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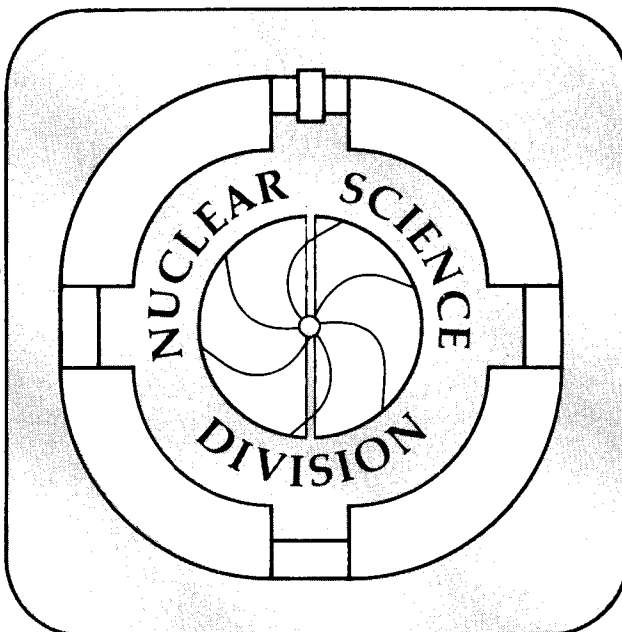
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# Energy Loss of a Heavy Quark in the Quark-Gluon Plasma<sup>1</sup>

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## Abstract

The energy loss  $dE/dx$  for a heavy quark propagating through a quark-gluon plasma is calculated to leading order in the QCD coupling constant. Simple formulae for  $dE/dx$  are obtained in the regions  $E \ll M_Q^2/T$  and  $E \gg M_Q^2/T$ , where  $M_Q$  is the mass of the heavy quark and  $T$  is the temperature. The crossover energy between the two regions is determined to be approximately  $1.8M_Q^2/T$ . Under conditions relevant to ultrarelativistic heavy ion collisions, charm quarks and bottom quarks lie on opposite sides of the crossover energy and therefore experience significantly different energy losses.

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Jets caused by high energy quarks and gluons in ultrarelativistic heavy ion collisions might provide a probe for the existence of a quark-gluon plasma (QGP). High energy partons coming from initial hard collisions lose energy by propagating through the dense matter formed between the nuclei after collision. The energy loss is expected to be greater in  $AA$  collisions compared to  $pp$  or  $pA$  collisions, a phenomenon known as jet quenching. Gyulassy and Plümer<sup>1</sup> suggested that jet quenching should be suppressed if the dense matter consists of a QGP instead of hadrons. Their observation was based on an estimate of the energy loss of high energy partons in a QGP by Bjorken<sup>2</sup> ( $-dE/dx \sim 0.2$  GeV/fm for a 20 GeV quark at a temperature of about  $T = 0.25$  GeV), which is considerably smaller than in hadronic matter ( $-dE/dx \sim 1$  GeV/fm). Bjorken considered the collisional energy loss of a massless quark due to elastic scattering off the quarks and gluons in the QGP. At tree level, there is a logarithmic infrared singularity in the integral over the 3-momentum transfer  $q$ . Bjorken estimated the energy loss by keeping only the logarithmic term with physically reasonable upper and lower limits  $q_{min}$  and  $q_{max}$ . In a complete calculation, the upper and lower cutoffs on  $q$  should be provided automatically by the physics of the energy loss process. The purpose of this paper is to present a complete calculation of the energy loss of an energetic heavy quark to leading order in the QCD coupling constant.

Thoma and Gyulassy<sup>3</sup> eliminated the ambiguity due to the lower limit  $q_{min}$  of the momentum transfer by properly including the screening effects of the QGP. The logarithmic infrared divergence that arises in naive calculations is cut off at the *soft* momentum scale  $g_s T$ , where  $g_s$  is the QCD coupling constant. Unfortunately their calculation was incomplete in the region of *hard* momentum transfer ( $q \sim T$ ) and therefore required the imposition of an upper limit  $q_{max}$ . The resulting ambiguity was avoided in a complementary calculation carried out by Svetitsky<sup>4</sup> in a study of the diffusion of charm quarks in the QGP. Svetitsky's drag coefficient is directly related to the energy loss:  $A(p^2) = (-dE/dx)/p$ , where  $E$  and  $p$  are the energy and momentum of the heavy quark. In a straightforward tree level calculation of the energy loss due to elastic scattering, the kinematics of the scattering process automatically sets an upper limit on the momentum transfer. To cut off the infrared divergence in the tree level calculation, Svetitsky used the ad hoc prescription of introducing a gluon mass which violates gauge invariance and introduces an ambiguity equivalent to Bjorken's choice of  $q_{min}$ .

To obtain the energy loss to leading order in  $g_s$ , the hard momentum transfer contribution can be calculated at tree level, but in the soft  $q$  region it is necessary to use a resummed perturbation expansion developed by Braaten and Pisarski<sup>5</sup>. The *hard thermal loop* corrections to the propagator of the exchanged gluon must be resummed in order to include the screening effects of the plasma, and this resummation reproduces the result of Thoma and Gyulassy<sup>3</sup> in the soft region. The contributions from the hard and soft regions must then be matched together consistently to give the complete energy loss to leading order in  $g_s$ . A general method for carrying out this matching has been developed by Braaten and Yuan<sup>6</sup>. One introduces an arbitrary intermediate momentum scale  $q^*$  satisfying  $g_s T \ll q^* \ll T$ , which is always possible in the weak coupling limit  $g_s \rightarrow 0$ . For moderate values of  $g_s$ , this should be interpreted as merely a mathematical device for isolating all terms of leading order in  $g_s$ . The contribution from hard  $q > q^*$  is calculated using tree level Feynman diagrams while ignoring any screening due to the plasma. The logarithmic infrared divergences of the tree level calculation manifest themselves as logarithms of  $q^*$ . The contribution from soft momentum transfers  $q < q^*$  is calculated using the resummed perturbation expansion to take into account the effects of screening, and it also depends logarithmically on  $q^*$ . Adding the hard and soft contributions, the dependence on the arbitrary scale  $q^*$  cancels.

In Ref. [7], a quantum field theoretic formulation of the energy loss was developed. The method of Braaten and Yuan<sup>6</sup> was then used to compute the energy loss of a muon propagating through a plasma of electrons, positrons, and photons to leading order in the QED coupling constant  $e$  and in  $T/M_\mu$ . In this paper, that calculation will be extended to obtain the energy loss of a relativistic heavy quark propagating through a QGP to leading order in  $g_s$  and in  $T/M_Q$ , where  $M_Q$  is the mass of the heavy quark. We assume that the mass  $M_Q$  and the momentum  $p$  of the heavy quark are both much larger than the temperature  $T$  of the plasma. Simple results can be obtained in two energy regimes,  $E \ll M_Q^2/T$  and  $E \gg M_Q^2/T$ . The maximum momentum transfer  $q$  from elastic scattering off a thermal quark or gluon with energy  $k$  is  $q_{max} = 2k(1 + k/E)/(1 - v + 2k/E)$ , where  $v$  is the velocity of the heavy quark. In the region  $E \ll M_Q^2/T$ , we can set  $q_{max} = 2k/(1 - v)$ , while for  $E \gg M_Q^2/T$ , the maximum momentum transfer is the energy of the heavy quark:  $q_{max} = E$ . In the region  $E \sim M_Q^2/T$ , the formula for  $dE/dx$  is very complicated because the general expression for  $q_{max}$  must be used. A good approximation to  $dE/dx$  in this

crossover region can be obtained by switching from the formula for  $E \ll M_Q^2/T$  to the formula for the ultrarelativistic region  $E \gg M_Q^2/T$  at a crossover energy  $E_{cross}$  which is determined by continuity.

At leading order in  $g_s$ , the energy loss of a heavy quark comes from elastic scattering from thermal quarks ( $Qq$  scattering) and thermal gluons ( $Qg$  scattering). We first consider the energy loss in the region  $E \ll M_Q^2/T$ . We assume that the heavy quark has a kinetic energy much greater than  $T$ . Some of the contributions to  $dE/dx$  can be obtained from the corresponding QED calculation in Ref. [7] by simple substitution. The soft contribution from  $Qg$  and  $Qq$  scattering is obtained from the QED case by replacing  $e$  by the QCD coupling constant  $g_s$ , multiplying by a color factor  $4/3$ , and replacing the thermal photon mass  $m_\gamma = eT/3$  by the thermal gluon mass<sup>8</sup>  $m_g = (g_s T/\sqrt{3})\sqrt{1+n_f/6}$ , where  $n_f$  is the number of active flavors in the QGP. The result is

$$\begin{aligned} \left(-\frac{dE}{dx}\right)_{soft}^{Qg+Qq} &= \frac{g_s^4 T^2}{6\pi} \left(1 + \frac{n_f}{6}\right) \left\{ \left[ \frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right] \log \frac{q^*}{m_g} \right. \\ &\quad - \frac{1}{v^2} \int_0^v dx x^2 \left[ \log \frac{3\pi x}{2} + \frac{1}{2} \log(1+Q_\ell(x)^2) + Q_\ell(x) \left( \frac{\pi}{2} - \arctan Q_\ell(x) \right) \right] \\ &\quad \left. - \frac{1}{2v^2} \int_0^v dx x^2 \frac{v^2 - x^2}{1-x^2} \left[ \log \frac{3\pi x}{4} + \frac{1}{2} \log(1+Q_t(x)^2) + Q_t(x) \left( \frac{\pi}{2} - \arctan Q_t(x) \right) \right] \right\}, \end{aligned} \quad (1)$$

where  $Q_\ell(x) = [-\log((1+x)/(1-x)) + 2x]/\pi$  and  $Q_t(x) = [\log((1+x)/(1-x)) + 2x/(1-x^2)]/\pi$ . The hard contribution to  $dE/dx$  from  $Qq$ -scattering is obtained from the QED calculation in Ref. [7] by replacing  $e$  by  $g_s$ , multiplying by a color factor  $2/3$ , and summing over the  $n_f$  flavors of the initial thermal quark:

$$\begin{aligned} \left(-\frac{dE}{dx}\right)_{hard}^{Qq} &= \frac{g_s^4 T^2 n_f}{6\pi \cdot 6} \left\{ \left[ \frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right] \left( \log \frac{4TE}{q^* M_Q} + \frac{3}{2} - \gamma + \frac{\zeta'(2)}{\zeta(2)} \right) \right. \\ &\quad \left. - \frac{1-v^2}{4v^2} \left[ Sp\left(\frac{1+v}{2}\right) - Sp\left(\frac{1-v}{2}\right) + \frac{1}{2} \log \frac{1+v}{1-v} \log \frac{1-v^2}{4} \right] - \frac{2}{3}v \right\}, \end{aligned} \quad (2)$$

where  $Sp(x) = -\int_0^x dt(1/t) \log(1-t)$  is the Spence function,  $\gamma = 0.57722$  is Euler's constant, and  $\zeta(z)$  is the Riemann zeta function:  $\zeta'(2)/\zeta(2) = -0.56996$ . Note that the logarithm of  $q^*$  in (2) cancels against the  $\log(q^*)$  term proportional to  $n_f$  in (1).

The only new calculation that is required to obtain  $dE/dx$  for a heavy quark is the hard contribution from  $Qg$  scattering. The rate of energy loss  $-dE/dx$  is obtained by computing the interaction rate of the heavy quark weighted by the factor  $(E - E')/v$ , where  $v$  is the velocity of the heavy quark and  $E'$  is its energy after the collision. The hard contribution can be isolated by imposing a constraint  $q > q^*$  on the magnitude of the 3-momentum transfer  $\vec{q} = \vec{k}' - \vec{k}$ . The hard contribution from  $Qg$  scattering is

$$\left(-\frac{dE}{dx}\right)^{Qg} = \frac{1}{2E} \int \frac{d^3p'}{(2\pi)^3 2E'} \int \frac{d^3k}{(2\pi)^3 2k} n_B(k) \int \frac{d^3k'}{(2\pi)^3 2k'} (1 + n_B(k'))$$

$$(2\pi)^4 \delta^4(P + K - P' - K') 16 \langle |\mathcal{M}|^2 \rangle \frac{E - E'}{v} \theta(q - q^*), \quad (3)$$

where  $P = (E, \vec{p})$  and  $K = (k, \vec{k})$  are the 4-momenta of the incoming quark and gluon and  $P'$  and  $K'$  are the momenta of the outgoing quark and gluon. The phase space is weighted by a Bose distribution  $n_B(k) = (e^{k/T} - 1)^{-1}$  for the incoming gluon and a Bose-enhancement factor  $1 + n_B(k')$  for the outgoing gluon. The factor 16 is the number of spin and color states of the thermal gluon, and  $\langle |\mathcal{M}|^2 \rangle$  is the square of the matrix element, averaged over initial and summed over final spins and colors.

The matrix element  $\mathcal{M} = \mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u$  for  $Qg$  scattering is given by the sum of the 3 Feynman diagrams in Figure 1. There are  $s$ - and  $u$ -channel diagrams that correspond to Compton scattering in QED and a  $t$ -channel diagram involving the 3-gluon vertex. In QED, Compton scattering does not contribute to  $dE/dx$  for a heavy lepton at leading order in  $T/M_\mu$  due to a cancellation between the  $s$ - and  $u$ -channel diagrams. In QCD, the cancellation is upset by the nonabelian coupling of quarks to gluons. The square of the amplitude  $\mathcal{M}_s + \mathcal{M}_u$  reduces in the limit  $k, k' \ll E$  to

$$\langle |\mathcal{M}_s + \mathcal{M}_u|^2 \rangle = g_s^4 \frac{M_Q^4}{(P \cdot K)(P \cdot K')}. \quad (4)$$

The interference term  $Re\langle \mathcal{M}_t(\mathcal{M}_s + \mathcal{M}_u)^* \rangle$  between the  $t$ -channel and the  $s$ - and  $u$ -channel amplitudes vanishes in this limit. In the Feynman gauge, the square of the  $t$ -channel amplitude reduces in the limit  $k, k' \ll E$  to

$$\langle |\mathcal{M}_t|^2 \rangle = 8g_s^4 \frac{(P \cdot K)(P \cdot K') - M_Q^2(K \cdot K')}{Q^4}, \quad (5)$$

where  $Q = K' - K$  is the four momentum of the exchanged gluon. One must take care in computing (5) to sum only over the physical polarization states of the gluons.



If a covariant expression is used for the polarization sum, then a contribution to (5) from quark-ghost scattering must be included in order to cancel that of unphysical gluon polarization states.<sup>9</sup>

The squared matrix elements (4) and (5) must be inserted into (3). The integrals can be evaluated analytically using the methods of Ref. [7]. Scattering in the  $s$ - and  $u$ -channels does not give rise to an infrared divergence from the small  $q$  region, so the infrared cutoff  $q^*$  can be set to zero in this calculation. The result is:

$$\left(-\frac{dE}{dx}\right)_{hard}^{Qg(s+u)} = \frac{g_s^4 T^2}{12\pi} \left[ \frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right]. \quad (6)$$

There is an infrared divergence from the  $t$ -channel diagram, so the cutoff  $q > q^*$  is necessary. Using  $q^* \ll T$ , the result is

$$\begin{aligned} \left(-\frac{dE}{dx}\right)_{hard}^{Qg(t)} &= \frac{g_s^4 T^2}{6\pi} \left\{ \left[ \frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right] \left( \log \frac{2TE}{q^* M_Q} + 1 - \gamma + \frac{\zeta'(2)}{\zeta(2)} \right) \right. \\ &\quad \left. - \frac{1-v^2}{4v^2} \left[ Sp\left(\frac{1+v}{2}\right) - Sp\left(\frac{1-v}{2}\right) + \frac{1}{2} \log \frac{1+v}{1-v} \log \frac{1-v^2}{4} \right] - \frac{2}{3}v \right\}. \quad (7) \end{aligned}$$

The complete result for  $dE/dx$  for a relativistic heavy quark with energy  $E \ll M_Q^2/T$  is obtained by adding the soft contribution (1) and the hard contributions (2), (6) and (7). It can be written in the form

$$-\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left(1 + \frac{n_f}{6}\right) \left[ \frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v} \right] \log \left( 2^{\frac{n_f}{6+n_f}} B(v) \frac{ET}{m_g M_Q} \right), \quad (8)$$

where  $B(v)$  is a smooth function of the velocity that increases monotonically from  $B(0) = 0.604$  at  $v = 0$  to a maximum of 0.731 at  $v = 0.88$ , and then decreases to  $B(1) = 0.629$  at  $v = 1$ .

We next consider scattering in the ultrarelativistic region  $E \gg M_Q^2/T$ . Here again several of the contributions can be extracted from the QED calculation published in Ref. [7]. The soft contribution from  $Qg$  and  $Qq$  scattering is

$$\left(-\frac{dE}{dx}\right)_{soft}^{Qg+Qq} = \frac{g_s^4 T^2}{6\pi} \left(1 + \frac{n_f}{6}\right) \left[ \log \frac{q^*}{m_g} - 0.843 \right]. \quad (9)$$

The hard contribution from  $Qq$  scattering is

$$\left(-\frac{dE}{dx}\right)_{hard}^{Qq} = \frac{g_s^4 T^2 n_f}{12\pi \cdot 6} \left[ \log \frac{2TE}{(q^*)^2} + \frac{8}{3} - \gamma + \frac{\zeta'(2)}{\zeta(2)} \right]. \quad (10)$$

The hard contribution from  $Qg$  scattering requires a new calculation. The  $s$ - and  $u$ -channel contributions vanish, and the  $t$ -channel gives

$$\left(-\frac{dE}{dx}\right)_{hard}^{Qg} = \frac{g_s^4 T^2}{12\pi} \left[ \log \frac{TE}{(q^*)^2} + \frac{8}{3} - \gamma + \frac{\zeta'(2)}{\zeta(2)} \right]. \quad (11)$$

Adding up the contributions to  $dE/dx$  in (9)-(11), the total energy loss for energies  $E \gg M_Q^2/T$  is

$$-\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left(1 + \frac{n_f}{6}\right) \log \left(2^{\frac{n_f}{2(6+n_f)}} 0.920 \frac{\sqrt{ET}}{m_g}\right), \quad (12)$$

In the region  $E \sim M_Q^2/T$ , the formula for the energy loss must cross over from the  $v \rightarrow 1$  limit of (8) to (12). It should be a good approximation to simply use (8) up to some crossover energy  $E_{cross}$  and then switch to (12). By demanding that  $dE/dx$  remain continuous at  $E = E_{cross}$ , we determine the crossover energy to be  $E_{cross} = 1.80M_Q^2/T$  for  $n_f = 2$  active flavors of quarks.

The energy loss formulae (8) and (12) are illustrated by the solid lines in Figures 2 and 3 for conditions of relevance to ultrarelativistic heavy ion collisions. We take the temperature of the plasma to be  $T = 250$  MeV, and we take the strong coupling constant at that temperature to be  $\alpha_s = 0.2$ , which is in accordance with some lattice QCD calculations.<sup>10</sup> In Figure 2, we show the energy loss of the charm quark, assuming a mass  $M_c = 1.5$  GeV. The crossover energy is  $E_{cross} = 16$  GeV, so that most of the range of energy relevant for jets in heavy ion collisions is covered by the ultrarelativistic formula (12). The discontinuity in slope at the crossover point could be avoided by a more complete calculation for the region  $E \sim M_Q^2/T$ . In Figure 3, we show the energy loss of a bottom quark with mass  $M_b = 5.0$  GeV. The crossover energy is  $E_{cross} = 180$  GeV, so the entire range of energy is covered by the formula (8). Note that the energy loss for a bottom quark in a QGP has a different momentum dependence from the charm quark, and is significantly smaller in magnitude. For example, for a momentum of 20 GeV, we find  $-dE/dx = 0.30$  GeV/fm for a charm quark and  $-dE/dx = 0.15$  GeV/fm for a bottom quark, both

of which are considerably smaller than the energy loss in hadronic matter of about 1 GeV/fm. Bottom quark jets should therefore suffer significantly less quenching than charm quark jets if a QGP is created in a heavy ion collision.

Also shown for comparison in Figures 2 and 3 are the results of previous calculations of the energy loss. The dotted curves are the estimates of Bjorken<sup>2</sup> for light quarks adapted to the case of heavy quarks:

$$-\frac{dE}{dx} = \frac{8\pi\alpha_s^2 T^2}{3} \left(1 + \frac{n_f}{6}\right) \left[\frac{1}{v} - \frac{1-v^2}{2v^2} \log \frac{1+v}{1-v}\right] \log \frac{q_{max}}{q_{min}}. \quad (13)$$

For the upper limit on the momentum transfer, we follow Bjorken in using  $q_{max} = \sqrt{4TE}$ , while for the lower limit we use the Debye screening mass  $q_{min} = \sqrt{3}m_g$ . The dashed curves in Figures 2 and 3 are the calculations of Thoma and Gyulassy<sup>3</sup>. For the charm quark in Figure 2, the complete calculation is in reasonable agreement with that of Thoma and Gyulassy at low energy, but then crosses over to a form that is closer to Bjorken's estimate (13) at high energies.

The formula (8) for the energy loss breaks down at thermal energies  $v \sim \sqrt{T/M_Q}$ . The energy loss must change sign in this region, because a quark with  $v = 0$  can only gain energy in a collision. The methods used above to compute  $dE/dx$  at high energies can also be used to compute it in the limit  $v \rightarrow 0$ . For weak coupling  $g_s$ ,  $-dE/dx$  is negative corresponding to energy gain and it has a kinematic  $1/v$  divergence:

$$-\frac{dE}{dx} = -\frac{16\pi\alpha_s^2 T^3}{3M_Q v} \left(1 + \frac{n_f}{6}\right) \log \left(2^{\frac{n_f}{6+n_f}} 0.604 \frac{T}{m_g}\right). \quad (14)$$

Note that the coefficient inside the logarithm agrees with the one in (8) in the limit  $v \rightarrow 0$ . A comparison of (14) with the  $v \rightarrow 0$  limit of (8) can be used to make a semiquantitative estimate of the velocity at which the energy loss changes sign:  $v \simeq \sqrt{3T/M_Q}$ . With a plasma temperature of 250 MeV as used in Figures 2 and 3, this corresponds to a momentum of 1.5 GeV for charm quarks and 2.1 GeV for bottom quarks.

The formula (8) for  $dE/dx$  also breaks down at low energies for another reason. For coupling constants  $g_s > 1.08$ , the argument of the logarithm in (8) is less than 1 for  $E < 0.93g_s M_Q$  and the energy loss is negative, implying a gain of energy. If  $M_Q$  is sufficiently large, the energy below which (8) turns negative is much greater than the thermal energy. This unphysical behavior is due to a failure of the extrapolation

from the weak coupling limit  $g_s \rightarrow 0$  to the physical value of  $g_s$ . This failure is not of great practical importance for the properties of heavy quark jets in ultrarelativistic heavy ion collisions. For the charm quark as shown in Figure 2, (8) turns negative at a momentum of about 1.2 GeV, while for the bottom quark as illustrated in Figure 3, it turns negative around 4.1 GeV. These are both lower than the energies of experimental interest. Nevertheless, this result serves as a warning that for properties of the QGP that have a logarithmic sensitivity to different energy scales, calculations to leading order in  $g_s$  may be meaningless. The usefulness of such a calculation can be determined only after computing the multiplicative constant inside the logarithm. In particular, any calculation that keeps only the leading order  $\log(1/g_s)$  term is meaningless until the constant is also calculated.

The negative values of  $-dE/dx$  predicted by (8) arise from isolating the terms of leading order in  $g_s$ , without any contamination from higher orders in  $g_s$ . This difficulty can be circumvented by using an effective propagator for soft gluons not only at soft momentum transfers  $q$  as in Ref. [7], but for all  $q$ . In the hard  $q$  region, this corresponds to including a subset of corrections that are higher order in  $g_s$ . Consistency and gauge invariance then demand that the effective vertices of Braaten and Pisarski<sup>5</sup> be used for the coupling of the virtual gluon to the thermal quarks and gluons and that the thermal quarks and gluons be placed on the mass shells of their respective effective propagators. This would of course greatly increase the complexity of the calculation of  $dE/dx$ .

We discuss briefly the energy loss of a high energy light quark in the QGP. The energy loss of an ultrarelativistic heavy quark in (12) was calculated under the assumption  $T \ll M_Q \ll E$ . To obtain the energy loss of a light quark to leading order in  $g_s$ , the calculation should be repeated under the assumption  $m_q \ll T \ll E$ . A recent calculation by Mrowczynski<sup>11</sup> improves on previous calculations but is incomplete. Mrowczynski recognized the need to include contributions from both hard and soft momentum transfer. Mrowczynski's separation of the integral over the transverse component  $q_T$  of the momentum transfer into 2 regions  $q_T < k_0$  and  $q_T > k_0$  is similar to our separation of  $q$  into soft ( $q < q^*$ ) and hard ( $q > q^*$ ) regions. In contrast to our calculation of  $dE/dx$  for a heavy quark where the dependence on  $q^*$  cancels, Mrowczynski's final result depends on  $k_0$ . He makes the arbitrary choice of setting  $k_0$  equal to the Debye screening mass  $\sqrt{3}m_g$ . The failure of the  $k_0$ -dependence to cancel in the final answer is a symptom of an incomplete calculation. Mrowczyn-

ski's calculation also suffers an additional ambiguity in the choice of the maximum energy transfer, which should be imposed automatically by the physics in a complete calculation. Very large energy transfers arise from the energy of the quark being transferred to a collinear gluon, in which case the energy is not dispersed into the plasma. For light particles, it is most appropriate to think in terms of the energy of the jet rather than that of individual particles. Energy transfer to a collinear particle should then be interpreted not as energy loss, but instead as evolution of the jet due to its interactions with the plasma. A complete calculation of  $dE/dx$  for the jet will necessarily depend on the definition of a jet.

In this paper, we have only considered the collisional energy loss due to elastic  $Qg$  and  $Qq$  scattering. In addition, there is radiative energy loss due to bremsstrahlung processes, such as  $Qq$  scattering into  $Qqg$  with exchange of a virtual gluon. While a naive tree level calculation of the radiative energy loss gives a result that is higher order by a factor of  $g_s^2$ , it suffers from a quadratic infrared divergence in the integral over the momentum transferred through the virtual gluon. When the screening of the plasma at the scale  $g_s T$  is taken into account by resumming hard thermal loop corrections to the gluon propagator, the quadratic divergence is replaced by a factor of  $1/g_s^2 T^2$ . Thus the radiative process contributes to the energy loss at the same order in  $g_s$  as elastic scattering. Unfortunately, as is the case with the damping rate for a high energy particle<sup>12,3</sup>, the screening of the plasma at the scale  $g_s T$  only softens the quadratic infrared divergence of the tree level calculation into a logarithmic one. Due to the lack of screening of the static magnetic interaction at the scale  $g_s T$ , the radiative energy loss has a logarithmic infrared sensitivity to the smaller momentum scale  $g_s^2 T$ . A complete leading order calculation of the radiative energy loss must therefore await the development of more powerful resummation techniques than those needed to calculate the energy loss from elastic scattering.

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## Figure Captions

1. Tree level Feynman diagrams for  $Qg$  scattering ( $s$ -channel,  $u$ -channel, and  $t$ -channel).
2. Energy loss  $dE/dx$  of a charm quark as a function of its momentum for  $T = 250\text{MeV}$  and  $\alpha_s = 0.2$ . The complete result to leading order in  $g_s$  (solid curve) is compared to previous calculations by Thoma and Gyulassy (dashed curve) and Bjorken (dotted curve).
3. Energy loss  $dE/dx$  of a bottom quark as a function of its momentum for  $T = 250\text{MeV}$  and  $\alpha_s = 0.2$ . The complete result to leading order in  $g_s$  (solid curve) is compared to previous calculations by Thoma and Gyulassy (dashed curve) and Bjorken (dotted curve).

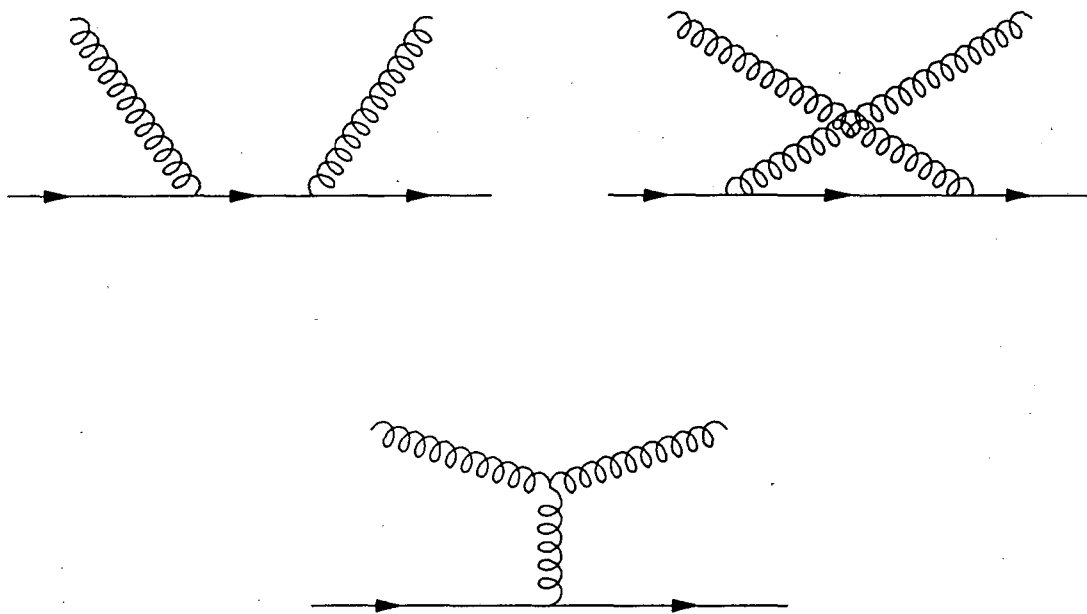


Figure 1



Figure 2

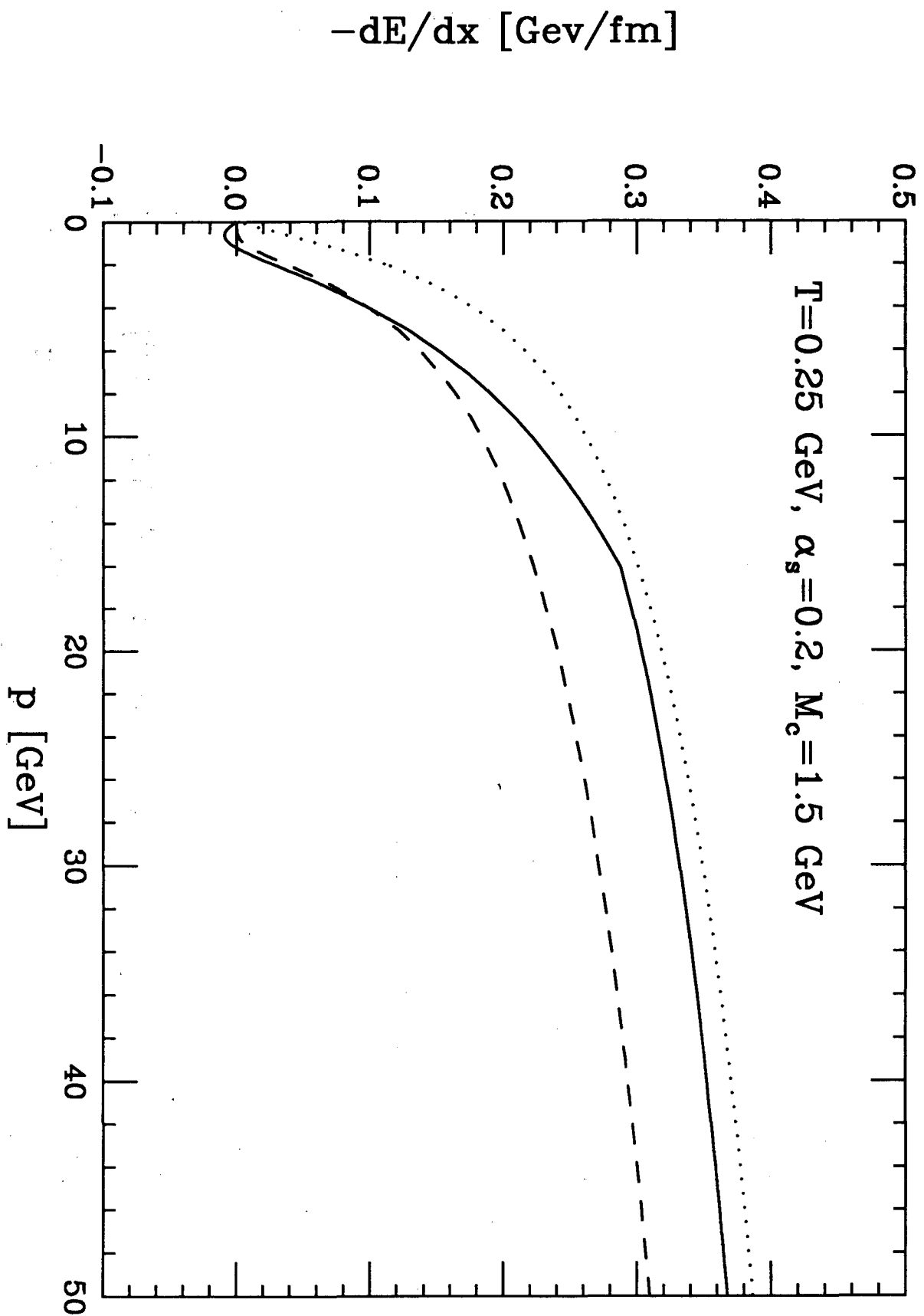
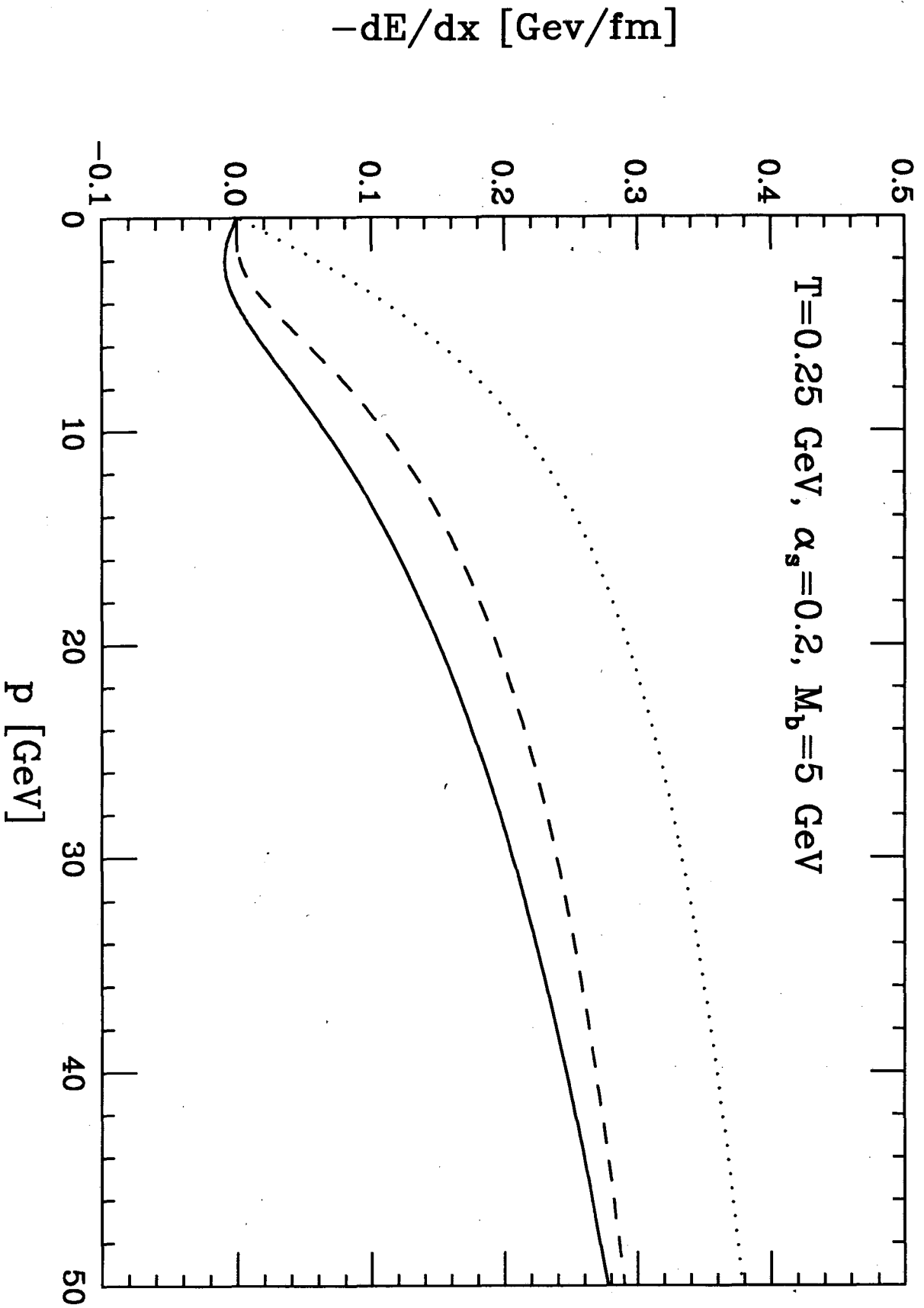


Figure 3



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