Exponential Stabilization of Inertial Memristive Neural Networks With Multiple Time Delays

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***Abstract*—This article investigates the global exponential stabilization (GES) of inertial memristive neural networks with discrete and distributed time-varying delays (DIMNNs). By introducing the inertial term into memristive neural networks (MNNs), DIMNNs are formulated as the second-order differential equations with discontinuous right-hand sides. Via a variable transformation, the initial DIMNNs are rewritten as the first-order differential equations. By exploiting the theories of differential inclusion, inequality techniques, and the comparison strategy, the pth moment GES ($p \geq 1$) of the addressed DIMNNs is presented in terms of algebraic inequalities within the sense of Filippov, which enriches and extends some published results. In addition, the global exponential stability of MNNs is also performed in the form of an M-matrix, which contains some existing ones as special cases. Finally, two simulations are carried out to validate the correctness of the theories, and an application is developed in pseudorandom number generation.**

***Index Terms*—Comparison approach, exponential stability, exponential stabilization, inertial memristive neural networks (IMNNs), time delays.**

I. Introduction

MEMRISTOR, which was conceived by Chua in 1971 [1], has experienced an explosive increase during recent decades. The significant advantages of the memristor compared to conventional circuit elements are the nonvolatility and parallel computation, which makes it a potential technology to revolutionize dynamical storage and information processing [2]. In recent years, the memristor has been employed in associative memory, neuromorphic computing, image processing, and sparse coding [3], [4]. On the other hand, the synapse of neuron in the classical neural-network circuit is modeled by the resistor. It is worth mentioning that the synaptic weight in the human brain is variable depending upon the stimulus and inhibition effects between neurons. Fortunately, the memristor can be adopted to perfectly simulate a synapse because of the nonvolatility of the memristor and the adjustability of memristance [5]–[7].

With the development of the memristive technology [8], theoretical and practical investigations of memristive neural networks (MNNs) have received more attention over these years. In [9], by replacing the resistors with memristors in the conventional Hopfield neural network circuit, a class of MNNs was built, due to the characteristics of the state-based switched memristive connection weights, under the framework of the Filippov differential inclusions, the exponential synchronization of MNNs was analyzed therein via matrix inequalities. Attractivity of cellular MNNs was discussed in [10] by adopting the analysis approach and state-space decomposition. Via transforming MNNs into fuzzy neural networks, exponential lag synchronization of MNNs was researched in [11] by employing the adaptive strategies. The finite-time stabilization of MNNs was explored in [12] by resorting to theories of M-matrix and a nonlinear discontinuous controller. In [13], the finite-time stabilization of complex-valued MNNs was researched by separating the initial model into real and imaginary parts. Multistability of Cohen–Grossberg MNNs with nonmonotonic piecewise linear activation functions was of concern in [14] by utilizing the nonsmooth analysis and fixed-point theorem. Furthermore, the results illustrated that the storage capacity of MNNs was extensively improved compared with general neural networks. Passivity and passification of MNNs with additive time-varying delays were analyzed in [15] by using the characteristic function approach and linear matrix inequalities (LMIs). Asymptotic and finite-time synchronization of MNNs were discussed in [16] via the robust analysis method. In [17], by means of the event-triggered sampling control strategy, the LMI-based criteria were obtained to ensure the stability and stabilization of MNNs with communication delays. The quasisynchronization of MNNs was studied in [18] through using the region-partitioning-dependent intermittent control. The stability of fractional-order MNNs was

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discussed in [19] by resorting to the Razumikhin method. By designing MNNs in charge-flux domain, asymptotic stability of the addressed MNNs was performed in [20], which provided a novel perspective to investigate MMNs. The finite-time stabilization of MNNs was also explored in [21] via the comparison method.

Common RNNs are differential equations with first-order derivatives of neuronic states. (Please refer to [22]–[34].) In reality, it is of significant importance to introduce the inertial term into MNNs from the points of theory and application. As indicated in [35], the incorporation of the inertial term is a critical instrument to generate complicated bifurcation behavior and chaos. By incorporating the inertial term into MNNs, inertial MNNs (IMNNs) are formulated as differential equations with second-order derivatives of neuronic states. In [36], exponential stability and synchronization of IMNNs were analyzed by virtue of the matrix measure method. Finite-time synchronization of IMNNs was researched in [37] by adopting a discontinuous controller and inequality techniques. Global dissipativity of IMNNs was discussed in [38] by employing the Lyapunov method and analysis techniques. Exponential stability of IMNNs with impulses was investigated in [39] via the matrix inequality and generalized delayed impulsive differential inequality. The exponential stability of IMNNs was studied in [40] by exploiting the Lyapunov approach in terms of 1-norm. Based upon [40], global exponential stabilization (GES) of IMNNs was analyzed in [41] under a state-feedback control law. By designing a delay-dependent controller, algebraic results were performed in [42] to ascertain the finite-time synchronization of IMNNs. The exponential synchronization of IMNNs was also discussed in [43] via constructing a nonlinear controller, including the sign term and diffusive term. By adopting adaptive control and the nonreduced-order method, asymptotic stabilization of IMNNs was of concern in [44]. Synchronization of IMNNs with linear coupling was studied in [45] by using the matrix measure approach and LMIs. Passivity of IMNNs and fuzzy IMNNs on time scales was researched in [46] and [47], respectively, by employing the LMIs.

It should be emphasized that the fundamental results in [36]–[47] provide general theoretical approaches to analyze the dynamical properties of IMNNs. Yet, there are few existing outcomes about the dynamical behaviors of IMNNs in terms of the pth moment ($p \geq 1$). On the other hand, time-varying delays frequently occur in neural networks since the finite rate of signal transmission [48], [49]. The problem on the stabilization of IMNNs with time delays was considered in [41] and [44] by exploiting the Lyapunov method in the form of 1-norm, which generalized some existing results. To perform dynamical analysis, time-varying delays are assumed to be bounded and differentiable, and the upper bounds of derivatives are required to be smaller than 1, besides, time-delay terms are tackled by constructing the integral-type Lyapunov functionals, which implies that if time delays are not differentiable, or the derivatives are bigger than 1, then the outcomes in [41] and [44] are invalid to investigate the corresponding IMNNs. How to consider the pth moment GES ($p \geq 1$) of IMNNs with discrete and distributed time-varying delays (DIMNNs) without assuming differentiability of multiple time delays is challenging.

Based upon the arguments, in this article, we attempt to research the pth moment GES ($p \geq 1$) of DIMNNs. Via the variable transformation method, the addressed DIMNNs are transferred as the first-order differential equations. Then, by employing the comparison strategy and inequality techniques, algebraic conditions are performed to ascertain the pth moment GES ($p \geq 1$) of the underlying DIMNNs in Filippov's sense, which enrich and extend the results in [41] and [44]. Moreover, mixed time delays are assumed to be continuous and bounded, and they are analyzed through the comparison method without assuming differentiability and constructing integral-type Lyapunov functionals. As a byproduct of the main result, global exponential stability of MNNs is carried out, which includes the ones in ([50, Theor. 2], [51, Corollary 2], and [52, Corollary 1]), if as special cases.

The remainder of this article is organized as follows. The preliminaries are given in Section II. The theoretical outcomes are performed in Section III. The simulations are carried out in Section IV. The conclusions are made in Section V.

Notations: In this article, $\mathbf{R}$ represents the set of real values. $\min\{c_1, c_2, \ldots, c_n\}$ and $\max\{c_1, c_2, \ldots, c_n\}$, respectively, denote the minimum and maximum values of $c_1, c_2, \ldots, c_n$. $n^\diamond$ is the set $\{1, 2, \ldots, n\}$. $D = [d_{ij}]_{n\times n}$ stands for a square matrix with elements d_{ij}, $i, j \in n^\diamond$, especially, if a diagonal matrix E is denoted by $\mathrm{diag}\{e_1, e_2, \ldots, e_n\}$.

II. Model and Problem Description

Consider DIMNNs as

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} = &-a_i\frac{dx_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{n} c_{ij}(x_i(t))f_j\big(x_j(t)\big)\\ &+\sum_{j=1}^{n} d_{ij}(x_i(t))g_j\big(x_j\big(t-\tau_j(t)\big)\big)\\ &+\sum_{j=1}^{n} e_{ij}(x_i(t))\int_{t-\delta_j(t)}^{t} h_j\big(x_j(s)\big)ds + \digamma_i(t) \quad (1)\end{aligned}$$

in which $i \in n^\diamond$, $x_i(t)$ is the activity of the ith neuron; the second-order derivative of $x_i(t)$ is the inertial term; $a_i > 0$ is a constant; $b_i > 0$ denotes the self-feedback coefficient; $\tau_j(t)$ and $\delta_j(t), j \in n^\diamond$ are, respectively, discrete and distributed time-varying delays; $f_j(\cdot)$, $g_j(\cdot)$, and $h_j(\cdot)$ represent the activation functions; $\digamma_i(t)$ is the controller to be designed; and $c_{ij}(x_i(t))$, $d_{ij}(x_i(t))$, and $e_{ij}(x_i(t))$ are the state-based switched memristive connection weights with

$$\begin{aligned}c_{ij}(x_i(t)) &= \frac{\mathbb{W}_{f,ij}}{\mathbb{C}_i} \times \mathrm{sgn}_{ij}\\ d_{ij}(x_i(t)) &= \frac{\mathbb{W}_{g,ij}}{\mathbb{C}_i} \times \mathrm{sgn}_{ij}\\ e_{ij}(x_i(t)) &= \frac{\mathbb{W}_{h,ij}}{\mathbb{C}_i} \times \mathrm{sgn}_{ij} \quad (2)\end{aligned}$$

where $\mathrm{sgn}_{ij} = -1$ if $i = j$; otherwise, $\mathrm{sgn}_{ij} = 1$, and $\mathbb{W}_{f,ij}$, $\mathbb{W}_{g,ij}$, and $\mathbb{W}_{h,ij}$ stand for the memductances of memristors

$\mathbb{M}_{f,ij}$, $\mathbb{M}_{g,ij}$, and $\mathbb{M}_{h,ij}$, respectively. According to the features of memristors

$$c_{ij}(x_i(t)) = \begin{cases} c_{ij}^{\Delta}, & |x_i(t)| \leq \Theta_i \\ c_{ij}^{\nabla}, & |x_i(t)| > \Theta_i \end{cases}$$
$$d_{ij}(x_i(t)) = \begin{cases} d_{ij}^{\Delta}, & |x_i(t)| \leq \Theta_i \\ d_{ij}^{\nabla}, & |x_i(t)| > \Theta_i \end{cases}$$
$$e_{ij}(x_i(t)) = \begin{cases} e_{ij}^{\Delta}, & |x_i(t)| \leq \Theta_i \\ e_{ij}^{\nabla}, & |x_i(t)| > \Theta_i \end{cases} \tag{3}$$

where the switching jump $\Theta_i > 0$, and $c_{ij}^{\Delta}, c_{ij}^{\nabla}, d_{ij}^{\Delta}, d_{ij}^{\nabla}, e_{ij}^{\Delta}$, and e_{ij}^{∇}, $i, j \in n^{\diamond}$ are real values.

To investigate the GES of DIMNNs (1), two assumptions are given.

Assumption 1: In DIMNNs (1), mixed time-varying delays $\tau_j(t)$ and $\delta_j(t)$ are continuous, and there are $\tau > 0$ and $\delta > 0$ such that $0 \leq \tau_j(t) \leq \tau$ and $0 \leq \delta_j(t) \leq \delta$, $j \in n^{\diamond}$, respectively. Denote $\delta_0 = \max\{\tau, \delta\}$.

Assumption 2: In DIMNNs (1), activation functions $f_i(\cdot)$, $g_i(\cdot)$, and $h_i(\cdot)$, $i \in n^{\diamond}$ are Lipschitz continuous, and there are $F_i > 0$, $G_i > 0$, and $H_i > 0$ such that for $z_1, z_2, \ldots, z_6 \in \mathbf{R}$

$$|f_i(z_1) - f_i(z_2)| \leq F_i|z_1 - z_2| \tag{4}$$
$$|g_i(z_3) - g_i(z_4)| \leq G_i|z_3 - z_4| \tag{5}$$
$$|h_i(z_5) - h_i(z_6)| \leq H_i|z_5 - z_6| \tag{6}$$

respectively. In addition, define $f_i(0) = g_i(0) = h_i(0) = 0$, $i \in n^{\diamond}$.

Remark 1: In DIMNNs (1), the inertial term, memristor, and multiple time delays are all considered, which formulates the second-order differential equations with discontinuous right-hand sides. DIMNNs (1) contain the models in [36], [51], and [53]–[55] as special cases.

Remark 2: Distributed time delay is not considered in [36]–[39], [42], [43], [45], and [47]; hence, the results therein are invalid to investigate the dynamical properties of DIMNNs (1). Meanwhile, mixed time delays in [40] and [41] are required to be bounded and differentiable. Besides, the upper bounds of derivatives are restricted to be less than 1. This article attempts to relax the restriction on differentiablity of multiple time delays.

The initial values of DIMNNs (1) are

$$x_i(s) = \theta_i(s), \quad \frac{dx_i(s)}{ds} = \vartheta_i(s), \quad -\delta_0 \leq s \leq 0 \tag{7}$$

where $i \in n^{\diamond}$, $\theta_i(s)$, and $\vartheta_i(s)$ are continuous and bounded.

By virtue of theories of differential inclusions [56], from DIMNNs (1)

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} \in& -a_i\frac{dx_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{n}\overline{\text{co}}\big[c_{ij}(x_i(t))\big]f_j\big(x_j(t)\big) \\ &+ \sum_{j=1}^{n}\overline{\text{co}}\big[d_{ij}(x_i(t))\big]g_j\big(x_j\big(t-\tau_j(t)\big)\big) \\ &+ \sum_{j=1}^{n}\overline{\text{co}}\big[e_{ij}(x_i(t))\big]\int_{t-\delta_j(t)}^{t}h_j\big(x_j(s)\big)ds + \digamma_i(t)\end{aligned} \tag{8}$$

where

$$\overline{\text{co}}\big[c_{ij}(x_i(t))\big] = \begin{cases} c_{ij}^{\Delta}, & |x_i(t)| < \Theta_i \\ [c_{ij}^{\flat}, c_{ij}^{\natural}], & |x_i(t)| = \Theta_i \\ c_{ij}^{\nabla}, & |x_i(t)| > \Theta_i \end{cases}$$
$$\overline{\text{co}}\big[d_{ij}(x_i(t))\big] = \begin{cases} d_{ij}^{\Delta}, & |x_i(t)| < \Theta_i \\ [d_{ij}^{\flat}, d_{ij}^{\natural}], & |x_i(t)| = \Theta_i \\ d_{ij}^{\nabla}, & |x_i(t)| > \Theta_i \end{cases}$$
$$\overline{\text{co}}\big[e_{ij}(x_i(t))\big] = \begin{cases} e_{ij}^{\Delta}, & |x_i(t)| < \Theta_i \\ [e_{ij}^{\flat}, e_{ij}^{\natural}], & |x_i(t)| = \Theta_i \\ e_{ij}^{\nabla}, & |x_i(t)| > \Theta_i \end{cases}$$

in which $c_{ij}^{\flat} = \min\{c_{ij}^{\Delta}, c_{ij}^{\nabla}\}$, $c_{ij}^{\natural} = \max\{c_{ij}^{\Delta}, c_{ij}^{\nabla}\}$, $d_{ij}^{\flat} = \min\{d_{ij}^{\Delta}, d_{ij}^{\nabla}\}$, $d_{ij}^{\natural} = \max\{d_{ij}^{\Delta}, d_{ij}^{\nabla}\}$, $e_{ij}^{\flat} = \min\{e_{ij}^{\Delta}, e_{ij}^{\nabla}\}$, and $e_{ij}^{\natural} = \max\{e_{ij}^{\Delta}, e_{ij}^{\nabla}\}$, $i, j \in n^{\diamond}$. Equivalently, there are $c_{ij}^{\diamond}(t) \in \overline{\text{co}}[c_{ij}(x_i(t))]$, $d_{ij}^{\diamond}(t) \in \overline{\text{co}}[d_{ij}(x_i(t))]$, and $e_{ij}^{\diamond}(t) \in \overline{\text{co}}[e_{ij}(x_i(t))]$, $i, j \in n^{\diamond}$ such that

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} =& -a_i\frac{dx_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{n}c_{ij}^{\diamond}(t)f_j\big(x_j(t)\big) \\ &+ \sum_{j=1}^{n}d_{ij}^{\diamond}(t)g_j\big(x_j\big(t-\tau_j(t)\big)\big) \\ &+ \sum_{j=1}^{n}e_{ij}^{\diamond}(t)\int_{t-\delta_j(t)}^{t}h_j\big(x_j(s)\big)ds + \digamma_i(t).\end{aligned} \tag{9}$$

Define

$$y_i(t) = \frac{dx_i(t)}{dt} + r_ix_i(t), \quad i \in n^{\diamond} \tag{10}$$

from DIMNNs (9)

$$\begin{cases} \frac{dx_i(t)}{dt} = -r_ix_i(t) + y_i(t) \\ \frac{dy_i(t)}{dt} = (r_i - a_i)y_i(t) + \big(r_ia_i - r_i^2 - b_i\big)x_i(t) \\ \qquad + \sum_{j=1}^{n}c_{ij}^{\diamond}(t)f_j\big(x_j(t)\big) \\ \qquad + \sum_{j=1}^{n}d_{ij}^{\diamond}(t)g_j\big(x_j\big(t-\tau_j(t)\big)\big) \\ \qquad + \sum_{j=1}^{n}e_{ij}^{\diamond}(t)\int_{t-\delta_j(t)}^{t}h_j\big(x_j(s)\big)ds + \digamma_i(t) \end{cases} \tag{11}$$

with initial values $x_i(s) = \theta_i(s)$, $y_i(s) = r_i\theta_i(s) + \vartheta_i(s) \triangleq \bar{\vartheta}_i(s)$, $-\delta_0 \leq s \leq 0$.

Remark 3: By employing the variable transformation (10), DIMNNs (9) can be rewritten as the first-order differential equations (11). The method is also adopted in [36], [43], and [45] with $r_i = 1$ to reduce the order of IMNNs. It is worth mentioning that with introducing the free weight r_i in (10), the obtained criteria can be flexibly adjusted.

To exponentially stabilize DIMNNs (11), the following state-feedback controller is designed:

$$\digamma_i(t) = -q_iy_i(t) \tag{12}$$

where $i \in n^\diamond$, $q_i > 0$ is the controller gain. Via embedding the controller (12) into DIMNNs (11), one obtains

$$\begin{cases} \frac{dx_i(t)}{dt} = -r_i x_i(t) + y_i(t) \\ \frac{dy_i(t)}{dt} = (r_i - a_i - q_i)y_i(t) + \big(r_i a_i - r_i^2 - b_i\big)x_i(t) \\ \qquad + \sum_{j=1}^{n} c_{ij}^{\diamond}(t) f_j\big(x_j(t)\big) \\ \qquad + \sum_{j=1}^{n} d_{ij}^{\diamond}(t) g_j\big(x_j\big(t-\tau_j(t)\big)\big) \\ \qquad + \sum_{j=1}^{n} e_{ij}^{\diamond}(t)\int_{t-\delta_j(t)}^{t} h_j\big(x_j(s)\big)ds. \end{cases} \tag{13}$$

The following work is to design suitable controller gains q_i, $i \in n^\diamond$ such that the controlled DIMNNs (13) are globally exponentially stable.

III. Main Results

In this part, the GES of DIMNNs (13) is investigated by employing the inequality techniques and the comparison strategy under the control law (12). In addition, two corollaries are given.

Theorem 1: Let $p \geq 2$, τ, δ, δ_0, F_i, G_i, H_i, and q_i be constants, under Assumptions 1 and 2, DIMNNs (13) can be exponentially stabilized under the control law (12), if there are positive constants $\rho_{k,ij}$, $\rho_{k,ij}^*$, $\sigma_{k,ij}$, $\sigma_{k,ij}^*$, $w_{k,ij}$, $w_{k,ij}^*$, $k \in p^\diamond$, $i,j \in n^\diamond$ with $\sum_{k=1}^{p}\rho_{k,ij} = \sum_{k=1}^{p}\rho_{k,ij}^* = \sum_{k=1}^{p}\sigma_{k,ij} = \sum_{k=1}^{p}\sigma_{k,ij}^* = \sum_{k=1}^{p} w_{k,ij} = \sum_{k=1}^{p} w_{k,ij}^* = 1$, and m_ℓ, $\ell \in (2n)^\diamond$ such that

$$r_i a_i - r_i^2 - b_i > 0 \tag{14}$$

$$(pr_i + 1 - p)m_i - m_{n+i} > 0 \tag{15}$$

$$\begin{aligned} &\Bigg[p(r_i - a_i - q_i) + (p-1)\Big(r_i a_i - r_i^2 - b_i\Big) \\ &\quad + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{c}_{ij}^{p\rho_{k,ij}} F_j^{p\rho_{k,ij}^*} + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{d}_{ij}^{p\sigma_{k,ij}} G_j^{p\sigma_{k,ij}^*} \\ &\quad + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{e}_{ij}^{pw_{k,ij}} H_j^{pw_{k,ij}^*}\delta\Bigg] m_{n+i} \\ &+ \Big(r_i a_i - r_i^2 - b_i\Big)m_i + \sum_{j=1}^{n}\hat{c}_{ij}^{p\rho_{p,ij}} F_j^{p\rho_{p,ij}^*} m_j \\ &+ \sum_{j=1}^{n}\hat{d}_{ij}^{p\sigma_{p,ij}} G_j^{p\sigma_{p,ij}^*} m_j + \sum_{j=1}^{n}\hat{e}_{ij}^{pw_{p,ij}} H_j^{pw_{p,ij}^*}\delta m_j < 0 \end{aligned} \tag{16}$$

in which $\hat{c}_{ij} = \max\{|c_{ij}^{\Delta}|, |c_{ij}^{\nabla}|\}$, $\hat{d}_{ij} = \max\{|d_{ij}^{\Delta}|, |d_{ij}^{\nabla}|\}$, and $\hat{e}_{ij} = \max\{|e_{ij}^{\Delta}|, |e_{ij}^{\nabla}|\}$.

Proof: Construct the following continuous functions:

$$\beth_1(\varepsilon_i) = (\varepsilon_i + p - 1 - pr_i)m_i + m_{n+i} \tag{17}$$

$$\begin{aligned} \beth_2(\epsilon_i) = &\Bigg[\epsilon_i + p(r_i - a_i - q_i) + (p-1)\Big(r_i a_i - r_i^2 - b_i\Big) \\ &\quad + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{c}_{ij}^{p\rho_{k,ij}} F_j^{p\rho_{k,ij}^*} + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{d}_{ij}^{p\sigma_{k,ij}} G_j^{p\sigma_{k,ij}^*} \\ &\quad + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{e}_{ij}^{pw_{k,ij}} H_j^{pw_{k,ij}^*}\delta\Bigg] m_{n+i} \\ &+ \Big(r_i a_i - r_i^2 - b_i\Big)m_i + \sum_{j=1}^{n}\hat{c}_{ij}^{p\rho_{p,ij}} F_j^{p\rho_{p,ij}^*} m_j \\ &+ \sum_{j=1}^{n}\hat{d}_{ij}^{p\sigma_{p,ij}} G_j^{p\sigma_{p,ij}^*} m_j e^{\epsilon_i\tau} \\ &+ \sum_{j=1}^{n}\hat{e}_{ij}^{pw_{p,ij}} H_j^{pw_{p,ij}^*}\delta m_j e^{\epsilon_i\delta} \end{aligned} \tag{18}$$

where $i \in n^\diamond$, $\varepsilon_i \geq 0$, and $\epsilon_i \geq 0$. First, considering (17), it follows from (15) that $\beth_1(0) < 0$ holds, besides, $\beth_1(\varepsilon_i)$ is monotonically increasing on $[0, +\infty)$, and $\lim_{\varepsilon_i \to +\infty}\beth_1(\varepsilon_i) = +\infty$; therefore, there is $\varepsilon_i^{\dagger} > 0$ such that $\beth_1(\varepsilon_i^{\dagger}) = 0$, $i \in n^\diamond$. Similarly, for (18), based upon (16), there also is $\epsilon_i^{\dagger} > 0$ such that $\beth_2(\epsilon_i^{\dagger}) = 0$, $i \in n^\diamond$. Select $0 < \lambda < \min_{i \in n^\diamond}\{\varepsilon_i^{\dagger}, \epsilon_i^{\dagger}\}$, one obtains $\beth_1(\lambda) < 0$ and $\beth_2(\lambda) < 0$.

Define

$$\mathbb{V}_i(t) = e^{\lambda t}|x_i(t)|^p, \quad \mathbb{W}_i(t) = e^{\lambda t}|y_i(t)|^p, \quad i \in n^\diamond \tag{19}$$

where $p \geq 2$ is an integer. From DIMNNs (13), for $i \in n^\diamond$

$$\begin{aligned} \dot{\mathbb{V}}_i(t) &= \lambda e^{\lambda t}|x_i(t)|^p + e^{\lambda t}p|x_i(t)|^{p-2}x_i(t)\dot{x}_i(t) \\ &= \lambda e^{\lambda t}|x_i(t)|^p + e^{\lambda t}p|x_i(t)|^{p-2}x_i(t)\big[-r_i x_i(t) + y_i(t)\big] \\ &\leq (\lambda + p - 1 - pr_i)e^{\lambda t}|x_i(t)|^p + e^{\lambda t}|y_i(t)|^p. \end{aligned} \tag{20}$$

Meanwhile, one obtains

$$\begin{aligned} \dot{\mathbb{W}}_i(t) &= \lambda e^{\lambda t}|y_i(t)|^p + e^{\lambda t}p|y_i(t)|^{p-2}y_i(t)\dot{y}_i(t) \\ &= \lambda e^{\lambda t}|y_i(t)|^p + e^{\lambda t}p|y_i(t)|^{p-2}y_i(t) \\ &\quad \times \Bigg[(r_i - a_i - q_i)y_i(t) + \Big(r_i a_i - r_i^2 - b_i\Big)x_i(t) \\ &\qquad + \sum_{j=1}^{n} c_{ij}^{\diamond}(t) f_j\big(x_j(t)\big) + \sum_{j=1}^{n} d_{ij}^{\diamond}(t) g_j\big(x_j\big(t-\tau_j(t)\big)\big) \\ &\qquad + \sum_{j=1}^{n} e_{ij}^{\diamond}(t)\int_{t-\delta_j(t)}^{t} h_j\big(x_j(s)\big)ds\Bigg] \\ &\leq \lambda e^{\lambda t}|y_i(t)|^p + p(r_i - a_i - q_i)e^{\lambda t}|y_i(t)|^p \\ &\quad + p\Big(r_i a_i - r_i^2 - b_i\Big)e^{\lambda t}|y_i(t)|^{p-1}|x_i(t)| \\ &\quad + e^{\lambda t}\sum_{j=1}^{n} p\hat{c}_{ij}F_j|y_i(t)|^{p-1}\big|x_j(t)\big| \\ &\quad + e^{\lambda t}\sum_{j=1}^{n} p\hat{d}_{ij}G_j|y_i(t)|^{p-1}\big|x_j\big(t-\tau_j(t)\big)\big| \\ &\quad + e^{\lambda t}\sum_{j=1}^{n} p\hat{e}_{ij}H_j\int_{t-\delta_j(t)}^{t}|y_i(t)|^{p-1}\big|x_j(s)\big|ds \end{aligned} \tag{21}$$

where $\hat{c}_{ij}$, $\hat{d}_{ij}$, and $\hat{e}_{ij}$, $i,j \in n^\diamond$ are shown in Theorem 1. Now, we tackle the cross terms in (21), first, in view of (14)

$$pe^{\lambda t}|y_i(t)|^{p-1}|x_i(t)| \leq (p-1)e^{\lambda t}|y_i(t)|^p + e^{\lambda t}|x_i(t)|^p. \tag{22}$$

Besides, since $\sum_{k=1}^{p}\rho_{k,ij} = \sum_{k=1}^{p}\rho^*_{k,ij} = \sum_{k=1}^{p}\sigma_{k,ij} = \sum_{k=1}^{p}\sigma^*_{k,ij} = \sum_{k=1}^{p} w_{k,ij} = \sum_{k=1}^{p} w^*_{k,ij} = 1$

$$\begin{aligned} &e^{\lambda t}\sum_{j=1}^{n} p\hat{c}_{ij}F_j|y_i(t)|^{p-1}\big|x_j(t)\big| \\ &= e^{\lambda t}\sum_{j=1}^{n} p\hat{c}_{ij}^{\rho_{1,ij}+\rho_{2,ij}+\cdots+\rho_{p,ij}} \times F_j^{\rho^*_{1,ij}+\rho^*_{2,ij}+\cdots+\rho^*_{p,ij}}|y_i(t)|^{p-1}\big|x_j(t)\big| \\ &\le e^{\lambda t}\sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{c}_{ij}^{p\rho_{k,ij}}F_j^{p\rho^*_{k,ij}}|y_i(t)|^p + e^{\lambda t}\sum_{j=1}^{n}\hat{c}_{ij}^{p\rho_{p,ij}}F_j^{p\rho^*_{p,ij}}\big|x_j(t)\big|^p \end{aligned} \tag{23}$$

$$\begin{aligned} &e^{\lambda t}\sum_{j=1}^{n} p\hat{d}_{ij}G_j|y_i(t)|^{p-1}\big|x_j\big(t-\tau_j(t)\big)\big| \\ &= e^{\lambda t}\sum_{j=1}^{n} p\hat{d}_{ij}^{\sigma_{1,ij}+\sigma_{2,ij}+\cdots+\sigma_{p,ij}} \times G_j^{\sigma^*_{1,ij}+\sigma^*_{2,ij}+\cdots+\sigma^*_{p,ij}} \\ &\quad\times|y_i(t)|^{p-1}\big|x_j\big(t-\tau_j(t)\big)\big| \\ &\le e^{\lambda t}\sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{d}_{ij}^{p\sigma_{k,ij}}G_j^{p\sigma^*_{k,ij}}|y_i(t)|^p + e^{\lambda t}\sum_{j=1}^{n}\hat{d}_{ij}^{p\sigma_{p,ij}}G_j^{p\sigma^*_{p,ij}} \\ &\quad\times|x_j\big(t-\tau_j(t)\big)|^p \end{aligned} \tag{24}$$

and

$$\begin{aligned} &e^{\lambda t}\sum_{j=1}^{n} p\hat{e}_{ij}H_j\int_{t-\delta_j(t)}^{t}|y_i(t)|^{p-1}|x_j(s)|ds \\ &= e^{\lambda t}\sum_{j=1}^{n} p\hat{e}_{ij}^{w_{1,ij}+w_{2,ij}+\cdots+w_{p,ij}} \\ &\quad\times H_j^{w^*_{1,ij}+w^*_{2,ij}+\cdots+w^*_{p,ij}}\int_{t-\delta_j(t)}^{t}|y_i(t)|^{p-1}\big|x_j(s)\big|ds \\ &\le e^{\lambda t}\sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{e}_{ij}^{pw_{k,ij}}H_j^{pw^*_{k,ij}}\delta|y_i(t)|^p \\ &\quad+ e^{\lambda t}\sum_{j=1}^{n}\hat{e}_{ij}^{pw_{p,ij}}H_j^{pw^*_{p,ij}}\int_{t-\delta_j(t)}^{t}\big|x_j(s)\big|^p ds. \end{aligned} \tag{25}$$

Substituting (22)–(25) into (21) yields

$$\begin{aligned} \dot{\mathbb{W}}_i(t) \le &\Bigg[\lambda + p(r_i - a_i - q_i) + (p-1)\Big(r_ia_i - r_i^2 - b_i\Big) \\ &+\sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{c}_{ij}^{p\rho_{k,ij}}F_j^{p\rho^*_{k,ij}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{d}_{ij}^{p\sigma_{k,ij}}G_j^{p\sigma^*_{k,ij}} \\ &+\sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{e}_{ij}^{pw_{k,ij}}H_j^{pw^*_{k,ij}}\delta\Bigg]e^{\lambda t}|y_i(t)|^p \\ &+ \Big(r_ia_i - r_i^2 - b_i\Big)e^{\lambda t}|x_i(t)|^p \\ &+ \sum_{j=1}^{n}\hat{c}_{ij}^{p\rho_{p,ij}}F_j^{p\rho^*_{p,ij}}e^{\lambda t}|x_j(t)|^p \\ &+ \sum_{j=1}^{n}\hat{d}_{ij}^{p\sigma_{p,ij}}G_j^{p\sigma^*_{p,ij}}e^{\lambda(t-\tau_j(t))}\big|x_j\big(t-\tau_j(t)\big)\big|^p e^{\lambda\tau} \\ &+ \sum_{j=1}^{n}\hat{e}_{ij}^{pw_{p,ij}}H_j^{pw^*_{p,ij}}e^{\lambda\delta}\int_{t-\delta_j(t)}^{t}e^{\lambda s}\big|x_j(s)\big|^p ds. \end{aligned} \tag{26}$$

Define

$$\xi = \frac{1+\alpha}{\min\limits_{\ell\in(2n)^\diamond}\{m_\ell\}}\sum_{i=1}^{n}\left[\sup_{-\delta_0\le s\le 0}|\theta_i(s)|^p + \sup_{-\delta_0\le s\le 0}|\bar{\vartheta}_i(s)|^p\right]$$

and

$$\xi_0 = \frac{1}{\min\limits_{\ell\in(2n)^\diamond}\{m_\ell\}}\sum_{i=1}^{n}\left[\sup_{-\delta_0\le s\le 0}|\theta_i(s)|^p + \sup_{-\delta_0\le s\le 0}|\bar{\vartheta}_i(s)|^p\right]$$

where $\alpha > 0$ is a real value. Notice that for $s \in [-\delta_0, 0]$, $i \in n^\diamond$, one has $\mathbb{V}_i(s) = e^{\lambda s}|x_i(s)|^p < m_i\xi$ and $\mathbb{W}_i(s) = e^{\lambda s}|y_i(s)|^p < m_{n+i}\xi$ in light of the definition of ξ. The following work is to show that $\mathbb{V}_i(t) < m_i\xi$ and $\mathbb{W}_i(t) < m_{n+i}\xi$ hold for $t > 0$, $i \in n^\diamond$; otherwise, there exist $\beta \in n^\diamond$ and $\bar{t} \ge 0$ such that

$$\begin{aligned} &\mathbb{V}_\beta(\bar{t}) = m_\beta\xi, \quad \dot{\mathbb{V}}_\beta\big(\bar{t}\big) \ge 0 \\ &\mathbb{V}_j(s) \le m_j\xi, \quad \mathbb{W}_j(s) \le m_{n+j}\xi \end{aligned} \tag{27}$$

or

$$\begin{aligned} &\mathbb{W}_\beta(\bar{t}) = m_{n+\beta}\xi, \quad \dot{\mathbb{W}}_\beta\big(\bar{t}\big) \ge 0 \\ &\mathbb{W}_j(s) \le m_{n+j}\xi, \quad \mathbb{V}_j(s) \le m_j\xi \end{aligned} \tag{28}$$

in which $s \in [-\delta_0, \bar{t}]$ and $j \in n^\diamond$. GES of DIMNNs (13) is now ascertained by utilizing the contradiction method. For the first case, consider $\dot{\mathbb{V}}_\beta(\bar{t})$, from (20)

$$\begin{aligned} \dot{\mathbb{V}}_\beta\big(\bar{t}\big) &\le \big(\lambda + p - 1 - pr_\beta\big)e^{\lambda\bar{t}}\big|x_\beta\big(\bar{t}\big)\big|^p + e^{\lambda\bar{t}}\big|y_\beta\big(\bar{t}\big)\big|^p \\ &\le \big(\lambda + p - 1 - pr_\beta\big)m_\beta\xi + m_{n+\beta}\xi < 0 \end{aligned} \tag{29}$$

which contradicts (27).

For the second case, consider $\dot{\mathbb{W}}_\beta(\bar{t})$, from (26)

$$\begin{aligned} \dot{\mathbb{W}}_\beta\big(\bar{t}\big) \le &\Bigg[\lambda + p\big(r_\beta - a_\beta - q_\beta\big) + (p-1)\Big(r_\beta a_\beta - r_\beta^2 - b_\beta\Big) \\ &+ \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{c}_{\beta j}^{p\rho_{k,\beta j}}F_j^{p\rho^*_{k,\beta j}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{d}_{\beta j}^{p\sigma_{k,\beta j}}G_j^{p\sigma^*_{k,\beta j}} \\ &+ \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{e}_{\beta j}^{pw_{k,\beta j}}H_j^{pw^*_{k,\beta j}}\delta\Bigg]e^{\lambda\bar{t}}\big|y_\beta\big(\bar{t}\big)\big|^p \\ &+ \Big(r_\beta a_\beta - r_\beta^2 - b_\beta\Big)e^{\lambda\bar{t}}\big|x_\beta\big(\bar{t}\big)\big|^p \\ &+ \sum_{j=1}^{n}\hat{c}_{\beta j}^{p\rho_{p,\beta j}}F_j^{p\rho^*_{p,\beta j}}e^{\lambda\bar{t}}|x_j(\bar{t})|^p \\ &+ \sum_{j=1}^{n}\hat{d}_{\beta j}^{p\sigma_{p,\beta j}}G_j^{p\sigma^*_{p,\beta j}}e^{\lambda(\bar{t}-\tau_j(\bar{t}))}\big|x_j\big(\bar{t}-\tau_j\big(\bar{t}\big)\big)\big|^p e^{\lambda\tau} \\ &+ \sum_{j=1}^{n}\hat{e}_{\beta j}^{pw_{p,\beta j}}H_j^{pw^*_{p,\beta j}}e^{\lambda\delta}\int_{\bar{t}-\delta_j(\bar{t})}^{\bar{t}}e^{\lambda s}\big|x_j(s)\big|^p ds \end{aligned}$$

$$\leq \Bigg[\lambda + p\big(r_\beta - a_\beta - q_\beta\big) + (p-1)\Big(r_\beta a_\beta - r_\beta^2 - b_\beta\Big)$$
$$+ \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{c}_{\beta j}^{p\rho_{k,\beta j}} F_j^{p\rho^*_{k,\beta j}} + \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{d}_{\beta j}^{p\sigma_{k,\beta j}} G_j^{p\sigma^*_{k,\beta j}}$$
$$+ \sum_{j=1}^{n}\sum_{k=1}^{p-1}\hat{e}_{\beta j}^{pw_{k,\beta j}} H_j^{pw^*_{k,\beta j}}\delta\Bigg] m_{n+\beta}\xi$$
$$+ \Big(r_\beta a_\beta - r_\beta^2 - b_\beta\Big) m_\beta \xi + \sum_{j=1}^{n}\hat{c}_{\beta j}^{p\rho_{p,\beta j}} F_j^{p\rho^*_{p,\beta j}} m_j\xi$$
$$+ \sum_{j=1}^{n}\hat{d}_{\beta j}^{p\sigma_{p,\beta j}} G_j^{p\sigma^*_{p,\beta j}} e^{\lambda\tau} m_j\xi$$
$$+ \sum_{j=1}^{n}\hat{e}_{\beta j}^{pw_{p,\beta j}} H_j^{pw^*_{p,\beta j}} e^{\lambda\delta} m_j\xi\delta < 0 \quad (30)$$

which contradicts (28). Combining (29) and (30), one obtains $\mathbb{V}_i(t) < m_i\xi$ and $\mathbb{W}_i(t) < m_{n+i}\xi$ for $t \geq -\delta_0$ and $i \in n^\diamond$, that is

$$|x_i(t)|^p < m_i\xi e^{-\lambda t}, \quad |y_i(t)|^p < m_{n+i}\xi e^{-\lambda t} \quad (31)$$

let $\alpha \to 0^+$

$$|x_i(t)|^p \leq m_i\xi_0 e^{-\lambda t}, \quad |y_i(t)|^p \leq m_{n+i}\xi_0 e^{-\lambda t} \quad (32)$$

which indicates that DIMNNs (13) can be exponentially stabilized via the controller (12). ■

Remark 4: Different from the conventional MNNs in [7], [9], [11], and [12], DIMNNs (1) are discussed in this article where the inertial term and multiple time-varying delays are taken into consideration. From this perspective, the results in [7], [9], [11], and [12] are invalid to investigate the theoretical properties of DIMNNs (1).

Remark 5: To investigate the pth moment GES ($p \geq 2$) of DIMNNs (9), first, via the variable substitution approach, the initial DIMNNs (9) are transformed as the first-order DIMNNs (13). Second, by constructing the non-negative functions in (19), we estimate the derivatives in (20) and (26). Third, to illustrate the GES of DIMNNs (13), the analysis approach is utilized.

Remark 6: Recently, some results on the dynamical analysis of IMNNs have been established by resorting to the matrix measure approach [36], [45] and the Lyapunov method [37], [38], [40]–[44], [46], [47]. Those strategies have special advantages to investigate the dynamical properties of IMNNs. Different from the results therein, the comparison method is employed in Theorem 1 to discuss the GES of DIMNNs (13). If there are appropriate constants such that conditions (14)–(16) hold, then the GES of DIMNNs (13) can be ascertained under the designed controller (12).

Remark 7: Compared with the discontinuous controller in [43], the continuous state-feedback controller (12) is designed in this article.

Corollary 1: Let τ, δ, δ_0, F_i, G_i, H_i, and q_i be constants, under Assumptions 1 and 2, DIMNNs (13) can be exponentially stabilized via the controller (12), if there are $\tilde{m}_\ell > 0$, $\ell \in (2n)^\diamond$ such that

$$r_i a_i - r_i^2 - b_i > 0 \quad (33)$$
$$r_i\tilde{m}_i - \tilde{m}_{n+i} > 0 \quad (34)$$
$$(r_i - a_i - q_i)\tilde{m}_{n+i} + \Big(r_i a_i - r_i^2 - b_i\Big)\tilde{m}_i + \sum_{j=1}^{n}\Big[\hat{c}_{ij}F_j\tilde{m}_j + \hat{d}_{ij}G_j\tilde{m}_j + \hat{e}_{ij}H_j\delta\tilde{m}_j\Big] < 0. \quad (35)$$

Proof: First, similar to (17) and (18), there exists $\tilde{\lambda} > 0$ such that

$$\Big(\tilde{\lambda} - r_i\Big)\tilde{m}_i + \tilde{m}_{n+i} < 0 \quad (36)$$

and

$$\Big(\tilde{\lambda} + r_i - a_i - q_i\Big)\tilde{m}_{n+i} + \Big(r_i a_i - r_i^2 - b_i\Big)\tilde{m}_i + \sum_{j=1}^{n}\Big[\hat{c}_{ij}F_j\tilde{m}_j + \hat{d}_{ij}G_j\tilde{m}_j e^{\tilde{\lambda}\tau} + \hat{e}_{ij}H_j\delta\tilde{m}_j e^{\tilde{\lambda}\delta}\Big] < 0. \quad (37)$$

Then, define

$$V_i(t) = e^{\lambda t}|x_i(t)|, \quad W_i(t) = e^{\lambda t}|y_i(t)|, \quad i \in n^\diamond. \quad (38)$$

The remaining proof is omitted. ■

Remark 8: In [41] and [44], the GES of IMNNs has been investigated via the Lyapunov method and nonsmooth analysis. Note that the outcomes therein are in terms of 1-norm, meanwhile, time-varying delays are required to be bounded and differentiable, and the upper bounds of derivatives are required to be smaller than 1. Besides, to resolve the time-delay terms, some integral-type Lyapunov functionals are constructed. In Theorem 1 and Corollary 1, the pth moment ($p \geq 2$) and 1th moment GES of DIMNNs (13) are investigated, respectively. Combining Theorem 1 with Corollary 1, the pth moment GES ($p \geq 1$) of DIMNNs (13) is developed. In dynamical analysis, the differentiability of multiple time-varying delays is removed herein. In addition, without adopting the Lyapunov approach, the comparison strategy is exploited to concern the GES of DIMNNs (13), and time delays are handled via analysis techniques.

It should be pointed out that the approach in Theorem 1 can be employed to explore the theoretical properties of general MNNs without the inertial term. To illustrate this point, corresponding to DIMNNs (1), consider

$$\frac{dx_i(t)}{dt} = -b_i x_i(t) + \sum_{j=1}^{n} c_{ij}(x_i(t)) f_j\big(x_j(t)\big) + \sum_{j=1}^{n} d_{ij}(x_i(t)) g_j\big(x_j\big(t - \tau_j(t)\big)\big) + \sum_{j=1}^{n} e_{ij}(x_i(t)) \int_{t-\delta_j(t)}^{t} h_j\big(x_j(s)\big)ds \quad (39)$$

where $i \in n^\diamond$, and the initial value is $x_i(s) = \check{\theta}_i(s)$, $-\delta_0 \leq s \leq 0$.

Corollary 2: Let τ, δ, δ_0, F_i, G_i, and H_i be constants, under Assumptions 1 and 2, MNNs (39) are globally exponentially stable provided that $B - C\mathcal{F} - D\mathcal{G} - E\mathcal{H}\delta$ is an

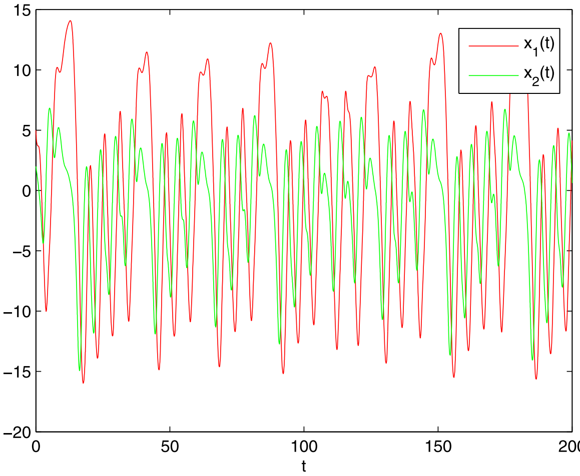

Fig. 1. Dynamics of $x_1(t)$ and $x_2(t)$ in DIMNNs (41).

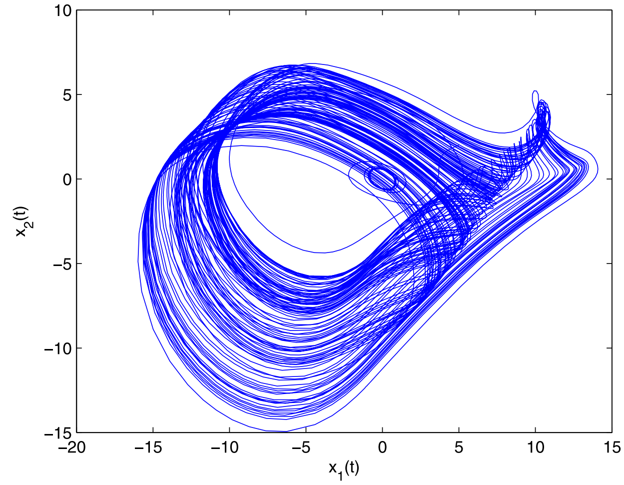

Fig. 2. Phase curves of $x_1(t)$ and $x_2(t)$ in DIMNNs (41).

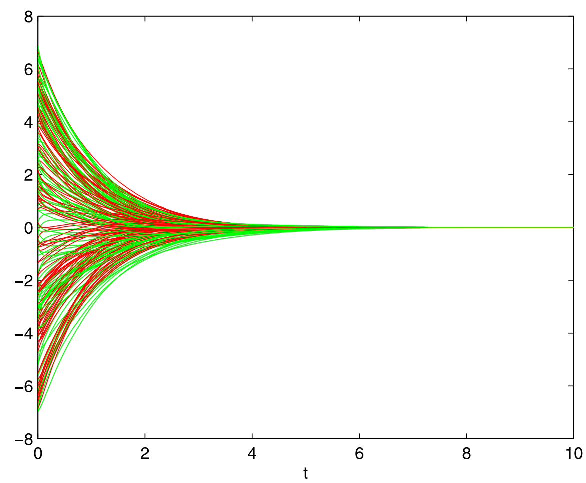

Fig. 3. Dynamics of $x_1(t)$ and $x_2(t)$ in DIMNNs (41) via control law (42).

M-matrix, where $B = \text{diag}\{b_1, b_2, \ldots, b_n\}$, $C = [\hat{c}_{ij}]_{n\times n}$, $D = [\hat{d}_{ij}]_{n\times n}$, $E = [\hat{e}_{ij}]_{n\times n}$, $\mathcal{F} = \text{diag}\{F_1, F_2, \ldots, F_n\}$, $\mathcal{G} = \text{diag}\{G_1, G_2, \ldots, G_n\}$, and $\mathcal{H} = \text{diag}\{H_1, H_2, \ldots, H_n\}$.

Proof: Since $B - C\mathcal{F} - D\mathcal{G} - E\mathcal{H}\delta$ is an M-matrix, there is $\check{m}_i > 0$ such that

$$-b_i + \sum_{j=1}^{n} \frac{\check{m}_j}{\check{m}_i}\Big[\hat{c}_{ij}F_j + \hat{d}_{ij}G_j + \hat{e}_{ij}H_j\delta\Big] < 0, \quad i \in n^{\diamond}. \tag{40}$$

The remaining proof is omitted. ■

Remark 9: Based upon Theorem 1, global exponential stability of MNNs (39) is analyzed in Corollary 2. Note that if distributed time delay is disregarded in MNNs (39), then the outcome in Corollary 2 reduces to the ones in ([50, Theor. 2], [51, Corollary 2], and [52, Corollary 1]).

IV. Numerical Examples

Example 1: Consider DIMNNs as

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} &= -a_i\frac{dx_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{2} c_{ij}(x_i(t))f_j\big(x_j(t)\big)\\ &\quad + \sum_{j=1}^{2} d_{ij}(x_i(t))g_j\big(x_j\big(t-\tau_j(t)\big)\big)\\ &\quad + \sum_{j=1}^{2} e_{ij}(x_i(t))\int_{t-\delta_j(t)}^{t} h_j\big(x_j(s)\big)ds + \digamma_i(t)\end{aligned} \tag{41}$$

where $i = 1, 2$, $a_1 = 1.9$, $a_2 = 1.97$, $b_1 = 0.03$, and $b_2 = 0.05$, switching jump $\Theta_i = 1$, $c_{11}^{\Delta} = -2.1$, $c_{11}^{\nabla} = -2$, $c_{12}^{\Delta} = 9.3$, $c_{12}^{\nabla} = 9.5$, $c_{21}^{\Delta} = -1.2$, $c_{21}^{\nabla} = -1.1$, $c_{22}^{\Delta} = 0.3$, $c_{22}^{\nabla} = 0.32$, $d_{11}^{\Delta} = -3.5$, $d_{11}^{\nabla} = -3.54$, $d_{12}^{\Delta} = -1.75$, $d_{12}^{\nabla} = -1.7$, $d_{21}^{\Delta} = 1.65$, $d_{21}^{\nabla} = 1.6$, $d_{22}^{\Delta} = -4.5$, $d_{22}^{\nabla} = -4.8$, $e_{11}^{\Delta} = -0.9$, $e_{11}^{\nabla} = -0.7$, $e_{12}^{\Delta} = -1.6$, $e_{12}^{\nabla} = -1.62$, $e_{21}^{\Delta} = 2.3$, $e_{21}^{\nabla} = 2$, and $e_{22}^{\Delta} = -0.01$, $e_{22}^{\nabla} = -0.02$, time delays $\tau_i(t) = \delta_i(t) = e^t/(1+e^t)$, and activation functions $f_i(z) = g_i(z) = h_i(z) = \tanh(z)$. Choosing initial values $x_1(s) = 5$, $x_2(s) = 2$, and $s \in [-1, 0]$, the state behaviors and phase behaviors of DIMNNs (41) are depicted in Figs. 1 and 2, respectively, which illustrate that $x_1(t)$ and $x_2(t)$ are not convergent as time goes by.

To consider the GES of DIMNNs (41), first, define $y_i(t) = dx_i(t)/dt + x_i(t)$, $i = 1, 2$, then select $p = 2$, $\rho_{k,ij} = \rho_{k,ij}^{*} = \sigma_{k,ij} = \sigma_{k,ij}^{*} = w_{k,ij} = w_{k,ij}^{*} = 0.5$, $k = 1, 2$, $m_1 = m_2 = 1$, $m_3 = m_4 = 0.8$, meanwhile, design

$$\digamma_i(t) = -q_iy_i(t) \tag{42}$$

with $q_1 = 22$ and $q_2 = 12$, one can compute that conditions (14)–(16) hold true; therefore, DIMNNs (41) can be exponentially stabilized via the controller (42). With 100 random initial values in $[-7, 7]$, Fig. 3 shows the state behaviors of DIMNNs (41) under controller (42).

Example 2: Consider MNNs as

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -b_ix_i(t) + \sum_{j=1}^{3} c_{ij}(x_i(t))f_j\big(x_j(t)\big)\\ &\quad + \sum_{j=1}^{3} d_{ij}(x_i(t))g_j\big(x_j\big(t-\tau_j(t)\big)\big)\\ &\quad + \sum_{j=1}^{3} e_{ij}(x_i(t))\int_{t-\delta_j(t)}^{t} h_j\big(x_j(s)\big)ds\end{aligned} \tag{43}$$

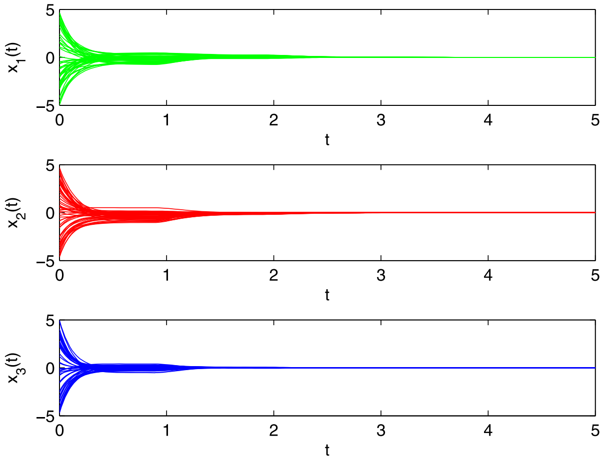

Fig. 4. Dynamics of $x_1(t)$, $x_2(t)$, and $x_3(t)$ in MNNs (43).

where $i = 1, 2, 3$, $b_1 = 7$, $b_2 = 7.5$, and $b_3 = 8$, switching jump $\Theta_i = 2$, $c_{11}^{\Delta} = 0.2$, $c_{11}^{\nabla} = 0.3$, $c_{12}^{\Delta} = -0.61$, $c_{12}^{\nabla} = -0.58$, $c_{13}^{\Delta} = 1$, $c_{13}^{\nabla} = 0.9$, $c_{21}^{\Delta} = -0.4$, $c_{21}^{\nabla} = -0.5$, $c_{22}^{\Delta} = -1.5$, $c_{22}^{\nabla} = -1.54$, $c_{23}^{\Delta} = 0.7$, $c_{23}^{\nabla} = 0.76$, $c_{31}^{\Delta} = 0.24$, $c_{31}^{\nabla} = 0.26$, $c_{32}^{\Delta} = -0.65$, $c_{32}^{\nabla} = -0.66$, $c_{33}^{\Delta} = 0.72$, $c_{33}^{\nabla} = 0.73$, $d_{11}^{\Delta} = -1.2$, $d_{11}^{\nabla} = -1.21$, $d_{12}^{\Delta} = 0.33$, $d_{12}^{\nabla} = 0.35$, $d_{13}^{\Delta} = 0.7$, $d_{13}^{\nabla} = 0.8$, $d_{21}^{\Delta} = -0.25$, $d_{21}^{\nabla} = -0.28$, $d_{22}^{\Delta} = 0.82$, $d_{22}^{\nabla} = 0.83$, $d_{23}^{\Delta} = 0.9$, $d_{23}^{\nabla} = 0.91$, $d_{31}^{\Delta} = 0.7$, $d_{31}^{\nabla} = 0.6$, $d_{32}^{\Delta} = 1.4$, $d_{32}^{\nabla} = 1.52$, $d_{33}^{\Delta} = -0.52$, $d_{33}^{\nabla} = -0.53$, $e_{11}^{\Delta} = 0.36$, $e_{11}^{\nabla} = 0.37$, $e_{12}^{\Delta} = -0.58$, $e_{12}^{\nabla} = -0.6$, $e_{13}^{\Delta} = -1.3$, $e_{13}^{\nabla} = -1.4$, $e_{21}^{\Delta} = 0.6$, $e_{21}^{\nabla} = 0.7$, $e_{22}^{\Delta} = 0.28$, $e_{22}^{\nabla} = 0.25$, $e_{23}^{\Delta} = -0.9$, $e_{23}^{\nabla} = -0.96$, $e_{31}^{\Delta} = 0.45$, $e_{31}^{\nabla} = 0.55$, $e_{32}^{\Delta} = -1.1$, $e_{32}^{\nabla} = -1.15$, $e_{33}^{\Delta} = 0.7$, $e_{33}^{\nabla} = 0.8$, $\tau_i(t) = \delta_i(t) = 0.5 + 0.5|\sin(t)|$, and $f_i(z) = g_i(z) = h_i(z) = \tanh(z)$.

It is clear that $\tau_i(t)$ and $\delta_i(t)$ are not differentiable, thus, the results in [41] and [44] are invalid to concern the GES of MNNs (43).

From Corollary 2, one has

$$B - C\mathcal{F} - D\mathcal{G} - E\mathcal{H}\delta = \begin{bmatrix} 5.12 & -1.56 & -3.2 \\ -1.48 & 4.85 & -2.63 \\ -1.51 & -3.33 & 5.94 \end{bmatrix}. \quad (44)$$

The three eigenvalues are 0.7709, 6.6673, and 8.4718, respectively, which implies that $B - C\mathcal{F} - D\mathcal{G} - E\mathcal{H}\delta$ is an M-matrix. Hence, MNNs (43) are globally exponentially stable. With 50 random initial values in $[-5, 5]$, Fig. 4 presents the time behaviors of $x_1(t)$, $x_2(t)$, and $x_3(t)$ in MNNs (43).

In the following text, we consider an application in the pseudorandom number generator (PRNG) [11] by means of DIMNNs (41). Define a pseudorandom number sequence $k(t) = h(\pi_1(t), \pi_2(t))$, $t \in [t_1, t_2]$, $[t_1, t_2]$ is the time span, and

$$h(\pi_1(t), \pi_2(t)) = \begin{cases} 0, & \pi_1(t) > \pi_2(t) \\ 1, & \pi_1(t) \le \pi_2(t) \end{cases} \quad (45)$$

where $\pi_1(t) = [x_1(t)/(\sup_{t\in[t_1,t_2]}\{x_1(t)\})]$ and $\pi_2(t) = [x_2(t)/(\sup_{t\in[t_1,t_2]}\{x_2(t)\})]$. Fig. 5 shows the PRNG produced by DIMNNs (41). Let $s(t)$ be the initial transmitted signal depicted in Fig. 6; thus, one can obtain the encrypted signals via $p(t) = s(t) \otimes k(t)$, which is depicted in Fig. 7. It is clear that the encrypted signals are quite different from the initial signals.

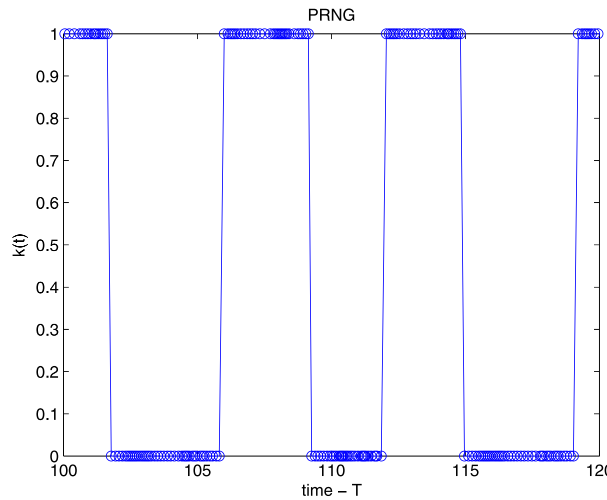

Fig. 5. PRNG produced by DIMNNs (41).

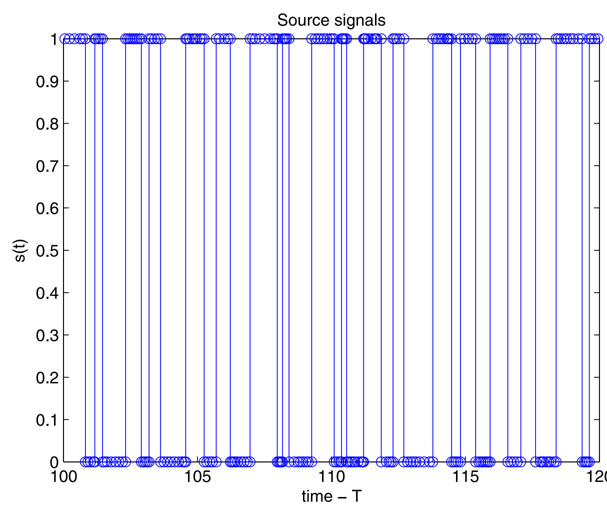

Fig. 6. Original signal.

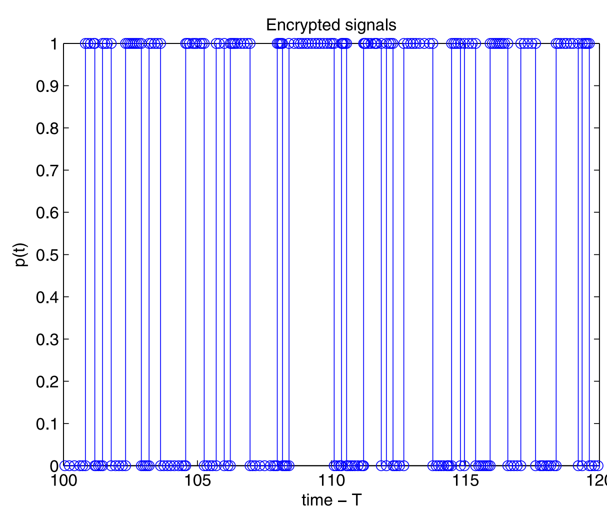

Fig. 7. Encrypted signals by DIMNNs (41) and the initial signals.

V. Conclusion

The investigation on the stabilization of MNNs has received considerable attention during recent decades. In this article, we discussed the GES of DIMNNs. First, by the variable transformation approach, DIMNNs were converted as the first-order differential equations. Then, by adopting the comparison approach and inequality techniques in Filippov's sense, algebraic criteria were presented to guarantee the pth moment GES ($p \geq 1$) of the addressed DIMNNs via designing a compatible state-feedback controller. Besides, an M-matrix-based condition was carried out to substantiate the global exponential stability of MNNs, which generalized some published results. The validness of the derived outcomes was illuminated by two simulations, and an application was developed in PRNG.

In future work, we attempt to utilize the comparison approach to investigate the multistability and multisynchronization of MNNs.

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