Title
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Here we first encounter quantities that do not follow from the properties of space-time; they are "extraspatial." Also, they are not absolutely conserved, and thus do not lead to absolute selection rules.

It was noticed some time ago that mirror nuclei, that is, two nuclei such that the number of $\begin{Bmatrix}\text{protons}\\ \text{neutrons}\end{Bmatrix}$ in one equals the number of $\begin{Bmatrix}\text{neutrons}\\ \text{protons}\end{Bmatrix}$ in the other, had very nearly the same energy levels, considering (a) the difference in Coulomb energy between the two nuclei, and (b) the $n \leftrightarrow p$ mass difference (which again is presumably a manifestation of electric forces). That is, with the neglect of electricity the Hamiltonian for interacting nucleons is invariant with respect to the interchange of protons and neutrons: this statement is called the hypothesis of charge symmetry. [Under this interchange of protons and neutron, positive and negative nuclear force mesons must also be interchanged, so that a process $p \longrightarrow n\pi^+$ is transformed to $n \longrightarrow p\pi^-$.] This transformation, called variously the charge-symmetry transformation, the mirror transformation, or transposition (T), is thus a constant. The eigenvalue of T is called charge symmetry parity: $T = \pm 1$. Of course, only systems transforming into themselves under T can be eigenstates of T: thus a nucleus has a T-parity only if it contains equal numbers of protons and neutrons.

Since the neutron and proton are equivalent (with the neglect of electricity) it is natural to regard them as different states of the same particle, the nucleon. The nucleon thus carries a two-valued quantum number, analogous to the z component of a $j = \frac{1}{2}$ spin: the nucleon is said to carry an isotopic spin (i-spin) $i = \frac{1}{2}$, the state $i_3 = \tau_3/2 = \left\{ {+ \atop -} \right\} \frac{1}{2}$ being the $\left\{ {\text{proton} \atop \text{neutron}} \right\}$. Two nucleons have four i-spin states, just as they have four spin states, which can be grouped into $i = 1$, $i_3 = 1, 0, -1$; $i = 0$, $i_3 = 0$. Because of the requirement of over-all antisymmetry, the spin-space state of the two nucleons determines their i-spin states, e.g., an odd-space even-spin state implies an even i-spin state, i.e., $i = 1$. In short, $(-)^{\ell + s + i} = -1$.

It is now well established that in a given spin-space state not only do two protons or two neutrons have equal nuclear forces, but so also do a neutron and a proton. That is, the nuclear energy of the nucleons is independent of the total i_3; this statement, generalized to any nuclear system, is called the hypothesis of charge independence. Since if the Hamiltonian does not depend on i_3 (nor on the other components i_1, i_2) $\vec{I}$ commutes with H, an equivalent way of stating charge independence is to say that $\vec{I}$ is conserved. This last implies that the Hamiltonian is invariant to notations in "isotopic spin space," the three-dimensional space in which i spin is an angular momentum. We can now see that T is a special rotation in i-spin space, namely 180° around an axis in the 1-2 plane, so that the 3 axis is turned around. Clearly, a state transforms into itself only if $i_3 = 0$; its charge symmetry parity is then $T = (-)^i$.

The charge independence of nuclear forces implies that, corresponding to charged nuclear-force mesons, there must be a neutral meson of the same mass, which is coupled oppositely to protons and neutrons (i.e. thru τ_3) with a coupling constant $1/\sqrt{2}$ times the charged-meson-nucleon coupling constant. Such a meson field is said to be coupled "charge symmetrically," and has an i spin $i = 1$; the states $i_3 = \{1, 0, -1\}$ are the $\{$positive, neutral, and negative$\}$ mesons. The observed π^0 <u>is</u> the neutral nuclear-force meson; it has very nearly the same mass as the charged π's, the difference being of a reasonable sign and size for an electric effect. Since for the π^0 $i = 1$ and $i_3 = 0$, it has $T_{\pi^0} = -1$.

Charge independence would of course still hold if an additional neutral meson having $i = 0$ and $T = +1$ were coupled equally to neutron and proton; such a meson has never been seen. Even if it had the same mass and decayed into two γ's it could not pass as a π^0; the meson production reactions $P\begin{pmatrix} p \\ n \end{pmatrix} \longrightarrow D \begin{pmatrix} \pi^+ \\ "\pi^0" \end{pmatrix}$ are observed to obey charge independence, whereas the second process would be enhanced if such an additional neutral meson were produced.

The π meson carries no extraspatial quantum number besides i spin; therefore the charge conjugate of the π^+ is a negative π meson with $i_3 = -1$, i.e., the π^-; and conversely, of course. The operator T also interchanges π^+ and π^-; therefore the product of charge conjugation and charge symmetry, CT, brings the $\pi^\pm$ back to itself. Thus the charged π mesons are eigenstates of "CT parity," just as the π^0 is: $CT_{\pi^0} = (C_{\pi^0})(T_{\pi^0}) = (+1)(-1) = -1$. The <u>value</u> of $CT_{\pi^\pm}$ depends on the particular axis about which the T rotation is made. If we use C as defined above in Section V, then making the T rotation around the 2 axis yields the convenient result $CT_{\pi^\pm} = CT_{\pi^0} = -1$. (This is a convention of precisely the same sort as calling the π^+ pseudoscalar.) Thus we have the selection rule: a state with $\begin{Bmatrix} \text{even} \\ \text{odd} \end{Bmatrix}$ CT parity can decay only into an $\begin{Bmatrix} \text{even} \\ \text{odd} \end{Bmatrix}$ number of π's. In general, a meson field with i-spin i, whose neutral component has charge parity C, has $CT = (-)^i C$.

The above selection rules, based on conservation of I^2, T, and CT, are valid only if electromagnetic interaction is neglected. The charge of i-spin-carrying particles is proportional to I_3: for a nucleon $Q = e(I_3 + \frac{1}{2})$, for a π meson $Q = eI_3$. Thus the term of the Hamiltonian that couples a particle to the electromagnetic field commutes with I_3, but not with I^2*, T, or CT. (See Example 1, below.) Thus I_3 is conserved,

* I^2 commutes with H in the exceptional case that only the <u>total</u> I_3 appears in H, e.g., if there is only one i-spin-carrying particle present.

but not I^2, T, or CT, in the presence of electromagnetic interaction; an immediate example in $\pi^0 \rightarrow \gamma\gamma$. In general, if a decay is forbidden only by i spin, then the decay can occur if photons are added. These photons need not appear in the final state; they can be reabsorbed and thus be purely virtual. But the reabsorption of a photon means another factor e in the matrix element, and $e^2 = 1/137$ in the rate, so that the nonradiative decay will be slower than the radiative (unless the kinetic energy in the final state is very small, so that the phase space of the final state is suppressed by the additional photons). Example 2 shows concretely how emission of real or virtual photons "breaks" i-spin selection rules.

The concept of i spin can be abstracted from nucleons and nuclear force mesons; in fact, it is believed that i spin is conserved in all strong interactions, i.e., interactions stronger than electromagnetic. Conversely, it seems to be a general principle that any interaction that conserves i spin is strong. For instance, since $N \rightarrow Pe^- \nu$ and $\pi^+ \rightarrow \mu^+ \nu$ are very weak interactions, one says that they do not conserve i spin, perhaps simply because leptons may not carry i spin.

Example 1:

A nucleus in an $i = 0$ state cannot make an electric-dipole transition to another $i = 0$ state. The matrix element for the current would be $\left\langle f \left| \sum_j e(\frac{1}{2} + I_3^{(j)}) \frac{p_{(j)}}{m} \right| i \right\rangle$ where the sum is over the nucleons in the nucleus; we shall neglect the neutron-proton mass difference, and thus there is no subscript on m. Now the first term is proportional to

$\left\langle f \left| \sum_j p_{(j)} \right| i \right\rangle = P \langle f | i \rangle = P\delta_{if}$, if the states are taken to be eigenstates of the total momentum. Thus only the second term remains:

$$\frac{e}{m} \left\langle f \left| \sum_j I_3^{(j)} \, p_{(j)} \right| i \right\rangle ;$$

since

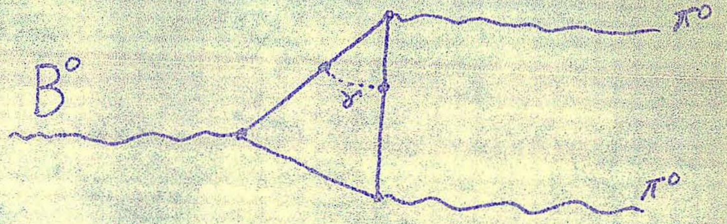
$$\left\langle i = 0 \left| I_3^{(j)} \right| i = 0 \right\rangle = 0 ,$$

this is zero if initial and final states are both $i = 0$.

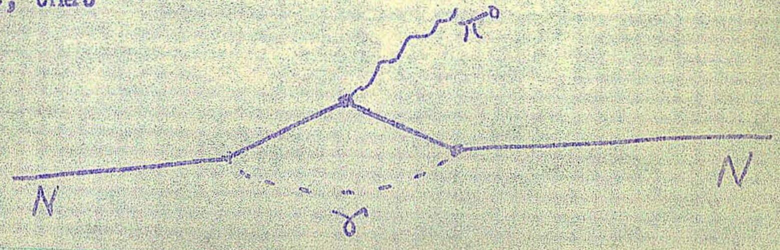
Example 2:

Consider a $\{0+\}$ heavy meson, having (like the pion) $i = 1$ ($Q = i_3$) and $CT = -1$. Then CT parity does not allow it to decay into two pions. But decay into two pions plus two photons is possible, as is decay into two pions with the aid of a virtual photon. To see explicitly how a virtual photon "breaks" the i-spin selection rule, consider the lowest-order Feynman diagram for the decay of the neutral $\{0+\}$ meson into π^0's through a nucleon loop:

Since the meson B^0 has $i = 1$, it is coupled to the nucleon with τ_3, as are the π^0's. Since there is an odd number of τ_3's on the loop, the matrix elements arising from a virtual neutron have opposite sign to those from a virtual proton; thus the sum over the two cases vanishes. But the following diagram, in which a virtual photon is inserted,



exists only if the virtual nucleon is a proton, thus there is no cancellation in this case. This, however, is not quite the end of the story, for this same fact, that



exists only for the proton, and not for the neutron, means that the π^0 is coupled with different strength to proton and neutron, contrary to the hypothesis of charge symmetry. To regain charge symmetry, one renormalizes the proton-π^0 coupling to make it equal to the neutron-π^0 coupling; but this equality would be true only for a particular situation, and would not be true in others. For instance, if the small-angle NN scattering is really charge-symmetric, the B^0 decay is not; that is, the matrix elements for proton and neutron loops are not exactly equal but opposite, and so the $B^0 \rightarrow \pi^0 \pi^0$ matrix element is not exactly zero.