross section measurement might be made.

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