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SLOSHING OF WATER IN TORUS PRESSURE-SUPPRESSION POOL OF BOILING WATER REACTORS UNDER EARTHQUAKE GROUND MOTIONS

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SLOSHING OF WATER IN TORUS
PRESSURE-SUPPRESSION POOL OF BOILING WATER REACTORS
UNDER EARTHQUAKE GROUND MOTIONS

A Report of an Analytical and Experimental Study
of Sloshing Response in Axisymmetric Tanks
Under Earthquake Ground Motions

by

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LIST OF SYMBOLS

a	Outer radius of tank
b	Inner radius of tank
B1	Liquid-solid interface boundary
B2	Free surface boundary
EB1	Element boundary B1
EB2	Element boundary B2
EV	Element Volume
$\underline{F}$	Load vector (Solid-liquid interface matrix)
g	Acceleration of gravity
h	Water depth
$\underline{J}$	Jacobian matrix
$\underline{K}$	Fluid matrix (equivalent to stiffness matrix)
$\underline{M}$	Free surface matrix (equivalent to mass matrix)
N	Shape functions or a number
p	Pressure
r	Local coordinate
s	Local coordinate or surface
t	Time
v_n	Normal velocity
X,Y,Z	Rectangular coordinates
ϕ	Velocity potential
$\dot{\phi}$	$d\phi/dt$
Δt	Time step
ρ	Mass density
δ	Free surface displacement or a parameter

ABSTRACT

This report presents an analytical and experimental investigation into the sloshing of water in torus tanks under horizontal earthquake ground motions. This study was motivated because of the use of torus tanks for pressure-suppression pools in Boiling Water Reactors. Such a pressure-suppression pool would typically have 80 ft and 140 ft inside and outside diameters, a 30 ft diameter section, and a water depth of 15 ft.

A general finite element analysis was developed for all axisymmetric tanks and a computer program was written to obtain time-history plots of sloshing displacements of water and dynamic pressures. Tests were carried out on a 1/60th scale model under sinusoidal as well as simulated earthquake ground motions. Tests and analytical results regarding natural frequencies, surface water displacements, and dynamic pressures were compared and a good agreement was found within the range of displacements studied. The computer program gave satisfactory results within a maximum range of sloshing displacements in the full-size prototype of 30 in. which is greater than the value obtained under the full intensity of the El Centro earthquake (N-S component 1940). The range of linear behavior was studied experimentally by subjecting the torus model to increasing intensities of the El Centro earthquake. The general computer program was also used for comparison with a previous study on the sloshing of water in annular tanks, and the previous annular tank solution was also used as an approximate solution in the torus tank problem showing that sloshing response is not very sensitive to the precise cross-section geometry of an axisymmetric tank.

Tests were also conducted to study the effect of vertical ground motions on the dynamic response of the fluid. These showed a negligible effect on sloshing displacements.

KEY WORDS

Sloshing Response, Pressure-suppression pool, Torus tanks, Annular tanks, Earthquake response, Dynamic pressures, Finite element solution.

1. INTRODUCTION

1.1 Objective

This report presents the results of a study into the sloshing response of water in torus tanks under the action of earthquake-induced ground motions, and is a continuation of a previous investigation into the sloshing response of water in annular tanks [1].

Torus tanks are used as pressure-suppression pools in certain designs of boiling water reactors (BWR)(e.g., General Electric Mark I), and a knowledge of the dynamic response of the water and particularly of the resulting water surface elevations is important in evaluating the effectiveness of the system under earthquake conditions. A typical torus suppression pool as used in the GE Mark I reactor has an outside diameter of 140 ft and a section height of 30 ft. A 1/60th scale idealized model of such a pool is shown in Fig. 2-1a.

Rather than deriving a particular analytical solution to the torus tank problem, it was decided at the outset to undertake a general study into the sloshing of water in axisymmetric tanks; this would be applicable to the torus tank, the annular tank previously studied, and to all tanks with a constant section of revolution. References [1] through [16] are some of the previous studies related to the sloshing of fluids in tanks.

1.2 Scope of the Investigation

This study includes the testing of a 1/60th scale model of GE Mark 1 torus, the development of a general finite element theory for axisymmetric tanks, and a computer code to implement the finite element theory. A comparison of test results with an approximate analysis based on the annular tank theory is also given in Chapter 2.

The test model was constructed by cementing together wedge-shaped lengths of 6 in. lucite tubing as shown in Fig. 2-1. This was tested under harmonic and simulated earthquake ground motions. The quantities measured included the sloshing frequencies and free surface displacements.

The finite element equations (Chapter 3) were derived using the Galerkin principle and a linearized small displacement theory was used. The velocity potential ϕ was taken as the field variable and the sloshing displacements as well as impulsive pressures were derived from ϕ. The finite element equations were first derived for a general three dimensional sloshing problem under arbitrary ground motions, and then it was specialized to an axisymmetric tank subjected to horizontal earthquake ground motions only.

A computer code named 'SLOSH2' was developed to implement the finite element theory, and a comparison of sloshing displacements and pressures as predicted by the computer program was made with the test results from annular [1] and torus tanks. These comparisons show that the finite element program can successfully predict the sloshing displacements as well as impulsive pressures in an axisymmetric tank under horizontal ground motions within the range of linear behavior.

2. TORUS TANK MODEL TESTS AND COMPARISON WITH APPROXIMATE ANALYSIS

2.1 Introduction

The objective of the tests on the torus tank model was to obtain experimental data to compare with the results obtained from approximate analysis based on a previous study on annular tanks [1], and also to check the accuracy of the finite element solution developed and described in Chapter 3.

Test data on sloshing displacements was obtained for harmonic as well as simulated seismic-type ground motions. Details of the test procedure, experimental data, and comparison with results of approximate analysis based on annular tank theory are discussed in this chapter.

2.2 Model Description and Instrumentation

A simplified 1/60th scale model of a Mark 1 suppression pool was constructed as shown in Figs. 2-1a and 2-1b. The model was fabricated from 16 short lengths of 6 in. diameter clear plastic tubing cemented together to approximate complete torus. The internal details of the prototype, including the 'Headers' and 'Downcomers', were not reproduced in the model and not taken into account in the analysis. However, for reference purposes, the Header and Downcomer configuration is shown in Fig. 2-1C for a 1/30th scaled model designed for proof-tests by the reactor manufacturer. The mean diameter of the 1/60th scale model was 22 in. The normal operating water depth was 3 in.; that is, the water surface at the section diameter, though water surface elevations above and below this level were also studied. The tank was mounted on a plywood base which was in turn prestressed to the shaking table.

The model was instrumented with one displacement gage located at a distance of 3/8 in. from the inside wall. This gage was of the same type as used in the annular tank tests [1].

2.3 Test Procedure and Experimental Data

2.3.1 Tests on the small shaking table

Sloshing frequencies of the 1/60th scale model of the Mark 1 torus and the steady state sloshing response under sinusoidal table motions were measured using the 3×4 ft shaking table described in Ref. [1] Sec. 4.4. The test set-up is shown in Fig. 2-2 and is similar to that described in Ref. [1] Sec. 4.5, the only difference being that this time a spectrum analyzer was used to determine the sloshing frequencies.

The test procedure to determine the steady state sloshing response under sinusoidal motions was the same as described in Ref. [1] Sec. 4.5 and was measured at the gage location shown in Fig. 2-1a. To determine the sloshing frequencies, the frequency of the table motion was continuously changed and the water displacement signal was fed to the spectrum analyzer which was arranged to produce an averaged frequency spectrum. In this way the sloshing frequencies were read directly.

Tests on the small shaking table were not only conducted at the normal depth of 3 in., but also at depths of 2, 2.5, 3.5 and 4 in.

Test data regarding the sloshing frequencies and the wave heights are presented in Tables 2-1 and 2-2 respectively. Table 2-1 shows the mode number, the depth of water, and the measured as well as the analytical sloshing frequencies as approximated by using annular tank theory. Table 2-2 gives the steady-state response for harmonic ground motion, and tabulates the frequency of table motion, depth of water in the tank, amplitude of the table acceleration, and measured as well as the predicted values of the amplitudes of wave heights at the gage location. Two approximate solutions are given based on annular tank theory and are described in Sec. 2.4.

2.3.2 Tests under simulated earthquakes

Tests under simulated earthquake motion using the El Centro (1940) record were made at the University of California's Earthquake Simulator Laboratory. The test set-up of the 1/60th scale model of the Mark 1 torus is shown in Fig. 2-3 where the model is shown mounted on the 20 × 20 ft shaking table. Details of the shaking table facility and data acquisition system are given in Ref. [1] Chapter 5. The testing procedure was similar to that described in Ref. [1] Sec. 5.6. The prototype of the El Centro earthquake record was reduced by a factor of $\sqrt{60}$ to meet the similitude requirements of a 1/60th scale model.

In each test the quantities that were recorded, digitized and stored on magnetic tape included the horizontal and vertical table accelerations and displacements, and the water surface displacements in the model at the gage location shown in Fig. 2-1. The peak values of these quantities together, with the test numbers are shown in Table 2-3. The maximum and minimum water displacements in this table represents the peak upward and downward displacements respectively. These tests were carried out both with and without the vertical component of the recorded ground motion. They were made at increasing amplitudes of ground motion in order to determine the range of linear response. Some selected results are plotted in Figs. 2-4 through 2-8. The center graph in Figs. 2-4 through 2-7 also shows a comparison of the measured displacement with approximate results from annular tank theory under horizontal ground motion only. Figure 2-8 shows that the vertical ground motion alone does not produce sloshing displacements as could be expected.

It should be remembered that a shaking table acts as a low-pass filter and will not fully reproduce motions at frequencies above the natural frequency of the system. The small time scale ($T_r = \sqrt{60} = 7.75$) used in this study resulted in the acceleration peaks associated with high frequency ground motion being filtered out by the table. Hence the intensity of the earthquake given in Table 2-3, and measured by peak table acceleration, cannot be compared directly with the intensity of the original El Centro record. However the sloshing response is produced mainly by the low frequency components of the ground motion and these are correctly reproduced. It is estimated that the simulated earthquake ground motion in Table 2-3 with a peak acceleration of 0.34 g is equivalent to approximately 2.0 times the actual intensity of the El Centro earthquake in the significant low frequency components.

(Note: The recorded peak acceleration in the actual 1940 El Centro earthquake is 0.32 g.)

2.4 Comparison of Test Results with Approximate Theory

Approximate analyses were carried out using the computer program 'SLOSH' which is based on the annular tank theory developed in Ref. [1]. For carrying out the analyses the measured table acceleration was used and the tank radii a and b were taken as the horizontal distances from the axis of symmetry to the outer and inner walls where the free water surfaces come in contact with the solid boundaries. Two approximations were then made for the water depth as follows:

(1) In the first approximation the depth of the water (h) in the torus tank solution was taken as the actual depth from the bottom of the torus to the free surface. This is called the 'Annular Tank' approximation in Tables 2-1 and 2-2.

(2) In the second approximation, the depth of water used in the annular solution was adjusted to make the volume of water in the annular tank (with the same values of a and b as the torus tank) equal to the actual volume in the torus. This is called the 'Equivalent Volume' approximation in Tables 2-1 and 2-2.

Using these two approximations,analyses were carried out using the SLOSH program and the comparison of test and computer results for the sloshing frequencies and displacements is given.

2.4.1 Comparison of natural sloshing frequencies

Table 2-1 shows the comparison between the approximate results based on annular tank theory and the test values. It can be seen that for the first four modes, and within the range of water depth considered, the two approximations give very similar results. This could be expected as in an annular tank over this range the natural frequencies are not very sensitive to water depth. Also, the approximate solutions compare well with the measured torus values in the 2 in. to 4 in. depth range indicating that frequencies are not very sensitive to the actual shape of cross-section of the tank.

2.4.2 Comparison of sloshing displacements under steady state harmonic ground motion

Table 2-2 gives a comparison of the measured and computed results from approximate annular tank theory for water depths ranging from 2.5 in. to 3.5 in. under steady state sinusoidal table motions varying in frequency from 1.50 Hz to 2.55 Hz. It can be observed in Table 2-2 that the annular tank theory gives satisfactory results for this range of water depth and the 'Equivalent Volume' approximation gives better results in most cases. It was also found that when the torus tank is nearly empty or nearly full (water depths less than 2 in. or greater than 4 in. in the model) as could be expected the approximate theory does not give satisfactory results and should not be used.

2.4.3 Comparison of sloshing displacements under simulated earthquake ground motions

Figures 2-4 through 2-7 show the time-history plots of the water surface displacements at a distance of 3/8" from the inside wall under increasing intensities of the simulated El Centro earthquake motion (horizontal component only). The depth of water in the torus tank was 3 in. in all these tests and the comparison of measured and approximate analytical displacements is given in the middle plot in each case. In Fig. 2-4 the analysis was carried out using approximation (1) (i.e., depth of water in the annular solution was taken as 3 in.). It can be seen that the measured and computed results differ by about 30% in displacement amplitude, and the measured frequency of response oscillation is somewhat lower. In Fig. 2-5 the 'Equivalent Volume' approximation was used (h = 2.36 in.) for the same ground motion and it can be seen that the agreement between the test and approximate analysis is better as regards displacement amplitude.

Figures 2-6 and 2-7 show a comparison at higher intensities of the ground motion where the analysis in both cases was done using the 'Equivalent Volume' annular tank approximation. It can be seen that even at these relatively large displacements the approximate theory, where a modified depth based on an equivalent volume is used in the annular tank solution, gives reasonably satisfactory results. This approximate theory should however be used with caution especially when the water depth is outside the range of that used in these tests.

2.5 Dynamic Pressures

The dynamic pressures were not measured on account of their small values and therefore no comparison is available between measured and analytical results. It is however anticipated that the annular tank theory should not be expected to give satisfactory results for the dynamic impulsive pressures in a torus tank and should not be used for this purpose. If the pressures are required in the torus, the Finite Element Method described in the next chapter should be used.

2.6 Linearity Range

The range of linear behavior of sloshing response was tested experimentally by subjecting the model to increasing intensities of the simulated El Centro earthquake (horizontal component only), and measuring the maximum (upward) and minimum (downward) peak water surface displacements. The displacements associated with this ground motion are primarily in the first radial mode, and hence the following comments on linearity are primarily related to displacements in this mode. The results are plotted in Fig. 2-9 and are tabulated in Table 2-3, together with some tests which included the vertical component of ground motion.

For displacements less than 0.1 in. in the model, linearity holds within 1%. But as the amplitude of displacement increases, nonlinearity also increases and becomes approximately 10% at displacements in the order of 0.4 in. estimated on the basis of both upward and downward displacements. For practical purposes it may be assumed that the linear theory gives satisfactory results as long as the displacements are less than 0.5 in. in the model (or 30 in. in the prototype Mark 1 suppression pool). It may be seen in Fig. 2-6 that the modified annular tank theory (Equivalent Volume approximation) gives quite satisfactory results although the displacements are of the order of 0.5 in.

2.7 Summary of Important Observations on Sloshing in Torus Tank Model

1) The overall sloshing behavior of water in the torus tank model studied in this report was very similar to that in the annular tank described in Ref. [1].

2) The effect of vertical ground motion on the sloshing displacements is negligible in the linear range (compare Test No. 220378.2 with Test No. 220378.10 in Table 2-3) but becomes more significant in the nonlinear range (compare Test No. 220378.6 with Test No. 220378.13 in Table 2-3).

3) Modified annular tank theory gives satisfactory results for sloshing displacements in the torus tank. However this approximate theory cannot be applied for the determination of dynamic pressures.

4) A non-linearity of approximately 10% could be expected at displacements of the order of 0.4 in. in the model (or 24 in. in the prototype). Linear theory gives satisfactory results even under strong ground motions such as the El Centro earthquake of 1940.

TABLE 2-1. Natural sloshing frequencies of small scale model (1/60-scale model).

Mode No.	Depth of Water (in.)	Natural Frequencies (Hz)		
		Test	Theory	
			Annular Tank	Equivalent Volume
1	3	0.45	0.49	0.44
2	3	2.15	2.18	2.12
3	3	3.02	3.20	3.19
4	3	3.95	3.93	3.92
1	4	0.55	0.56	0.52
2	4	2.37	2.34	2.31
3	4	3.15	3.31	3.30
4	4	4.15	4.04	4.04
1	2	0.35	0.40	0.34
2	2	2.00	2.13	1.91
3	2	3.20	3.27	3.16
4	2	3.92	4.04	4.00

TABLE 2-2. Sloshing response of water in torus tank under sinusoidal ground acceleration (1/60th scale model) a = 14", b = 8".

Frequency (Hz)	Depth (in.)	Table Acceleration (g)	Displacement at Inner Wall (in.)		
			Test	Theory	
				(Equivalent Volume)	Annular Tank
1.5	3.0	.0109	.069	.069	.0685
1.8	3.0	.00392	.040		
1.8	3.0	.00785	.079		
1.8	3.0	.0118	.119		
1.8	3.0	.0157	.157	.157	.139
1.8	3.0	.0196	.179		
1.9	3.0	.00438	.058		
1.9	3.0	.00875	.116		
1.9	3.0	.0131	.179		
1.9	3.0	.0175	.240	.240	.190
1.9	3.0	.0218	.271		
2.4	3.0	.0242	.246	.205	.319
1.50	2.5	.0118	.083	.078	.073
1.50	2.5	.0233	.165		
1.80	2.5	.0163	.216	.210	.148
1.80	2.5	.0326	.444		
1.80	2.5	.0245	.330		
1.50	3.5	.0118	.072		.074
1.80	3.5	.0163	.150	.140	.132
1.80	3.5	.0326	.310		
2.55	3.5	.0325	.252	.226	.275

TABLE 2-3. Extreme values in torus tank model tests under simulated El Centro 1940 earthquake (time scale (= $\sqrt{60}$ = 7.75, depth of water = 3 inches).

Test No	Peak Table Acceleration (g)		Peak Table Displacement (inches)		Water Displacement (inches)	
	Horizontal	Vertical	Horizontal	Vertical	Maximum (up)	Minimum (down)
220378.2	0.115	0.0	0.048	0.0	0.246	0.231
220378.3	0.164	0.0	0.073	0.0	0.409	0.346
220378.4	0.237	0.0	0.100	0.0	0.614	0.458
220378.5	0.284	0.0	0.125	0.0	0.826	0.563
220378.6	0.338	0.0	0.149	0.0	0.999	0.667
220378.7	0.049	0.0	0.023	0.0	0.109	0.111
220378.8	0.0	0.083	0.0	0.05	0.003	0.006
220378.9	0.0	0.260	0.0	0.160	0.014	0.012
220378.10	0.107	0.067	0.046	0.036	0.249	0.245
220378.11	0.170	0.103	0.075	0.063	0.485	0.378
220378.12	0.232	0.163	0.100	0.098	0.758	0.552
220378.13	0.358	0.217	0.151	0.136	1.26	0.924
230378.1	0.477	0.0	0.201	0.0	1.22	0.921

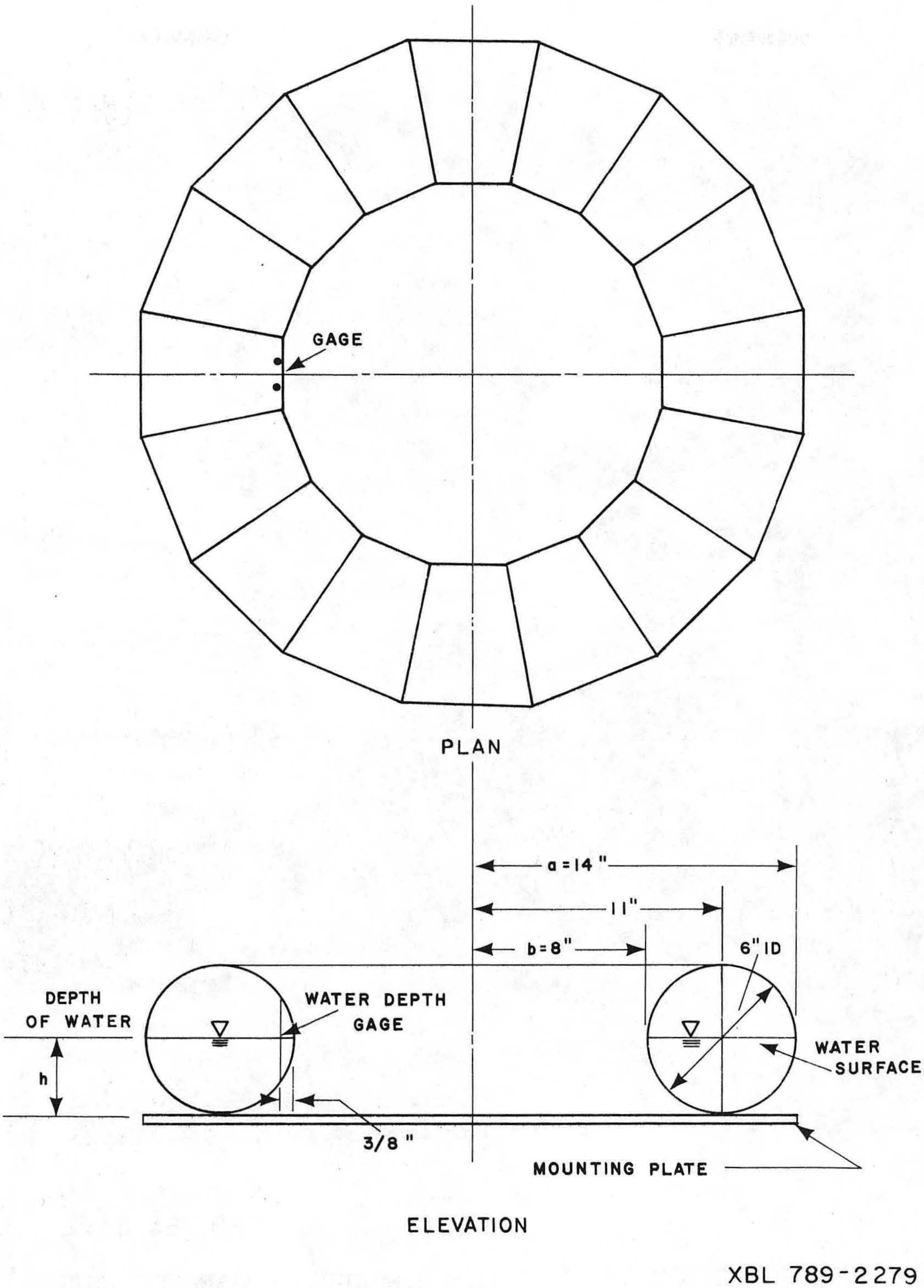

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FIG. 2-1a 1/60 SCALE MODEL OF MARK I PRESSURE SUPPRESSION POOL

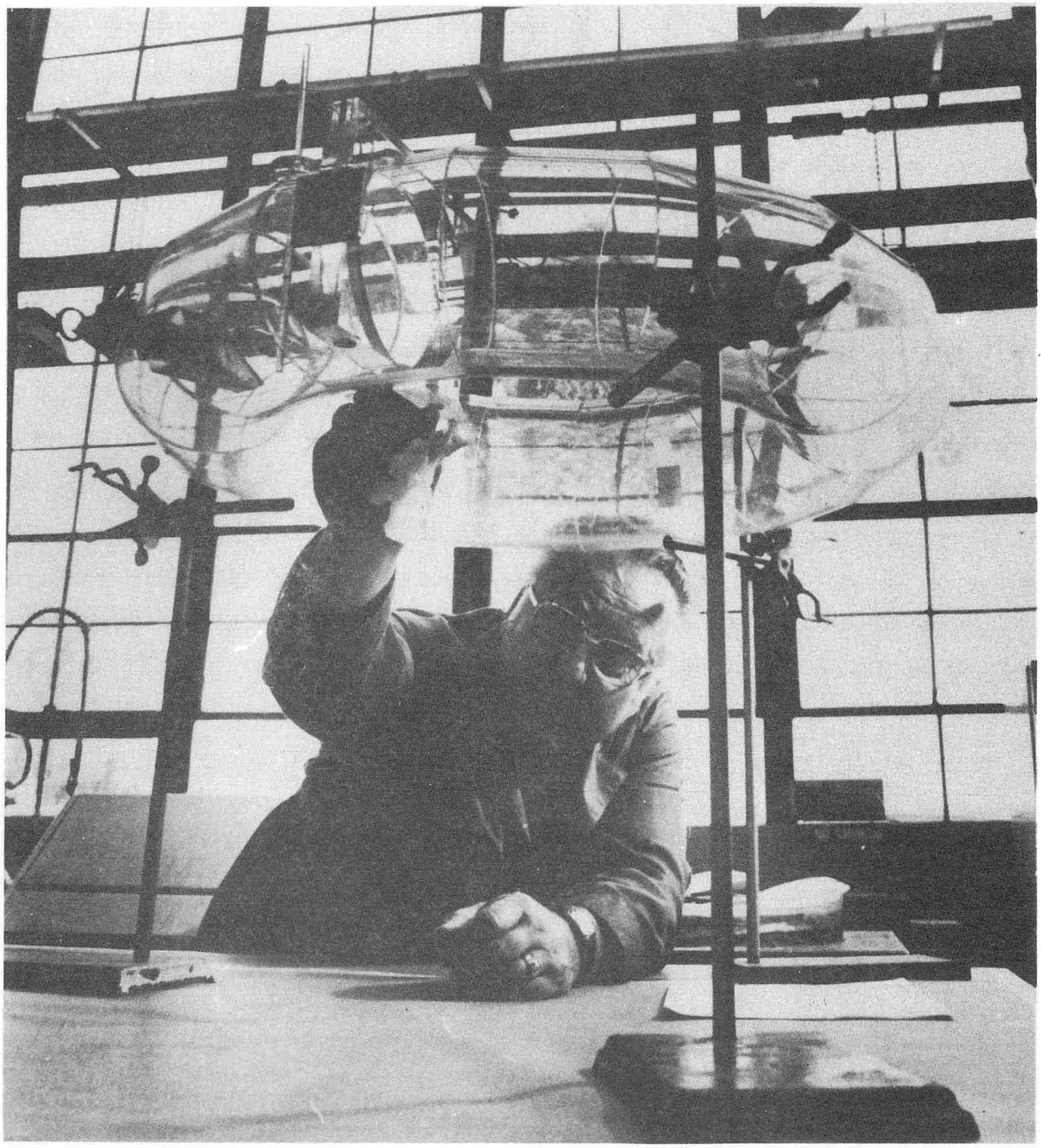

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FIG. 2-1b CONSTRUCTION OF 1/60 SCALE MODEL OF MARK I TORUS PRESSURE SUPPRESSION POOL

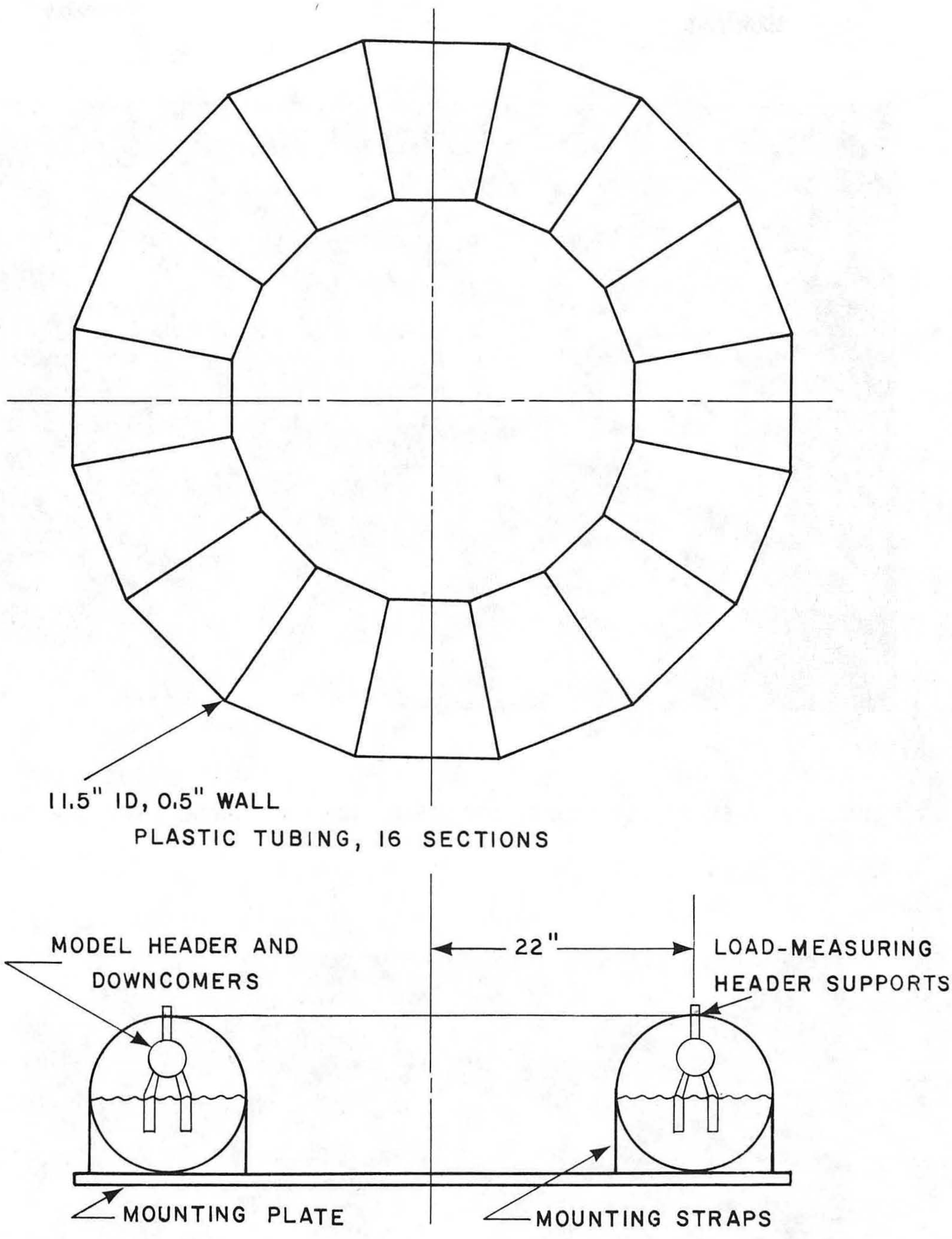

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FIG. 2-1c 1/30 SCALE MODEL OF MARK I SUPPRESSION POOL TORUS WITH HEADERS AND DOWNCOMERS

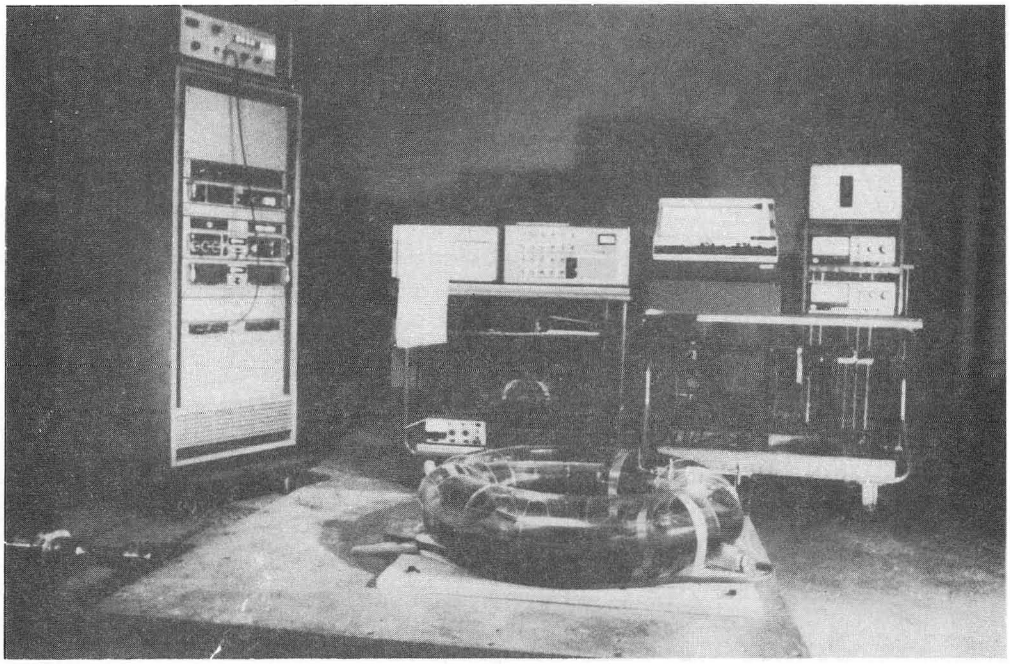

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FIG. 2-2 1/60 SCALE MODEL OF TORUS TANK ON SMALL SHAKING TABLE

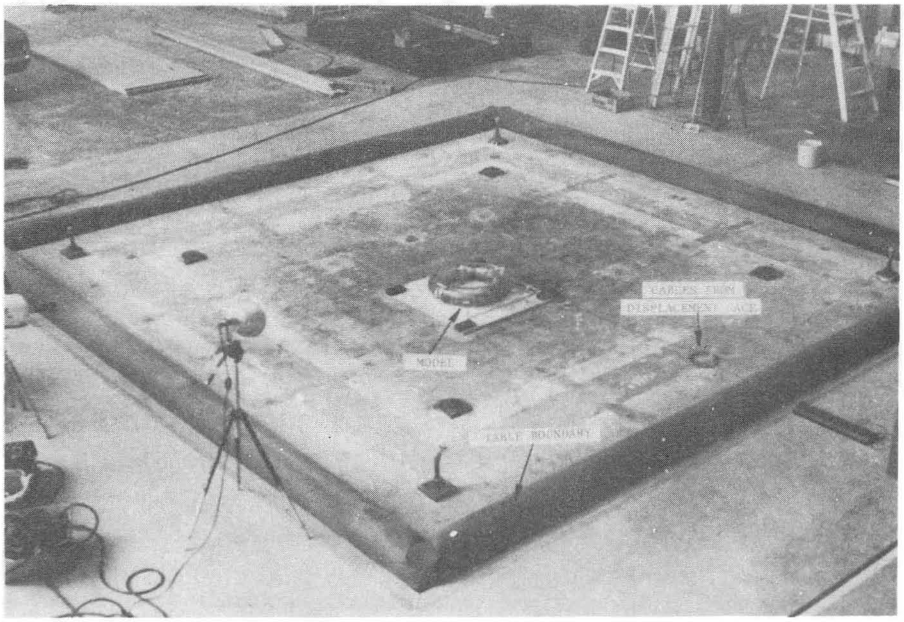

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FIG. 2-3 1/60 SCALE MODEL ON 20ft X 20ft SHAKING TABLE

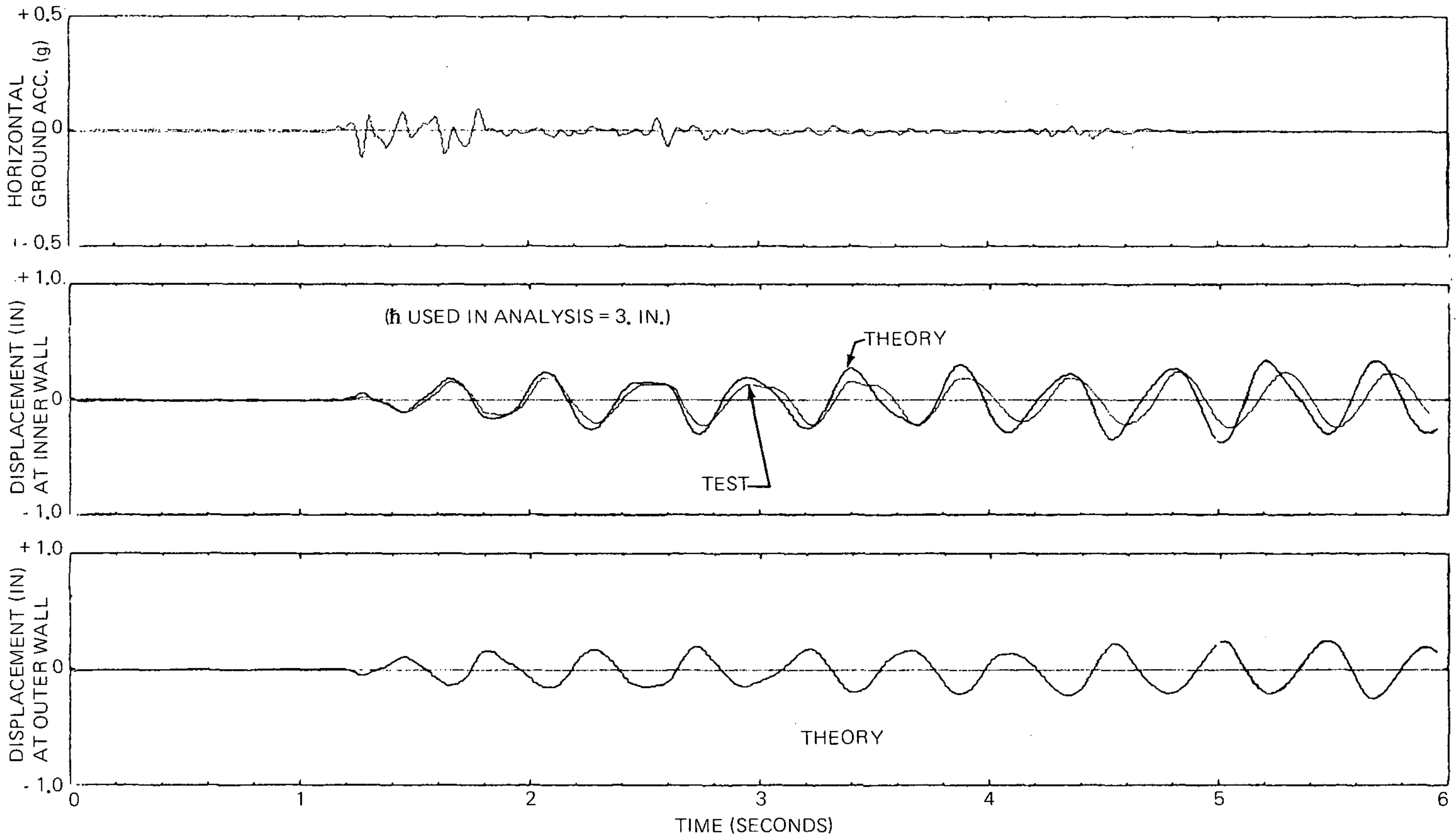

FIG. 2-4 SLOSHING RESPONSE OF WATER IN A TORUS TANK (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.) UNDER SIMULATED EL CENTRO 1940 EARTHQUAKE, TIME SCALE = 7.7, TEST NO. 220378.2. PEAK SHAKING TABLE ACCELERATION = 0.115g HORIZONTAL, 0.0g VERTICAL.

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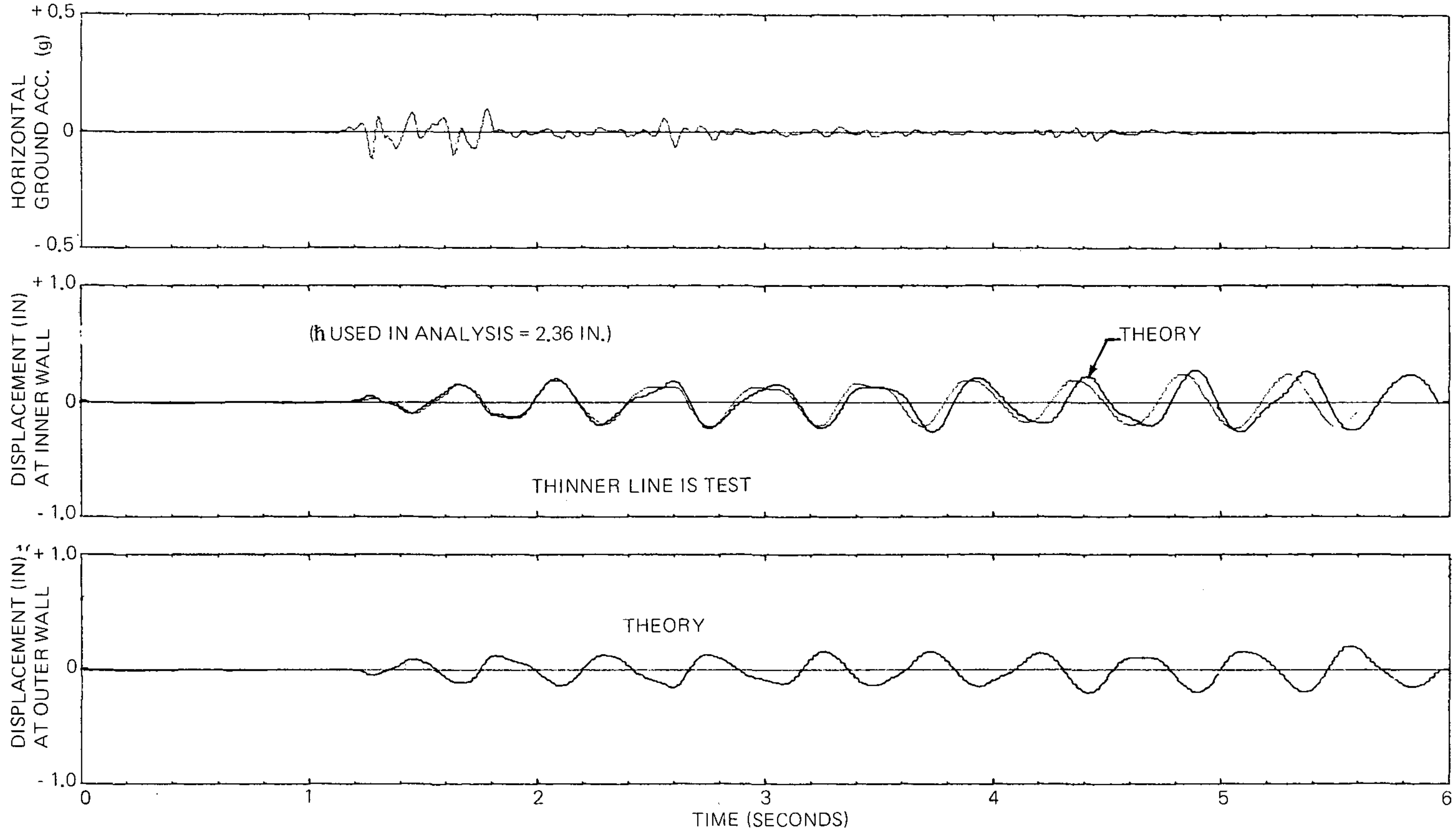


FIG. 2-5 SLOSHING RESPONSE OF WATER IN A TORUS TANK (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.) UNDER SIMULATED EL CENTRO 1940 EARTHQUAKE, TIME SCALE = 7.7, TEST NO. 220378.2. PEAK SHAKING TABLE ACCELERATION = 0.115g HORIZONTAL, 0.0g VERTICAL.

XBl 789-11310

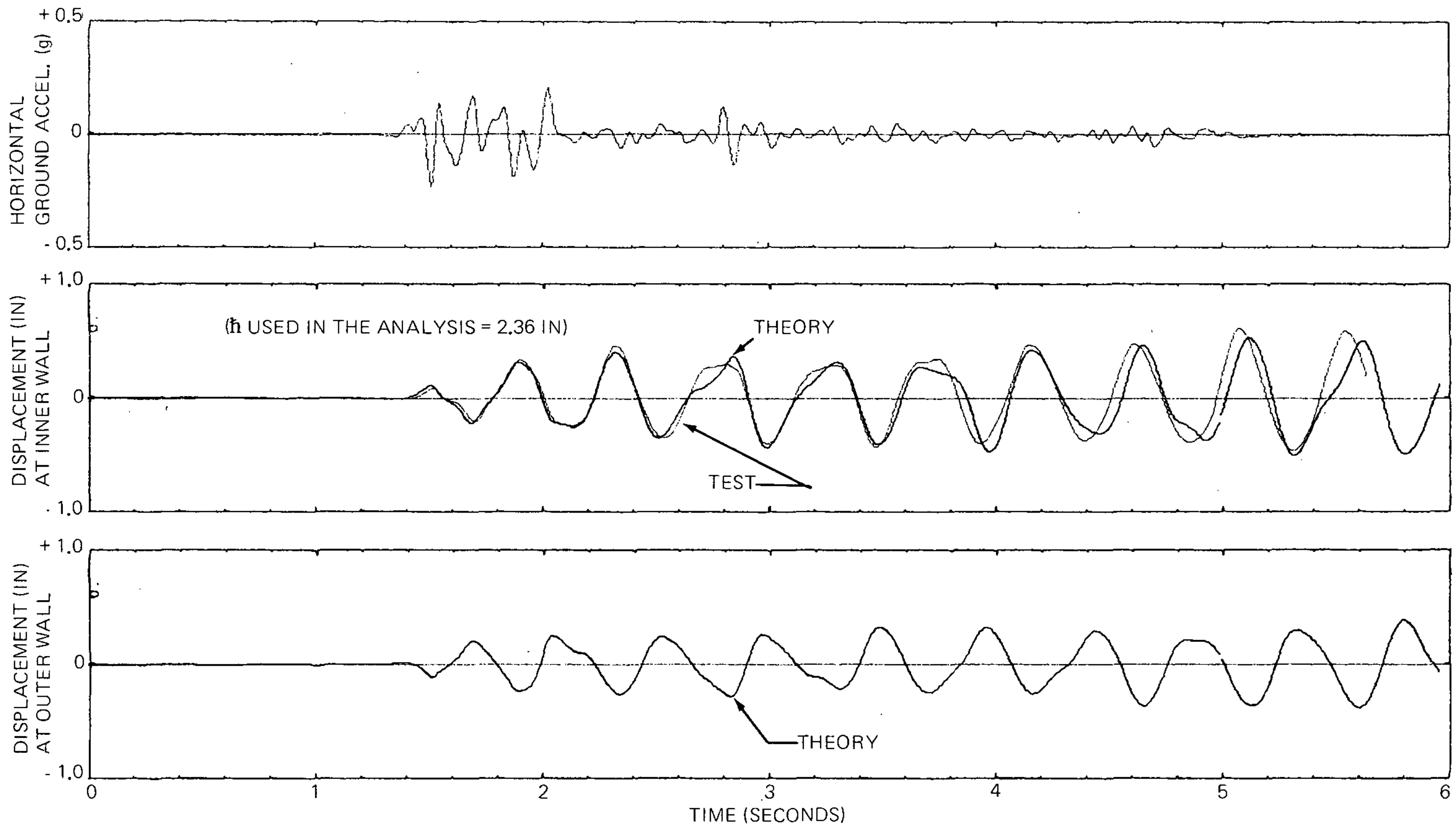


FIG. 2-6 SLOSHING RESPONSE OF WATER IN A TORUS TANK (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.) UNDER SIMULATED EL CENTRO 1940 EARTHQUAKE, TIME SCALE = 7.7, TEST NO. 220378.4. PEAK SHAKING TABLE ACCELERATION = 0.237g HORIZONTAL, 0.0g VERTICAL.

XBL 789-11311

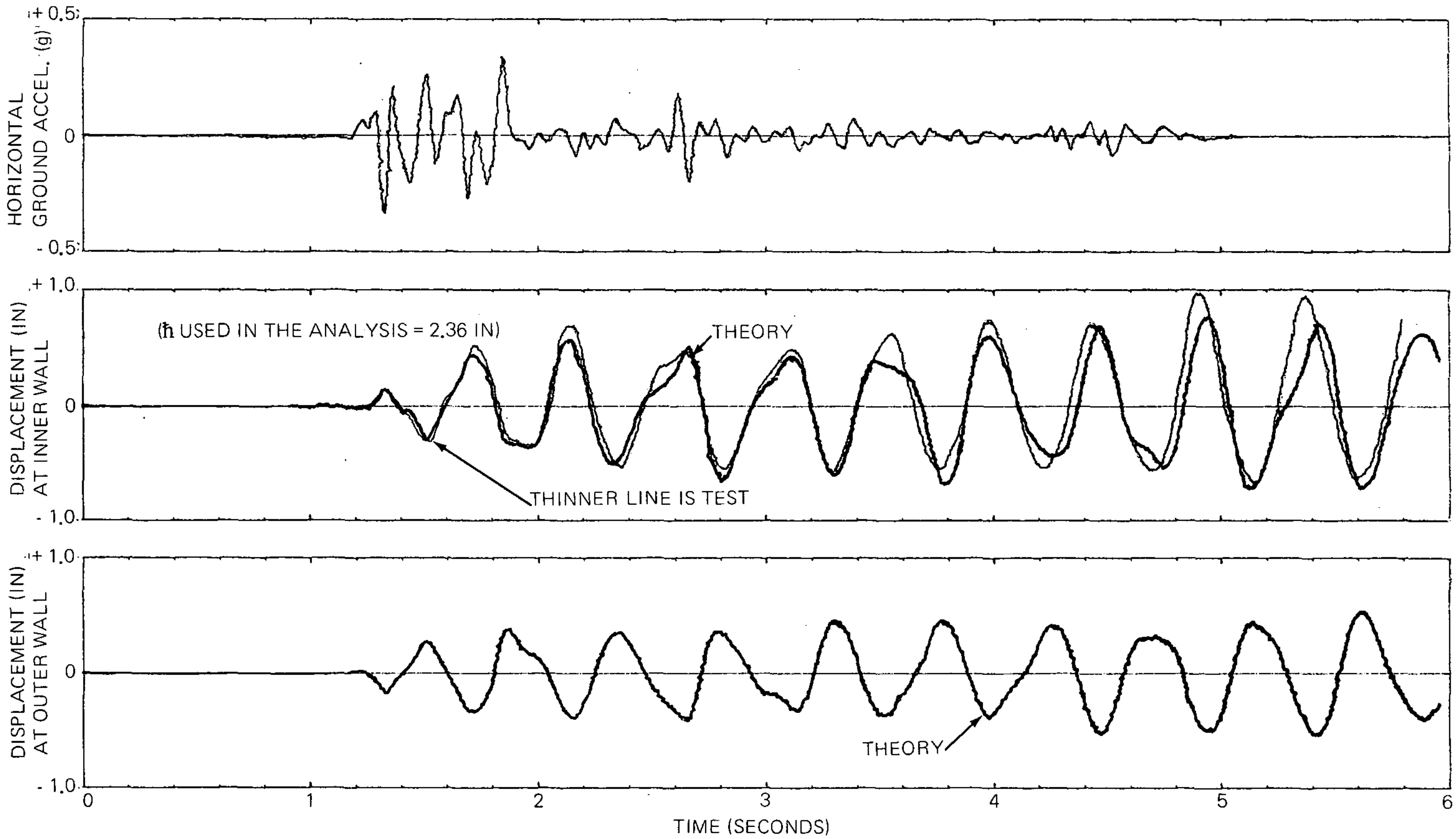


FIG. 2-7 SLOSHING RESPONSE OF WATER IN A TORUS TANK (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.) UNDER SIMULATED EL CENTRO 1940 EARTHQUAKE, TIME SCALE = 7.7, TEST NO. 220378.6. PEAK SHAKING TABLE ACCELERATION = 0.338g HORIZONTAL, 0.0g VERTICAL.

XBL 789-11312

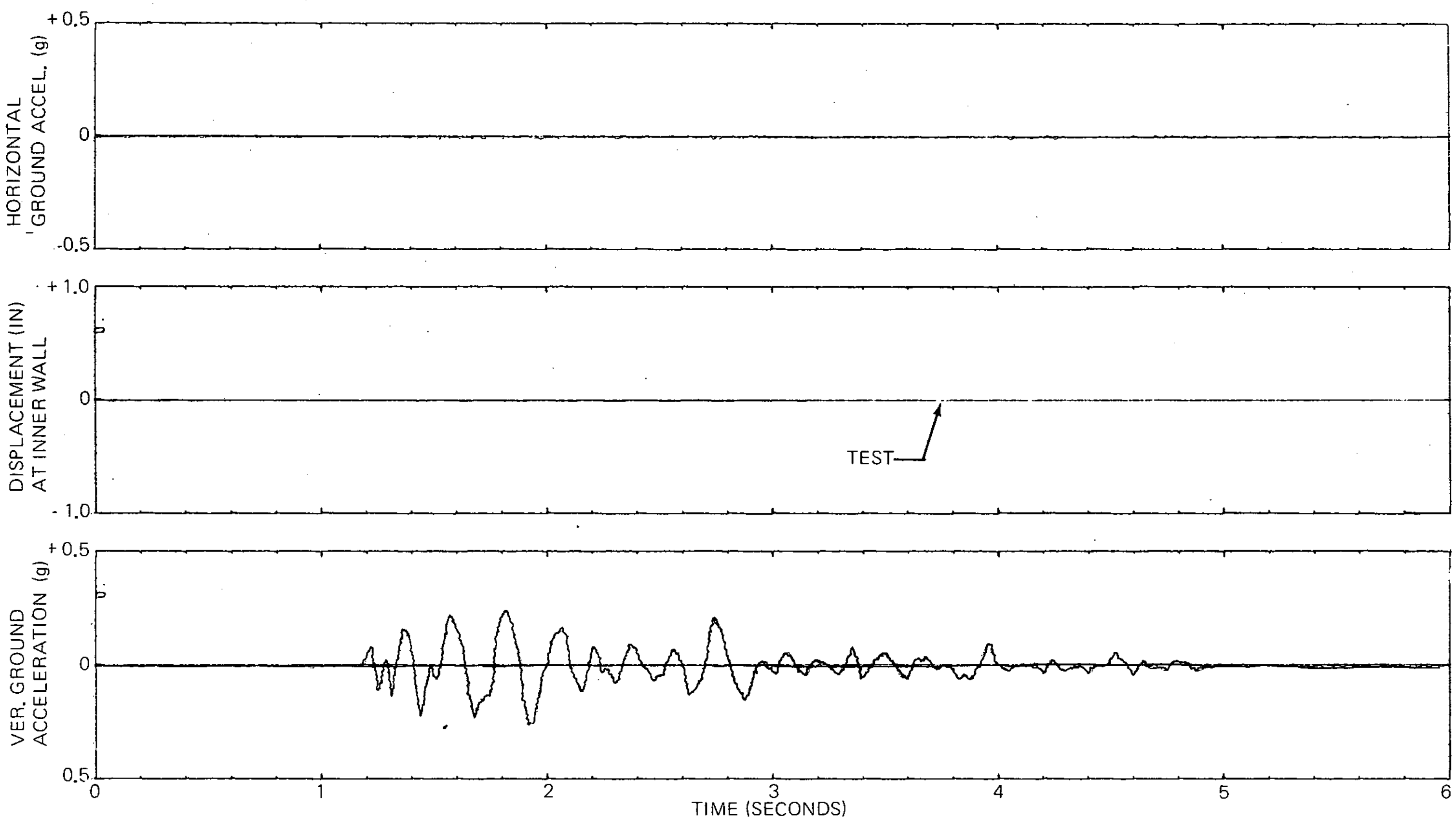


FIG. 2-8 SLOSHING RESPONSE OF WATER IN A TORUS TANK (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.) UNDER SIMULATED EL CENTRO 1940 EARTHQUAKE, TIME SCALE = 7.7, TEST NO. 220378.9. PEAK SHAKING TABLE ACCELERATION = 0.0g HORIZONTAL, 0.260g VERTICAL.

XBL 789-11313

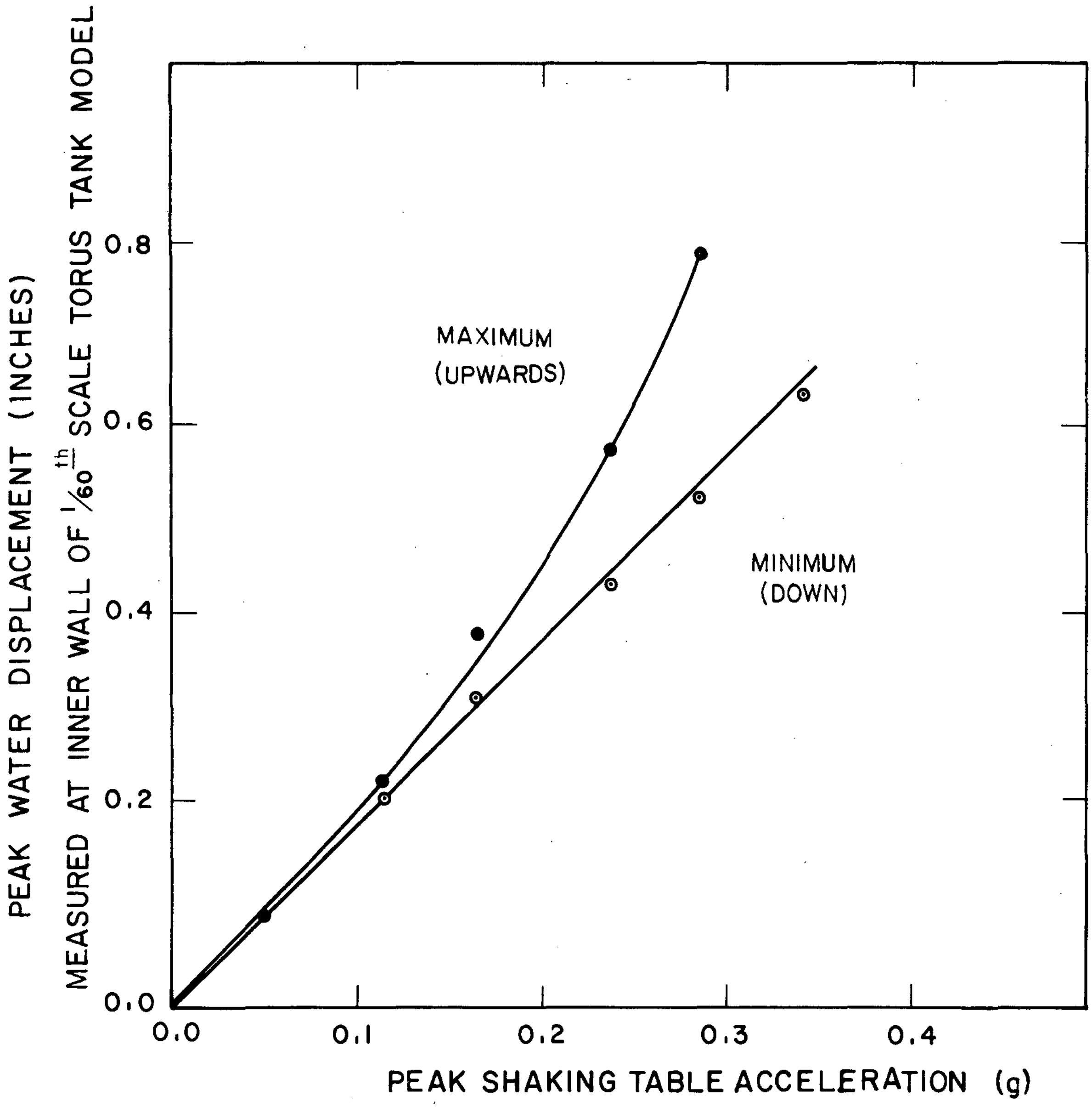


FIG. 2-9 VARIATION OF PEAK WATER DISPLACEMENTS WITH INTENSITY OF SIMULATED EL CENTRO 1940 EARTHQUAKE GROUND MOTION, TIME SCALE = $\sqrt{60}$

XBL 789-2276

3. FINITE ELEMENT ANALYSIS OF EARTHQUAKE INDUCED SLOSHING IN AXISYMMETRIC TANKS

3.1 Introduction

The finite element analysis [18] has become a powerful tool in solving complex engineering problems. Since the finite element method is completely general, it was decided that instead of looking for a closed form solution to predict the sloshing displacements and hydrodynamic pressures due to earthquake ground motions in a torus tank, the finite element method would be a better alternative in that it would be more general and thus can be applied to tank shapes other than cylindrical and toroidal.

Previous work on the finite element analysis of sloshing in tanks was done by Edwards [19] in which the shell theory was used for the prediction of seismic stresses and displacements in a cylindrical tank filled with liquid, but the sloshing was not considered in this analysis.

Finite element analysis for liquid sloshing problems by Luck [20] gives only the mode shapes and frequencies in an elastic container. His analysis is based on the variational principle suggested by Tong [21].

In this investigation our main concern is to study the sloshing effects in pressure-suppression pools of boiling water reactors, namely the Mark I torus and the Mark III annular suppression pools. Such structures can be considered as effectively rigid for the sloshing problem and thus the coupled effect of water-structure interaction is neglected in this analysis. Also the nonlinear sloshing problem has been linearalized [13] for this analysis.

The finite element equations were first derived for a completely general three dimensional problem and then were specialized to an axisymmetric tank subjected to arbitrary horizontal ground motions. The finite element equations were derived using the Galerkin principle [22]. More background information on Galerkin method may be found in References [18] and [23-26].

3.2 Mathematical Formulation:

3.2.1 Equation of motion

Consider a tank of arbitrary shape with rigid walls filled with a liquid whose free surface area is B2 as shown in Fig. 3-1. B1 represents the surface area of liquid in contact with the solid boundary of the container. V is the volume of the liquid and δ is the surface water displacement with respect to the undisturbed liquid surface. Using the same assumptions as in Ref. [1] Sec. 2.2, the velocity potential ϕ exists at every point in V and must satisfy the Laplace equation which in rectangular coordinates can be written as:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (3\text{-}1)$$

where $\phi = \phi\ (x, y, z, t)$.

Equation (3-1) will be solved by the finite element method subject to the appropriate time dependent boundary conditions as specified below.

3.2.2 Boundary conditions

Let $v_n(t)$ be the velocity of the tank wall along its outward normal to the boundary at any pount, then:

$$\frac{\partial \phi}{\partial n} = v_n(t) \quad \text{on B1} \tag{3-2}$$

where n is the outward normal to the solid boundary and v_n is a function of time t.

It can also be shown that a liquid particle on the free surface B2 must satisfy the following two conditions [13]

$$\frac{\partial \phi}{\partial x}\frac{\partial \delta}{\partial x} - \frac{\partial \phi}{\partial z} + \frac{\partial \phi}{\partial y}\frac{\partial \delta}{\partial y} + \frac{\partial \delta}{\partial t} = 0 \tag{3-3}$$

and

$$g\delta + \frac{\partial \phi}{\partial t} + \frac{1}{2}\left[(\frac{\partial \phi}{\partial x})^2 + (\frac{\partial \phi}{\partial y})^2 + (\frac{\partial \phi}{\partial z})^2\right] = 0 \tag{3-4}$$

where g is the acceleration of gravity.

Equations (3-3) and (3-4) which represent the non-linear free surface boundary conditions can be simplified and combined into one boundary condition by neglecting higher order terms and eliminating δ. This single linearized boundary condition [13] can be written as follows.

$$\frac{1}{g}\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial \phi}{\partial z} = 0 \quad \text{on B2.} \tag{3-5}$$

3.3 Finite Element Formulation

3.3.1 Derivation of finite element equations

In the finite element analysis, the continuum is divided into discrete elements or subregions which are interconnected at a finite number of points called nodes. The necessary formulation follows the Galerkin principle where we let the unknown field variable ϕ, throughout the solution domain, be approximated as

$$\phi = \sum_{1}^{N} N_j(x, y, z)\ \phi_j(t) \tag{3-6}$$

in which N_j are the shape functions defined piecewise, element by element, and $\phi_j(t)$ are the time dependent nodal values of the field variable i.e., the velocity potential in this case. In the summation process an appropriate function for the particular point in space must be used. The N nodal values ϕ_j are obtained by solving a set of N simultaneous equations each derived by equating the boundary and interior residuals calculated by multiplying with a weighting function and integrating over the domain. In the Galerkin approach, the shape functions are taken as the weighting functions and for a typical node i substituting Eq. (3-6) into Eqs. (3-1), (3-2) and (3-5) and equating the weighted and integrated interior and boundary residual, we have:

$$\begin{aligned}\int_V N_i\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right\}\sum_{1}^{N} N_j\phi_j dv &= \int_{B1} N_i \frac{\partial}{\partial n}\sum_{1}^{N} N_j\phi_j ds \\ &- \int_{B1} N_i v_n ds + \int_{B2}\left\{\frac{N_i}{g}\ \frac{\partial^2}{\partial t^2}\sum_{1}^{N} N_j\phi_j + N_i\frac{\partial}{\partial z}\sum_{1}^{N} N_j\phi_j\right\} ds\end{aligned} \tag{3-7}$$

in which $\int dv$ and $\int ds$ represent the integrals over the volume and appropriate surfaces respectively. Consider the first term on the left hand side of Eq. (3-7) and write it in the following form

$$\int_v N_i \sum_1^N \frac{\partial^2 N_j}{\partial x^2} \phi_j \, dv = \int_v \frac{\partial}{\partial x}\left(N_i \sum_1^N \frac{\partial N_j}{\partial x}\right) \phi_j dv - \int_v \frac{\partial N_i}{\partial x} \sum_1^N \frac{\partial N_j}{\partial x} \phi_j dv . \tag{3-8}$$

Applying the Divergence theorem on the first integral on the right hand side of Eq. (3-8), we can rewrite it in the following form.

$$\int_v N_i \sum_1^N \frac{\partial^2 N_j}{\partial x^2} \phi_j \, dv = \int_B N_i \sum_1^N \frac{\partial N_j}{\partial x} \ell_x \phi_j \, ds - \int_v \frac{\partial N_i}{\partial x} \sum_1^N \frac{\partial N_j}{\partial x} \phi_j \, dv \tag{3-9}$$

in which B = B1 + B2 and ℓ_x is the direction cosine in the x-direction of the outward normal n. Similar expression can be written for the second and third terms in Eq. (3-7). Substituting Eq. (3-9) and similar expressions into the right hand side of Eq. (3-7), we get

$$\begin{aligned}
&\int_B N_i\left(\sum_1^N \frac{\partial N_j}{\partial x} \ell_x \phi_j + \sum_1^N \frac{\partial N_j}{\partial y} \ell_y \phi_j + \sum_1^N \frac{\partial N_j}{\partial z} \ell_z \phi_j\right) ds \\
&- \int_v\left(\frac{\partial N_i}{\partial x} \sum_1^N \frac{\partial N_j}{\partial x} \phi_j + \frac{\partial N_i}{\partial y} \sum_1^N \frac{\partial N_j}{\partial y} \phi_j + \frac{\partial N_i}{\partial z} \sum_1^N \frac{\partial N_j}{\partial z} \phi_j\right) dv \qquad (3\text{-}10) \\
&= \int_{B1} N_i \sum_1^N \frac{\partial N_j}{\partial n} \phi_j ds - \int_{B1} N_i v_n ds + \frac{1}{g} \int_{B2} N_i \sum_1^N N_j \ddot{\phi} \, ds + \int_{B2} N_i \sum_1^N \frac{\partial N_j}{\partial z} \phi_j ds
\end{aligned}$$

in which $\ddot{\phi} = d^2\phi/dt^2$.

The boundary integral on the left hand side of Eq. (3-10) can be substituted by

$$\int_B N_i \sum_1^N \frac{\partial N_j}{\partial n} \phi_j \, ds \qquad (3\text{-}11)$$

or by

$$\int_{B1} N_i \sum_1^N \frac{\partial N_j}{\partial n} \phi_j \, ds + \int_{B2} N_i \sum_1^N \frac{\partial N_j}{\partial n} \phi_j \, ds \, . \qquad (3\text{-}12)$$

Replacing the boundary integral of left hand side of Eq. (3-10) with Eq. (3-12) and using the approximation (small slopes)

$$\frac{\partial N_i}{\partial z} \approx \frac{\partial N_i}{\partial n} \qquad \text{on B2, Eq. (3-10)}$$

can be simplified to the following form.

$$\int_v \left[\frac{\partial N_i}{\partial x} \sum_1^N \frac{\partial N_j}{\partial x} \phi_j + \frac{\partial N_i}{\partial y} \sum_1^N \frac{\partial N_j}{\partial y} \phi_j + \frac{\partial N_i}{\partial z} \sum_1^N \frac{\partial N_j}{\partial z} \phi_j \right] dv$$

$$+ \frac{1}{g} \int_{B2} N_i \sum_1^N N_j \ddot{\phi}_j \, ds = \int_{B1} N_i v_n \, ds \qquad (3\text{-}13)$$

or

$$\underline{M} \, \ddot{\underline{\phi}} + \underline{K} \, \underline{\phi} = \underline{F} \qquad (3\text{-}14)$$

in which the elements of $\underline{M}$, $\underline{K}$ and $\underline{F}$ are given by

$$M_{ij} = \frac{1}{g} \sum \int_{EB2} N_i N_j \, ds \qquad (3\text{-}15)$$

$$K_{ij} = \sum \int_{EV} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right) dv \qquad (3\text{-}16)$$

$$F_i = \sum \int_{EB1} N_i v_n \, ds \qquad (3\text{-}17)$$

where summation for M_{ij} covers only the elements on the free surface boundary and the integral is carried out on the free surface of each element EB2. Summation for K_{ij} covers the contribution of each element and EV is the element region. EB1 refers only to the elements which lie on the solid boundary, B1, and the loading term thus is associated with the elements that lie on the tank wall boundary.

The free surface matrix $\underline{M}$ and the fluid matrix $\underline{K}$ are comparable to the mass and stiffness matrices respectively used in structural dynamics. It is interesting to note that the free surface matrix $\underline{M}$ gets the contribution only from the free surface elements. $\underline{M}$ and $\underline{K}$ are symmetric matrices and Eq. (3-14) is a set of second order linear differential equations which can be solved either by direct integration or by mode superposition.

3.3.2 Isoparametric formulation for axisymmetric tank under arbitrary horizontal ground motions

The following analysis will be restricted to rigid tanks which are symmetrical about the z - axis and are subjected to arbitrary horizontal ground motions alone. Since Eq. (3-13) involves only the first derivatives of shape functions, a 4-node quadrilateral element with linear interpolation functions will satisfy the convergence requirements. However, the more recently developed 4-to-8 variable node isoparametric element [27] has greater flexibility in accommodating the curved boundaries and is convenient for numerical integration. Therefore a variable 4-to-8 node, 2-dimensional isoparametric element will be used in the present formulation. Such an element can be used to model an axisymmetric problem or a two dimensional problem such as sloshing in a rectangular tank.

Figure 3-2(a) shows such a 4-to-8 variable node element lying in the x-z plane where z is the axis of symmetry. Any of the mid-side nodes 5 through 8 may or may not be present and can be eliminated if desired. Such a curvilinear 4-to-8 node element can be obtained by using an iso-parametric mapping from a bi-unit square which has a local r-s coordinate system as shown in Fig. 3-2(b). The local coordinates r and s vary between -1 and +1. The nodes 1 through 4 are the corner nodes and the nodes 5 through 8 are the mid-side nodes corresponding to Fig. 3-2(a). Node 1 has the coordinates (1,1), The mapping between the local coordinate system (r,s) and the global (x,z) coordinate system must be unique in order to carry out the transformations properly. The coordinate transformation between the bi-unit square and the curvilinear element is given by;

$$x_m(r,s) = \sum_{i=1}^{8} h_i(r,s)\, x_{im} \tag{3-18}$$

$$z_m(r,s) = \sum_{i=1}^{8} h_i(r,s)\, z_{im} \tag{3-19}$$

in which (x_{im}, z_{im}) are the global coordinates of node i in element m and h_i are the interpolation functions in local coordinates corresponding to node m. The interpolation functions h_i for any element m are defined as follows:

$$\begin{aligned}
h_1 &= \frac{1}{4}(1+r)(1+s) - \frac{1}{2}h_5 - \frac{1}{2}h_8 \\
h_2 &= \frac{1}{4}(1-r)(1+s) - \frac{1}{2}h_5 - \frac{1}{2}h_6 \\
h_3 &= \frac{1}{4}(1-r)(1-s) - \frac{1}{2}h_6 - \frac{1}{2}h_7 \\
h_4 &= \frac{1}{4}(1+r)(1-s) - \frac{1}{2}h_7 - \frac{1}{2}h_8 \\
h_5 &= \frac{1}{2}(1-r^2)(1+s) \\
h_6 &= \frac{1}{2}(1-r)(1-s^2) \\
h_7 &= \frac{1}{2}(1-r^2)(1-s) \\
h_8 &= \frac{1}{2}(1+r)(1-s^2).
\end{aligned} \tag{3-20}$$

Since a horizontal ground motion will excite only the antisymmetric modes (in the linearized case) of sloshing in an axisymmetric tank, we can approximate the distribution of the velocity potential within an element m in terms of the velocity potential at node 1 through 8 and the interpolation functions given by Eq. (3-20)

$$\phi_m(r,s,\theta,t) = \sum_{n=1}^{\infty}\sum_{i=1}^{8} h_i(r,s)\cos n\theta\,\phi_{im}(t) \tag{3-21}$$

in which ϕ_{im} is the value of the velocity potential at node i of element m.

It is obvious that if this shape function given by Eq. (3-21) is used in Eq. (3-17) to calculate the loading vector, the integral between the limits 0 and 2π will be non-zero only when n = 1 in the case of horizontal ground motion only, because v_n in such a case varies as a function of cosθ and the $\int_0^{2\pi} \cos\theta \cdot \cos n\theta \, d\theta = 0$ for any $n \neq 1$. Therefore Eq. (3-21) can be written as

$$\phi_m(r,s,\theta,t) = \sum_{i=1}^{8} h_i(r,s) \cos\theta \cdot \phi_{im}(t) \tag{3-22}$$

and

$$\frac{\partial}{\partial r} \phi_m(r,s,\theta,t) \sum_{i=1}^{8} \frac{\partial}{\partial r} h_i \cos\theta \cdot \phi_{im}(t) \tag{3-23}$$

$$\frac{\partial}{\partial s} \phi_m(r,s,\theta,t) = \sum_{i=1}^{8} \frac{\partial}{\partial s} h_i \cos\theta \cdot \phi_{im}(t) \tag{3-24}$$

or in matrix form, the above two equations can be written as

$$\begin{Bmatrix} \frac{\partial \phi}{\partial r} \\ \frac{\partial \phi}{\partial s} \end{Bmatrix}_m = \cos\theta \begin{bmatrix} \underset{2\times 8}{\underline{p_m}}(r,s) \end{bmatrix} \underset{8\times 1}{\underline{\phi_m}}(t) \tag{3-25}$$

in which $\underline{p_m}(r,s)$ contains the derivatives of interpolation functions derived from Eq. (3-20). Using the chain rule of differentiation, we can relate the global derivatives to the local derivatives.

$$\begin{bmatrix} \frac{\partial \phi}{\partial r} \\ \frac{\partial \phi}{\partial s} \end{bmatrix}_m = \underbrace{\begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial z}{\partial s} \end{bmatrix}}_{\text{Jacobian matrix}} \begin{bmatrix} \frac{\partial \phi}{\partial x} \\ \frac{\partial \phi}{\partial z} \end{bmatrix}_m \tag{3-26}$$

by inverting

$$\begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \\ \frac{\partial\phi}{\partial z} \end{bmatrix}_m = \underline{J}_m^{-1} \begin{bmatrix} \frac{\partial\phi}{\partial r} \\ \\ \frac{\partial\phi}{\partial s} \end{bmatrix}_m \tag{3-27}$$

in which $\underline{J}_m^{-1}$ is the inverse of the Jacobian matrix in Eq. (3-26). Substituting for $\partial\phi/\partial r$ and for $\partial\phi$ ∂s from Eq. (3-25) into Eq. (3-27), we obtain the relationship between the global derivatives and nodal values of ϕ.

$$\begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \\ \frac{\partial\phi}{\partial z} \end{bmatrix}_m = \underset{2\times2}{\underline{J}_m^{-1}} \times \underset{2\times8}{\underline{p}_m} (r,s) \; \underset{8\times1}{\underline{\phi}_m}(t) \cdot \cos\theta \tag{3-28}$$

or

$$\begin{bmatrix} \frac{\partial\phi}{\partial x} \\ \\ \frac{\partial\phi}{\partial z} \end{bmatrix}_m = \underline{B}_m \; \underline{\phi}_m(t) \; \cos\theta \tag{3-29}$$

in which $\underline{B}_m = \underline{J}_m^{-1} \; \underline{p}_m$. (3-30)

Thus

$$\begin{Bmatrix} \frac{\partial N}{\partial x} \\ \\ \frac{\partial N}{\partial y} \end{Bmatrix}_m = \underline{B}_m \cos\theta \tag{3-31}$$

3.3.3 Free surface (mass) matrix for axisymmetric element

The complete free surface matrix for the system is formed by direct summation of individual element matrices i.e.,

$$\underline{M} = \frac{1}{g} \sum_{m=1}^{n} \underline{M}_m \tag{3-32}$$

where n is the total number of free surface elements, and the element matrix $\underline{M}_m$ is a 2 × 2 matrix given by

$$M_m = \int_{EB2} \underline{N}_m^T \underline{N}_m \, ds . \tag{3-33}$$

In case of axisymmetric case the surface integral can be transformed to a line integral. Consider a free surface element with nodes i and j (Fig. 3-3) and let

r = local coordinates for free surface boundary element

R(r) = radius to any point r between node i and j

L = length of the elements

$$L = \sqrt{(x_i - x_j)^2 + (z_i - z_j)^2} . \tag{3-34}$$

Assuming that x-axis coincides with the horizontal plane, we can write the transformation.

$$R(r) = \begin{bmatrix} (1 - r/L) & r/L \end{bmatrix} \begin{Bmatrix} x_i \\ x_j \end{Bmatrix} . \tag{3-35}$$

$$ds = R(r) \, dr d\theta \tag{3-36}$$

where θ is defined in Fig. 3-2

$$ds = \begin{bmatrix} 1 - r/L & r/L \end{bmatrix} \begin{Bmatrix} x_i \\ x_j \end{Bmatrix} dr d\theta . \tag{3-38}$$

Thus

$$\underline{N} = \left[1 - r/L \quad r/L \right] \cos\theta. \tag{3-39}$$

Therefore

$$\underline{M}_m = \int_0^{2\pi} \int_0^{L} \begin{Bmatrix} 1-r/L \\ r/L \end{Bmatrix} \left[(1-r/L) \quad r/L \right] R(r) \cos^2\theta \cdot dr d\theta. \tag{3-40}$$

$$\underline{M}_m = \pi \int_0^{L} \begin{Bmatrix} 1-r/L \\ r/L \end{Bmatrix} \left[(1-r/L) \quad r/L \right] R(r) \; dr d\theta. \tag{3-41}$$

Evaluation of the above matrix gives

$$M_{ii} = \frac{\pi L}{4} \left[x_i + \frac{x_j}{3} \right] \tag{3-42}$$

$$M_{jj} = \frac{\pi L}{4} \left[\frac{x_i}{3} + x_j \right] \tag{3-43}$$

$$M_{ij} = \frac{\pi L}{12} \left[x_i + x_j \right] \tag{3-44}$$

3.3.4 Evaluation of fluid (stiffness) matrix for axisymmetric element

The complete system fluid matrix is formed by direct summation of element matrices

$$\underline{K} = \sum_{m=1}^{n} \underline{K}_m \tag{3-45}$$

where n is the number of liquid elements and the element matrix $\underline{K}_m$ is obtained by using Eq. (3-31) and Eq. (3-16).

$$\underline{K}_m = \int_{EV} \cos^2\theta \; \underline{B}_m^T \underline{B}_m \; dv \tag{3-46}$$

In case of an axisymmetric element, dv = R dθdr, the volume integral then becomes

$$\underline{K}_m = \int_0^{2\pi} \int_{A_m} \cos^2\theta \quad R\, \underline{B}_m^T \underline{B}_m \, dA d\theta \tag{3-47}$$

where R is the radius of any point and A is the area.

In the natural coordinate system $dA = |J_m|\, dsdr$ where $|J_m|$ is the determinant of the Jacobian matrix, therefore

$$\underline{K}_m = \pi \int_{-1}^{1} \int_{-1}^{1} R\, \underline{B}_m^T \underline{B}_m \, |J_m| \, dsdr. \tag{3-48}$$

The above integral can be evaluated numerically using Gaussian quadrature as follows [18]

$$\underline{K}_m = \sum_{j=1}^{N} \sum_{i=1}^{N} H_i H_j \underline{f} (r_i, s_i) \tag{3-49}$$

where N refers to the order of integration; H_i and H_j are the weighting factors and

$$\underline{f} (r_i, s_i) = \underline{B}_m^T (r_i, s_i)\, \underline{B}_m (r_i, s_i) \left| J_m (r_i, s_i) \right| R\, (r_i, s_i) \tag{3-49}$$

3.3.5 Load vector for axisymmetric element

The loading vector $\underline{F}$ for the complete system is the sum of the contribution of individual elements and using Eq. (3-17) we can write

$$\underline{F} = \sum_{m=1}^{n} \int_{EB1} N_m^T v_n \, ds = \sum_{m=1}^{n} \underline{F}_m \tag{3-50}$$

where

$$\underline{F}_m = \int_{EB1} N_m^T v_n \, ds \tag{3-51}$$

where summation is over all the n elements which are at the liquid-solid interface.

Consider a typical liquid-solid boundary element with nodes i and j and let r be a local coordinate as shown in Fig. 3-4. Let R(r) be the radius at any point r and L be the length of the element where $L = \sqrt{(x_i - x_j)^2 + (z_i - z_j)^2}$.

$$R(r) = \left[(1-r/L) \quad r/L \right] \begin{Bmatrix} X_i \\ x_j \end{Bmatrix} . \tag{3-52}$$

$$\underline{N} = \left[(1-r/L) \quad r/L \right] \cos\theta \tag{3-53}$$

$$ds = R(r)\, dr\, d\theta .$$

If v_x = horizontal ground velocity in the x-direction then $v_n = v_x \cos\theta \cdot \cos\psi$ where $\psi = \tan^{-1} \left(\frac{x_i - x_j}{z_i - z_j} \right)$; then

$$\underline{F}_m = v_x \int_0^L \int_0^{2\pi} \begin{Bmatrix} (1-r/L \\ r/L \end{Bmatrix} \left[(1-r/L) \quad r/L \right] \begin{Bmatrix} x_i \\ x_j \end{Bmatrix} \cos^2\theta \cdot \cos\psi \, d\theta \, dr \tag{3-54}$$

integrating we get the following element load vector

$$\underline{F}_m = \frac{\pi L v_x \cos\psi}{6} \begin{Bmatrix} 2x_i + x_j \\ x_i + 2x_j \end{Bmatrix} = \begin{Bmatrix} F_i \\ F_j \end{Bmatrix} . \tag{3-55}$$

The contribution to the loading matrix comes from those elements which lie at the liquid-solid interface.

3.4 Numerical Solution of Finite Element Equations

The discretization of the continuum into finite elements and the assemblage of free surface, liquid and loading element matrices results in a set of linear, coupled, second order ordinary differential equations. Since these equations are linear they can be uncoupled by an orthogonal transformation and the solution can be obtained using mode superposition [28, 29] or they can be solved by direct step-by-step integration. In this study Newmark's step by step integration method [30,29] which is based on the following expressions was used.

$$\dot{\phi}_{t+\Delta t} = \dot{\phi}_t + \Delta t \; (1-\delta) \; \ddot{\phi}_t + \Delta t \, \delta \, \ddot{\phi}_{t+\Delta t} \tag{3-56}$$

$$\phi_{t+\Delta t} = \phi_t + \Delta t \dot{\phi}_t + \Delta t^2 \left(\frac{1}{2} - \alpha\right) \ddot{\phi}_t + \Delta t^2 \; \alpha \; \ddot{\phi}_{t+\Delta t} \tag{3-57}$$

in which Δt is step size. α and δ are parameters which are selected to produce the desired stability and accuracy. In all the sample analyses carried out in this investigation, Newmark's constant-average-acceleration method ($\delta = 1/2$ and $\alpha = 1/4$) was used, which is an unconditionally stable method without numerical damping.

This is an implicit method and satisfies the equilibrium equations at time $t + \Delta t$, i.e.,

$$\underline{M} \, \underline{\ddot{\phi}}_{t+\Delta t} + \underline{K} \, \underline{\phi}_{t+\Delta t} = \underline{F}_{t+\Delta t} \tag{3-58}$$

The above three equations can be combined into a step-by-step algorithm which involves the solution of a set of equations at each time step of the form.

$$\underline{K}^* \, \underline{\phi}_{t+\Delta t} = \underline{F}^* \tag{3-59}$$

In this analysis $\underline{K}^*$ is independent of time and is formed and triangularized only once. To make the numerical algorithm more general, the option of combining the Wilson Theta method [31] with the Newmark method was incorporated in the computer program. The Wilson Theta method was first applied to Newmark's linear accelerator method in order to improve stability and to damp out high frequency oscillations which often develop in step by step integration. A summary of the Newmark-Wilson algorithm used in the computer program is given below.

3.4.1 The Newmark-Wilson algorithm for linear step-by-step integration

INITIAL CALCULATIONS

1. Initialize ϕ_0 and $\dot{\phi}_0$ (taken to be zero) (ϕ is a vector)
2. Form the free surface and fluid matrices ($\underline{M}$, $\underline{K}$)
3. Specify algorithm parameters α, δ and θ
4. Calculate integration constants.

$$\tau = \theta\Delta t \qquad a_3 = \frac{1}{2\alpha} - 1 \qquad a_7 = \Delta t\delta$$

$$a_0 = \frac{1}{\alpha\tau^2} \qquad a_4 = \frac{\delta}{\alpha} - 1 \qquad a_8 = \Delta t^2 \left(\frac{1}{2} - \alpha\right)$$

$$a_1 = \frac{\delta}{\alpha\tau} \qquad a_5 = \frac{\tau}{2} (\delta/\alpha - 2) \qquad a_9 = \alpha\Delta t^2$$

$$a_2 = \frac{1}{\alpha\tau} \qquad a_6 = \Delta t\ (1-\delta)$$

5. Form $\underline{K}^* = \underline{K} + a_0\,\underline{M}$
6. Triangularize $\underline{K}^*$: $\underline{K}^* = \underline{LDL}^T$

FOR EACH TIME STEP

1. Calculate the effective load vector $\underline{F}^*$ at time $t+\tau$

$$\underline{F}^* = \underline{F}_{t+\tau} + \underline{M}\ (a_0\phi_t + a_2\dot{\phi} + a_3\ddot{\phi}_t).$$

2. Solve for velocity potential ϕ at $t + \tau$:

$$\underline{LDL}^T\ \phi_{t+\tau} = F^*$$

3. Calculate the velocity potential ϕ and its derivatives at time $t+\Delta t$:

$$\ddot{\phi}_{t+\tau} = a_0\ (\phi_{t+\tau} - \phi_t) - a_2\dot{\phi}_t - a_3\ddot{\phi}_t$$

$$\ddot{\phi}_{t+\Delta t} = \ddot{\phi}_t + \frac{1}{\theta}\ (\ddot{\phi}_{t+\tau} - \ddot{\phi}_t)$$

$$\dot{\phi}_{t+\Delta t} = \dot{\phi}_t + a_6\ddot{\phi}_t + a_7\ddot{\phi}_{t+\Delta t}$$

$$\phi_{t+\Delta t} = \phi_t + \Delta t\dot{\phi}_t + a_8\ddot{\phi}_t + a_9\ddot{\phi}_{t+\Delta t}$$

4. Determine the sloshing displacements and hydrodynamic impulsive pressures at time t + Δt

   Sloshing displacement $\delta_{t+\Delta t} = -\frac{1}{g}\dot{\phi}_{t+\Delta t}$

   Impulsive pressure $p_{t+\Delta t} = -\rho\,\dot{\phi}_{t+\Delta t}$

3.5 Computer Program 'SLOSH2'

The program 'SLOSH2' developed to implement the finite element theory of the sloshing phenomenon in axisymmetric tanks is coded in standard FORTRAN IV language. The basic set up of SLOSH2 is the same as that of the computer program DOT [32] because of the fact that the finite element formulation of the sloshing problem is similar in certain respects to that of the heat conduction equations.

The earthquake input can either be as an accelerogram or a displacement-time history, digitized in the appropriate format. The program derives the velocity-time history by integration or differentiation depending upon the type of ground motion input. The earthquake input must be properly adjusted for base-line correction such that at the end of earthquake as acceleration goes to zero, the ground velocity and displacement also go to zero.

The 'effective' equilibrium equations (Eq. (3-59) are solved using the linear equation solver COLSOL [33]. This subroutine processes only those elements which are within the skyline of $\underline{K}^*$, thus minimizing the storage requirements as well as the number of operations. This subroutine is based on Gauss elimination and requires a symmetrical positive-definite system of equations.

A compact storage scheme is used in the computer program whereby a one-dimensional array is used to store only those elements of K^* which are within its skyline. In the actual implementation of the computer program a lumped mass parameter system was used which not only simplifies the analysis considerably, but also minimizes the storage requirements. The three finite element groups namely, the free surface elements, the fluid elements and the liquid-solid interface elements, are processed in blocks and then stored on the disc for later use in order to increase the maximum capacity of the program.

It should be noted that the free surface elements and the liquid-solid interface elements which contribute to $\underline{M}$ and $\underline{F}$ respectively, are only two node elements for the axisymmetric case, whereas the liquid continuum itself is discretized by two-dimensional 4-to-8 node isoparametric elements which contribute to the fluid matrix $\underline{K}$.

In this computer program, a variable dimension is used for dynamic allocation of primary storage into a single array in blank common. The lower primary storage locations are used for the storage of each block of element group data which is read in from the secondary storage (disc) as required during the solution phase. The user has to supply the maximum estimated number of storage locations required to store any individual element group in the lowest primary storage. Usually this number is determined by the number of liquid elements that form the $\underline{K}$ matrix and not the free surface or liquid-solid interface elements. An estimation of the C.P.U. time required to run the program on CDC 6400 will be indicated in the next article. A user's manual and Fortran listing for SLOSH2 are given in Appendix A1 and A2 respectively.

3.6 Sample Analyses and Comparison with Test Data from Annular and Torus Tanks

Figures 3-5 and 3-6 show the finite element mesh layout for the 8 ft diameter annular tank and the 28 inches diameter torus tank models respectively which were studied in Ref. [1] and in Chapter 2 of this report. The annular tank chosen for this analysis is a simplified 1/15th scale model of pressure-suppression pool of Boiling Water Reactor GE Mark III (see Ref. [1] Figs. 5-3 and 5-7a) whereas the torus tank represents the 1/60th scale model (see Fig. 2-1) of the GE Mark I pressure-suppression pool. These analyses were carried out to check the accuracy of the finite element model against precise test data. A finer mesh size has been used near the free water surfaces because that is where the maximum sloshing displacements occur.

3.6.1 Annular tank

In Fig. 3-5 the x-axis is taken at the bottom of the tank and z-axis as the axis of symmetry of the annular tank. There are a total of 25, 10 and 17 elements in group Nos. 1, 2 and 3 respectively. Group No.1 contains 4-to-8 node elements whereas group Nos. 2 and 3 contain the free surface elements and liquid-solid interface elements respectively with two nodes each. There are a total of 52 nodes in this mesh. The finite element analysis was done using the digitized accelerogram recorded in Test No. 211276.1 of Ref. [1] Chapter 5 and a comparison of measured and predicted results is given in Figs. 3-7 and 3-8.

Figure 3-7 shows a comparison of the sloshing displacements at node 2 between the finite element solution and the test data under the same ground motion. The results are shown for the first six seconds and it can be seen that there is close agreement between the test and finite element results.

Figure 3-8 shows a comparison of impulsive pressures between the measured and finite element results, and it can be seen again that the agreement between the two is excellent with test results consistently 5-10% higher compared with the finite element solution indicating a possible error of calibration of pressure gage and some error due to electrical noise. This comparison is given at node 52 (Fig. 3.5).

3.6.2 Torus tank

The finite element mesh layout for the torus tank model (Fig. 3-6) has a total of 24, 12 and 24 elements in group Nos. 1, 2 and 3 respectively. The z-axis is taken as the axis of symmetry of the torus and the x-axis at the bottom of the tank. The total number of nodes in this case is 73.

The earthquake input used for the finite element analysis was the recorded shaking table displacement instead of acceleration because the displacements gave zero velocity and acceleration on differentiation at the end of the earthquakes whereas the integration of the accelerogram usually gives finite amount of velocity and displacement without a base line correction.

In comparing the finite element solution with the test data for the torus tank it should be remembered that the two systems are not exactly the same: the finite element solution is for a perfectly round tank, whereas the test tank was made of a set of 16 straight segments as shown in Fig. 2-1. Figures 3-9, 3-10 and 3-11 show the comparison of sloshing displacements at node No.2 for increasing intensity of ground motion. In Figs. 3-9 and 3-10 the agreement between the two results during the first half of the earthquake is excellent and for the rest of the earthquake quite satisfactory. In Fig. 3-11, although the sloshing response is relatively large, the linear finite element analysis still gives satisfactory results for practical purposes. The hydrodynamic pressures in this case were not measured on account of their relatively small magnitude in this small scale model.

The C.P.U. time required by the CDC 6400 to obtain the time-history response in the torus tank model analysis (Fig. 3-9) with the mesh size shown in Fig. 3-6 was 126 seconds.

3.7 Sample Analysis of Mark I Prototype Torus Tank Under El Centro 1940 Earthquake

Figure 3-12 shows the sloshing response in the prototype of Mark I torus under the full intensity of the 1940 El Centro earthquake (N - S component). The mesh layout for this analysis was similar to that shown in Fig. 3-6 except the overall dimensions which in this case are 60 times larger. The sloshing displacement shown in Fig. 3-12 is at node #2 with a maximum value of 24.3".

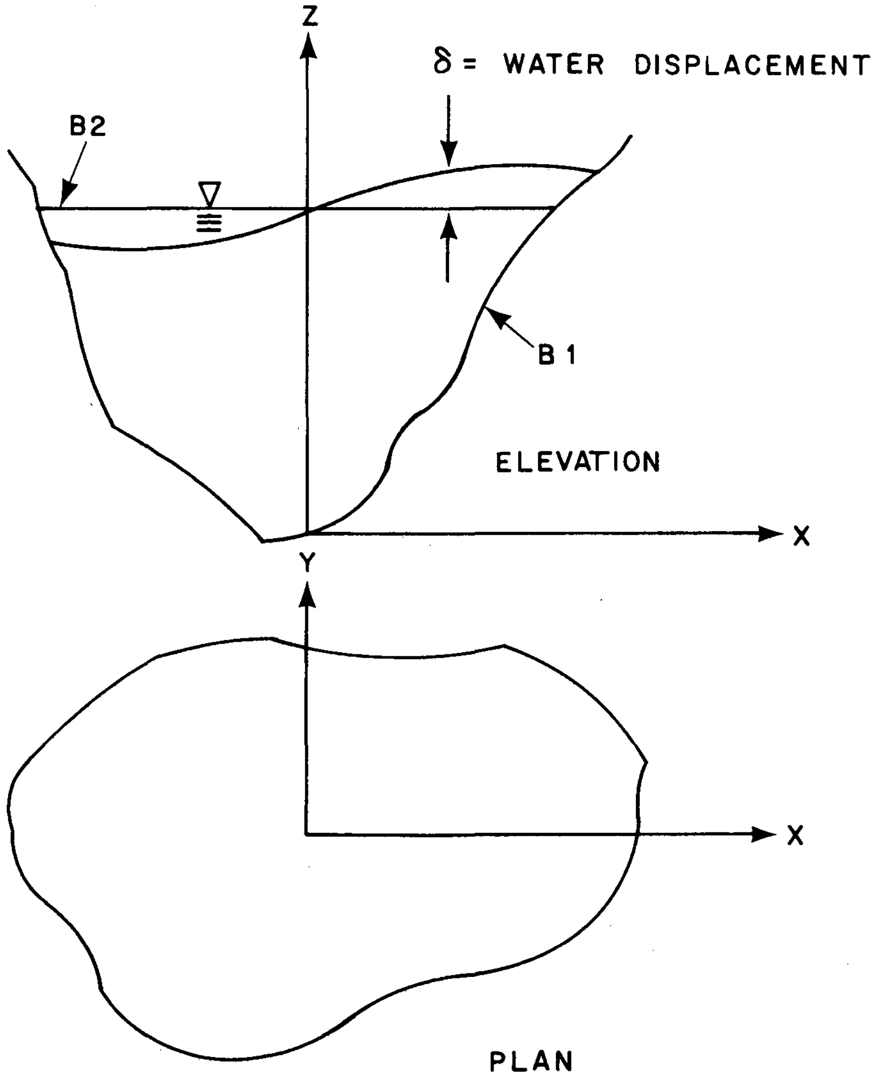
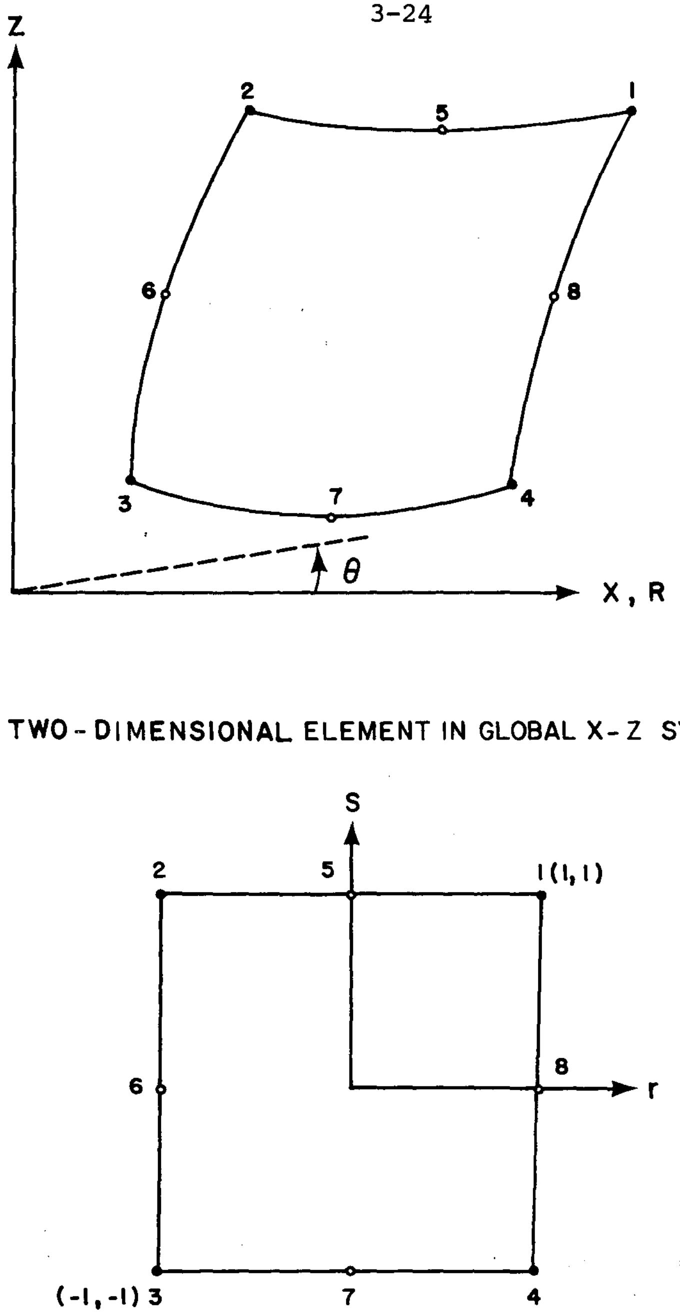


FIG. 3-1 TANK OF ARBITRARY SHAPE FILLED WITH LIQUID

XBL 789-2282



(a) TWO-DIMENSIONAL ELEMENT IN GLOBAL X-Z SYSTEM

(b) BI-UNIT SQUARE IN LOCAL r-S SYSTEM

FIG. 3-2 TWO-DIMENSIONAL MAPPING OF AN ISOPARAMETRIC ELEMENT

XBL 789-2275

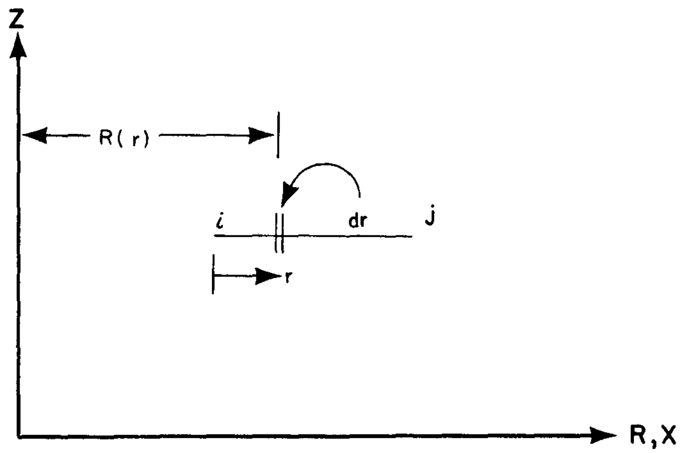


FIG. 3-3 FREE SURFACE ELEMENT (AXISYMMETRIC CASE)

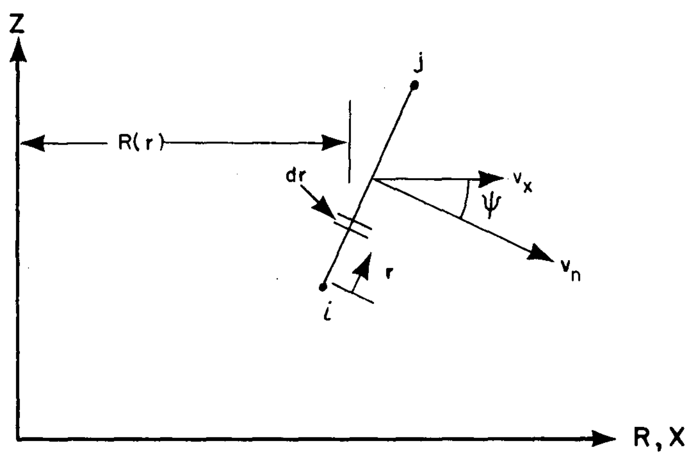


FIG. 3-4 BOUNDARY ELEMENT AT LIQUID-SOLID INTERFACE
(AXISYMMETRIC CASE)

XBL 789-2277

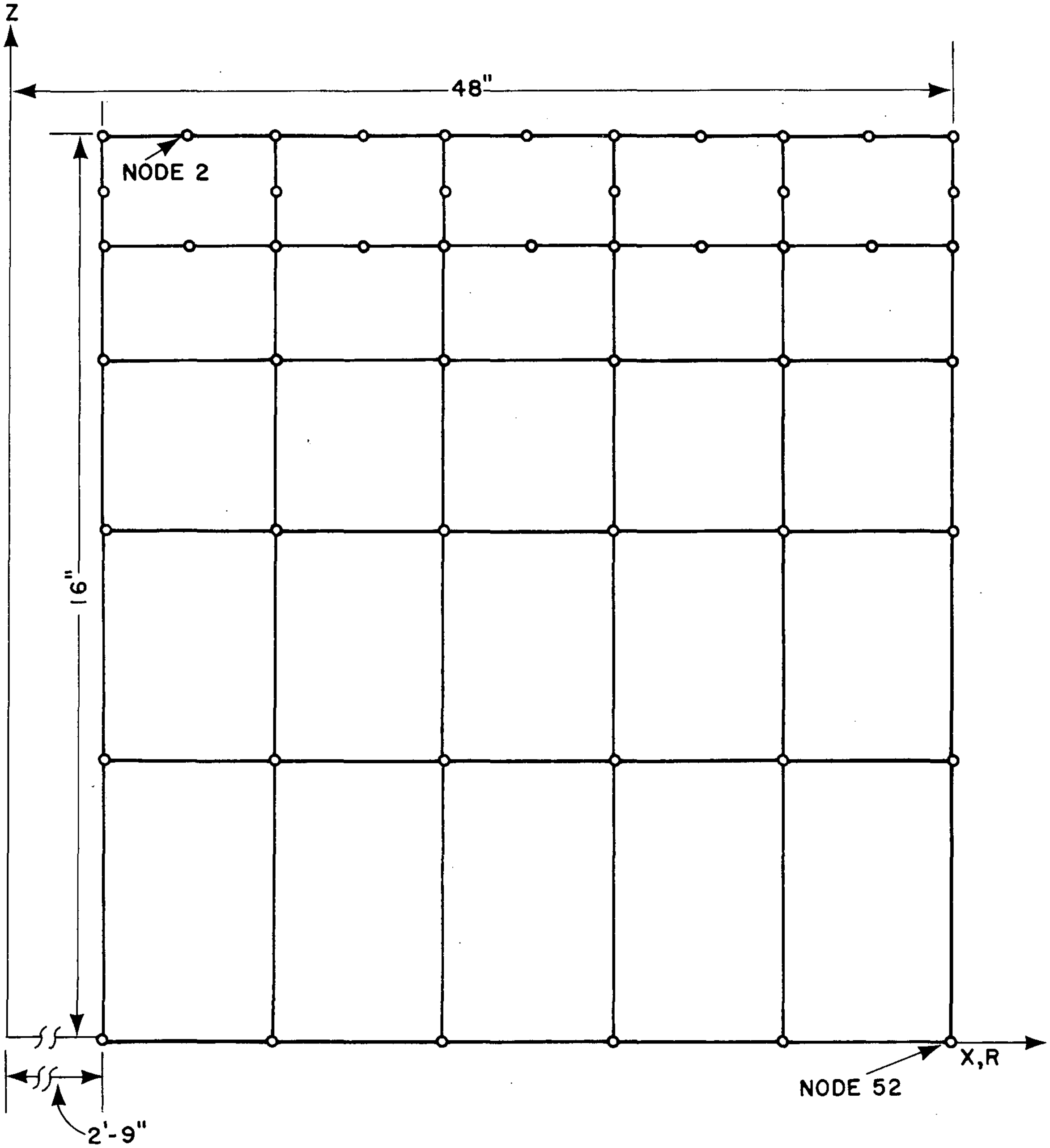


FIG. 3-5 FINITE ELEMENT MESH FOR ANNULAR TANK MODEL

XBL 789-2281

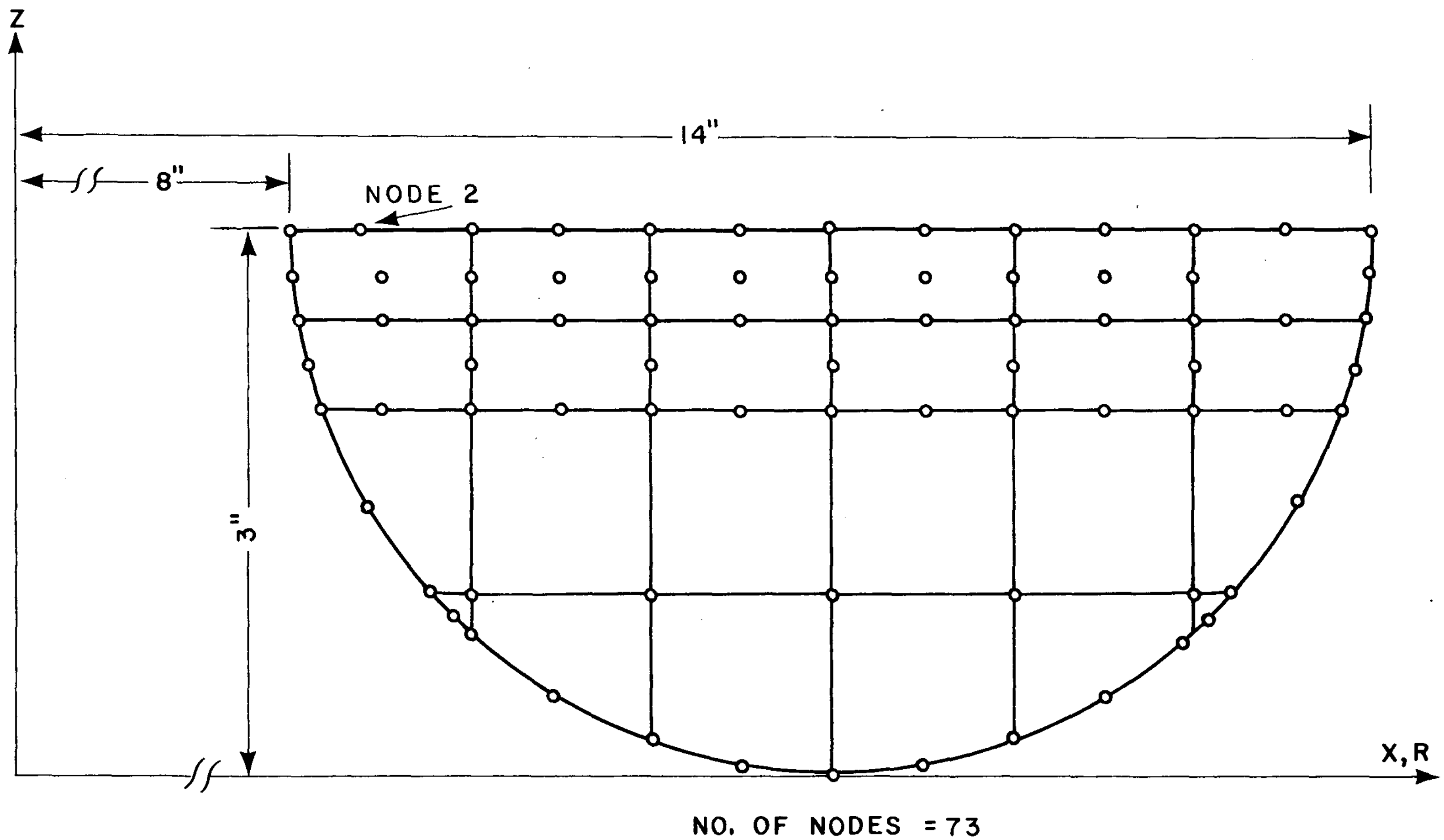


FIG. 3-6 FINITE ELEMENT MESH LAYOUT FOR TORUS TANK MODEL

XBL 789-2278

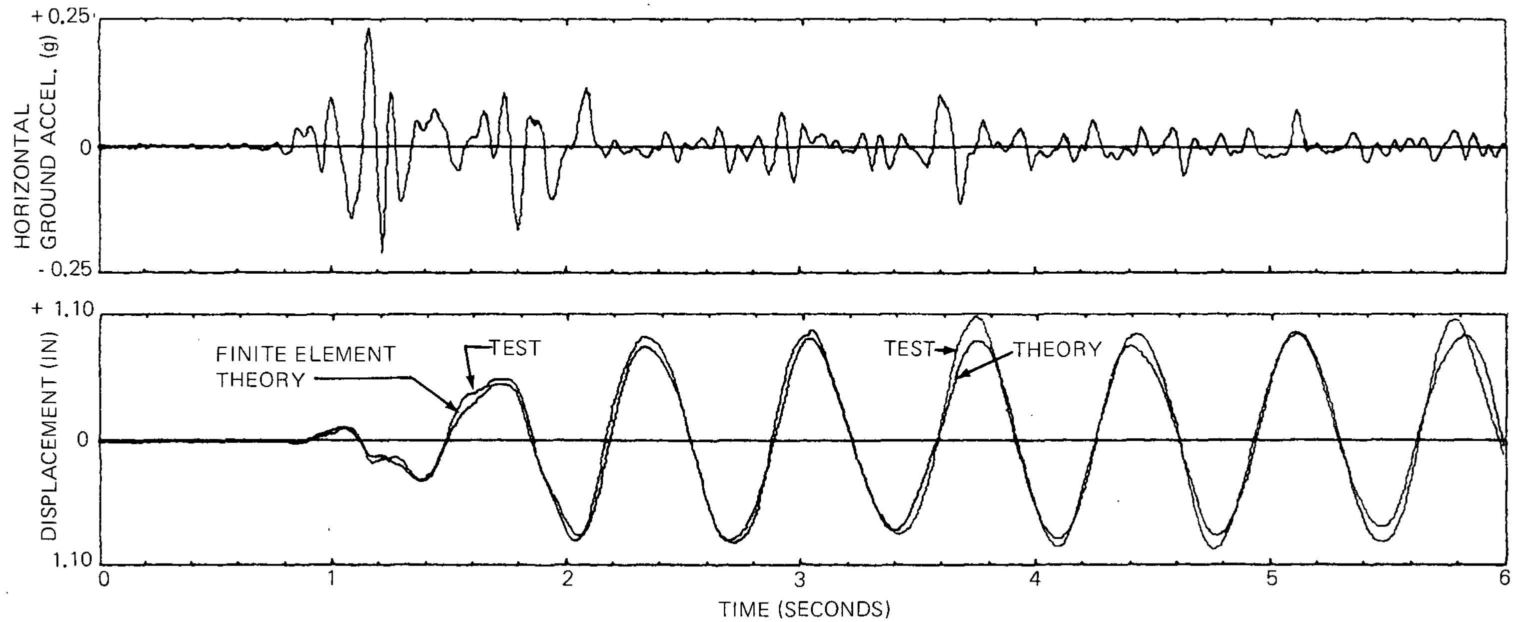


FIG. 3-7 COMPARISON OF SLOSHING DISPLACEMENTS BETWEEN THE TEST AND FINITE ELEMENT RESULTS AT NODE #2 (TEST #211276.1) IN ANNULAR TANK MODEL (INNER RADIUS = 33.2 IN., OUTER RADIUS = 48.0 IN., DEPTH OF WATER = 16 IN.) UNDER SIMULATED EL CENTRO 1940 EARTHQUAKE, TIME SCALE = $\sqrt{15}$ = 3.9, PEAK SHAKING TABLE ACCELERATION = 0.24g HORIZONTAL, 0.0g VERTICAL.

XBL 789-11314

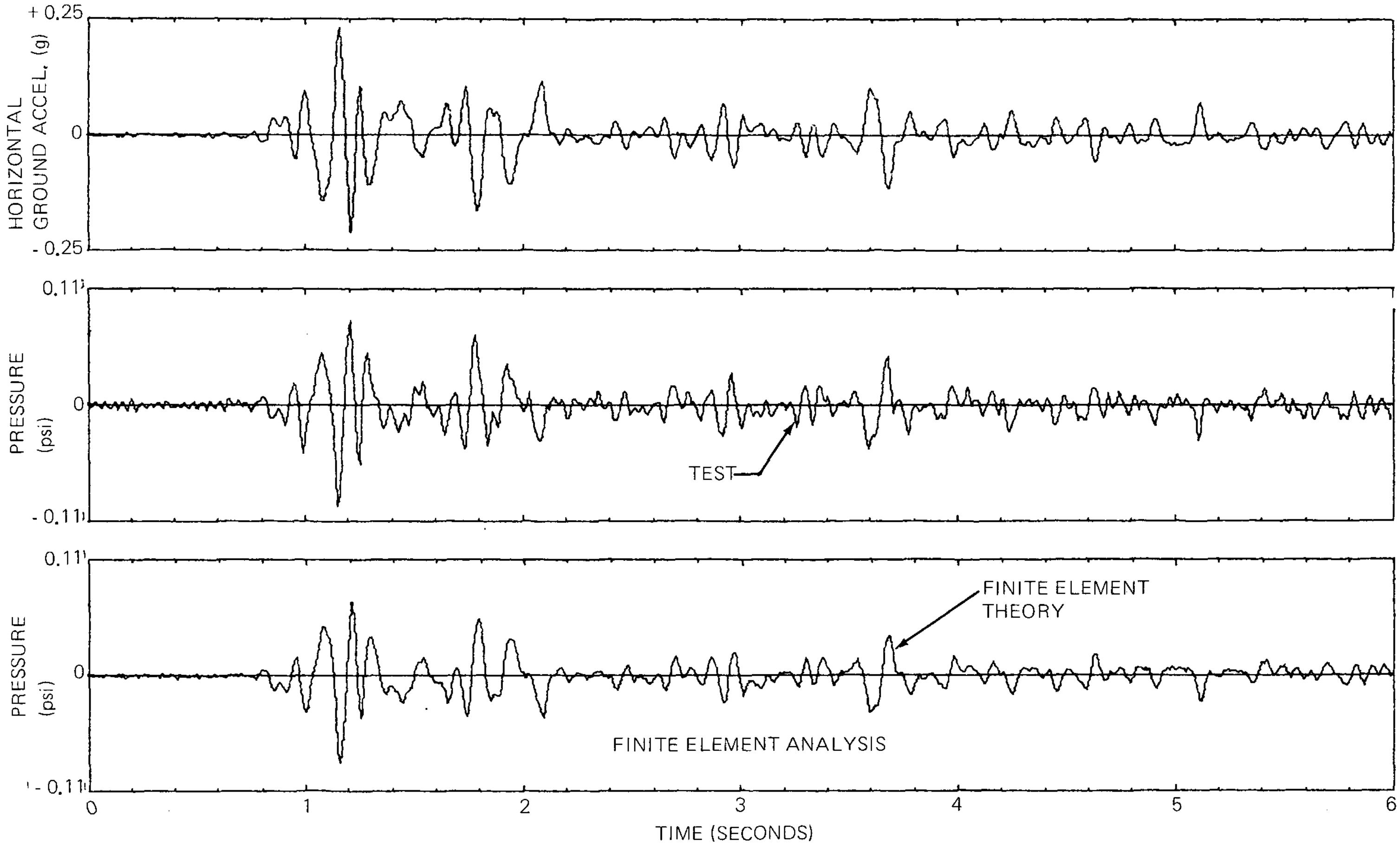


FIG. 3-8 COMPARISON OF IMPULSIVE PRESSURES BETWEEN THE TEST AND FINITE ELEMENT RESULTS AT NODE #52 (TEST #211276.1) IN ANNULAR TANK MODEL.

XBL 789-11316

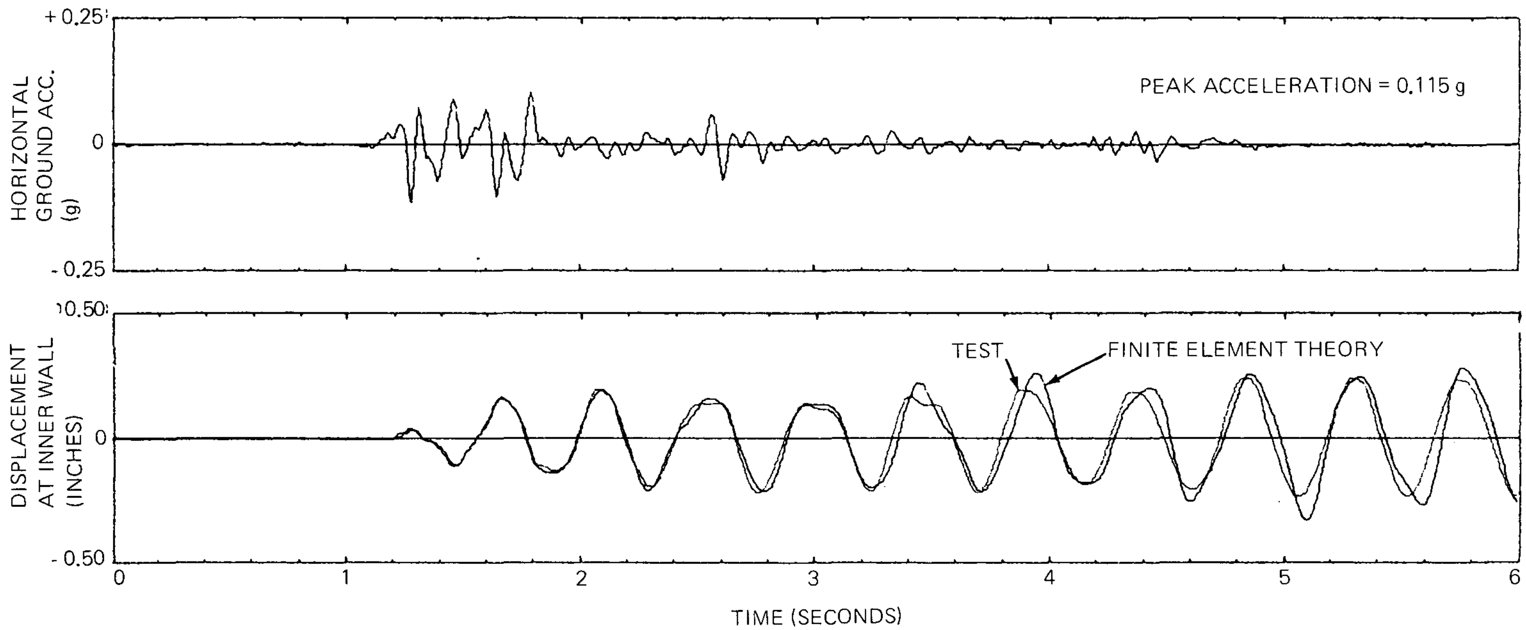


FIG. 3-9 COMPARISON OF SLOSHING DISPLACEMENTS BETWEEN THE TEST AND FINITE ELEMENT RESULTS AT NODE #2 (TEST #220378.2) IN TORUS TANK MODEL (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.)

XBL 789-11315

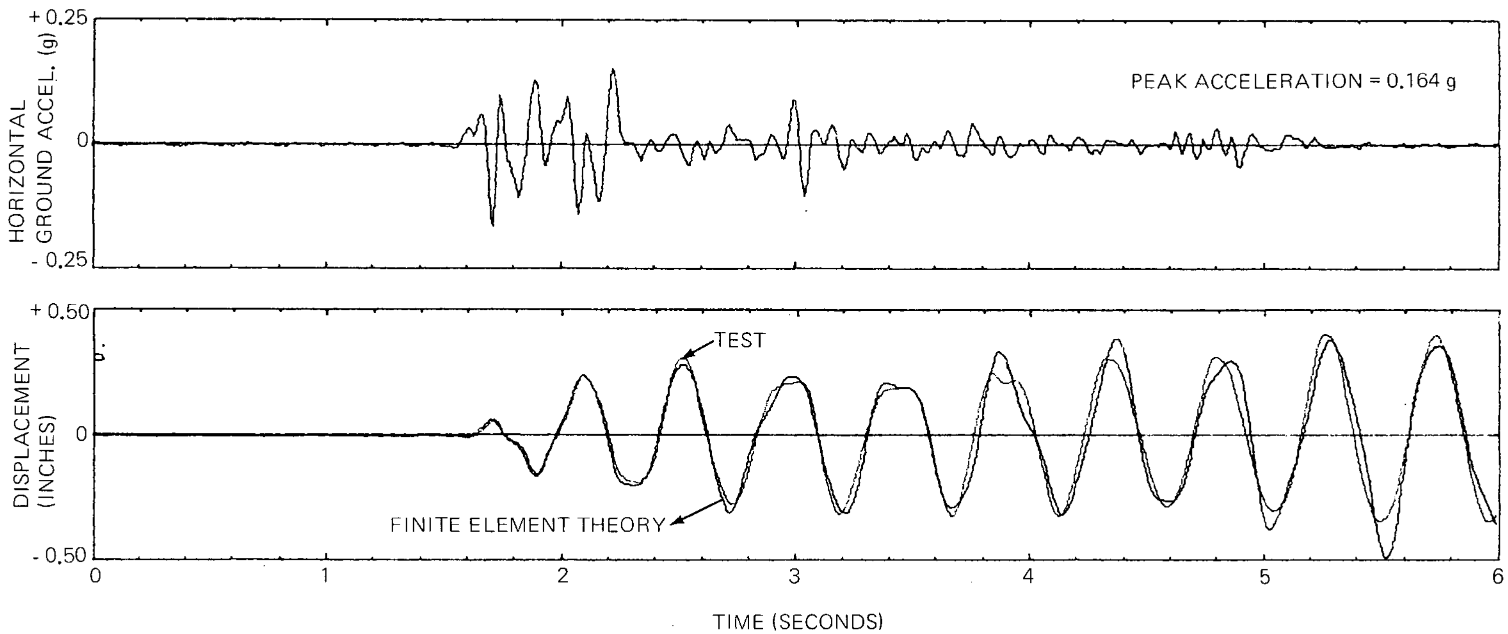


FIG. 3-10 COMPARISON OF SLOSHING DISPLACEMENTS BETWEEN THE TEST AND FINITE ELEMENT RESULTS AT NODE #2 (TEST #220378.3) IN TORUS TANK MODEL (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.)

XBL 789-11317

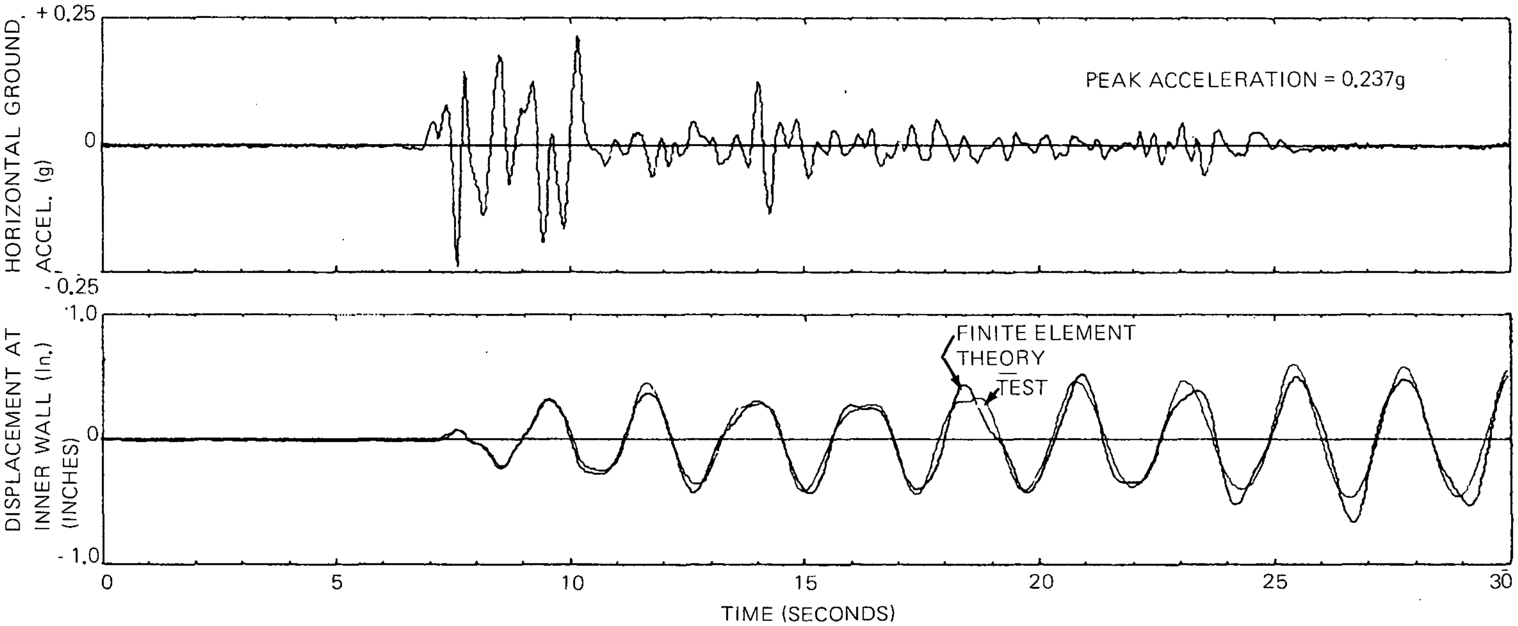


FIG. 3-11 COMPARISON OF SLOSHING DISPLACEMENTS BETWEEN THE TEST AND FINITE ELEMENT RESULTS AT NODE #2 (TEST #220378.4) IN TORUS TANK MODEL (INNER RADIUS = 8 IN., OUTER RADIUS = 14 IN., DEPTH OF WATER = 3 IN.)

XBL 789-11318

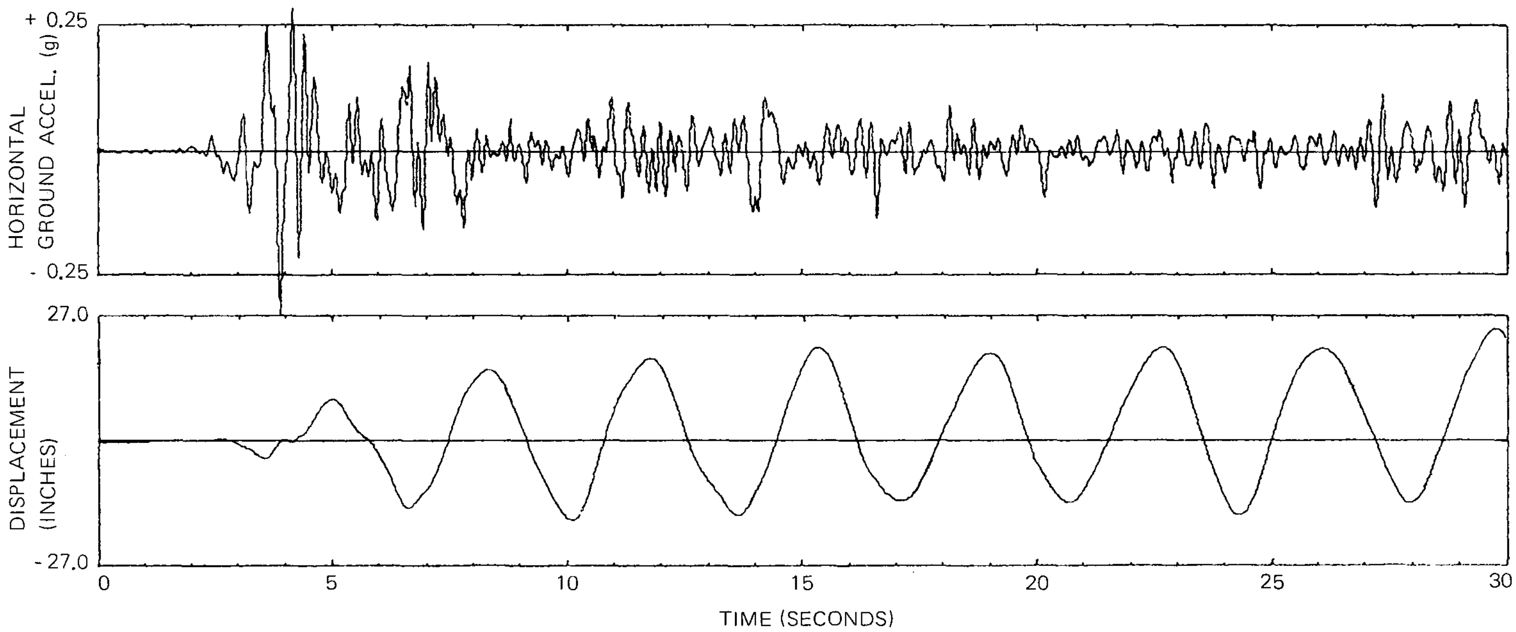


FIG. 3-12 SLOSHING DISPLACEMENTS IN PROTOTYPE TORUS TANK AT NODE #2 UNDER ELCENTRO 1940 EARTHQUAKE (INNER RADIUS = 40 FT., OUTER RADIUS = 70FT, DEPTH OF WATER = 15 FT.)

XBL 789-11319

4. CONCLUSIONS

(1) A comparison of data from shaking table tests on annular and torus tanks confirms that the finite element analysis presented in this report can successfully predict hydrodynamic pressures and free surface displacement in rigid axisymmetric tanks under strong motion earthquakes. The finite element program is applicable not only to axisymmetric tanks but may also have possible application to offshore structures under seismic conditions.

(2) As sloshing response is not very sensitive to the precise geometry of the tank section, a modified annular tank solution gives satisfactory results in predicting the sloshing frequencies and displacements in torus tanks under horizontal ground motions.

(3) The validity of the theory developed herein is independent of the size of the model that was analyzed and tested. In the 1/60-scale torus model the sloshing response is produced mainly by the low frequency components of the reference earthquake ground motion, and these are correctly reproduced on the 20-foot shaking table.

ACKNOWLEDGMENTS

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APPENDIX A1

SLOSH2 USER'S MANUAL

SLOSH2: A linear finite element program which determines the sloshing displacements and impulsive pressures in axisymmetric rigid tanks under arbitrary horizontal ground motions.

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CONTENTS

A1.1 Program Description

The computer program SLOSH2 has been developed to predict sloshing displacements and impulsive pressures in a liquid filled axi-symmetric container subjected to only horizontal ground motion. The tank is assumed to be rigid and fixed at the base. The program requires the following three types of elements.

1. Two dimension 4-to-8 node axisymmetric elements idealizing the liquid.
2. Two node free surface elements.
3. Two node elements representing the liquid-solid interface.

A1.2 Program Capacity

The program uses a variable dimensioning in order to make an optimum use of high speed storage. Element group data is stored block wise on the disc. The program capacity can be varied through two Fortran statements in the main program.

```
COMMON (n)
MTOT =  n
```

The total memory n required can be estimated by the following formula.

n = M + 2 * NPTM + 10 * NUMNP

in which

M	=	NEL1 * (4 * MXNODS - 2)
NEL1	=	Number of elements in group 1
MXNODS	=	Maximum nodes in any element of group 1
NPTM	=	Number of points of earthquake input
NUMNP	=	Total number of node points

A1.3 Program Input Data

The following format should be followed for the necessary input data.

I. Problem Initiation and Title (A5, 3X, 18A4) - one card

Columns	Variable	Description
1 - 5	MODE	Punch the word 'START'
9 - 80	HED	Title of the problem

II. Master Control Card (4I5) - one card

Columns	Variable	Description
1 - 5	NUMNP	Total number of nodes
6 - 10	NEG	Number of element groups
11 - 15	NUMEST	Estimated number of storage locations required (M_1) for element group 1. Zero or blank: defaults to 3000
16 - 20	MODEX	Execution mode. Specify (a) Zero: data check only (b) 1: execution

III. Nodal Coordinates (I5, 5X, 2F 10.0, I5)

As many cards as needed to generate total number of nodes NUMNP and their coordinates

Columns	Variable	Description
1 - 5	N	Node number See Note 1
11 - 20	X(N)	X coordinate
21 - 30	Y(N)	Z coordinate
35 - 35	KN	Node number difference between successive generated nodes (given on first card in a sequence). Specify. Zero: No generation. See Note 2.

Note:

(1) Node cards may not be in numerical order. Eventually, however, all nodes must be identified.

(2) The mesh generation parameter KN must appear on the first card of a series of nodal points to be generated. The intermediate nodes to be generated between nodes (say N1 and N2) will be located at equal intervals along the straight line joining the two nodes. KN is the nodal increment to be added to previous node number. The node difference N2-N1 must be exactly divisible by KN.

IV. Solution Time and Step Size (2I5, 3F10.0, 3I5)

Columns	Variable	Description
1 - 5	NDT	Number of solution time steps. Specify Zero: defaults to 1 step
6 - 15	DT	Step size
16 - 20	NPRINT	Time interval for printout of nodal displacements and pressures expressed as a multiple of the integration time step. Specify Zero: defaults to 1

V. Earthquake Input

A. Control Information (2I5) - one card

Columns	Variable	Description
1 - 5	NBCF	Number of ground input components (use 1)
6 - 10	NPTM	Maximum number of points to describe the earthquake input. See Note 1.

Note:

(1) NPTM is the number of [f(t),t] pairs used to define the earthquake ground motion which could be either an acceleration or displacement-time history record. At least two points are required to describe the input.

B. Earthquake Input Data

For one component of ground motion (horizontal in this case) a control card followed by as many cards as needed to define the earthquake.

1. Control Card (2I5, F10.0, I5) - First card

Columns	Variable	Description
1 - 5	NC	Function number. Specify equal to 1 in this case
6 - 10	NPTS(NC)	Number of time points used to describe the earthquake input (GE.2 and EQ. to NPTM)
11 - 20	FOM	Multiplication factor for conversion to right units. See Note 1
21 - 25	INPUT	Specify 1 - If acceleration is ground input 2 - If displacement is the given ground input.

2. [f(t), t] Earthquake Data (8F10.0)

As many cards as needed to define NPTS (NC) pairs of points [TFN(NC,I), FN(NC,I)]; four pairs per card. See Note 2.

Columns	Variable	Description
1 - 10	TFN(NC,1)	Time at point 1 : t_1
11 - 20	FN(NC,1)	Acceleration or displacement value at point 1 : $f(t_1)$
21 - 30	TFN(NC,2)	t_2
31 - 40	FN(NC,2)	$f(t_2)$
41 - 50	TFN(NC,3)	t_3
51 - 60	FN(NC,3)	$f(t_3)$
61 - 70	TFN(NC,4)	t_4
71 - 80	FN(NC,4)	$f(t_4)$
Next card(s)	-	as many as needed to define the earthquake input.

Note:

(1) Factor of multiplication if necessary to make the units of ground input compatible with the units of the tank dimensions. This option is available only if input is in the form of accelerogram. In case of displacement history the units must be in inches, seconds and pounds.

(2) Time values at successive points are assumed to increase in magnitude. Values of ground input other than TFN(NC,I) are calculated within the program using a linear interpolation.

VI. Element Data

Elements are divided into three groups (NEG). An element group is a series of elements of a particular type.

The elements in a particular group must be numbered sequentially starting with the number of the first element as specified on the element group control card.

Following are the three types of element groups used in this program.

Type 1 - Two Dimensional Finite Elements

These are 4-to-8 node axisymmetric isoparametric elements which lie in the global X-Z plane and are used to model the liquid continuum. Z-axis has been taken as the axis of revolution for the axisymmetric tank.

Type 2 - Free Surface Elements

These are 2 node axisymmetric elements which have been used to represent the free surface of the liquid. These elements lie in the X-Z global plane where Z is the axis of revolution of the tank. These elements contribute to mass matrix.

Type 3 - Liquid-Solid Interface Elements

These are two node axisymmetric elements and lie in the global X-Z plane. These elements contribute to the loading vector.

Type 1 - Two Dimensional Finite Elements

A. Control Information (6I5) - one card.

Columns	Variable	Description
1 - 5	NGR	Element group indicator. Punch the number "1".
6 - 10	NEL1	Number of elements in group 1.
11 - 15	MFST	Element number of the first element in this. See Note 1.
16 - 20	ITYP2D	Element type code. Specify Zero: axisymmetric
21 - 25	MXNODS	Maximum number of nodes used to describe any one element. Specify Zero: defaults to 4. (GE.4 and LE.8)
26 - 30	NINT	Numerical integration order to be used in Gaussian quadrature. Specify Zero: defaults to 2 (GE.2 and LE.4) See Note 2.

Note:

(1) Element numbers in any group may not start from 1 if MFST is specified.

(2) For rectangular elements, an integration order of 2 is sufficient. For non rectangular elements a higher order should be used.

B. Element Data (11I5)

As many data cards as are needed in order to generate the element data for the elements (NEL1) in this group.

Columns	Variable	Description
1 - 5	M	Element number See Note 1
6 - 10	NOD(1)	Global node number of element node 1.
11 - 15	NOD(2)	Global node number of element node 2.
16 - 20	NOD(3)	Global node number of element node 3.
21 - 25	NOD(4)	Global node number of element node 4.
26 - 30	NOD(5)	Global node number of element node 5.
31 - 35	NOD(6)	Global node number od element node 6.
36 - 40	NOD(7)	Global node number of element node 7.
41 - 45	NOD(8)	Global node number of element node 8. See Note 2
46 - 50	IEL	Number of nodes in the element. Zero: defaults to MXNODS
51 - 55	KG	Node number increment for element generation (given on 1st card in a sequence) Zero: defaults to 1 See Note 3

Note:

(1) Elements must be input in increasing sequence, with MFST being the 1st element. Cards for the first and last element must be included.

(2) If an element has less than 8 nodes (i.e., IEL.LT.8), input a zero or blank corresponding to the missing node location. For example, for a 6 node element with nodes 6 and 8 missing, the element node number array would be NOD(I) = [X X X X X 0 X 0] where X entries represent the global node numbers.

(3) The node generation parameter KG must appear on the first element card of a sequence and is used to determine the node numbers for a group of missing elements in that sequence. If M is the first element of the sequence and the elements [M + 1, M + 2, ... M + J] are the missing J elements, then the node numbers of the successive J elements are automatically incremented by the value KG given for the element M. Only the nonzero node numbers appearing on the M-th element card are incremented in this automatic generation. In the printout of the element data, generated elements are marked with an asterisk.

Type 2 - Free Surface Boundary Elements

A. Control Information (4I5) - one card.

Columns	Variable	Description
1 - 5	NGR	Element group number. Punch the number '2'.
6 - 10	NEL2	Number of elements in group 2
11 - 15	MFST	Number of the first element in group 2 (need not start with 1)
16 - 20	ITYP	Element type. Specify Zero: axisymmetric this is the only option available.

B. Element Data (4I5)

As many cards as needed to generate NEL2 elements.

Columns	Variable	Description
1 - 5	M	Element number See Note (1)
6 - 10	NOD(1)	Global node number of element node I
11 - 15	NOD(2)	Global node number of element node J
16 - 20	KG	Node number increment for element generation. Zero: defaults to 1. See Note (2)

Note:

(1) All elements must be input in ascending numerical order, starting with element number MFST. Cards for the first and last element must be included.

(2) The node generation parameter KG must be given on the first element card prior to a group of missing elements. In the print out of the element data, generated elements are prefixed by an asterisk.

Type 3 - Liquid-Solid Interface Elements

A. Control Information (4I5) - one card.

Columns	Variable	Description
1 - 5	NGR	Element group number Punch the number '3'
6 - 10	NEL3	Number of elements in group 3
11 - 15	MFST	Number of first element in group 3
16 - 20	ITYP	Element type : Specify Zero: axisymmetric (the only option available)

B. Element Data (3I5, 5X, F10.0)

As many cards as the number of elements NEL3 in group 3

Columns	Variable	Description
1 - 5	M	Element number
6 - 10	NOD(1)	Global node number of element node I
11 - 15	NOD(2)	Global node number of element node J
21 - 30	COSS	X-direction cosine of the outward normal to the element.

VII. New Problem Data

A new problem may now be solved by adding data starting with Section I. Any number of problems can be solved in one run.

VIII. Termination Card (A4) - one card

Columns	Variable	Description
1 - 4	MODE	Punch the word 'STOP'.

A1.4 Output

Output includes the nodal displacements and impulsive pressures. Displacements are meaningful only for the free surface nodes.

```
C     ************************************************************************
C
      PROGRAM SLOSH2(INPUT,OUTPUT,PUNCH,TAPE5=INPUT,TAPE6=OUTPUT,         SLOSH  1
     +       TAPE1,TAPE2,TAPE3=PUNCH)                                      SLOSH  2
C                                                                          SLOSH  3
C     *************************************************************************SLOSH  4
C                                                                          SLOSH  5
C        SLOSH2----A FINITE ELEMENT PROGRAM TO DETERMINE THE SLOSHING      SLOSH  6
C                  RESPONSE UNDER EARTHQUAKE GROUND MOTIONS IN AN          SLOSH  7
C                  AXI-SYMMETRIC RIGID TANK                                SLOSH  8
C        DEVELOPED BY--- MOHAMMAD ASLAM, DEPARTMENT OF CIVIL               SLOSH  8
C                  ENGINEERING, UNIVERSITY OF CALIFORNIA,BERKELEY          SLOSH  8
C                  AUGUST 1978                                             SLOSH  8
C                                                                          SLOSH  9
C     *************************************************************************SLOSH 10
C                                                                          SLOSH 11
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                      SLOSH 12
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP         SLOSH 13
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15SLOSH 14
      COMMON /ELSTOR/ NUMEST,MIDEST,MAXEST                                 SLOSH 15
      COMMON /JUNK  / HED(18),MTOT,NLINE                                   SLOSH 16
      COMMON /NBC   / NNBC,NBCF,NPTM                                       SLOSH 17
      COMMON /WORK  / WORK(200)                                            SLOSH 18
      COMMON/CONST /A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,THETA,DELTA,ALPHA,PI,GSLOSH 19
     +,RO                                                                  SLOSH 20
      DIMENSION SD(2000),PB(2000)                                          SLOSH 21
      COMMON  A(10000)                                                     SLOSH 22
C                                                                          SLOSH 23
      MTOT   = 10000                                                       SLOSH 24
  200 MAXEST = 0                                                           SLOSH 25
C                                                                          SLOSH 26
C     ===========                                                          SLOSH 27
C     INPUT PHASE                                                          SLOSH 28
C     ===========                                                          SLOSH 29
C                                                                          SLOSH 30
C     PROGRAM MASTER CONTROL DATA                                          SLOSH 31
      CALL DOTI                                                            SLOSH 32
C     INPUT ELEMENT INFORMATION                                            SLOSH 33
C                                                                          SLOSH 34
      CALL ELCAL                                                           SLOSH 35
C                                                                          SLOSH 36
C     ==============                                                       SLOSH 37
C     SOLUTION PHASE                                                       SLOSH 38
C     ==============                                                       SLOSH 39
C                                                                          SLOSH 40
C     *************************************************************************SLOSH 41
C                                                                          SLOSH 42
C     B L A N K   C O M M O N   S T O R A G E   A L L O C A T I O N        SLOSH 43
C                                                                          SLOSH 44
C           ARRAY   -----------DESCRIPTION-----------     DIMENSION        SLOSH 45
C     N1    TFN     TIME VALUES AT POINTS                 NPTM*NBCF        SLOSH 46
C     N2    FN      FUNCTION VALUES AT POINTS             NPTM*NBCF        SLOSH 47
C     N3    NPTS    NUMBER OF FUNCTION INPUT POINTS       NBCF             SLOSH 48
C     N4    TD      FIRST DERIVATIVE OF VEL POT.          NUMNP            SLOSH 48
C     N5    TDD     2ND DERIVATIVE OF VEL. POT.           NUMNP            SLOSH 49
C     N6    TTAU                                          NUMNP            SLOSH 50
C     N7    P       PRESSURE(DYNAMIC IMPULSIVE)           NUMNP            SLOSH 51
C     N8    T       VELOCITY POTENTIAL                    NUMNP            SLOSH 52
C     N9    MAXA    ADDRESSES OF XK DIAGONAL ELEMENTS     NUMNP+1          SLOSH 53
C     N10   XK      EFFECTIVE STIFFNESS MATRIX            NWK              SLOSH 54
C     N11   Q       LOADING VECTOR                        NUMNP            SLOSH 55
C     N12   C       MASS MATRIX                           NUMNP            SLOSH 56
C     N13   E       WATER DISPLACEMENTS AT SURFACE        NUMNP            SLOSH 57
```

```
C     N14   TT      VELOCITY POTENTIAL AT (T-DT)        NUMNP
C
C     **********************************************************************
C
      CALL ADRSK (A(N11),A(N12),NUMNP,NWK,MB)
C
C     SHIFT STORAGE TO ELIMINATE NODAL COORDINATE DATA
C
    5 I = 1 + MAXEST
      N12M = N12 - 1
      DO 10 J=N3,N12M
      A(I) = A(J)
   10 I = I + 1
C
      N1  =  1   +  MAXEST
      N2  =  N1  +  NPTM*NBCF
      N3  =  N2  +  NPTM*NBCF
      N4  =  N3  +  NBCF
      N5=N4  +  NUMNP
      N6=N5  +  NUMNP
      N7=N6  +  NUMNP
      N8=N7  +  NUMNP
      N9  =  N8  +  NUMNP
      N10 =  N9  +  NUMNP + 1
      N11 =  N10 +  NWK
      N12 =  N11 +  NUMNP
      N13 =  N12 +  NUMNP
      N14 =  N13 +  NUMNP
      N15 =  N14 +  NUMNP
      IF(N15.GT.MTOT) CALL ERROR (N15-MTOT)
C
      IF(MODEX.EQ.0) GO TO 200
C
C     INITIALIZE STIFFNESS MATRIX (XK) AND LOADING VECTOR Q
C
      N12M = N12 - 1
      DO 15 I=N10,N12M
   15 A(I) = 0.0
C
C     INITIALIZE VELUCITY POTENTIAL VECTOR AT  AT  TT(0)=T(0)
C
      DO 20 I=1,NUMNP
      IT  = N8  + I - 1
      ITT = N14 + I - 1
   20 A(ITT) = A(IT)
C
C     INITIALIZE THE TIME STEP COUNTER
C
   22 KSTEP = 0
      TIME=0.
C
C     INITIALIZE MASS MATRIX (LUMPED MASS SYSTEM)
C
      N13M = N13 - 1
      DO 25 I=N12,N13M
   25 A(I) = 0.0
C
C     CALCULATE CONSTANTS OF INTEGRATION
C
      PI=3.141592654
      G=386.18
      RO=0.00009351
      THETA=1.0
```

```
      DELTA=0.50
      ALPHA=0.25
      TAU=THETA*DT
      A0=1.0/(ALPHA*TAU*TAU)
      A1=DELTA/(ALPHA*TAU)
      A2=1./(ALPHA*TAU)
      A3=1./(2.*ALPHA)-1.
      A4=DELTA/ALPHA-1.
      A5=TAU*(DELTA/ALPHA-2.0)/2.
      A6=DT*(1.-DELTA)
      A7=DT*DELTA
      A8=DT*DT*(0.5-ALPHA)
      A9=ALPHA*DT*DT
C
C     ASSESBLE THE EFFEECTIVE SYSTEM STIFFNESS MATRIX(K*)
C
   30 CALL ASSEMK
C
C     FORM THE EFFECTIVE K AND CALL IT XK
C
      CALL KSTAR(A(N9),A(N10),A(N12))
C
C     INITIALIZE VELOCITY POTENTIAL AND ITS DERIVATIVES
      N6M=N6-1
      DO 36 I=N4,N6
36    A(I)=0.
C     TRIANGULARIZE THE EFFECTIVE CONDUCTIVITY MATRIX. (K*)
C
   40 KTR = 0
      CALL COLSOL (A(N10),A(N11),A(N9),NUMNP,MB,NUK,KTR)
C
C     ==================
C     TIME MARCHING LOOP
C     INITIALIZE Q
      N12M=N12-1
      DO 44 I=N11,N12M
 44   A(I)=0.
      TX=THETA*DT
C
C     ==================
C
  100 KSTEP = KSTEP + 1
      TTH=TIME+TX
      TIME  = TIME  + DT
C
C     FORM THE LOAD VECTOR
C
75    CALL FORMQC(TTH)
C
C     COMPUTE EFFECTIVE LOAD VECTOR
C
      CALL QEFF(A(N11),A(N12),A(N8),A(N4),A(N5),NUMNP)
C
C     UPDATE (TT) VECTOR
C
      DO 82 I=1,NUMNP
      IT  = N8  + I - 1
      ITT = N14 + I - 1
   82 A(ITT) = A(IT)
C
C     SOLVE THE EQUILIBRIUM EQUATIONS FOR VELOCITY POTENTIAL
C
   84 KTR = 2
```

```
      CALL COLSOL (A(N10),A(N11),A(N9),NUMNP,MB,NWK,KTR)
C
C     Q-VECTOR IS NOW T-VECTOR. SET T(I)=Q(I) AND Q(I)=0.
C
      DO 85 I=1,NUMNP
      IT = N8  + I - 1
      IQ = N11 + I - 1
      A(IT) = A(IQ)
   85 A(IQ) = 0.0
C
C     CALCULATE VEL. POTENTIAL AND ITS DERIVITIVE AT  TIME+DT
      CALL CALCU(A(N8),A(N14),A(N4),A(N5),A(N7),A(N13),NUMNP)
C
C     PRINT AND/OR PUNCH THE NODAL DISPLACEMENTS  AND PRESSURES,
C     IF REQUESTED, AT THIS TIME STEP
C
      K = MOD(KSTEP,NPRINT)
      IF(K.NE.0) GO TO 90
      CALL OUT(A(N13),NUMNP,TIME,KSTEP)
      CALL OUP(A(N7),NUMNP,TIME,KSTEP)
   90 IF(KP.EQ.0) GO TO 92
      L = MOD(KSTEP,KP)
      IF(L.NE.0) GO TO 92
      CALL PTEMP (A(N8),TIME,NUMNP)
C
92    CONTINUE
      SD(1)=0.
      NN=KSTEP+1
C     CHECK FOR FINAL TIME STEP
C
      IF(KSTEP.LT.NDT) GO TO 100
      GO TO 200
C
      END
      SUBROUTINE DOTI
C
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15
      COMMON /ELSTOR/ NUMEST,MIDEST,MAXEST
      COMMON /JUNK  / HED(18),MTOT,NLINE
      COMMON /NBC   / NNBC,NBCF,NPTM
      COMMON  A(1)
      DIMENSION  MOD(2)
      DATA  MOD/5HSTART,5HSTOP /
C
C     =======================
C     READ CONTROL INFORMATION
C     =======================
C
   10 READ (5,1000) MODE,HED
      IF(MODE.EQ.MOD(2)) STOP
      IF(MODE.EQ.MOD(1)) GO TO 20
      WRITE(6,3000)
      GO TO 10
C
   20 READ (5,1001) NUMNP,NEG,NUMEST,MODEX
C
      IF(NUMEST.EQ.0) NUMEST = 4000
      IF(NUMNP.GT.0) GO TO 30
      WRITE(6,3001)
      STOP
   30 IF(NEG.GT.0) GO TO 40
```

```
      WRITE(6.3002)                                                      DOTI  30
      STOP                                                               DOTI  31
C                                                                        DOTI  32
   40 CALL TITLE (HED)                                                   DOTI  33
      WRITE(6.2000) NUMNP.NEG.NUMEST.MODEX                               DOTI  34
      NLINE = 17                                                         DOTI  35
C                                                                        DOTI  36
C                                                                        DOTI  37
C     *******************************************************************DOTI  38
C                                                                        DOTI  39
C     B L A N K   C O M M O N   S T O R A G E   A L L O C A T I O N      DOTI  40
C                                                                        DOTI  41
C           ARRAY  -----------DESCRIPTION-----------     DIMENSION       DOTI  42
C     N1    X      NODAL X-COORDINATES                   NUMNP           DOTI  43
C     N2    Y      NODAL Y-COORDINATES                   NUMNP           DOTI  44
C     N3    TFN    TIME VALUES AT POINTS                 NPTM*NBCF       DOTI  45
C     N4    FN     FUNCTION VALUES AT POINTS             NPTM*NBCF       DOTI  46
C     N5    NPTS   NUMBER OF FUNCTION INPUT POINTS       NBCF            DOTI  47
C     N6    TD     FIRST DERIVATIVE OF VEL POT.          NUMNP           DOTI  48
C     N7    TDD    2ND DERIVATIVE OF VEL. POT.           NUMNP           DOTI49
C     N8    TTAU                                         NUMNP           DOTI50
C     N9    P      PRESSURE(DYNAMIC IMPULSIVE)           NUMNP           DOTI51
C     N10   T      VELOCITY  POTENTIAL                   NUMNP           DOTI  52
C     N11   MAXA   ADDRESSES OF (XK) DIAGONAL ELTS.      NUMNP+1         DOTI  53
C     N12   MHT    ACTIVE COLUMN HEIGHTS                 NUMNP           DOTI  54
C                                                                        DOTI  55
C     *******************************************************************DOTI  56
C                                                                        DOTI  57
C     ================================                                   DOTI  58
C     READ NODAL POINT COORDINATE DATA                                   DOTI  59
C     ================================                                   DOTI  60
C                                                                        DOTI  61
      N1  =  1   +  NUMEST                                               DOTI  62
      N2  =  N1  +  NUMNP                                                DOTI  63
      N3  =  N2  +  NUMNP                                                DOTI  64
      IF(N3.GT.MTOT) CALL ERROR (N3-MTOT)                                DOTI  65
C                                                                        DOTI  66
      CALL COORD (A(N1),A(N2),NUMNP)                                     DOTI  67
C                                                                        DOTI  68
C     =======================================================            DOTI  69
C     READ SOLUTION TIME AND GROUND ACCELERATION                         DOTI  70
C     =======================================================            DOTI  71
C                                                                        DOTI  72
      READ(5,1002)NDT,DT,NPRINT,KP                                       DOTI- 73
      KST=0 $ TSTART=0.                                                  DOTI- 73
      TAMB=0. $ NTSREF=0                                                 DOTI- 73
C                                                                        DOTI  74
      IF(NDT   .EQ.0) NDT    = 1                                         DOTI  75
      IF(NPRINT.EQ.0) NPRINT = 1                                         DOTI  76
      CALL TITLE (HED)                                                   DOTI  77
      WRITE(6.2001) NDT.DT.NPRINT.KP                                     DOTI- 78
      NLINE = 22                                                         DOTI  79
C                                                                        DOTI  80
C     ====================================                               DOTI  81
C     READ THE EARTHQUAKE ACCELEROGRAMS                                  DOTI- 82
C     ====================================                               DOTI  83
C                                                                        DOTI  84
      READ (5.1003) NBCF.NPTM.NNBC                                       DOTI  85
      WRITE(6.2002) NBCF.NPTM                                            DOTI  86
      NLINE = NLINE + 9                                                  DOTI  87
   50 N4  =  N3  +  NPTM*NBCF                                            DOTI  91
      N5  =  N4  +  NPTM*NBCF                                            DOTI  92
      N6  =  N5  +  NBCF                                                 DOTI  93
```

```
      IF(N6.GT.MTOT) CALL ERROR (N6-MTOT)                                DOTI  94
C                                                                        DOTI  95
      CALL FUNC (A(N3),A(N4),A(N5),NPTM)                                 DOTI  96
C                                                                        DOTI 101
   60 CALL TITLE (HED)                                                   DOTI 102
      NLINE = 10                                                         DOTI 104
70    N7=N6+NUMNP                                                        DOTI-105
      N8=N7+NUMNP                                                        DOTI-106
      N9=N8+NUMNP                                                        DOTI-107
      N10=N9+NUMNP                                                       DOTI-108
      IF(N10.GT.MTOT) CALL ERROR (N10-MTOT)                              DOTI 112
C                                                                        DOTI 115
C     ========================================                           DOTI 116
C     INITIALISE VEL. POTENTIAL                                          DOTI-117
C     ========================================                           DOTI 118
C                                                                        DOTI 119
   80 N11 =  N10 +  NUMNP                                                DOTI 120
      IF(N11.GT.MTOT) CALL ERROR (N11-MTOT)                              DOTI 121
C                                                                        DOTI 122
      CALL INITAL (A(N10),TAMB,NUMNP)                                    DOTI 123
C                                                                        DOTI 124
C     FORMAT STATEMENTS                                                  DOTI 125
C                                                                        DOTI 126
 1000 FORMAT(A5,3X,18A4)                                                 DOTI 127
 1001 FORMAT(4I5)                                                        DOTI 128
1002  FORMAT(I5,F10.0,2I5)                                               DOTI-129
 1003 FORMAT(3I5)                                                        DOTI 130
 2000 FORMAT(20(1H*)/20H CONTROL INFORMATION/20(1H*)///                  DOTI 131
     1        34H    NUMBER OF NODAL POINTS ..... = I5/                  DOTI 132
     2        34H    NUMBER OF ELEMENT GROUPS ... = I5/                  DOTI 133
     3        34H    MAX. ELEMENT GROUP STORAGE . = I5//                 DOTI 134
     4        34H    SOLUTION MODE .............. = I5/                  DOTI 135
     4        23H       EQ. 0, DATA CHECK/6X,16HEQ. 1, EXECUTION///)     DOTI 136
 2001 FORMAT(50(1H*)/38H SOLUTION TIME AND PRINT,PUNCH                   DOTI-137
     1        12H INFORMATION/50(1H*)///                                 DOTI 138
     5        48H    NUMBER OF SOLUTION TIME STEPS ........... = I5/     DOTI 142
     6        48H    SOLUTION TIME STEP INCREMENT ............ = F10.4//DOTI 143
     9        48H    OUTPUT PRINT INTERVAL ................... = I5/     DOTI 146
     B        48H    OUTPUT PUNCH INTERVAL ................... = I5///) DOTI 148
 2002 FORMAT(25(1H*)/25H TIME DEPENDENT FUNCTIONS/25(1H*)///             DOTI 149
     1        48H    NUMBER OF TIME DEPENDENT FUNCTIONS ....... = I5/    DOTI 150
     2        48H    MAXIMUM NUMBER OF (F(T),T) POINTS ........ = I5//)  DOTI 151
 3000 FORMAT(//51H **ERROR**  PROBLEM DECK MUST BEGIN WITH START CARD)   DOTI 155
 3001 FORMAT(//49H **ERROR**  NO. OF NODAL POINTS MUST BE .GT. ZERO)     DOTI 156
 3002 FORMAT(//51H **ERROR**  NO. OF ELEMENT GROUPS MUST BE .GT. ZERO)   DOTI 157
C                                                                        DOTI 158
      RETURN                                                             DOTI 159
      END                                                                DOTI 160
      SUBROUTINE COORD (X,Y,NUMNP)                                       COOR   1
C                                                                        COOR   2
C     *******************************************************************COOR   3
C     THIS ROUTINE READS AND GENERATES THE GLOBAL NODAL POINT            COOR   4
C     COORDINATE DATA FOR 4- TO 8-NODE ISOPARAMETRIC ELEMENTS            COOR   5
C     *******************************************************************COOR   6
C                                                                        COOR   7
      DIMENSION  X(1),Y(1)                                               COOR   8
      COMMON /JUNK  / HED(18),MTOT,NLINE                                 COOR   9
C                                                                        COOR  10
C     READ OR GENERATE NODAL POINT DATA                                  COOR  11
C                                                                        COOR  12
      WRITE(6,2000)                                                      COOR  13
      WRITE(6,2001)                                                      COOR  14
      NLINE = NLINE + 12                                                 COOR  15
```

```
      NOLD  = 0
C
   10 READ (5,1000) N,X(N),Y(N),KN,JPR
      IF(N.EQ.1) IPR=JPR
      IF(NLINE.LT.55) GO TO 15
      CALL TITLE (HED)
      WRITE(6,2001)
      NLINE = 10
C
   15 WRITE(6,2002) N,X(N),Y(N),KN
      NLINE = NLINE + 1
      IF(NOLD.EQ.0) GO TO 30
C
C     CHECK IF GENERATION IS REQUIRED
C
      IF(KNOLD.EQ.0) GO TO 30
      NUM  = (N-NOLD)/KNOLD
      NUMN = NUM - 1
      RNUM = NUM
      DX   = (X(N)-X(NOLD))/RNUM
      DY   = (Y(N)-Y(NOLD))/RNUM
      K    = NOLD
      DO 20 J=1,NUMN
      KK   = K
      K    = K + KNOLD
      X(K) = X(KK) + DX
   20 Y(K) = Y(KK) + DY
C
   30 NOLD = N
      KNOLD= KN
      IF(N.NE.NUMNP) GO TO 10
C
      IF(IPR.EQ.1) GO TO 200
C
C     PRINT ALL NODAL POINT DATA
C
      CALL TITLE (HED)
      WRITE(6,2003)
      NLINE = 9
      NROW = NUMNP/3 + 1
      NR = 0
C
      DO 100 I=1,NUMNP,3
      NR = NR + 1
      IP = I + 2
      IF(NR.EQ.NROW) IP = NUMNP
      IF(NLINE.LT.55) GO TO 50
      CALL TITLE (HED)
      WRITE(6,2003)
      NLINE = 9
   50 WRITE(6,2004) (N,X(N),Y(N),N=I,IP)
  100 NLINE = NLINE + 1
C
C     FORMAT STATEMENTS
C
 1000 FORMAT(I5,5X,2F10.0,I5,I1)
 2000 FORMAT(28(1H*)/28H NODAL POINT COORDINATE DATA/28(1H*)/)
 2001 FORMAT(19(1H*)/19H A. INPUT NODE DATA/19(1H*)///
     1       4X,4HNODE,5X,7HX-COORD,5X,7HY-COORD,5X,4HDIFF/)
 2002 FORMAT(3X,I5,2F12.3,3X,I5)
 2003 FORMAT(23(1H*)/23H B. GENERATED NODE DATA/23(1H*)///
     1       3(4X,4HNODE,5X,7HX-COORD,5X,7HY-COORD,5X)/)
 2004 FORMAT(3(3X,I5,2F12.3,5X))
```

```
C
  200 RETURN
      END
      SUBROUTINE FUNC (TFN,FN,NPTS,NPTM1)
C
C     ***********************************************************************
C     DEFINE ALL BOUNDARY CONDITION FUNCTIONS
C     ***********************************************************************
C
      DIMENSION  TFN(NPTM1,1),FN(NPTM1,1),NPTS(1)
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP
      COMMON /JUNK  / HED(18),MTOT,NLINE
      COMMON /NBC   / NNBC,NBCF,NPTM
      COMMON /WORK  / FORM(4),WORK(196)
C
      WRITE(6,2001)
      NLINE = NLINE + 3
C
      DO 100 LL=1,NBCF
      READ (5,1000) NC,NPTS(NC),FOM,INPUT
      WRITE(6,2002) NC,NPTS(NC),FOM
      NLINE = NLINE + 1
      IF(NPTS(NC).GE.2.AND.NPTS(NC).LE.NPTM) GO TO 20
      WRITE(6,3000)
      STOP
C
C     READ TIME FUNCTION VERSUS TIME TABLE
C
   20 NT    = NPTS(NC)
      READ (5,1001) (TFN(K,NC),FN(K,NC),K=1,NT)
C
C     CHECK THAT TIME POINTS ARE IN INCREASING ORDER
C
      TOLD = -1.
      DO 30 K=1,NT
      IF(TFN(K,NC).GT.TOLD) GO TO 30
      WRITE(6,3001)
      STOP
   30 TOLD = TFN(K,NC)
C
C
   50 DO 70 K=1,NT
      IF(NLINE.LT.55) GO TO 60
      CALL TITLE (HED)
      WRITE(6,2000)
      WRITE(6,2001)
      NLINE = 10
   60 WRITE(6,2003) K,TFN(K,NC),FN(K,NC)
   70 NLINE = NLINE + 1
C
C
C     INTEGRATE THE ACCELEROGRAM TO OBTAIN THE VELOCITY
C
      FI=FN(1,NC)
      FN(1,NC)=0.
      IF(INPUT.EQ.2)  GO TO 55
      DO 80 K=2,NT
      L=K-1
      FI1=FN(K,NC)
      FN(K,NC)=FN(L,NC)+(FI+FI1)*(TFN(K,NC)-TFN(L,NC))*FOM/2.
      FI=FI1
      WRITE(6,81)K,TFN(K,NC),FN(K,NC)
81    FORMAT(60X,I5,2F15.5)
```

```
80     CONTINUE                                                        FUNC-67
       GO TO 100                                                       FUNC-68
C                                                                      FUNC-68
C      DIFFERENTIATE DISPLACEMENT TO GET VELOCITY                      FUNC-69
C                                                                      FUNC-70
55     DO 110 I=2,NT                                                   FUNC-71
       J=I-1                                                           FUNC-72
       IF(I.EQ.NT) GO TO 111                                           FUNC-73
       K=I+1                                                           FUNC-74
       A=TFN(I,NC)-TFN(J,NC)                                           FUNC-75
       B=TFN(K,NC)-TFN(I,NC)                                           FUNC-76
       C=FN(I,NC)                                                      FUNC-77
       D=(C-FI)/A                                                      FUNC-78
       E=(FN(K,NC)-C)/B                                                FUNC-79
       FN(I,NC)=(D+E)/2.                                               FUNC-80
       FI=C                                                            FUNC-81
       WRITE(6,81)I,TFN(I,NC),FN(I,NC)                                 FUNC-82
110    CONTINUE                                                        FUNC-83
111    FN(NT,NC)=(FN(NT,NC)-C)/(TFN(NT,NC)-TFN(J,NC))                  FUNC-84
       WRITE(6,81)K,TFN(K,NC),FN(K,NC)                                 FUNC-85
  100 CONTINUE                                                         FUNC  55
C                                                                      FUNC  56
C                                                                      FUNC  57
C      FORMAT STATEMENTS                                               FUNC  58
C                                                                      FUNC  59
 1000 FORMAT(2I5,F10.0,I5)                                             FUNC- 60
 1001 FORMAT(8F10.0)                                                   FUNC  61
 2000 FORMAT(25(1H*)/25H TIME DEPENDENT FUNCTIONS/25(1H*)//)           FUNC  62
 2001 FORMAT(4X,8HFUNCTION,4X,9HNUMBER OF,6X,10HTIME POINT,4X,4HTIME,  FUNC  63
     1       4X,8HFUNCTION/5X,6HNUMBER,4X,11HTIME POINTS,7X,6HNUMBER,  FUNC  64
     2       5X,5HVALUE,5X,5HVALUE/)                                   FUNC  65
 2002 FORMAT(4X,I5,7X,I5,40X,*MULTIPLICATION FACTOR=*,F10.4)           FUNC- 66
 2003 FORMAT(31X,I6,F12.3,F12.6)                                       FUNC- 67
 3000 FORMAT(//49H **ERROR**  (NPTS) MUST BE .GE. 2 AND .LE. (NPTM))   FUNC  68
 3001 FORMAT(//52H **ERROR**  BC FUNCTION TIME POINTS ARE OUT OF ORDER) FUNC  69
C                                                                      FUNC  72
       RETURN                                                          FUNC  73
       END                                                             FUNC  74
       SUBROUTINE INITAL (T,TAMB,NUMNP)                                INIT   1
C                                                                      INIT   2
C      *****************************************************************INIT   3
C      INITIALISE V.P.      (VELOCITY POTENTIAL)                       UNUT-  4
C      *****************************************************************INIT   5
C                                                                      INIT   6
       DIMENSION  T(NUMNP)                                             INIT   7
C                                                                      INIT   8
       ICON=0                                                          INIT-  9
       TAMB=0.                                                         INIT- 10
       DO 100 I=1,NUMNP                                                INIT  12
  100 T(I) = TAMB                                                      INIT  13
  300 RETURN                                                           INIT  35
       END                                                             INIT  36
       SUBROUTINE ELCAL                                                ELCL   1
C                                                                      ELCL   2
       COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                 ELCL   3
       COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15ELCL   4
       COMMON /ELSTOR/ NUMEST,MIDEST,MAXEST                            ELCL   5
       COMMON /JUNK  / HED(18),MTOT,NLINE                              ELCL   6
       COMMON /WORK  / NST(10),WORK(190)                               ELCL   7
       COMMON  A(1)                                                    ELCL   8
       DIMENSION  LABEL(2,2)                                           ELCL   9
       DATA  LABEL/6HAXISYM,6HMETRIC,6HP L A ,6HN A R /                ELCL  10
C                                                                      ELCL  11
```

```
C     ************************************************************************
C     THIS ROUTINE CALLS THE APPROPRIATE ELEMENT ROUTINES FOR READING,
C     GENERATING AND STORING THE ELEMENT DATA
C
C     TAPE ALLOCATION0
C          TAPE 1 - STORES ELEMENT GROUP DATA
C     ************************************************************************
C     T W O   D I M E N S I O N A L   F I N I T E   E L E M E N T S
C
C        NPAR(1) = 1
C        NPAR(2) = NUMBER OF TWO DIMENSIONAL ELEMENTS (NEL1)
C        NPAR(3) = NUMBER OF FIRST ELEMENT IN THIS GROUP (MFST)
C        NPAR(4) = ELEMENT TYPE CODE (ITYP2D)
C                  EQ.0, AXISYMMETRIC
C        NPAR(5) = MAXIMUM NUMBER OF NODES (MXNODS)
C        NPAR(6) = NUMERICAL INTEGRATION ORDER (NINT)
C     ************************************************************************
C     FREE SURFACE ELEMENTS
C
C        NPAR(1) = 2
C     NPAR(2)= NUMBER OF FREE SURFACE ELEMENTS(NEL2)
C        NPAR(3) = NUMBER OF FIRST ELEMENT IN THIS GROUP (MFST)
C        NPAR(4) = ELEMENT TYPE CODE (ITYP)
C                  EQ.0, AXISYMMETRIC FREE SURFACE ELEMENT
C     ************************************************************************
C     SOLID BOUNDRY ELEMENTS
C
C        NPAR(1) = 3
C     NPAR(2)=NUMBER OF SOLID BOUNDARY ELEMENTS(NEL3)
C        NPAR(3) = NUMBER OF FIRST ELEMENT IN THIS GROUP (MFST)
C        NPAR(4) = ELEMENT TYPE CODE (ITYP)
C                  EQ.0, AXISYMMETRIC SOLID-LIQUID BOUNDARY ELEMENTS
C     ************************************************************************
C
C     ZERO ACTIVE COLUMN HEIGHT ARRAY (MHT)
C
      N12 = N11 + NUMNP + 1
      N13 = N12 + NUMNP
      IF(N13.GT.MTOT) CALL ERROR (N13-MTOT)
C
      DO 5 I=N12,N13
    5 A(I) = 0.0
C
      REWIND 1
C
C     LOOP OVER ALL ELEMENT GROUPS
C
      DO 100 NG=1,NEG
      CALL TITLE (HED)
      WRITE(6,2000) NG
      NLINE = 7
C
      READ (5,1000) NPAR
C
      NGR  = NPAR(1)
C
      GO TO (1,2,3) NGR
C
C     ---------------
C     ELEMENT GROUP 1
C     ---------------
C
    1 IF(NPAR(2).GT.0) GO TO 10
```

```
      WRITE(6,3000)
      STOP
   10 IF(NPAR(6).LE.4) GO TO 20
      WRITE(6,3001)
      STOP
   20 IF(NPAR(3).EQ.0) NPAR(3) = 1
      IF(NPAR(5).EQ.0) NPAR(5) = 4
      IF(NPAR(6).EQ.0) NPAR(6) = 2
      IF(NPAR(7).EQ.0) NPAR(7) = 1
      IF(NPAR(8).EQ.0) NPAR(8) = 1
      IT = NPAR(4) + 1
C
      WRITE(6,2001) NGR,(LABEL(I,IT),I=1,2),NPAR(2),NPAR(3),NPAR(5),
     1              NPAR(6)
      CALL ADRS1
      GO TO 50
C
C     ---------------
C     ELEMENT GROUP 2
C     ---------------
C
    2 IF(NPAR(2).GT.0) GO TO 30
      WRITE(6,3000)
      STOP
30    IF(NPAR(5).EQ.0) NPAR(5) = 1
      IT = NPAR(4) + 1
C
      WRITE(6,2002) NGR,(LABEL(I,IT),I=1,2),NPAR(2),NPAR(3)
      CALL ADRS2
      GO TO 50
C
C     ---------------
C     ELEMENT GROUP 3
C     ---------------
C
    3 IF(NPAR(2).GT.0) GO TO 40
      WRITE(6,3000)
      STOP
   40 IF(NPAR(3).EQ.0) NPAR(3) = 1
C
      IT=NPAR(4)+1
      WRITE(6,2002) NGR,(LABEL(I,IT),I=1,2),NPAR(2),NPAR(3),NPAR(5)
      CALL ADRS3
C
   50 IF(MIDEST.GT.MAXEST) MAXEST = MIDEST
      IF(MIDEST.LE.NUMEST) GO TO 60
      GO TO 100
C
C     STORE ALL ELEMENT GROUP INFORMATION ONTO TAPE 1
C
   60 WRITE(1) MIDEST,NPAR,NST,(A(I),I=1,MIDEST)
C
  100 CONTINUE
C
      IF(MAXEST.LE.NUMEST) GO TO 300
      WRITE(6,3002) MAXEST
      STOP
C
C     FORMAT STATEMENTS
C
 1000 FORMAT(10I5)
 2000 FORMAT(23(1H*)/20H ELEMENT DATA, GROUP,I3/23(1H*)//)
 2001 FORMAT(26H ELEMENT GROUP INDICATOR =I3,18H (TWO DIMENSIONAL ,2A6,
```

```
    1         10H ELEMENTS)///                                            ELCL 143
    2         38H    NUMBER OF ELEMENTS ............ = I5/                ELCL 144
    3         38H    NO. OF FIRST ELEMENT IN GROUP .. = I5/               ELCL 145
    4         38H    MAX. NO. OF NODES PER ELEMENT .. = I5/               ELCL 146
    5         38H    NUMERICAL INTEGRATION ORDER .... = I5/)              ELCL-147
 2002 FORMAT(26H ELEMENT GROUP INDICATOR =I3,                             ELCL 153
    1         2H (,2A6,36H FREE SURFACE ELEMENTS            )///          ELCL 154
    2         38H    NUMBER OF ELEMENTS ............ = I5/                ELCL 155
    3         38H    NO. OF FIRST ELEMENT IN GROUP .. = I5/)              ELCL 156
 3000 FORMAT(//51H **ERROR**  NO. OF ELTS. IN GROUP MUST BE .GT. ZERO)   ELCL 162
 3001 FORMAT(//53H **ERROR**  NO. OF INTEGRATION PTS. MUST BE .LE. FOUR)ELCL 163
 3002 FORMAT(//41H **ERROR**  (NUMEST) MUST BE INCREASED TO I6)          ELCL 164
C                                                                         ELCL 165
  300 RETURN                                                              ELCL 166
      END                                                                 ELCL 167
      SUBROUTINE ADRS1                                                    ADS1   1
C                                                                         ADS1   2
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                     ADS1   3
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15ADS1   4
      COMMON /ELSTOR/ NUMEST,MIDEST,MAXEST                                ADS1   5
      COMMON /WORK  / M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,WORK(190)            ADS1   6
      COMMON  A(1)                                                        ADS1   7
C                                                                         ADS1   8
C     ******************************************************************ADS1   9
C                                                                         ADS1  10
C     B L A N K   C O M M O N   S T O R A G E   A L L O C A T I O N      ADS1  11
C                                                                         ADS1  12
C           ARRAY  -----------DESCRIPTION-----------   DIMENSION          ADS1  13
C     M1    LM     ELEMENT CONNECTIVITY ARRAY          MXNODS*NEL1        ADS1  14
C     M2    XY     ELEMENT COORDINATES                 2*MXNODS*NEL1      ADS1  15
C     M3    IELT   NO. OF NODES DESCRIBING ELEMENT     NEL1               ADS1  16
C     M4    NOD5   MIDSIDE NODES LOCATION ARRAY        ND5DIM*NEL1        ADS1  17
C                                                                         ADS1  25
C     ******************************************************************ADS1  26
C                                                                         ADS1  27
      NEL1   = NPAR(2)                                                    ADS1  28
      MXNODS = NPAR(5)                                                    ADS1  29
      NDM    = 2*MXNODS                                                   ADS1  32
      ND5DIM = MXNODS-4                                                   ADS1  33
C                                                                         ADS1  34
      M1     = 1                                                          ADS1  35
      M2     = M1   + MXNODS*NEL1                                         ADS1  36
      M3     = M2   + NDM*NEL1                                            ADS1  37
      M4     = M3   + NEL1                                                ADS1  38
      M5     = M4   + ND5DIM*NEL1                                         ADS1  39
      NLAST=M5-1                                                          ADS1- 40
C                                                                         ADS1  47
      WRITE(6,2000) NLAST                                                 ADS1  48
      MIDEST = NLAST                                                      ADS1  49
      IF(ND5DIM.EQ.0) ND5DIM = 1                                          ADS1  50
C                                                                         ADS1  51
      CALL ELGR1 (A(N1),A(N2),A(N12),A(M1),A(M2),A(M3),A(M4),MXNODS,NDM,ADS1- 52
     1ND5DIM)                                                             ADS1- 53
C                                                                         ADS1  55
 2000 FORMAT(38H    LENGTH OF ELEMENT INFORMATION .. = I5///)             ADS1  56
C                                                                         ADS1  57
      RETURN                                                              ADS1  58
      END                                                                 ADS1  59
      SUBROUTINE ELGR1 (X,Y,MHT,LM,XY,IELT,NOD5,MXNODS,NDM,ND5DIM)        ELG-   1
C                                                                         ELG1   3
C     ******************************************************************ELG1   4
C     INPUT INFORMATION FOR 4- TO 8-NODE ISOPARAMETRIC ELEMENTS           ELG1   5
C     ******************************************************************ELG1   6
```

```
C                                                                         ELG1    7
      DIMENSION  X(1),Y(1),MHT(1),LM(MXNODS,1),XY(NDM,1),IELT(1),         ELG1    8
     1           NOD5(ND5DIM,1)                                           ELG1-   9
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                     ELG1   11
      COMMON /JUNK  / HED(18),MTOT,NLINE                                  ELG1   12
      COMMON /WORK  / DUM(10),NOD(8),NODM(8),NOD5M(8),WORK(166)           ELG1   13
      DIMENSION  AST(2)                                                   ELG1   14
      DATA  AST/2H  ,2H */                                                ELG1   15
C                                                                         ELG1   16
      NEL1   = NPAR(2)                                                    ELG1   17
      MFST   = NPAR(3)                                                    ELG1   18
C                                                                         ELG1   19
C     ======================================                              ELG1   20
C     ======================================                              ELG1   22
C                                                                         ELG1   23
      CALL TITLE (HED)                                                    ELG1   24
C                                                                         ELG1   50
C     ====================================                                ELG1   51
C     READ AND GENERATE ELEMENT INFORMATION                               ELG1   52
C     ====================================                                ELG1   53
C                                                                         ELG1   54
      CALL TITLE (HED)                                                    ELG1   55
      WRITE(6,2003) NG                                                    ELG1   56
      WRITE(6,2004) (I,I=1,8)                                             ELG1   57
      NLINE = 10                                                          ELG1   58
      N     = 1                                                           ELG1   59
      IMEM  = MFST                                                        ELG1   60
      NLAST = MFST + NEL1 - 1                                             ELG1   61
C                                                                         ELG1   62
  100 READ (5,1002) M,NOD,IEL,KG                                          ELG-   63
C                                                                         ELG1   64
      IF(MTYP.EQ.0) MTYP = 1                                              ELG1   65
      IF(IEL .EQ.0) IEL  = MXNODS                                         ELG1   66
      IF(KG  .EQ.0) KG   = 1                                              ELG1   67
      IF(MXNODS.GE.IEL) GO TO 110                                         ELG1   68
      WRITE(6,3002) M                                                     ELG1   69
      STOP                                                                ELG1   70
C                                                                         ELG1   71
  110 IF(M-IMEM) 200,120,200                                              ELG1   72
C                                                                         ELG1   73
C     SAVE ELEMENT INFORMATION FOR GENERATION OF ADDITIONAL ELEMENTS      ELG1   74
C                                                                         ELG1   75
  120 DO 130 I=1,8                                                        ELG1   76
  130 NODM(I)=NOD(I)                                                      ELG1   77
      IF(IEL.EQ.4) GO TO 150                                              ELG1   78
      II=0                                                                ELG1   79
      DO 140 I=5,8                                                        ELG1   80
      NN=NOD(I)                                                           ELG1   81
      IF(NN.EQ.0) GO TO 140                                               ELG1   82
      II=II + 1                                                           ELG1   83
      NOD5M(II)=I                                                         ELG1   84
  140 CONTINUE                                                            ELG1   85
C                                                                         ELG1   86
150   IELM  = IEL                                                         ELG1-  87
      KKK   = KG                                                          ELG1   89
      ASTT  = AST(1)                                                      ELG1   90
C                                                                         ELG1   91
C     STORE PERMANENT ELEMENT INFORMATION                                 ELG1   92
C                                                                         ELG1   93
  200 I2 = 0                                                              ELG1   94
      DO 230 I=1,IELM                                                     ELG1   95
      IF(I.LE.4) GO TO 210                                                ELG1   96
      JJ = NOD5M(I-4)                                                     ELG1   97
```

```
      II = NODM(JJ)
      GO TO 220
  210 II = NODM(I)
  220 LM(I,N) = II
      I2 = I2 + 2
      XY(I2-1,N) = X(II)
  230 XY(I2  ,N) = Y(II)
C
      IELT(N) = IELM
      IF(IELM.EQ.4) GO TO 250
      NN=IELM - 4
      DO 240 I=1,NN
  240 NOD5(I,N)=NOD5M(I)
C
C     UPDATE COLUMN HEIGHTS AND BANDWIDTH
C
  250 CALL COLHT (MHT,IELM,LM(1,N))
C
      IF(NLINE.LT.55)  GO TO 260
      CALL TITLE (HED)
      WRITE(6,2003) NG
      WRITE(6,2004) (I,I=1,8)
      NLINE = 10
C
  260 WRITE(6,2005) ASTT,IMEM,NODM,IELM
      NLINE = NLINE + 1
      IF(IMEM.EQ.NLAST)  GO TO 300
C
      N = N + 1
      IMEM = IMEM + 1
C
C     CHECK IF ELEMENT DATA IS TO BE STORED FOR CURRENT ELEMENT
C
      IF(IMEM.EQ.M) GO TO 120
C
C     GENERATE NODE NUMBERS FOR NEXT ELEMENT
C
      DO 270 I=1,8
      IF(NODM(I).EQ.0) GO TO 270
      NODM(I)=NODM(I)+KKK
  270 CONTINUE
C
C     CHECK IF NEXT ELEMENT CARD IS TO BE READ
C
      ASTT=AST(2)
      IF(IMEM.GT.M) GO TO 100
C
C     GENERATE INFORMATION FOR NEXT ELEMENT
C
      GO TO 200
C
  280 WRITE(6,3003) M
      STOP
C
C     FORMAT STATEMENTS
C
 1002 FORMAT(11I5)
 2003 FORMAT(30(1H*)/27H ELEMENT INFORMATION, GROUP,I3/30(1H*)//)
 2004 FORMAT(4X,4HELT.,3X,10(1H-),12HNODE NUMBERS,10(1H-),3X,5H      ,3X,
     1       6HNO. OF/5X,3HNO.,3X,8(3X,I1),4X,3HNO.,5X,5HNODES/)
 2005 FORMAT(A2,I5,4X,8I4,10X,I5)
 3002 FORMAT(//10H **ELEMENT,I5,34H EXCEEDS MAXIMUM NUMBER OF NODES**)
 3003 FORMAT(//26H **ERROR**  ELEMENT CARD =I5,16H OUT OF SEQUENCE)
```

```
C                                                                       ELG1 176
  300 RETURN                                                            ELG1 177
      END                                                               ELG1 178
      SUBROUTINE ADRS2                                                  ADS2   1
C                                                                       ADS2   2
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                   ADS2   3
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15ADS2   4
      COMMON /ELSTOR/ NUMEST,MIDEST,MAXEST                              ADS2   5
      COMMON /WORK  / M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,WORK(190)          ADS2   6
      COMMON  A(1)                                                      ADS2   7
C                                                                       ADS2   8
C     *****************************************************************ADS2   9
C                                                                       ADS2  10
C     B L A N K   C O M M O N   S T O R A G E   A L L O C A T I O N     ADS2  11
C                                                                       ADS2  12
C           ARRAY   -----------DESCRIPTION-----------     DIMENSION     ADS2  13
C     M1    LM      ELEMENT CONNECTIVITY ARRAY            2*NEL2        ADS2  14
C     M2    XX      ELEMENT X-COORDINATES                 2*NEL2        ADS2  15
C     M3    CL      ELEMENT LENGTHS                       NEL2          ADS2  16
C                                                                       ADS2  23
C     *****************************************************************ADS2  24
C                                                                       ADS2  25
      NEL2  = NPAR(2)                                                   ADS2  26
C                                                                       ADS2  28
      M1    = 1                                                         ADS2  29
      M2    = M1   + 2*NEL2                                             ADS2  30
      M3    = M2   + 2*NEL2                                             ADS2  31
      M4    = M3   + NEL2                                               ADS2  32
      NLAST=M4-1                                                        ADS2- 33
C                                                                       ADS2  40
      WRITE(6,2000) NLAST                                               ADS2  41
      MIDEST = NLAST                                                    ADS2  42
C                                                                       ADS2  43
      CALL ELGR2 (A(N1),A(N2),A(M1),A(M2),A(M3))                        ADS2  44
C                                                                       ADS2  46
 2000 FORMAT(38H    LENCTH OF ELEMENT INFORMATION .. = I5///)           ADS2  47
C                                                                       ADS2  48
      RETURN                                                            ADS2  49
      END                                                               ADS2  50
      SUBROUTINE ELGR2 (X,Y,LM,XX,CL)                                   ELG2-  2
C                                                                       ELG2   2
C     *****************************************************************ELG2   3
C     INPUT INFORMATION FOR FREE SURFACE ELEMENTS                       ELG2 - 4
C     *****************************************************************ELG2   5
C                                                                       ELG2   6
      DIMENSION  X(1),Y(1),LM(2,1),XX(2,1),CL(1)                        ELG2-  7
C                                                                       ELG2   9
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                   ELG2  10
      COMMON /JUNK  / HED(18),MTOT,NLINE                                ELG2  11
      COMMON /NBC   / NNBC,NBCF,NPTM                                    ELG2  12
      COMMON /WORK  / DUM(10),NOD(2),NODM(2),WORK(186)                  ELG2  13
      DIMENSION  AST(2)                                                 ELG2  14
      DATA  AST/2H  ,2H */                                              ELG2  15
C                                                                       ELG2  16
      NEL2 = NPAR(2)                                                    ELG2  17
      MFST = NPAR(3)                                                    ELG2  18
C                                                                       ELG2  19
C     ======================================                            ELG2  20
      KBC = 0                                                           ELG2  24
C                                                                       ELG2  44
C     =====================================                             ELG2  45
C     READ AND GENERATE ELEMENT INFORMATION                             ELG2  46
C     =====================================                             ELG2  47
```

```
C
      N      = 1
      IMEM  = MFST
      NLAST = MFST + NEL2 - 1
      CALL TITLE (HED)
      WRITE(6,2003) NG
      WRITE(6,2004)
      NLINE = 10
C
  100 READ (5,1003) M,NOD,KG
C
      IF(KG  .EQ.0) KG   = 1
      II = NOD(1)
      JJ = NOD(2)
C
      IF(M-IMEM) 280,120,200
C
C     SAVE ELEMENT INFORMATION FOR GENERATION OF ADDITIONAL ELEMENTS
C
  120 NODM(1) = II
      NODM(2) = JJ
      KKK     = KG
      ASTT    = AST(1)
      XL      = SQRT((X(JJ)-X(II))**2 + (Y(JJ)-Y(II))**2)
C
C     STORE PERMANENT ELEMENT INFORMATION
C
  200 DO 230 I=1,2
      IJ = NODM(I)
      LM(I,N) = IJ
  230 XX(I,N) = X(IJ)
C
      CL(N)   = XL
C
      IF(NLINE.LT.55) GO TO 250
      CALL TITLE (HED)
      WRITE(6,2003) NG
      WRITE(6,2004)
      NLINE = 10
C
  250 WRITE(6,2005) ASTT,IMEM,NODM
      NLINE = NLINE + 1
      IF(IMEM.EQ.NLAST) GO TO 300
C
      N = N + 1
      IMEM = IMEM + 1
C
C     CHECK IF ELEMENT DATA IS TO BE STORED FOR CURRENT ELEMENT
C
      IF(IMEM.EQ.M) GO TO 120
C
C     GENERATE NODE NUMBERS FOR NEXT ELEMENT
C
      DO 270 I=1,2
  270 NODM(I) = NODM(I) + KKK
C
C     CHECK IF NEXT ELEMENT CARD IS TO BE READ
C
      ASTT = AST(2)
      IF(IMEM.GT.M) GO TO 100
C
C     GENERATE INFORMATION FOR NEXT ELEMENT
C
```

```
      GO TO 200                                                         ELG2 114
C                                                                       ELG2 115
  280 WRITE(6,3002) M                                                   ELG2 116
      STOP                                                              ELG2 117
C                                                                       ELG2 118
C     FORMAT STATEMENTS                                                 ELG2 119
C                                                                       ELG2 120
 1003 FORMAT(4I5)                                                       ELG2-121
 2003 FORMAT(30(1H*)/27H ELEMENT INFORMATION, GROUP,I3/30(1H*)//)       ELG2 133
 2004 FORMAT(4X,4HELT.,4X,6HI-NODE,4X,6HJ-NODE,4X,5H      /5X,          ELG2 134
     1       3HNO.,25X,3HNO./)                                          ELG2 135
 2005 FORMAT(A2,I5,5X,I5,5X,I5)                                         ELG2-136
 3002 FORMAT(//26H **ERROR**  ELEMENT CARD =I5,16H OUT OF SEQUENCE)     ELG2 139
C                                                                       ELG2 140
  300 RETURN                                                            ELG2 141
      END                                                               ELG2 142
      SUBROUTINE ADRS3                                                  ADS3   1
C                                                                       ADS3   2
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                   ADS3   3
      COMMON /ELSTOR/ NUMEST,MIDEST,MAXEST                              ADS3   4
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15ADS2   4
      COMMON /WORK  / M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,WORK(190)          ADS3   6
      COMMON  A(1)                                                      ADS3   7
C                                                                       ADS3   8
C     ******************************************************************ADS3   9
C                                                                       ADS3  10
C     B L A N K   C O M M O N   S T O R A G E   A L L O C A T I O N     ADS3  11
C                                                                       ADS3  12
C           ARRAY  -----------DESCRIPTION-----------    DIMENSION       ADS3  13
C     M1    LM     ELEMENT LOCATION ARRAY               NEL3            ADS3  14
C     M2    XX     ELEMENT X-COORDINATES                2*NEL3          ADS3- 15
C     M3    CL     ELEMENT LENGTHS                      NEL3            ADS3- 16
C     M4    SINS   SINE OF ANGLE SI                     NEL3            ADS3- 17
C     M5    COSS   COSINE OF ANGLE SI                   NEL3            ADS3- 17
C                                                                       ADS3  20
C     ******************************************************************ADS3  21
C                                                                       ADS3  22
      NEL3 = NPAR(2)                                                    ADS3  23
C                                                                       ADS3  24
      M1   = 1                                                          ADS3  25
      M2=M1+2*NEL3                                                      ADS3- 26
      M3=M2+2*NEL3                                                      ADS3- 27
      M4=M3+NEL3                                                        ADS3- 28
      M5=M4+NEL3                                                        ADS3- 28
      M6=M5+NEL3                                                        ADS3- 29
      NLAST=M6-1                                                        ADS3- 30
      WRITE(6,2000) NLAST                                               ADS3- 31
      MIDEST = NLAST                                                    ADS3- 32
      CALL ELGR3 (A(N1),A(N2),A(M1),A(M2),A(M3),A(M4),A(M5))            ADS3- 33
 2000 FORMAT(38H    LENGTH OF ELEMENT INFORMATION .. = I5///)           ADS3- 34
      RETURN                                                            ADS3  42
      END                                                               ADS3  43
      SUBROUTINE ELGR3(X,Y,LM,XX,CL,SINS,COSS)                          ELG3---1
C                                                                       ELG2   2
C     ******************************************************************ELG2   3
C     INPUT INFORMATION FOR SOLID BOUNDARY ELEMENT                      ELG3-  4
C     ******************************************************************ELG2   5
C                                                                       ELG2   6
      DIMENSION  X(1),Y(1),LM(2,1),XX(2,1),CL(1),SINS(1),COSS(1)        ELG3-  7
C                                                                       ELG2   9
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                   ELG2  10
      COMMON /JUNK  / HED(18),MTOT,NLINE                                ELG2  11
      COMMON /NBC   / NNBC,NBCF,NPTM                                    ELG2  12
```

```
      COMMON /WORK  / DUM(10),NOD(2),NODM(2),WORK(186)               ELG2  13
      DIMENSION  AST(2)                                              ELG2  14
      DATA  AST/2H  ,2H */                                           ELG2  15
C                                                                    ELG2  16
      NEL3=NPAR(2)                                                   ELG2--17
      MFST = NPAR(3)                                                 ELG2  18
C                                                                    ELG2  19
C     ======================================                         ELG2  20
      KBC = 0                                                        ELG2  24
C                                                                    ELG2  44
C     =====================================                          ELG2  45
C     READ AND GENERATE ELEMENT INFORMATION                          ELG2  46
C     =====================================                          ELG2  47
C                                                                    ELG2  48
      N      = 1                                                     ELG2  49
      IMEM   = MFST                                                  ELG2  50
      NLAST=MFST+NEL3-1                                              ELG2- 51
      CALL TITLE (HED)                                               ELG2  52
      WRITE(6,2003) NG                                               ELG2  53
      WRITE(6,2004)                                                  ELG2  54
      NLINE = 10                                                     ELG2  55
C                                                                    ELG2  56
100   READ(5,1003)M,NOD,KG,CS,SS                                     ELG2- 57
C                                                                    ELG2  58
      IF(KG  .EQ.0) KG   = 1                                         ELG2  60
      II = NOD(1)                                                    ELG2  61
      JJ = NOD(2)                                                    ELG2  62
C                                                                    ELG2  63
      IF(M-IMEM) 280,120,200                                         ELG2  64
C                                                                    ELG2  65
C     SAVE ELEMENT INFORMATION FOR GENERATION OF ADDITIONAL ELEMENTS ELG2  66
C                                                                    ELG2  67
  120 NODM(1) = II                                                   ELG2  68
      NODM(2) = JJ                                                   ELG2  69
      YD=ABS(Y(II)-Y(JJ))                                            ELG2- 70
      IF(YD.GT.0.00000001) GO TO 5                                   ELG2- 71
      SI=999999999.9                                                 ELG2- 72
      GO TO 6                                                        ELG2- 73
5     SI=(X(II)-X(JJ))/(Y(II)-Y(JJ))                                 ELG2- 74
6     KKK=KG                                                         ELG2- 75
      ASTT     = AST(1)                                              ELG2  72
      XL       = SQRT((X(JJ)-X(II))**2 + (Y(JJ)-Y(II))**2)           ELG2- 73
C                                                                    ELG2  74
C     STORE PERMANENT ELEMENT INFORMATION                            ELG2  75
C                                                                    ELG2  76
  200 DO 230 I=1,2                                                   ELG2  77
      IJ = NODM(I)                                                   ELG2  78
      LM(I,N) = IJ                                                   ELG2  79
  230 XX(I,N) = X(IJ)                                                ELG2  80
C                                                                    ELG2  81
      CL(N)    = XL                                                  ELG2  82
      SI=ATAN(SI)                                                    ELG3-83
      SINS(N)=SIN(SI)                                                ELG2- 84
      COSS(N)=COS(SI)                                                ELG2- 85
      COSS(N)=CS                                                     ELG2- 86
      SINS(N)=SS                                                     ELG2- 88
C                                                                    ELG2  84
      IF(NLINE.LT.55) GO TO 250                                      ELG2  85
      CALL TITLE (HED)                                               ELG2  86
      WRITE(6,2003) NG                                               ELG2  87
      WRITE(6,2004)                                                  ELG2  88
      NLINE = 10                                                     ELG2  89
  250 WRITE(6,2005) ASTT,IMEM,NODM                                   ELG2- 90
```

```
      NLINE = NLINE + 1                                                 ELG2  92
      IF(IMEM.EQ.NLAST) GO TO 300                                       ELG2  93
C                                                                       ELG2  94
      N = N + 1                                                         ELG2  95
      IMEM = IMEM + 1                                                   ELG2  96
C                                                                       ELG2  97
C     CHECK IF ELEMENT DATA IS TO BE STORED FOR CURRENT ELEMENT         ELG2  98
C                                                                       ELG2  99
      IF(IMEM.EQ.M) GO TO 120                                           ELG2 100
C                                                                       ELG2 101
C     GENERATE NODE NUMBERS FOR NEXT ELEMENT                            ELG2 102
C                                                                       ELG2 103
      DO 270 I=1,2                                                      ELG2 104
  270 NODM(I) = NODM(I) + KKK                                           ELG2 105
C                                                                       ELG2 106
C     CHECK IF NEXT ELEMENT CARD IS TO BE READ                          ELG2 107
C                                                                       ELG2 108
      ASTT = AST(2)                                                     ELG2 109
      IF(IMEM.GT.M) GO TO 100                                           ELG2 110
C                                                                       ELG2 111
C     GENERATE INFORMATION FOR NEXT ELEMENT                             ELG2 112
C                                                                       ELG2 113
      GO TO 200                                                         ELG2 114
C                                                                       ELG2 115
  280 WRITE(6,3002) M                                                   ELG2 116
      STOP                                                              ELG2 117
C                                                                       ELG2 118
C     FORMAT STATEMENTS                                                 ELG2 119
C                                                                       ELG2 120
1003  FORMAT(4I5,2F10.0)                                                ELG2-121
 2003 FORMAT(30(1H*)/27H ELEMENT INFORMATION, GROUP,I3/30(1H*)//)       ELG2 133
 2004 FORMAT(4X,4HELT.,4X,6HI-NODE,4X,6HJ-NODE,4X,5HDIREC/5X,           ELG1-121
     1        3HNO.,25X,6HCOSINE/)                                      ELG2-135
 2005 FORMAT(A2,I5,5X,I5,5X,I5)                                         ELG2-136
 3002 FORMAT(//26H **ERROR**  ELEMENT CARD =I5,16H OUT OF SEQUENCE)     ELG2 139
C                                                                       ELG2 140
  300 RETURN                                                            ELG2 141
      END                                                               ELG2 142
      SUBROUTINE COLHT (MHT,ND,LM)                                      CLHT   1
C                                                                       CLHT   2
      DIMENSION  LM(1),MHT(1)                                           CLHT   3
C                                                                       CLHT   4
C     FIND SMALLEST GLOBAL NODE NUMBER (LS) FOR ELEMENT                 CLHT   5
C                                                                       CLHT   6
      LS=100000                                                         CLHT   7
      DO 100 I=1,ND                                                     CLHT   8
      IF (LM(I)) 80,100,80                                              CLHT   9
   80 IF (LM(I)-LS) 90,100,100                                          CLHT  10
   90 LS=LM(I)                                                          CLHT  11
  100 CONTINUE                                                          CLHT  12
C                                                                       CLHT  13
C     COMPUTE COLUMN HEIGHT ABOVE DIAGONAL (ME) AND CHECK IF MAXIMUM    CLHT  14
C                                                                       CLHT  15
      DO 200 I=1,ND                                                     CLHT  16
      II=LM(I)                                                          CLHT  17
      IF (II.EQ.0) GO TO 200                                            CLHT  18
      ME=II - LS                                                        CLHT  19
      IF (ME.GT.MHT(II)) MHT(II)=ME                                     CLHT  20
 200  CONTINUE                                                          CLHT  21
C                                                                       CLHT  22
      RETURN                                                            CLHT  23
      END                                                               CLHT  24
      SUBROUTINE ERROR (N)                                              ERR    1
```

```
C                                                                         ERR    2
      WRITE(6,2000) N                                                     ERR    3
 2000 FORMAT(//31H **ERROR**  STORAGE EXCEEDED BY I6)                     ERR    4
      STOP                                                                ERR    5
      END                                                                 ERR    6
      SUBROUTINE TITLE (HED)                                              TITL   1
C                                                                         TITL   2
      DIMENSION  HED(18)                                                  TITL   3
C                                                                         TITL   4
C     THIS ROUTINE PRINTS THE TITLE CARD AT TOP OF OUTPUT PAGE            TITL   5
C                                                                         TITL   6
      WRITE(6,2000) HED                                                   TITL   7
 2000 FORMAT(1H1,18A4,39X,8HDOT 1976/)                                    TITL   8
      RETURN                                                              TITL   9
      END                                                                 TITL  10
      SUBROUTINE ADRSK (MAXA,MHT,NUMNP,NWK,MA)                            ADSK   1
C                                                                         ADSK   2
C     ******************************************************************ADSK   3
C     TO CALCULATE ADDRESSES OF DIAGONAL ELEMENTS IN A                    ADSK   4
C     BANDED MATRIX WHOSE COLUMN HEIGHTS ARE KNOWN.                       ADSK   5
C                                                                         ADSK   6
C     MA   = MAXIMUM BAND WIDTH                                           ADSK   7
C     MHT  = ACTIVE COLUMN HEIGHTS ABOVE DIAGONAL                         ADSK   8
C     MAXA = ADDRESSES OF DIAGONAL ELEMENTS                               ADSK   9
C     NWK  = MAXIMUM STORAGE REQUIRED                                     ADSK  10
C     ******************************************************************ADSK  11
C                                                                         ADSK  12
      DIMENSION  MAXA(1),MHT(1)                                           ADSK  13
C                                                                         ADSK  14
      MAXA(1) = 1                                                         ADSK  15
      MAXA(2) = 2                                                         ADSK  16
      MA = 0                                                              ADSK  17
      IF (NUMNP.EQ.1) GO TO 100                                           ADSK  18
      DO 10 I=2,NUMNP                                                     ADSK  19
      IF (MHT(I).GT.MA) MA = MHT(I)                                       ADSK  20
   10 MAXA(I+1) = MAXA(I) + MHT(I) + 1                                    ADSK  21
  100 MA  = MA + 1                                                        ADSK  22
      NWK = MAXA(NUMNP+1) - 1                                             ADSK  23
C                                                                         ADSK  24
      RETURN                                                              ADSK  25
      END                                                                 ADSK  26
      SUBROUTINE ASSEMK                                                   ASMK   1
C                                                                         ASMK   2
C     ******************************************************************ASMK   3
C     ASSEMBLE THE EFFECTIVE SYSTEM STIFFNESS MATRIX (K*)                 ASMK-  4
C     ******************************************************************ASMK   5
C                                                                         ASMK   6
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NC,KBC                     ASMK   7
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15ASMK   8
      COMMON /WORK  / M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,WORK(19)             ASMK   9
      COMMON  A(1)                                                        ASMK  10
      DIMENSION  NST(10)                                                  ASMK  11
      EQUIVALENCE  (NST(1),M1)                                            ASMK  12
C                                                                         ASMK  13
      REWIND 1                                                            ASMK  14
      REWIND 2                                                            ASMK  15
C                                                                         ASMK  16
C     LOOP OVER ALL ELEMENT GROUPS                                        ASMK  17
C                                                                         ASMK  18
      DO 100 NG=1,NEG                                                     ASMK  19
C                                                                         ASMK  20
      READ (1) MIDEST,NPAR,NST,(A(I),I=1,MIDEST)                          ASMK  21
C                                                                         ASMK  22
```

```
      NGR = NPAR(1)
C
      GO TO (1,2,3) NGR
C
C     ---------------
C     ELEMENT GROUP 1
C     ---------------
C
    1 MXNODS = NPAR(5)
      NDM    = 2*MXNODS
      ND5DIM = MXNODS-4
      IF(ND5DIM.EQ.0) ND5DIM = 1
C
      CALL COND1 (A(M1),A(M2),A(M3),A(M4),A(N8),MXNODS,NDM,ND5DIM)
      GO TO 100
C
C     ---------------
C     ELEMENT GROUP 2
C     ---------------
C
2     CALL COND2 (A(M1),A(M2),A(M3))
      GO TO 100
C
C     ---------------
C     ELEMENT GROUP 3
C     ---------------
3     CONTINUE
  100 CONTINUE
C
      RETURN
      END
      SUBROUTINE COND1 (LM,XY,IELT,NOD5,T,MXNODS,NDM,ND5DIM)
C
C     ******************************************************************
C     FORM THE EFFECTIVE SYSTEM STIFFNESS MATRIX (K*)
C     ******************************************************************
C
      DIMENSION  LM(MXNODS,1),XY(NDM,1),IELT(1),NOD5(ND5DIM, [illegible]
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSPEF,T [illegible]
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15
      COMMON /TODIM / NEL,NODS,MTYPE,NND5
      COMMON /WORK  / DUM(10),SK(64),SC(8),HF(8),TNOD(8),WOR[illegible]
      COMMON  A(1)
C
      NEL1  = NPAR(2)
C
      DO 100 N=1,NEL1
      NEL   = N
      NODS  = IELT(N)
      NND5  = NODS - 4
      NDOF  = NODS*NODS
C
C     ZERO ELEMENT STIFFNESS MATRIX SK(NODS,NODS)
C
      DO 10 I=1,NDOF
   10 SK(I) = 0.0
C
C     ZERO ELEMENT MASS VECTOE SC(NODS)
C
      DO 40 I=1,NODS
   40 SC(I) = 0.0
C
```

```
C     FORM ELEMENT STIFFNESS MATRIX USING GAUSS QUADRATURE
C
      CALL FORMK1 (SK,SC,XY(1,N),NOD5(1,N),NODS)
C
C     ASSEMBLE EFFECTIVE ELEMENT STIFFNESS    MATRIX (SK*)
C     INTO EFFECTIVE STRUCTURAL   STIFFNESS  MATRIX (K*)
C
   70 CALL ADDBAN (A(N10),A(N9),SK,LM(1,N),NODS)
  100 CONTINUE
C
      RETURN
      END
      SUBROUTINE FORMK1 (SK,SC,XY,NOD5,NODS)
C
C     ****************************************************************
C     ISOPARAMETRIC FORMULATION OF ELEMENT  STIFFNESS MATRIX
C     FOR  AXI-SYMMETRIC ELEMENT
C     ****************************************************************
C
      DIMENSION  SK(NODS,NODS),SC(NODS),XY(1),NOD5(1)
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP
      COMMON /WORK  / DUM(98),H(8),P(2,8),B(2,8),XJ(2,2),EK(3),WORK(55)
      DIMENSION  XG(4,4),WGT(4,4)
      DATA XG /       0.,               0.,               0.,               0.,
     1 -.5773502691896, .5773502691896,                  0.,               0.,
     2 -.7745966692415, .0000000000000, .7745966692415,                   0.,
     3 -.8611363115941,-.3399810435849, .3399810435849, .8611363115941/
      DATA WGT / 2.000,                 0.,               0.,               0.,
     1 1.0000000000000,1.0000000000000,                  0.,               0.,
     2  .5555555555556, .8888888888889, .5555555555556,                   0.,
     3  .3478548451375, .6521451548625, .6521451548625, .3478548451375/
C
      ITYP2D = NPAR(4)
      NINT   = NPAR(6)
C
C     LOOP OVER ALL  INTEGRATION POINTS
C
      VOL = 0.0
C
      DO 100 LX=1,NINT
      R  = XG(LX,NINT)
      DO 100 LY=1,NINT
      S  = XG(LY,NINT)
      WT = WGT(LX,NINT)*WGT(LY,NINT)
C
C     FIND INTERPOLATION FUNCTIONS (H) AND THEIR DERIVATIVES (P).
C     FIND JACOBIAN (XJ) AND ITS DETERMINANT (DETJ).
C
      CALL SHAPE1 (R,S,XY,H,P,NOD5,XJ,DETJ)
C
C     EVALUATE JACOBIAN INVERSE (XJI) AND GLOBAL DERIVATIVE OPERATOR (B)
C     AT EACH INTEGRATION POINT (R,S) WITHIN THE ELEMENT
C
      CALL DERIV1 (XY,H,P,B,XJ,DETJ,RAD,ITYP2D)
C
      FAC = WT*RAD*DETJ
      VOL = VOL + FAC
C
C     FORM  SK = B(TRANSPOSE)*EK*B FOR INTEGRATION POINT (R,S)
C
      DO 50 I=1,NODS
      BT1 = B(1,I)
```

```
      BT2 = B(2,I)
      DO 50 J=I,NODS
50    SK(I,J)=SK(I,J)+(BT1*B(1,J)+BT2*B(2,J))*FAC
C
  100 CONTINUE
C
      NODM = NODS - 1
      DO 200 I=1,NODM
      II = I + 1
      DO 200 J=II,NODS
  200 SK(J,I) = SK(I,J)
C
  600 RETURN
      END
      SUBROUTINE SHAPE1 (R,S,XY,H,P,NOD5,XJ,DETJ)
C
C     *****************************************************************
C     1. TO FIND INTERPOLATION FUNCTIONS ( H )
C        AND DERIVATIVES ( P ) CORRESPONDING TO THE NODAL POINTS
C        OF A 4- TO 8-NODE TWO DIMENSIONAL ISOPARAMETRIC ELEMENT
C     2. TO FIND JACOBIAN ( XJ ) AND ITS DETERMINANT ( DETJ )
C
C
C                        NODE NUMBERING CONVENTION
C
C
C                2                  5                  1
C
C                  0 . . . . . . . . 0 . . . . . . . . 0
C                  .                                   .
C                  .                                   .
C                  .                 S                 .
C                  .                 .                 .
C                  .                 .                 .
C                6 0                 . . . R           0 8
C                  .                                   .
C                  .                                   .
C                  .                                   .
C                  .                                   .
C                  0 . . . . . . . . 0 . . . . . . . . 0
C
C                3                  7                  4
C
C     *****************************************************************
C
      DIMENSION  XY(2,1),H(1),P(2,1),NOD5(1),XJ(2,2)
      COMMON /TODIM / NEL,NODS,MTYPE,NND5
      DIMENSION  IPERM(4)
      DATA IPERM/2,3,4,1/
C
C     INTERPOLATION FUNCTIONS (4-NODE ELEMENT)
C
      RP   = 1.0 + R
      SP   = 1.0 + S
      RM   = 1.0 - R
      SM   = 1.0 - S
      R2   = 1.0 - R*R
      S2   = 1.0 - S*S
C
      H(1) = 0.25*RP*SP
      H(2) = 0.25*RM*SP
      H(3) = 0.25*RM*SM
```

```
      H(4) = 0.25*RP*SM
C
C     LOCAL DERIVATIVES OF INTERPOLATION FUNCTIONS (4-NODE ELEMENT)
C
      P(1,1) = 0.25*SP
      P(1,2) = -P(1,1)
      P(1,3) = -0.25*SM
      P(1,4) = -P(1,3)
      P(2,1) = 0.25*RP
      P(2,2) = 0.25*RM
      P(2,3) = -P(2,2)
      P(2,4) = -P(2,1)
C
C     INTERPOLATION FUNCTIONS AND LOCAL DERIVATIVES FOR MIDSIDE NODES
C
      IF(NODS.EQ.4) GO TO 50
C
      I = 0
    2 I = I + 1
      IF (I.GT.NND5) GO TO 40
      NN = NOD5(I) - 4
      GO TO (5,6,7,8) NN
C
    5 H(5)   = 0.50*R2*SP
      P(1,5) = -R*SP
      P(2,5) = 0.50*R2
      GO TO 2
    6 H(6)   = 0.50*RM*S2
      P(1,6) = -0.50*S2
      P(2,6) = -RM*S
      GO TO 2
    7 H(7)   = 0.50*R2*SM
      P(1,7) = -R*SM
      P(2,7) = -0.50*R2
      GO TO 2
    8 H(8)   = 0.50*RP*S2
      P(1,8) = 0.50*S2
      P(2,8) = -RP*S
      GO TO 2
C
C     MODIFY INTERPOLATION FUNCTIONS H(1) TO H(4) AND LOCAL DERIVATIVES
C
   40 IH = 0
   41 IH = IH + 1
      IF(IH.GT.NND5) GO TO 50
      IN = NOD5(IH)
      I1 = IN - 4
      I2 = IPERM(I1)
      H(I1) = H(I1) - 0.5*H(IN)
      H(I2) = H(I2) - 0.5*H(IN)
      H(IH+4) = H(IN)
      DO 45 J=1,2
      P(J,I1) = P(J,I1) - 0.5*P(J,IN)
      P(J,I2) = P(J,I2) - 0.5*P(J,IN)
   45 P(J,IH+4) = P(J,IN)
      GO TO 41
C
C     EVALUATE THE JACOBIAN MATRIX AT POINT (R,S)
C
   50 DO 100 I=1,2
      DO 100 J=1,2
      SUM = 0.0
      DO 90 K=1,NODS
```

```
   90 SUM = SUM + P(I,K)* XY(J,K)
  100 XJ(I,J) = SUM
C
C     COMPUTE THE DETERMINANT OF THE JACOBIAN MATRIX AT POINT (R,S)
C
      DETJ = XJ(1,1)* XJ(2,2) - XJ(2,1)* XJ(1,2)
      DUM = ABS(DETJ)
      IF(DUM.GT.1.0E-8) GO TO 500
      WRITE (6,3000) NEL
      STOP
C
 3000 FORMAT(//49H **ERROR**  ZERO JACOBIAN DETERMINANT FOR ELEMENT,I5)
C
  500 RETURN
      END
      SUBROUTINE DERIV1 (XY,H,P,B,XJ,DETJ,RAD,ITYP2D)
C
C     ******************************************************************
C     EVALUATION OF THE GLOBAL DERIVATIVE OPERATOR (B) AT A POINT (R,S)
C     FOR A QUADRILATERAL ELEMENT HAVING PLANAR OR AXISYMMETRIC GEOMETRY
C     ******************************************************************
C
      DIMENSION  XY(2,1),H(1),P(2,1),B(2,1),XJ(2,2)
      COMMON /TODIM / NEL,NODS,MTYPE,NND5
      COMMON /WORK  / DUM(145),XJI(2,2),WORK(51)
C
C     COMPUTE INVERSE OF THE JACOBIAN MATRIX
C
      DETJI    = 1.0/DETJ
      XJI(1,1) =  XJ(2,2)* DETJI
      XJI(1,2) = -XJ(1,2)* DETJI
      XJI(2,1) = -XJ(2,1)* DETJI
      XJI(2,2) =  XJ(1,1)* DETJI
C
C     EVALUATE GLOBAL DERIVATIVE OPERATOR ( B-MATRIX )
C
      DO 10 K=1,NODS
      B(1,K) = XJI(1,1)*P(1,K) + XJI(1,2)*P(2,K)
   10 B(2,K) = XJI(2,1)*P(1,K) + XJI(2,2)*P(2,K)
C
      RAD = 1.0
      IF(ITYP2D.NE.0) GO TO 500
C
C     COMPUTE THE RADIUS AT POINT (R,S) FOR AXISYMMETRIC SOLID
C
      RAD = 0.0
      DO 50 K=1,NODS
   50 RAD = RAD + H(K)* XY(1,K)
C
      IF(RAD.GT.1.0E-8) GO TO 500
      WRITE(6,3000) NEL
      STOP
C
 3000 FORMAT(//50H **ERROR**  ZERO RADIUS ENCOUNTERED IN ELEMENT NO.,I5)
C
  500 RETURN
      END
      SUBROUTINE ADDBAN (A,MAXA,S,LM,NDOF)
C
C     ******************************************************************
C     ASSEMBLE ELEMENT STIFFNESS INTO COMPACTED GLOBAL STIFFNESS
C     ******************************************************************
C
```

```
      DIMENSION  A(1),MAXA(1),S(1),LM(1)                                ADBN   7
C                                                                       ADBN   8
      DO 200 J=1,NDOF                                                   ADBN   9
      JJ = LM(J)                                                        ADBN  10
      MJ = MAXA(JJ)                                                     ADBN  11
      DO 200 I=1,NDOF                                                   ADBN  12
      II = LM(I)                                                        ADBN  13
      IJ = JJ - II                                                      ADBN  14
      IF(IJ) 200,100,100                                                ADBN  15
  100 KK = MJ + IJ                                                      ADBN  16
      LS = (J-1)*NDOF + I                                               ADBN  17
      A(KK) = A(KK) + S(LS)                                             ADBN  18
  200 CONTINUE                                                          ADBN  19
C                                                                       ADBN  20
      RETURN                                                            ADBN  21
      END                                                               ADBN  22
      SUBROUTINE ADDC (A,SC,LM,IEL,NUMNP)                               ADDC-  1
C                                                                       ADDC   2
C     ******************************************************************ADDC   3
C     ADD ELEMENT MASS TO GLOBAL LUMPED MASS VECTOR                     ADDC-  4
C     ******************************************************************ADDC   5
C                                                                       ADDC   6
      DIMENSION  A(1),SC(1),LM(1)                                       ADDC   7
C                                                                       ADDC   8
      DO 100 I=1,IEL                                                    ADDC   9
      II=LM(I)                                                          ADDC  10
  100 A(II) = A(II) + SC(I)                                             ADDC  11
C                                                                       ADDC  12
      RETURN                                                            ADDC  13
      END                                                               ADDC  14
      SUBROUTINE COND2 (LM,XX,CL)                                       CON2-  1
C                                                                       CON2   2
C     ******************************************************************CON2   3
C     CALCULATE MASS MATRIX                                             CON2-  4
C     ******************************************************************CON2   5
C                                                                       CON2   6
      DIMENSION  LM(2,1),XX(2,1),CL(1)                                  CON2-  7
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NS,KBC                   CON2   8
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP      CON2   9
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15CON2  10
      COMMON/CONST /A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,THETA,DELTA,ALPHA,PI,GCON2- 11
     +,RO                                                               CON2- 12
      COMMON /WORK  / DUM(10),SC(2),WORK(188)                           CON2- 13
      COMMON  A(1)                                                      CON2  12
C                                                                       CON2  13
      NEL2 = NPAR(2)                                                    CON2  14
C                                                                       CON2  16
      DO 100 N=1,NEL2                                                   CON2  17
C                                                                       CON2  33
C     AXI-SYMMETRIC FREE SURFACE BOUNDARY ELEMENTS                      CON2- 34
C                                                                       CON2  35
   30 XI       = XX(1,N)                                                CON2  36
      XJ       = XX(2,N)                                                CON2  37
      WRITE(6,5) XI,XJ,CL(N),G,N                                        CON2- 38
    5 FORMAT(50X,4F15.5,I10)                                            CON2- 39
      SC(1)=(2.*XI+XJ)*CL(N)/(6.3*G)                                    CON2- 40
      SC(2)=(XI+2.*XJ)*CL(N)/(6.3*G)                                    CON2- 41
C                                                                       CON2  40
      CALL ADDC(A(N12),SC,LM(1,N),2,NUMNP)                              CON2- 42
  100 CONTINUE                                                          CON2  48
C                                                                       CON2  49
      RETURN                                                            CON2  50
      END                                                               CON2  51
```

```
      SUBROUTINE KSTAR(MAXA,XK,C)
C
C*********************************************************************
C     THIS SUBROUTINE CALCULATES  EFFECTIVE K
C*********************************************************************
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC
      COMMON/CONST /A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,THETA,DELTA,ALPHA,PI,G
     +,RO
      DIMENSION MAXA(1),XK(1),C(1)
      DO 100 I=1,NUMNP
      NNI=MAXA(I)
      XK(NNI)=XK(NNI)+C(I)*A0
100   CONTINUE
      RETURN
      END
      SUBROUTINE COLSOL (A,V,MAXA,NN,MA,NWA,KKK)
C
C     *****************************************************************
C     TO SOLVE SIMULTANEOUS EQUATIONS AX=V IN COPE, USING
C     COMPACTED STORAGE AND COLUMN REDUCTION SCHEME.
C
C     A     = MATRIX STORED IN COMPACTED FORM
C     V     = VECTOR TO BE REDUCED
C     MAXA = VECTOR CONTAINING ADDRESSES OF DIAGONAL ELEMENTS OF A
C
C     FLAG FOR TRIANGULARIZATION (A=LU) AND/OR SIMPLE FORWARD
C     REDUCTION (LY=V) AND BACKSUBSTITUTION (UX=Y)0
C
C        KKK=0  TRIANGULARIZATION ONLY
C        KKK=1  TRIANGULARIZATION PLUS SOLUTION
C        KKK=2  FORWARD REDUCTION AND BACKSUBSTITUTION ONLY
C        KKK=3  BACKSUBSTITUTION ONLY
C     *****************************************************************
C
      DIMENSION A(NWA),V(1),MAXA(1)
C
      MA1=MA - 1
      IF (KKK-2) 100,700,800
C
C     =================
C     TRIANGULARIZATION
C     =================
C
  100 IF(NN.EQ.1) GO TO 800
      N=1
      IF (A(1)) 80,85,110
   80 WRITE (6,3000) N
      STOP
   85 WRITE (6,3001) N
      STOP
C
  110 DO 200 N=2,NN
      KL=MAXA(N) + 1
      KU=MAXA(N+1) - 1
      IF (KU-KL) 200,210,210
 210  D=0.
C
      KN=MAXA(N)
      K=N
      DO 220 KK=KL,KU
      K=K - 1
      KI=MAXA(K)
      C=A(KK)/A(KI)
```

```
      B=B + C*A(KK)
 220  A(KK)=C
      A(KN)=A(KN) - B
C
      IF (A(KN)) 222,224,226
 222  WRITE (6,3000) N
      STOP
 224  WRITE (6,3001) N
      STOP
 226  MR=MIN0(MA1,NN-N)
      IF (MR) 200,200,228
 228  MN=KU - KL + 1
C
      DO 240 J=1,MR
      MJ=MAXA(N+J) + J
      MNJ=MAXA(N+J+1) - MJ - 1
      IF (MNJ) 240,240,230
 230  ND=MIN0(MN,MNJ)
      C=0.
      KU=KN + ND
      IC=MJ - KN
      DO 300 KK=KL,KU
 300  C=C + A(KK)*A(KK+IC)
      A(KN+IC)=A(KN+IC) - C
 240  CONTINUE
C
 200  CONTINUE
      IF(KKK.EQ.0) RETURN
C
C     =================
C     FORWARD REDUCTION
C     =================
C
 700  DO 400 N=2,NN
      KL=MAXA(N) + 1
      KU=MAXA(N+1) - 1
      IF (KU-KL) 400,410,410
 410  K=N
      C=0.
      DO 420 KK=KL,KU
      K=K - 1
 420  C=C + A(KK)*V(K)
      V(N)=V(N) - C
 400  CONTINUE
      GO TO 800
C
C     =================
C     BACK SUBSTITUTION
C     =================
C
 800  DO 480 N=1,NN
      K=MAXA(N)
 480  V(N)=V(N)/A(K)
      IF (NN.EQ.1) RETURN
      N=NN
      DO 500 L=2,NN
      KL=MAXA(N) + 1
      KU=MAXA(N+1) - 1
      IF (KU-KL) 500,510,510
 510  K=N
      DO 520 KK=KL,KU
      K=K - 1
 520  V(K)=V(K) - A(KK)*V(N)
```

```
 500  N=N - 1                                                           COLS 112
C                                                                       COLS 113
C     FORMAT STATEMENTS                                                 COLS 114
C                                                                       COLS 115
 3000 FORMAT(//45H **STOP**  STIFFNESS NOT POSITIVE DEFINITE.../,        COLS 116
     1          10X,27H NEGATIVE PIVOT IN POSITION I4)                  COLS 117
 3001 FORMAT(//33H **STOP**  ZERO PIVOT IN POSITION I4)                  COLS 118
C                                                                       COLS 119
      RETURN                                                            COLS 120
      END                                                               COLS 121
      SUBROUTINE FORMQC(TTH)                                            FMQC-  1
C                                                                       FMQC   2
C     ******************************************************************FMQC   3
C     CALCULATE THE LOADING VECTOR                                      FMQC-  4
C     ******************************************************************FMQC   6
C                                                                       FMQC   7
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                   FMQC   8
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP      FMQC   9
      COMMON /DIM   / N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15FMQC  10
      COMMON /NBC   / NNBC,NBCF,NPTM                                    FMQC  11
      COMMON /WORK  / M1,M2,M3,M4,M5,M6,M7,M8,M9,M10,WORK(190)          FMQC  12
      COMMON  A(1)                                                      FMQC  13
      DIMENSION  NST(10)                                                FMQC  14
      EQUIVALENCE  (NST(1),M1)                                          FMQC  15
C                                                                       FMQC  16
      REWIND 1                                                          FMQC  17
      REWIND 2                                                          FMQC  18
C                                                                       FMQC  19
C     LOOP OVER ALL ELEMENT GROUPS                                      FMQC  20
C                                                                       FMQC  21
      DO 100 NG=1,NEG                                                   FMQC  22
C                                                                       FMQC  23
      READ (1) MIDEST,NPAR,NST,(A(I),I=1,MIDEST)                        FMQC  24
C                                                                       FMQC  25
      NGR = NPAR(1)                                                     FMQC  26
C                                                                       FMQC  27
      IF(NGR.NE.3)  GO TO 100                                           FMQC- 28
      CALL FLUX2 (A(M1),A(M2),A(M3),A(M4),A(M5),A(N1),A(N2),A(N3),A(N8),FMQC- 29
     1A(N11),A(N14),NPTM,TTH)                                           FMQC- 30
  100 CONTINUE                                                          FMQC  45
C                                                                       FMQC  46
      RETURN                                                            FMQC  47
      END                                                               FMQC  48
      SUBROUTINE FLUX2 (LM,XX,CL,SINS,COSS,TFN,FN,NPTS,T,Q,TT,NPTM,TTH) FLX2-  1
      COMMON/CONST /A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,THETA,DELTA,ALPHA,PI,GFLX2-  2
     +,RO                                                               FLX2-  3
C                                                                       FLX2   3
C     ******************************************************************FLX2   4
C     CALCULATE THE LOADING VECTOR                                      FLX2-  5
C     ******************************************************************FLX2   6
C                                                                       FLX2   7
      COMMON /CNTRL1/ NUMNP,NEG,MODEX,NPAR(10),NG,KBC                   FLX2  10
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP      FLX2  11
      COMMON /NBC   / NNBC,NBCF,IDUMMY                                  FLX2- 12
      DIMENSION  LM(2,1),XX(2,1),CL(1),SINS(1),COSS(1),T(1),TFN(NPTM,1),FLX2- 13
     1FN(NPTM,1),NPTS(1),Q(1),TT(1)                                     FLX2- 14
C                                                                       FLX2  12
      NEL3=NPAR(2)                                                      FLX2- 13
      ITYP = NPAR(4)                                                    FLX2  14
C                                                                       FLX2  21
 10   DO 100 N=1,NEL3                                                   FLX2- 22
      II    = LM(1,N)                                                   FLX2  23
      JJ    = LM(2,N)                                                   FLX2  24
```

```
      XL    = CL(N)                                                     FLX2  25
      NPT=NPTS(1)                                                       FLX2- 26
      NC=1                                                              FLX2- 27
      CALL  INTERP(TFN(1,NC),FN(1,NC),NPT,TTH,VX)                       FLX2- 28
      IF(NBCF.NE.1) GO TO 21                                            FLX2- 29
      VZ=0.                                                             FLX2- 30
      GO TO 22                                                          FLX2- 31
 21   NC=2                                                              FLX2- 32
      NPT=NPTS(2)                                                       FLX2-33
      CALL  INTERP(TFN(1,NC),FN(1,NC),NPT,TTH,VZ)                       FLX2- 34
22    VN=VX*COSS(N)+VZ*SINS(N)                                          FLX2- 35
C                                                                       FLX2  59
C                                                                       FLX2  61
   25 XI = XX(1,N)                                                      FLX2  62
      XJ = XX(2,N)                                                      FLX2  63
      Q(II)=Q(II)+    VN*XL*(2.*XI+XJ)/6.0                              FLX2- 64
      Q(JJ)=Q(JJ)+    VN*XL*(XI+2.*XJ)/6.0                              FLX2- 65
  100 CONTINUE                                                          FLX2  99
C                                                                       FLX2 100
      RETURN                                                            FLX2 101
      END                                                               FLX2 102
      SUBROUTINE  INTERP (TFN,FN,NPT,TIME,VAL)                          INTP   1
C                                                                       INTP   2
C     ******************************************************************INTP   3
C     THIS ROUTINE  INTERPOLATES A GIVEN  TIME-DEPENDENT  FUNCTION  TO FIND INTP   4
C     THE VALUE  OF  THE FUNCTION  (VAL)  AT A  PARTICULAR  TIME POINT  (TIME) INTP   5
C     ******************************************************************INTP   6
C                                                                       INTP   7
      DIMENSION   TFN(1),FN(1)                                          INTP   8
C                                                                       INTP   9
      DO 10 N=1,NPT                                                     INTP  10
      DTIME = TFN(N) - TIME                                             INTP  11
      IF(DTIME.GT.0.) GO TO 15                                          INTP  12
   10 CONTINUE                                                          INTP  13
C                                                                       INTP  14
   15 DIFF = TFN(N) - TFN(N-1)                                          INTP  15
      VAL  = FN(N) - (FN(N) - FN(N-1))*DTIME/DIFF                       INTP  16
C                                                                       INTP  17
      RETURN                                                            INTP  18
      END                                                               INTP  19
      SUBROUTINE QEFF  (Q,C,T,TD,TDD,NUMNP)                             QEFF-  1
C                                                                       QEFF   2
C     ******************************************************************QEFF   3
C     FORM THE  EFFECTIVE  LOAD  VECTOR                                 QEFF-  4
C     ******************************************************************QEFF   5
C                                                                       QEFF   6
      COMMON/CONST /A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,THETA,DELTA,ALPHA,PI,GQEFF-  7
     +,RO                                                               QEFF-  8
      DIMENSION Q(1),C(1),T(1),TD(1),TDD(1)                             QEFF-  9
C                                                                       QEFF   8
      DO 10  I=1,NUMNP                                                  QEFF   9
10    Q(I)=Q(I)+C(I)*(A0*T(I)+A2*TD(I)+A3*TDD(I))                       QEFF- 10
C                                                                       QEFF  11
      RETURN                                                            QEFF  12
      END                                                               QEFF  13
      SUBROUTINE PTEMP  (T,TIME,NUMNP)                                  PTEM   1
C                                                                       PTEM   2
C     ******************************************************************PTEM   3
C     PUNCH  THE  NODAL  DISPLACEMENTS                                  PTEM-  4
C     ******************************************************************PTEM   5
C                                                                       PTEM   6
      DIMENSION   T(1)                                                  PTEM   7
C                                                                       PTEM   8
```

```
      WRITE(3,2000) TIME                                                PTEM   9
      NCARD = NUMNP/4 + 1                                               PTEM  10
      NC = 0                                                            PTEM  11
C                                                                       PTEM  12
      DO 100 I=1,NUMNP,4                                                PTEM  13
      NC = NC + 1                                                       PTEM  14
      IP = I + 3                                                        PTEM  15
      IF(NC.EQ.NCARD) IP = NUMNP                                        PTEM  16
  100 WRITE(3,2001) (N,T(N),N=I,IP)                                     PTEM  17
C                                                                       PTEM  18
 2000 FORMAT(35H NODAL POINT DISPLACEMENT AT TIME = F11.4)              PTEM- 19
 2001 FORMAT(4(I5,5X,F10.3))                                            PTEM  20
C                                                                       PTEM  21
      RETURN                                                            PTEM  22
      END                                                               PTEM  23
      SUBROUTINE CALCU(T,TT,TD,TDD,P,E,NUMNP)                           CAL    1
      COMMON/CONST /A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,THETA,DELTA,ALPHA,PI,GCAL    2
     +,RO                                                               CAL    3
      COMMON /CNTRL2/ KST,NDT,DT,TSTART,TAMB,NPRINT,NTSREF,TIME,KP      CAL    4
      DIMENSION T(1),TT(1),TD(1),TDD(1),P(1),E(1)                       CAL    5
C                                                                       CAL    6
C     T ABOVE HAS VALUES AT TIME + TAU                                  CAL    7
      DO 1000 I=1,NUMNP                                                 CAL    8
C     CALCULATE SECOND DERIVATVE OF V. P. AT TIME+ TAU(T1)              CAL    9
C                                                                       CAL   10
      T1=A0*(T(I)-TT(I))-A2*TD(I)-A3*TDD(I)                             CAL   11
C                                                                       CAL   13
C     FIND SECOND DERIVATIVE OF V.P. AT TIME+DT                         CAL   14
C                                                                       CAL   14
      T2=TDD(I)                                                         CAL   15
      TDD(I)=T2+(T1-T2)/THETA                                           CAL   16
C                                                                       CAL   17
C     FIND FIRST DERIVATIVE OF V.P. AT TIME+DT                          CAL   18
C                                                                       CAL   19
      T3=TD(I)                                                          CAL   20
      TD(I)=T3+A6*T2+A7*TDD(I)                                          CAL   21
C                                                                       CAL   22
C     FIND V.P. AT TIME+DT                                              CAL   23
      T(I)=TT(I)+DT*T3+A8*T2+A9*TDD(I)                                  CAL   24
C                                                                       CAL   25
C     FIND PRESSURE(P) AND SURFACE DISPLACEMENT    E                    CAL   26
C                                                                       CAL   27
      P(I)=-RO*TD(I)                                                    CAL   28
      E(I)=-TD(I)/G                                                     CAL   29
1000  CONTINUE                                                          CAL   30
  300 RETURN                                                            NPBC  92
      END                                                               NPBC  93
      SUBROUTINE OUT (T,NUMNP,TIME,KSTEP)                               OUT    1
C                                                                       OUT    2
C     ******************************************************************OUT    3
C     PRINT NODALDISPLACEMENTS FOR *TIME*                               OUT    4
C     ******************************************************************OUT    5
C                                                                       OUT    6
      DIMENSION  T(1)                                                   OUT    7
C                                                                       OUT    8
      WRITE(6,2000) KSTEP,TIME                                          OUT    9
      WRITE(6,2001) (N,T(N),N=1,NUMNP)                                  OUT   10
C                                                                       OUT   11
C     FORMAT STATEMENTS                                                 OUT   12
C                                                                       OUT   13
 2000 FORMAT(//28H DISPLACEMENT AT TIME STEP =I5,2X,7H(TIME =E11.4,1H)/)OUT   19
 2001 FORMAT(6(I6,E14.6))                                               OUT   15
C                                                                       OUT   16
```

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