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Journal

Combustion Science and Technology, 187(1-2)

ISSN

0010-2202 1563-521X

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Publication Date

2014-12-10

DOI

10.1080/00102202.2014.967397

Peer reviewed

FLAME-SURFACE ADVECTION IN TRANSIENT PERIODIC FLOW

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The advection of an interface by a known transient periodic excitation flow is investigated in the present study, the results of which are relevant to the propagation of nearly isothermal premixed flames in turbulent flow. The relation between the corrugated-surface area and the intensity of the excitation flow is obtained numerically and compared with exact analytically derived solutions. The results elucidate the relation between the surface area and the intensity of spatial and temporal flow-field variations at low, moderate, and large intensities and highlight the stabilizing influence of Huygens surface propagation (e.g., of premixed flames) on the rate of surface-area increase, which is found to be otherwise unbounded in time.

Keywords: Burning speed; Flame speed; Flame surface area; Interface advection; Interface tracking; Level-set; Premixed flames; SLAP; Surface propagation

INTRODUCTION

The equation below describes the evolution of the scalar distribution $G(\mathbf{x}, t)$ and the motion of its isoscalar surfaces, which have the local velocity $\mathbf{a}(\mathbf{x}, t)$ (Bjerknes et al., 1933):

$$\frac{\partial G}{\partial t} + \mathbf{a} \cdot \nabla G = 0 \quad (1)$$

Markstein and Squire (1955) used this equation to describe the advection and propagation of flame surfaces, with a particular isoscalar of G defining the spatial location of a flame surface propagating normal to itself into the reactants at the speed S_N while being advected by the flow $\mathbf{U}(\mathbf{x}, t)$. Specifically, $\mathbf{a} \equiv \mathbf{U} - S_N \mathbf{n}$ is the local velocity of the isoscalar surface as a result of both advection and propagation, where $\mathbf{n}(\mathbf{x}, t) \equiv \nabla G / |\nabla G|$ defines the local normal vector of the distribution G . Markstein and Squire's advection-propagation (AP) equation below, Eq. (2), has been employed in many analytical studies of flame-front stability (e.g., Markstein, 1964; Markstein and Squire, 1955; Williams, 1985) and used by others (Aldredge, 2006; Aldredge and Williams, 1991; Kerstein et al., 1988) to investigate

Received 22 August 2014; revised 12 September 2014; accepted 16 September 2014.

Published as part of the Special Issue in Honor of Professor Forman A. Williams on the Occasion of His 80th Birthday with Guest Editors Chung K. Law and Vigor Yang.

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interface propagation in turbulent flow. Osher and Sethian (1988) developed the first stable numerical methods for the solution of Eq. (2), also known as the level-set equation, for front propagation in quiescent flow:

$$\frac{\partial G}{\partial t} + (\mathbf{U} - S_N \mathbf{n}) \cdot \nabla G = 0 \quad (2)$$

In general, the local isoscalar-surface propagation speed S_N depends on local properties of the advection field $\mathbf{U}$ (e.g., the strain rate) and the scalar distribution G (e.g., the surface curvature) (Aldredge, 1992; Aldredge and Williams, 1991; Markstein, 1964; Markstein and Squire, 1955; Osher and Sethian, 1988; Williams, 1985).

When considering the problem of premixed-flame propagation through nonuniform transient flow, the surface area of the flame, which may become corrugated by advection or as a result of intrinsic flame instabilities, is of interest because it is related directly to the overall propagation rate of the flame and its burning rate. In this case, Eq. (2) provides a means of calculating the evolution of the area of the flame surface (defined by a particular isoscalar of the distribution G). For example, when the flame-surface location is defined by the locus of points (the level set) $\underline{x} = \underline{F}_0(s_1, s_2, t)$ for which $G(\underline{x}, t) = 0$ (parameterized by independent orthogonal-coordinate variables s_1 and s_2), it can be shown that the burning rate of reactants consumed by the flame $\dot{m}$ is given by:

$$\dot{m} \equiv \iint \rho S_N dA_{(G=0)} = \iint \rho S_N \left| \frac{\partial \underline{F}_0}{\partial s_1} \times \frac{\partial \underline{F}_0}{\partial s_2} \right| ds_1 ds_2 \quad (3)$$

where ρ is the density of the reactant flow along the flame surface. In the case of a weakly wrinkled quasi-planar flame having an average normal along the x axis, $s_1 = y$ and $s_2 = z$ so that $\underline{F}_0(s_1, s_2, t) = (f_0(y, z, t), y, z)$, hence, the expression of the last term of Eq. (3) gives:

$$\dot{m} = \iint (\rho S_N)_{(G=0)} (1 + |\nabla f_0|^2)^{1/2} dydz = \iint \rho S_N \left| \frac{\nabla G}{\partial G / \partial x} \right|_{(G=0)} dydz \quad (4)$$

The second equality of Eq. (4) follows from the relation $x = f_0(y, z, t)$ defining the location of the surface on which $G(\underline{x}, t) = 0$. Specifically,

$$\left. \begin{aligned} dG = 0 &\Rightarrow \frac{\partial G}{\partial x} dx + \frac{\partial G}{\partial y} dy + \frac{\partial G}{\partial z} dz + \frac{\partial G}{\partial t} dt = 0 \\ x = f_0(y, z, t) &\Rightarrow \left. \frac{\partial x}{\partial y} \right|_{z,t} = \frac{\partial f_0}{\partial y}, \quad \left. \frac{\partial x}{\partial z} \right|_{y,t} = \frac{\partial f_0}{\partial z}, \quad \left. \frac{\partial x}{\partial t} \right|_{y,z} = \frac{\partial f_0}{\partial t} \end{aligned} \right\} \quad (5)$$

and, therefore,

$$\frac{\partial f_0}{\partial y} = -\frac{\partial G / \partial y}{\partial G / \partial x}, \quad \frac{\partial f_0}{\partial z} = -\frac{\partial G / \partial z}{\partial G / \partial x}, \quad \frac{\partial f_0}{\partial t} = -\frac{\partial G / \partial t}{\partial G / \partial x} \quad (6)$$

Equation (4) demonstrates how the burning rate of the flame can be determined from the evolution of the scalar distribution G , obtained from the solution of the AP equation. When the reactant density is constant and S_N is equal to the constant laminar-flame speed S_L

with which an adiabatic planar flame would propagate through the same reactant mixture, then it follows from Eq. (4) that the burning speed of the quasi-planar flame along the x axis, U_T , is given by:

$$\left. \begin{aligned} U_T &\equiv \frac{\dot{m}}{\rho A_{yz}} = \frac{S_L}{A_{yz}} \iint (1 + |\nabla f_0|^2)^{1/2} dydz = \frac{S_L}{A_{yz}} \iint \left| \frac{\nabla G}{\partial G/\partial x} \right|_{(G=0)} dydz \\ &\Rightarrow \frac{U_T}{S_L} = \frac{A_{(G=0)}}{A_{yz}} = \frac{1}{A_{yz}} \iint (1 + |\nabla f_0|^2)^{1/2} dydz \\ &= \frac{1}{A_{yz}} \iint \left| \frac{\nabla G}{\partial G/\partial x} \right|_{(G=0)} dydz \end{aligned} \right\} \quad (7)$$

where A_{yz} is the area of the flame-surface projection onto the y - z plane. Hence, in this case the fractional increase in the burning speed of the quasi-planar flame above S_L is a result of and equal to the fractional increase in the flame-surface area above A_{yz} caused by the transient, nonuniform advection field $\mathbf{U}$, as described earlier (Williams, 1985). Equation provides for more generally applicable results beyond the case of Huygens propagation ($S_N = S_L$), accounting for variations of the reactant density and local normal propagation speed along the flame (e.g., caused by effects of local flow-field strain and flame-surface curvature) (Aldredge, 2005; Aldredge and Killingsworth, 2004; Aldredge and Williams, 1991; Clavin, 1985; Clavin and Garcia-Ybarra, 1983; Clavin and Williams, 1982; Joulin and Clavin, 1979; Kwon et al., 1992; Law, 1988; Lewis and von Elbe, 1961; Markstein, 1953, 1964; Markstein and Squire, 1955; Searby and Clavin, 1986; Sivashinsky, 1983; Tseng et al., 1993; Williams, 1985).

Consider now the propagation of a wrinkled, quasi-planar surface described by $x = f_0(y, z, t)$. The wrinkling of the surface by the advection field is considered to be sufficiently weak, so that the component of the surface normal along the x axis is always positive; i.e., $\partial G/\partial x > 0$. Using the relations in Eq. (6), the AP equation then provides the following equation for the level set $x = f_0(y, z, t)$:

$$\frac{\partial f_0}{\partial t} = u_0 - \mathbf{v}_0 \cdot \nabla f_0 - S_N (1 + |\nabla f_0|^2)^{1/2} \quad (8)$$

Here, u_0 and $\mathbf{v}_0$ are the longitudinal (x -axis) and transverse components of the advection velocity field $\mathbf{U}$ evaluated at the location $(x = f_0(y, z, t), y, z)$ on the surface. Theoretical modeling based on linearization of Eq. (8) has predicted quadratic dependence of the overall propagation rate U_T of a weakly wrinkled quasi-planar premixed-flame surface with constant local normal propagation speed $S_N = S_L$ on the fluctuation intensities of a stationary advection field (Aldredge, 2006; Clavin and Williams, 1979; Shchelkin, 1943). Specifically, these models predict that:

$$U_T/S_L - 1 = \lambda (u'/S_L)^2, \quad u'/S_L \ll 1 \quad (9)$$

where u' is the root mean square of the advection-field velocity fluctuations, assumed to be isotropic, and λ is a constant numerical factor equal to 1 (Clavin and Williams, 1979) or 2 (Aldredge, 2006; Shchelkin, 1943). However, the work of Kerstein and Ashurst (1992) suggests that the quadratic dependence of the flame speed on intensity given in Eq. (9) likely holds only for the earliest stage of flame wrinkling in turbulent flow and that quasi-steady front propagation is governed by a less sensitive dependence according to:

$$U_T/S_L - 1 \sim (u'/S_L)^{4/3}, \quad u'/S_L = O(1) \quad (10)$$

at a later stage where influences of Huygens front propagation are important.

For $u'/S_L \gg 1$, an essentially linear increase in the burning rate with increasing intensity has been predicted and found experimentally (Aldredge et al., 1998; Anand and Pope, 1987; Bedat and Cheng, 1995; Cheng and Shepherd, 1991; Yakhot, 1988), namely,

$$U_T/S_L - 1 = c(u'/S_L), \quad u'/S_L \gg 1 \quad (11)$$

In this relation, c is a parameter that depends on the extent of chemical-heat release by the flame (Anand and Pope, 1987).

The quadratic relation of Eq. (9) indeed follows from the completely linearized version of the level-set equation, Eq. (8), when the variation of the stationary or time-periodic advection velocity along the x axis is neglected. However, the advection fields considered in the earlier studies leading to Eq. (9) (Aldredge, 2006; Clavin and Williams, 1979; Shchelkin, 1943) exhibit an inherent x dependence, due to an assumed isotropic character and a stipulation of Taylor's hypothesis. It will be demonstrated in the present work that this inherent x dependence is non-negligible except at the earliest times of the evolution of an initially planar flame (when the departure of f_0 away from the y - z plane is very small). At intermediate times, the variation of the advection field with x consistent with Taylor's hypothesis makes the level-set equation for the flame surface nonlinear (through the dependence of the advection field therein on f_0) even when the higher-dimensional AP equation from which the level-set equation is derived is still linear; e.g., before the local-propagation term in Eq. (2) has become important. This nonlinearity of the level-set equation at intermediate times results in a steady increase in the extent of flame-surface wrinkling with time (at fixed u') until stabilized at later times by Huygens flame propagation.

SOLUTION OF THE LINEAR AP EQUATION

When only a zero-mean x component (u) of the advection field is nonzero and there is no propagation ($S_N = 0$), the AP equation is linear without approximation. Specifically, one has for this case (Case A):

$$\frac{\partial G}{\partial t} + u \frac{\partial G}{\partial x} = 0 \quad (12)$$

However, this same linear result is also obtained as a leading-order approximation of the AP equation when $S_N = S_L$, $\mathbf{U} \equiv (u + S_L, \mathbf{v})$, with $\mathbf{v}/S_L \ll 1$, and surface wrinkling is sufficiently weak so that $\partial G/\partial x > 0$ and the transverse gradients of G are negligibly small (Case B). Considering the last relation of Eq. (6), one obtains for both cases the first-order nonlinear level-set equation:

$$\frac{\partial f_0}{\partial t} = u_0 \quad (13)$$

which is exact for Case A and the leading-order approximation to Eq. (8) while the conditions for Case B remain valid.

The conditions of Case B are consistent with those considered in the investigations of Clavin and Williams (1979) and Aldredge (2006), both of which also assumed validity of Taylor's hypothesis in deriving Eq. (9). Hence, we shall also consider velocity fields that satisfy Taylor's hypothesis; the first of which is:

$$\left. \begin{aligned} u(\mathbf{x}, t) &\equiv \alpha(y, z) \cos(k_x(x - S_L t)), \quad \mathbf{v} = 0 \\ \Rightarrow u_0(y, z, t) &= \alpha(y, z) \cos(k_x(f_0 - S_L t)) \end{aligned} \right\} \quad (14)$$

The time-independent amplitude function α that characterizes the velocity fluctuations about the y - z plane, prescribed later, has a zero spatial mean so that the intensity u' of the velocity fluctuations (based on averaging over the transverse coordinates and time) is given by:

$$u' \equiv \frac{1}{\sqrt{2}} \left(\frac{1}{A_{yz}} \int_{A_{yz}} \alpha^2 dA_{yz} \right)^{1/2} \quad (15)$$

The following analytical solutions of Eqs. (13) and (14) are obtained for the level set f_0 of an initially planar surface, $f_0(y, z, 0) = 0$, its local speed $\partial f_0 / \partial t$ along the x axis and the local surface gradient ∇f_0 (in normalized units):

$$\left. \begin{aligned} \hat{f}_0 &= \hat{t} - 2 \tan^{-1} \left(\frac{1 - \hat{\alpha}}{\gamma} \tan \left(\frac{\gamma \hat{t}}{2} \right) \right) = \hat{t} - 2 \tan^{-1} \left(\frac{1 - \hat{\alpha}}{i\gamma} \tanh \left(\frac{i\gamma \hat{t}}{2} \right) \right) \\ \frac{\partial \hat{f}_0}{\partial \hat{t}} &= \hat{\alpha} \frac{\hat{\alpha} + \cos(\gamma \hat{t})}{1 + \hat{\alpha} \cos(\gamma \hat{t})} = \hat{\alpha} \frac{\hat{\alpha} + \cosh(i\gamma \hat{t})}{1 + \hat{\alpha} \cosh(i\gamma \hat{t})} \\ \hat{\nabla} \hat{f}_0 &= \frac{\hat{\alpha} \hat{t} + \sin(\gamma \hat{t}) / \gamma}{1 + \hat{\alpha} \cos(\gamma \hat{t})} \hat{\nabla} \hat{\alpha} = \frac{\hat{\alpha} \hat{t} + \sinh(i\gamma \hat{t}) / (i\gamma)}{1 + \hat{\alpha} \cosh(i\gamma \hat{t})} \hat{\nabla} \hat{\alpha} \end{aligned} \right\} \quad (16)$$

Here, $\hat{t} \equiv t k_x S_L$, $\hat{f}_0 \equiv k_x f_0$, $\hat{\nabla} \equiv \nabla / k_x$, $\hat{\alpha} \equiv \alpha / S_L$, and $\gamma \equiv \sqrt{1 - \hat{\alpha}^2}$. These solutions are valid for all real values of $\hat{\alpha}$, although only small amplitudes of $\hat{\alpha}$ are consistent with the conditions of Case B defined above. Table 1 summarizes values of the local position $\hat{f}_0$, speed $\partial \hat{f}_0 / \partial \hat{t}$, and surface gradient $\hat{\nabla} \hat{f}_0$ of the level set for selected values of the velocity perturbation amplitude $\hat{\alpha}$ (i.e., -1, 1, and 1). Where the perturbation of the advection field about the transverse plane is zero the level-set surface remains at its initial location $f_0 = 0$ for all $t > 0$, while its gradient oscillates sinusoidally in time with amplitude $\nabla \alpha$. On the other hand, where the magnitude of the advection velocity ($|\alpha|$) equals S_L (i.e., $\hat{\alpha} = \pm 1$), the local surface speed along the x axis also ultimately equals S_L (the speed of the traveling

Table 1 The local position $\hat{f}_0$, speed $\partial \hat{f}_0 / \partial \hat{t}$, and surface gradient ∇f_0 of the level set for selected values of the velocity perturbation amplitude $\hat{\alpha}$

$\hat{\alpha}$	$\hat{f}_0$	$\partial \hat{f}_0 / \partial \hat{t}$	∇f_0
-1	$\hat{t} - 2 \tan^{-1}(\hat{t}) \sim \hat{t} - \pi, \quad \hat{t} \gg 1$	$-(1 - \hat{t}^2) / (1 + \hat{t}^2) \sim 1, \quad \hat{t} \gg 1$	$\hat{t} \nabla \hat{\alpha}$
0	0	0	$\sin(\hat{t}) \nabla \hat{\alpha}$
1	$\hat{t}$	1	$\hat{t} \nabla \hat{\alpha}$

wave of frozen perturbations dictated by Taylor's hypothesis), while the local flame-surface gradient increases linearly with time in proportion with the local advection-field gradient $\nabla\alpha$.

The following advection-field amplitude distribution $\hat{\alpha}$ is now assumed for further demonstration of the solutions of the AP equation:

$$\hat{\alpha}(\hat{y}, \hat{z}) = \hat{a} \sin(\hat{y}/\phi) \sin(\hat{z}/\phi) \quad (17)$$

Here, $\hat{a} = \hat{u}'\sqrt{8} = (u'/S_L)\sqrt{8}$ in accordance with Eq. (15). The parameter ϕ may be set to 1 (2) for an example of a flow with identical (different) length scales along the longitudinal and transverse coordinates. However, it is clear from the first relation of Eq. that the influence of ϕ is only a rescaling of the domain of the level-set in the $\hat{y} - \hat{z}$ plane, with the distribution $\hat{f}_0$ at a given time $\hat{t}$ remaining otherwise changed. [Figures 1](#) shows the initially planar level-set surface $\hat{f}_0$ at $\hat{t} = 2\pi$, for $\hat{u}' = 3/4$ and $\phi = 1$ on the domain $(-\pi \leq \hat{y} \leq \pi, -\pi \leq \hat{z} \leq \pi)$. The surface protuberances become longer (shorter) and more (less) pronounced than those shown in [Figure 1](#) with an increase (decrease) in either $\hat{u}'$ or $\hat{t}$ for the same value of ϕ . The level-set surface obtained at $\hat{t} = 2\pi$ for the same intensity ($\hat{u}' = 3/4$), but with $\phi = 2$, looks identical to that in [Figure 1](#) when the scale of each of the transverse coordinates is increased by a factor of 2.

The increase in the level-set surface area for conditions under which Eq. (13) is valid—described above for Cases A and B—as a result of the transient nonuniform advection field defined by Eqs. (14), (15), and (17) may be calculated by integration using the last analytical expression in Eq. (16) in the second relation of the second line in Eq. (7). In this manner, it is found that at the smallest intensities ($\hat{u}' \ll 1$) and earliest times

Figure 1 The level-set surface $\hat{f}_0$ on the domain $(-\pi \leq \hat{y} \leq \pi, -\pi \leq \hat{z} \leq \pi)$ when $\hat{t} = 2\pi$, $\phi = 1$, and $u' = 3/4$.

($\hat{t} \ll 1$) the ratio of the level-set surface area to its initially planar value $\hat{A} \equiv A_{(G=0)}/A_{yz}$ is given by:

$$\hat{A} \sim 1 + 2(\hat{u}'/\phi)^2 \sin^2 \hat{t} \quad (\hat{u}' \ll 1, \hat{t} \ll 1) \quad (18)$$

Figures 2 and 3 show more general analytical results, including larger ranges of $\hat{u}'$ and $\hat{t}$ along with results from direct numerical integration of the AP equation for comparison. Numerical integration was achieved using the third-order SLAP algorithm developed by Aldredge (2010) with three-time-level interpolation of characteristic trajectories (Temperton and Staniforth, 1987). In Figure 2, the variation of $(\hat{A} - 1)/2(\hat{u}'/\phi)^2$ with $\hat{t}$ is shown over a range of small advection-field intensities: (a) $\hat{u}' = 1/16$, (b) $\hat{u}' = 1/8$, (c) $\hat{u}' = 3/16$, and (d) $\hat{u}' = 1/4$. The circle and triangle symbols represent the numerical results obtained for $\phi = 1$ and $\phi = 2$, respectively, using SLAP while the solid lines represent the analytical results obtained in the manner described above. In all cases there is excellent agreement between the numerical and analytical results, as reflected by the

Figure 2 A normalized measure of the ratio $\hat{A}$ of the level-set surface area to its initially planar value vs. normalized time for (a) $\hat{u}' = 1/16$, (b) $\hat{u}' = 1/8$, (c) $\hat{u}' = 3/16$, and (d) $\hat{u}' = 1/4$.

Figure 3 The ratio of the level-set surface area to its initially planar value vs. normalized time for $\hat{u}' = 1/2$, $\hat{u}' = 3/4$, and $\hat{u}' = 1$.

coincidence of the symbols with the solid lines. The result for the smallest intensity considered ($\hat{u}' = 1/16$) is in complete agreement with Eq. (18) for almost 1.5 cycles, after which the nonlinearity of the first-order level-set equation (Eq. (13)) (through the dependence of the advection field therein on $\hat{f}_0$) becomes apparent. The effect of this nonlinearity is an unbounded increase in surface area with time in the absence of the second-order stabilizing influence of Huygens surface propagation, which was neglected in the formulation of Eq. (13) from the general level-set equation given in Eq. (8), for both Cases A and B. The periodic variation of the surface-area ratio with time becomes less distinct with increasing intensity, consistent with the linear dependence of the surface gradient on $\hat{t}$ for $\hat{a}\hat{t} \gg 1$ apparent in Eq. (16). Figure 2 shows that the variation of $\hat{A}$ is in direct proportion with the inverse of ϕ^2 is valid beyond the range of $\hat{u}'$ and $\hat{t}$ for which the linear-response result of Eq. (13) is apparent, with $(\hat{A} - 1)/2(\hat{u}'/\phi)^2$ remaining independent of ϕ for the entire range of $\hat{t}$ considered, up to 5 cycles when $\hat{u}' = 1/16$ and for about 2.5 cycles when $\hat{u}' = 1/8$. For the larger intensities of $\hat{u}' = 3/16$ and $\hat{u}' = 1/4$, a less sensitive dependence of $\hat{A}$ on ϕ is indicated by the departure of the curves for $\phi = 1$ and $\phi = 2$ with increasing time. For all intensities, the surface area increases over each cycle ($\Delta \hat{t} = \pi/2$) of its variation with time. This is consistent with the fact that the surface is immobile at points where $\hat{a} = 0$ and moves with its maximum speed along the x axis at points on the surface where $\hat{a} = 1$, as discussed above.

Figure 3 shows the variation of the surface-area ratio $\hat{A}$ with $\hat{t}$ at higher advection-field intensities ($\hat{u}' = 3/4$, $\hat{u}' = 1/2$, and $\hat{u}' = 1$), where the time periodicity of the surface-area ratio is barely noticeable. At these intensities the surface area is found to increase

monotonically with time. At very large intensities ($\hat{u}' \rightarrow \infty$), $\hat{A}$ is found to become independent of intensity, inversely proportional to ϕ , and linearly dependent on $\hat{t}$ after a short time ($\hat{t} \approx \pi/4$).

EFFECT OF TRANVERSE ADVECTION VELOCITIES

When all three components of the advection-field velocity are considered, but there is still no propagation ($S_N = 0$) or mean flow (as in Case A defined above), the AP equation becomes (after nondimensionalization):

$$\frac{\partial \hat{G}}{\partial \hat{t}} + \hat{\mathbf{u}} \cdot \hat{\nabla} \hat{G} = 0 \quad (19)$$

In order to examine the influence of the transverse components on the evolution of the advected surface and its area, we now consider the zero-mean, three-component advection field $\hat{\mathbf{u}}$ defined as follows:

$$\hat{\mathbf{u}} = \begin{Bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{Bmatrix} \equiv 2\sqrt{2}u' \begin{Bmatrix} \sin(\hat{y}/\phi) \sin(\hat{z}/\phi) \cos(\hat{x} - \hat{t}) \\ -\cos(\hat{y}/\phi) \sin(\hat{z}/\phi) \sin(\hat{x} - \hat{t}) \\ -\sin(\hat{y}/\phi) \cos(\hat{z}/\phi) \sin(\hat{x} - \hat{t}) \end{Bmatrix} \quad (20)$$

For $\phi = 1$, this advection field consistent with those considered in the investigations of Clavin and Williams (1979) and Aldredge (2006), both of which assumed statistical isotropy and validity of Taylor's hypothesis. However, the advection field defined in Eq. (20) is solenoidal only for the two-scale case of $\phi = 2$. In Figure 4, the variation of $(\hat{A} - 1)/2(\hat{u}'/\phi)^2$ with $\hat{t}$ obtained numerically is shown for the three-component advection field of Eq. (20) and compared with that of the single-component flow over the range of small intensities considered earlier: (a) $\hat{u}' = 1/16$, (b) $\hat{u}' = 1/8$, (c) $\hat{u}' = 3/16$, and (d) $\hat{u}' = 1/4$. As in Figure 2, the circle and triangle symbols represent the numerical results obtained for $\phi = 1$ and $\phi = 2$, respectively, using the same spatial and temporal discretization as that used for the single-component flow ($64 \times 64 \times 64$ grid with 0.5 CFL). Since Eq. (19) is not analytically tractable for the three-component advection field specified in Eq. (20), no analytical solutions could be obtained for comparison. As for the single-component advection field considered earlier, it is clear that surface wrinkling by the single-scale three-component flow ($\phi = 1$) is always greater than that caused by the two-scale three-component flow ($\phi = 2$), at the same advection-field intensity $\hat{u}'$ and time $\hat{t}$. This is true even when $(\hat{A} - 1)/2(\hat{u}'/\phi)^2$ happens to be larger for $\phi = 2$ than for $\phi = 1$. For example, as presented in Figure 4d, $\hat{A} = 3.8$ (1.9) for $\phi = 1$ ($\phi = 2$) when $\hat{u}' = 1/4$ and $\hat{t}/2\pi = 2.5$ even though the values of $(\hat{A} - 1)/2(\hat{u}'/\phi)^2$ are 22.4 and 28.4 for $\phi = 1$ and $\phi = 2$, respectively. Figure 5 shows the variation of the surface-area ratio $\hat{A}$ with $\hat{t}$ at the higher advection-field intensities considered earlier for the single-component flow ($\hat{u}' = 3/4$, $\hat{u}' = 1/2$, and $\hat{u}' = 1$). As for the single-component advection field, the time-periodic variations of the area ratio for the three-component advection field are less pronounced at the higher intensities, especially when $\phi = 2$.

Figure 4 A normalized measure of the ratio $\hat{A}$ of the level-set surface area to its initially planar value vs. normalized time for (a) $\hat{u}' = 116$, (b) $\hat{u}' = 18$, (c) $\hat{u}' = 316$, and (d) $\hat{u}' = 1/4$.

It is clear from the results presented in [Figure 4](#), and from comparison of [Figures 3](#) and [5](#), that at the same overall intensity $\hat{u}'$ the effect of the additional, transverse components of the advection field is to increasingly enhance flame-surface wrinkling, for both $\phi = 1$ and $\phi = 2$, as the surface develops from its initially planar shape and the magnitude of the nonlinear transverse velocity and surface-gradient terms in Eq. (19) become non-negligible. This is apparent also in [Figure 6](#), where the level-set surface defined by $\hat{G} = 0$, obtained by numerical solution of the AP equation for the three-component advection field, is shown at $\hat{t} = \pi$ for $\hat{u}' = 3/4$ and $\phi = 1$. Four different views of the same surface (Views A, B, C, and D) are provided in [Figures 6a–6d](#). As is evident by comparison of [Figures 1](#) and [6](#), the collective effect of the three components of velocity is much more substantial surface corrugation in comparison with that of the single-component flow, to the extent that corrugated surface balls are created at sufficiently high intensities. These surface balls, which are created and shed at the leading edge of the surface front, are connected to the rest of the surface by only a vanishingly thin tube. It is expected that the surface balls and their connecting tubes will be smaller and in some cases nonexistent when Huygens surface propagation ($S_N = S_L$) and concomitant surface merging is considered.

Figure 5 The ratio of the level-set surface area to its initially planar value vs. normalized time for $\hat{u}' = 1/2$, $\hat{u}' = 3/4$, and $\hat{u}' = 1$.

Figure 6 (a) The level-set surface defined by $\hat{G} = 0$, obtained by numerical solution of the AP equation for the three-component advection field, evaluated for $\hat{u}' = 3/4$, $\phi = 1$, and $\hat{t} = \pi$ (View A).

Figure 6 (b) The level-set surface defined by $\hat{G} = 0$, obtained by numerical solution of the AP equation for the three-component advection field, evaluated for $\hat{u}' = 3/4$, $\phi = 1$, and $\hat{t} = \pi$ (View B).

Figure 6 (c) The level-set surface defined by $\hat{G} = 0$, obtained by numerical solution of the AP equation for the three-component advection field, evaluated for $\hat{u}' = 3/4$, $\phi = 1$, and $\hat{t} = \pi$ (View C).

SUMMARY AND DISCUSSION OF FUTURE WORK

The results of the present work have elucidated the relation between the area of an advected, nonpropagating surface and the intensity of spatial and temporal flow-field variations at low, moderate, and large intensities. Specifically, we have demonstrated both analytically and numerically that without Huygens surface propagation the area of a surface

Figure 6 (d) The level-set surface defined by $\hat{G} = 0$, obtained by numerical solution of the AP equation for the three-component advection field, evaluated for $\hat{u}' = 3/4$, $\phi = 1$, and $\hat{t} = \pi$ (View D, down the $\hat{z}$ axis).

advected by a temporally and spatially periodic excitation flow increases monotonically in time and without bound, under conditions consistent with those considered in the earlier investigations of Clavin and Williams (1979) and Aldredge (2006). Although a quadratic dependence of the surface area on excitation-flow intensity consistent with that of the previous studies, reflected in Eq. (9), was confirmed for the earliest times of advection; the nature of the dependence at later times when Huygens surface propagation becomes important (with S_N nonzero in Eqs. (2) and (8)) has not been demonstrated and will be the subject of future work.

FUNDING

The author is grateful for support from the National Science Foundation (No. CTS-0500505) during the course of the work presented herein.

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