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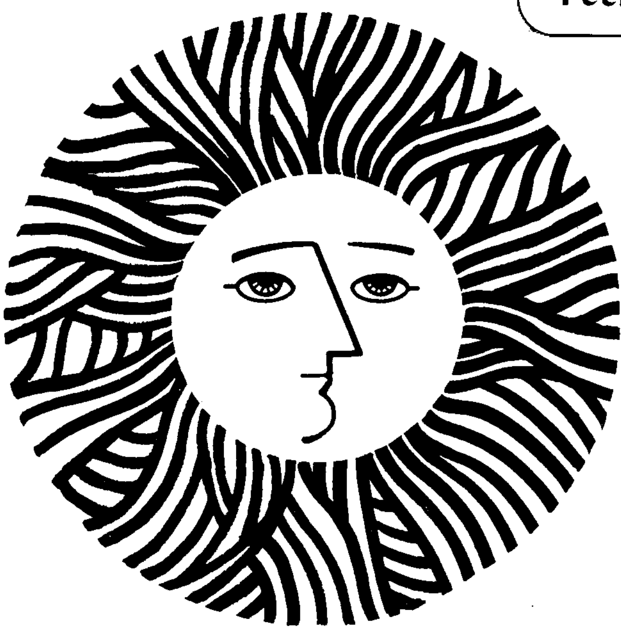

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THE DETERMINATION OF THE DYNAMIC PERFORMANCE OF WALLS

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ABSTRACT

Measuring the thermal performance of walls in-situ poses two separate problems: 1) how to measure time-varying surface temperatures and heat fluxes on both sides of the test wall and 2) how to reduce this data set into a minimal number of parameters that effectively characterize the wall. In this paper we present a methodology for interpreting field measurements of wall performance and describe an instrument developed for carrying out such measurements. The method is a simplified dynamic model that uses a small number of simplified thermal parameters (STP) — a steady-state conductance, a time constant and a few surface storage terms — to describe the termal performance of a wall. We demonstrate the ability of this model to simulate actual wall performance by comparing model predictions with results generated from conventional response-factor methods. The instrument developed for field measurements is the Envelope Thermal Test Unit (ETTU), which consists of two four-foot by six-foot blankets placed on either side of the test wall that are used to both measure and control the surface heat fluxes and surface temperatures of the wall. During a typical test, which lasts about 12 hours, one blanket imposes a specified flux through one surface of the test wall while the resulting heat flux on the other surface and the surface temperatures on both sides are measured. The model presented here can be used for both laboratory and field measurements and may be applied to any component of the building envelope.

Keywords: thermal performance, dynamic performance, field measurement, modeling.

The Determination of the Dynamic Performance of Walls

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INTRODUCTION

The thermal performance of building walls, in situ, is largely unknown. Until now, most wall performance measurements have been done in laboratories, typically by using large hot-boxes. In-situ performance is considerably more difficult to measure, for the experimenter usually has little control over temperature, solar radiation, or wind conditions. The task of accurately measuring surface temperatures and heat fluxes over significant lengths of time is not easy. Furthermore, once measurements have been obtained analysis of the data is not a trivial matter. As illustrated by a review of measurement techniques and wall performance models compiled by Carroll,[1] most existing models contain too many parameters to be suitable for direct analysis.

A simplified model of dynamic thermal performance that allows the characteristics of a wall to be quantified on the basis of measured surface temperatures and heat fluxes has been developed. The model uses a set of simplified thermal parameters (STPs) to characterize the thermal performance of walls from an arbitrary temperature history. In addition, the STPs can be used to arrive at a physical interpretation of the behavior of a wall. This model is applicable to any set of data. In this report, however, its applicaton to the analysis of data collected by the Envelope Thermal Test Unit (ETTU), is demonstrated. Accordingly, laboratory measurements using ETTU are included as part of the validation procedure.

BASIC HEAT-TRANSFER MODEL

Any model that purports to describe the transport of heat through walls must begin with the basic principles of heat conduction through solids. Accordingly, the derivation of a wall model will be begun with the fundamental equations of thermal conduction; the results will be specialized until the model has been endowed with sufficient richness to describe actual walls.

Heat conduction across any homogeneous slab of building material can be regarded as one-dimensional if corner effects and thermal bridges caused by studs, cavities, and other inhomogeneities are neglected. This common assumption, although not always reliable, will be adopted for this purpose.

The one-dimensional heat conduction equation,

$$\frac{\partial T(x,t)}{\partial t} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

where: $T(x,t)$ is the temperature distribution in the slab (°C)
α is the thermal diffusivity (m^2/s)
x is the position in the wall (m) and
t is the time (s)

governs heat transfer at any point of the wall at any time.

Many numerical methods exist for solving this equation for actual, multilayered walls; these include a variety of response-factor methods[2,3] and methods based on frequency transforms, sometimes referred to as admittance methods.[4-6] Most numerical methods were developed for calculating heat flows and/or temperatures at one or both surfaces of walls whose compositions are known. The field measurement of the thermal performance of a wall poses the opposite problem: temperatures and heat fluxes are measured, but thermal properties are unknown.

In principle, one could simply take any existing numerical model and fit its parameters to the measured data. The values of the parameters yielding the best fit would then be the experimentally determined thermal properties of the wall. Unfortunately, this approach usually fails because of the excessive number of parameters (or degrees of freedom) in most numerical methods.[7] In addition, parameter values determined by experiment fit the data but are, of themselves, unphysical (e.g., have the wrong sign, etc.). Notable exceptions are lumped-parameter models with undetermined values for the resistors and capacitors of which they are composed. Generally speaking, the order of these models is determined by the number of capacitors. For a limited range of boundary conditions, first order models are often sufficient to model heat flows in walls and entire buildings to satisfactory accuracy.[8]

The model presented here is particularly suitable for the analysis of measured heat flux and temperature data. Like lumped-parameter models, it uses digital filters (see definitions below), but it is not restricted to the class of filters that represent actual resistance-capacitor networks. The following paragraphs summarize the results derived in the appendix for both homogeneous and inhomogeneous (i.e., the general case) walls.

Homogeneous Walls

As shown in the appendix, Eq 1 can be solved for a homogeneous wall (i.e., single layer), whose thermal properties are independent of temperature, in the following integral form:

$$\underline{J}^1(t) = U\,(T^1(t) - T^2(t)\,) + 2U \sum_{n=1}^{n_o} F_n^1(t) - (-1)^n\, F_n^2(t) \tag{2.1}$$

$$\underline{J}^2(t) = U\,(T^2(t) - T^1(t)\,) + 2U \sum_{n=1}^{n_o} F_n^2(t) - (-1)^n F_n^1(t) \qquad (2.2)$$

where: $\underline{J}^1$, $\underline{J}^2$ are heat fluxes (W/m^2) at surfaces 1 and 2* of the homogeneous wall
T^1, T^2 are temperatures (K) at wall surfaces 1 and 2
F^1, F^2 are the normalized temperature filters (K)
U is the conductance of the slab(W/m^2-K)
n_o is a summation limit large enough to contain all the frequencies of interest

The "homogeneous" heat fluxes are defined as:

$$\underline{J}^1(t) = \underline{J}(0,t) = - \lim_{x \to 0} U\,L\,\nabla T(x,t) \qquad (3.1)$$

$$\underline{J}^2(t) = - \underline{J}(L,t) = \lim_{x \to L} U\,L\,\nabla T(x,t) \qquad (3.2)$$

where: L is the thickness of the slab (m).

The temperature filters are defined as:

$$F_n^{(1,2)}(t) = \frac{n^2}{\tau} \int_0^{\infty} e^{-\frac{t'}{\tau}n^2} \left(T^{(1,2)}(t) - T^{(1,2)}(t-t') \right) dt' \qquad (4)$$

where: τ is the time constant of the material (s) (see appendix).

The terms $F_n^{(1,2)}$ are called filters, because they filter the past history of the temperatures in such a manner as to eliminate "fast" frequency components and leave "slow" frequency components unchanged. (Filters having this property are called "low-pass filters.") The separation between "fast" and "slow" is determined for each filter by the frequency component with time constant τ/n^2. Note that the first term in Eq 3 above is the steady-state heat flux, ($U*\Delta T$). The second term represents a correction to the steady-state heat flux caused by thermal storage (it disappears for massless walls as the time constant approaches zero).

Inhomogeneous Walls

Because real walls can rarely be treated as homogeneous, more complex models are necessary to describe them. The classical approach is to break up the wall into homogeneous layers and to apply the homogeneous solution to each layer, being careful to match boundary conditions at each interface. Unfortunately, this cannot be done in closed form for arbitrary layers (or for multi-dimensional walls, materials with time-dependent properties, and nonlinear components). Therefore, an empirical generalization of the homogeneous solution's proposed; specifically, the coefficients in front of each filter are allowed to be free parameters (as opposed to being

* The surface heat fluxes have been defined as positive when they flow into the wall.

fixed as in the homogeneous case). This is equivalent to adding to the homogeneous solution terms that are proportional to the filters:

$$J^1(t) = \underline{J}^1(t) + 2U \sum_{n=1}^{n_m} a_n F_n^1(t) \tag{5.1}$$

$$J^2(t) = \underline{J}^2(t) + 2U \sum_{n=1}^{n_m} b_n F_n^2(t) \tag{5.2}$$

where: a_n, b_n are dimensionless surface storage parameters,
n_m is the "order" of the model
$J^{(1,2)}$ are the generalized fluxes.

The parameters a_n and b_n have been called surface storage parameters, because they describe the effective amount of thermal storage that takes place on each surface of the wall relative to a homogenous wall. Since the solutions $\underline{J}$ are the homogeneous solutions, the general solution must have all of the surface storage factors equal to zero; that is, for a homogeneous wall,

$$a_n = b_n = 0 \tag{6}$$

and the general solution becomes the solution for a homogeneous wall.

In an actual test wall, the further the test wall is from being homogeneous, more the the values for a_n, b_n will differ from zero. An example is a two-layer wall composed of light, very resistive material and another massive, but very conductive, material. As will be seen in a later section, these semi-empirical constants can be transformed into the more familiar response factors by applying a set of algebraic relations.

This completes our set of Simplified Thermal Parameters. There are two basic parameters (U and τ) and two additional ones for every additional order (i.e., a_n, b_n), making a total of $2+2n_m$ STPs.

Discrete Time Intervals

The equations so far derived are strictly valid only for temperatures and heat fluxes that are continuous functions of time. In any practical application, however, data will be obtained at discrete time intervals. Let us now transform these equations into discrete time-step equations:

$$J_k^1 = \underline{J}_k^1 + 2U \sum_{n=1}^{n_m} a_n F_{nk}^1 \tag{7.1}$$

$$J_k^2 = \underline{J}_k^2 + 2U \sum_{n=1}^{n_m} b_n F_{nk}^2 \tag{7.2}$$

where: $J_k^{(1,2)}$ are the measured discrete heat fluxes at $t=k*\Delta t$ (W/m^2)
$\underline{J}_k^{(1,2)}$ are the homogeneous discrete fluxes (W/m^2)
$\underline{F}_{nk}^{(1,2)}$ are the discrete filters(K) and
Δt is the time increment between measurements (s)

The discrete homogeneous fluxes are as follows:

$$J_k^1 = U\,(T_k^1 - T_k^2) + 2U \sum_{n=1}^{n_o} (\,F_{nk}^1 - (-1)^n\,F_{nk}^2\,) \tag{8.1}$$

$$J_k^2 = U\,(T_k^2 - T_k^1) + 2U \sum_{n=1}^{n_o} (\,F_{nk}^2 - (-1)^n\,F_{nk}^1\,) \tag{8.2}$$

In order to evaluate these digital filters, one must make some assumption about the behavior of the temperature during the time intervals separating measurements. A most reasonable assumption is that the temperature is linear between measured points; then, the filters become,

$$F_{nk}^{(1,2)} = \frac{(1 - \beta_n\,)}{\frac{\Delta t}{\tau} n^2} \sum_{j=0}^{\infty} (\,T_{k-j}^{(1,2)} - T_{k-j-1}^{(1,2)}\,)\beta_n{}^j \tag{9}$$

where:

$$\beta_n = e^{-\frac{\Delta t}{\tau} n^2}$$

$$\beta_n{}^j = \left[\,e^{-\frac{\Delta t}{\tau} n^2}\right]^j$$

Digital filters of this type are conventionally called infinite impulse response filters and can be represented by a recursive relation that allows the current filter value to be calculated from the current temperature and the previous value of the filter:

$$F_{nk}^{(1,2)} = \beta_n\,F_{n(k-1)}^{(1,2)} + \frac{(1 - \beta_n\,)}{\frac{\Delta t}{\tau} n^2}\,(T_k^{(1,2)} - T_{k-1}^{(1,2)}\,) \tag{10}$$

Relation to Response Factors

Many building simulation models calculate the dynamic performance of walls with so-called response factors. Response factors are a series of weighting factors that multiply past temperatures to obtain present heat fluxes:

$$J_k^1 = \sum_{j=0}^{\infty} X_j T_{k-j}^1 - Y_j T_{k-j}^2 \tag{11.1}$$

$$J_k^2 = \sum_{j=0}^{\infty} Z_j T_{k-j}^2 - Y_j T_{k-j}^1 \tag{11.2}$$

where: X_j, Y_j, Z_j are response factor series (W/m^2-K),

Remember that both heat fluxes are positive when heat flows into the wall. In practice, the summation stops long before $j=\infty$. Typically, 20 to 30 terms are sufficient, and several elegant mathematical shortcuts are available to further reduce the required number of terms.[9] Response factors for large values of j have constant common ratios:

$$\frac{X_{j+1}}{X_j} = \frac{Y_{j+1}}{Y_j} = \frac{Z_{j+1}}{Z_j} = R_c \qquad \text{for } j \gg 1 \tag{12}$$

where: R_c is the common ratio $(0 < R_c < 1)$

To find expressions for the response factors as functions of the coefficients of the model, one may start by rewriting the digital filters:

$$F^{(1,2)}_{nk} = \frac{1-\beta_n}{\frac{\Delta t}{\tau}n^2} T^{(1,2)}_{k} - \frac{(1-\beta_n)^2}{\beta_n \frac{\Delta t}{\tau}n^2} \sum_{j=1}^{\infty} \beta_n{}^j \, T^{(1,2)}_{k-j} \tag{13}$$

By inserting these expressions into Eqs 7, inverting the order of summation over j and n, and collecting terms in $T^{(1,2)}_{k-j}$, one find the desired relations separately for $j=0$ and for $j>0$

$$\begin{aligned}
X_{j=o} &= \underline{X}_{j=o} + 2U \sum_{n=1}^{n_m} a_n \frac{1-\beta_n}{\frac{\Delta t}{\tau}n^2} & \qquad X_{j>o} &= \underline{X}_{j>o} - 2U \sum_{n=1}^{n_m} a_n \frac{(1-\beta_n)^2}{\frac{\Delta t}{\tau}n^2} \beta_n{}^{j-1} \\
Y_{j=o} &= \underline{Y}_{j=o} & \qquad Y_{j>o} &= \underline{Y}_{j>o} \\
Z_{j=o} &= \underline{Z}_{j=o} + 2U \sum_{n=1}^{n_m} b_n \frac{1-\beta_n}{\frac{\Delta t}{\tau}n^2} & \qquad Z_{j>o} &= \underline{Z}_{j>o} - 2U \sum_{n=1}^{n_m} b_n \frac{(1-\beta_n)^2}{\frac{\Delta t}{\tau}n^2} \beta_n{}^{j-1}
\end{aligned}$$

where: $\underline{X}_j$, $\underline{Y}_j$, $\underline{Z}_j$ are the response factors for the "homogeneous" wall and

$$\begin{aligned}
\underline{X}_{j=o} = \underline{Z}_{j=o} &= U + 2U \sum_{n=1}^{n_o} \frac{1-\beta_n}{\frac{\Delta t}{\tau}n^2} & \qquad \underline{X}_{j>o} = \underline{Z}_{j>o} &= -2U \sum_{n=1}^{n_o} \frac{(1-\beta_n)^2}{\frac{\Delta t}{\tau}n^2} \beta_n{}^{j-1} \\
\underline{Y}_{j>o} &= U + 2U \sum_{n=1}^{n_o} (-1)^n \frac{1-\beta_n}{\frac{\Delta t}{\tau}n^2} & \qquad Y_{j>o} &= -2U \sum_{n=1}^{n_o} (-1)^n \frac{(1-\beta_n)^2}{\frac{\Delta t}{\tau}n^2} \beta_n{}^{j-1}
\end{aligned}$$

Note that the common ratio is related to the time constant:

$$R_c = e^{-\frac{\Delta t}{\tau}} \tag{14}$$

By using Eqs 14 with measured STPs, one can calculate response factors determined by measurement, as opposed to response factors determined by prediction, and use them in conventional building simulation models. Even though a given set of STPs is sufficient to calculate a consistent set of response factors, a given set of response factors may not be converted easily into STPs, except for the U-value and the time constant:

$$U = \sum_{j=0}^{\infty} X_j = \sum_{j=0}^{\infty} Y_j = \sum_{j=0}^{\infty} Z_j \tag{15}$$

$$\tau = -\frac{\Delta t}{\ln(R_c)} \tag{16.1}$$

SAMPLE APPLICATION

To summarize and demonstrate the model presented above, it will be applied to a wall whose thermal properties are known exactly. From these properties the flux for some suitable temperature history will be calculated. (This is an illustration only; if the composition of the wall to be tested was known, its response factors could be computed using conventional methods.) The hypothetical wall consists of (from outside to inside) an outdoor air film, 4 in. (102 mm) face brick, 3/4 in. (19 mm) air space, 2 in. (51 mm) insulation [R-4 per inch, 5.7 lb/ft^3 density], 3/8 in. (10 mm) gypsum board, 1/2 in. (13 mm) plaster, and an inside air film.

A temperature history has been generated, consisting of a white-noise spectrum added to some low frequencies, as shown in the top half of Fig. 1. Using the response-factor method, the heat fluxes at both sides of the wall have been calculated as a function of time. This "synthetic" data set for a wall is guaranteed to behave exactly as heat-conduction theory predicts (as opposed to an actual wall, in which air leakage, convection in cavities, temperature dependence, and lateral heat flow may significantly alter performance). Applying the model to this synthetic data set, one obtains the parameters shown below:

U[W/m^2-K]	τ[hr]	a_1	b_1	a_2	b_2
0.61	4.11	8.44	-0.48	-2.3	1.45

The U-value can be compared to the calculated value of U=.60 and the time constant to τ=4.22 as calculated from the common ratio of the wall's response factors.

Since this is a hypothetical wall, one can compare the response factors used in generating the data to the response factors derived from the model. The table below gives a representative sampling of the response factors; the left-hand set is calculated using conventional methods based on layer-by-layer thermal properties; the right-hand set is derived from the STPs extracted from heat fluxes and temperatures of the hypothetical wall.

TABLE 1
Sample Response Factors for Hypothetical Wall

Delay (hrs.)	Calculated[W/m^2-K] X	Y	Z	Predicted[W/m^2-K] X	Y	Z
1	-.122	.0007	-.096	-.102	-.0001	-.082
2	-.081	.0032	-.043	-.077	.0022	-.027
3	-.063	.0042	-.020	-.064	.0041	-.012
5	-.039	.0036	-.004	-.041	.0039	-.003
10	-.011	.0012	-.000	-.012	.0013	-.001

Delay=0 is the current point.

At first sight, the degree of correspondence between the two sets appears modest; however, when comparing sets of response factors, bear in mind that two term-by-term expansions of sets of response factors can look quite different yet produce very similar fluxes for a given temperature history — because of the redundancy inherent in the large number of terms involved.

The wide range of effectively similar response factors can be understood by considering the number of free parameters inherent in a response-factor approach. It is not uncommon to keep 100 sets of response factors for a wall, giving a total of 300 free parameters; by comparison, the STP approach always uses fewer than 10 free parameters. Thus, there is quite a bit of interdependency in the response-factor approach; that is, for any arbitrary degree of accuracy, there are many allowable combinations of response factors that will describe the same wall.

Possibly the most important fact resulting from this illustration is the kind of tracking displayed in Fig. 1. During nine hours in the early part of the test, temperature differences and heat fluxes were relatively large and slowly varying. At the outset and during the second half of the test,
the opposite is true: <u>fluctuations</u> of temperatures and heat fluxes dominate, while their <u>averages</u> are comparatively small. In other words, the parameters of this model can be used to predict the thermal performance of walls over a wide frequency band. This characteristic is important if the same model is to be used to calculate the effects of outdoor temperature, solar radiation, and indoor furnace pulses.

<u>Obtaining Model Parameters from Heat Flux and Temperature Data</u>

So far, a model whose parameters (between four and eight, depending on the wall) are determined from measured histories of surface temperatures and heat fluxes has been presented. In principle, the parameters are obtained by fitting model Eq 7 to the actual data. For example, an initial guess of the wall parameters could be used to compute heat fluxes from the measured temperature data. The error of this guess could be quantified by RMS deviation between computed and measured heat fluxes. The best-fitting values of the wall parameters would then be found by progressively varying the initial guesses until the smallest RMS deviation of heat flux was found.

Unfortunately, this procedure can only yield physically meaningful values if the measured time series points are mutually independent (i.e., if a measurement of heat flux and temperature at one time is independent of the same measurement at previous times). Since this independence does not exist in this application, the model parameters must be fitted in the frequency domain rather than in the time domain. Frequency components of time histories of heat flux and temperature are linearly independent and very quickly calculated using Fast Fourier Transform methods. Since these fitting methods are somewhat peripheral to the model itself, their description has been relegated to the appendix.

<u>MODEL INTERPRETATION</u>

The ability of a model to reproduce measured data is only one facet of its usefulness — the other is the ability to effect a physical interpretation of its parameters. The most important parameter of any wall is its steady-state U value, and, not surpriisingly, the most important STP for common use is U. The next most important parameter of a wall is the time constant. The time constant, τ, is a measure of how long it takes for a heat pulse on one side of the wall to be felt on the other side of the wall; it is related to the U-value and the thermal mass of the entire wall.

The time constant of a wall serves as a yardstick when one speaks of quickly or slowly varying temperatures: quickly varying temperatures complete one cycle in less than one-fifth of a time constant, while slowly varying temperatures take several time constants to complete one cycle. For slowly varying temperatures the thermal properties of the wall can be adequately approximated by a steady-state analysis.

The remaining STPs are the surface storage factors; these factors can be used to qualitatively estimate the variation of the wall surface from perfect homogeneity. That is, if the storage factor for one side of a wall is much larger than zero, that surface has more mass than does the wall as a whole; conversely, a negative storage factor means that there is more resistance (less mass) on that surface.

This effect is apparent in the hypothetical wall used to illustrate the model. On the outside face of that wall (side 1 in Fig. 1) is a four in. (102 mm) layer of face brick; since this layer comprises the bulk of the thermal mass, we expect the first surface storage factors to be positive on side one and negative on side 2. This is, in fact, the case: $a_1=8.44$ and $b_1=-.48$.

ENVELOPE THERMAL TEST UNIT

To measure time histories of temperature and heat flux of actual building walls, a portable apparatus has been developed, the envelope thermal test unit (ETTU). The design of this device has been described in an earlier article.[10] ETTU differs from a standard guarded hot-box in two respects: (1) it is portable and thus can be used for on-site testing of actual building walls; (2) it measures the wall temperature response to known heat flows, as opposed to measuring heat flows in response to given temperatures. The physical arrangement of ETTU is shown schematically in Fig. 2. Two identical "blankets" are placed in close thermal contact with the wall to be tested. Each blanket consists of a pair of 1.2 m by 1.8 m (4 ft.X 6 ft.) electric heaters separated by a low thermal mass insulating layer. The heater in contact with the wall is called the "primary," the other is the "secondary." Embedded in each heater layer is an array of temperature sensors. The blankets cover the wall section under test and are slightly flexible, so that they can be made to conform to minor irregularities in the wall surfaces. Although we recognize the problem of very uneven outer surfaces (e.g., shingles), the current version of ETTU does not attempt to address them; future versions of ETTU will consider these problems.

ETTU can be operated in two modes: in the first mode, the heat flux through one surface of the test wall can be specified accurately and a steady-state temperature difference can be created across the test wall. In this mode, the two blankets of ETTU play active and passive roles in the "active" blanket, heat flux is provided to the primary heater according to a user-selected, time-dependent function that covers the required frequency spectrum. At the same time, the secondary heater is used as a guard, with a control strategy that minimizes the temperature difference (and thus the heat loss) across the active blanket. The electrical power dissipated by each heater is controlled by adjusting the current flowing through the heater.

The passive blanket on the opposite side of the wall is used as a large-area heat-flux sensor: its heaters are not energized, but the difference between primary and secondary temperatures, in conjunction with the blanket thermal properties, is used to measure the heat flow of the wall on the passive side.

In the second mode of operation, the secondary heaters are unused and both primary heaters are independently driven. In this symmetric mode, there is little or no steady-state temperature difference between the two wall surfaces and, therefore, little information about the steady-state conductance; but, unlike the previous mode, a great deal of information is available about the transient thermal properties.

A microprocessor-based data acquisition system is used to drive ETTU. It drives the surface heaters, records all primary and secondary temperatures, and performs the necessary on-line heat-flux computations.[11] To eliminate the effects of lateral heat transfer, analysis is restricted to the central region of the blanket; in effect, the outer region of the blanket is used as a guard. Fig. 3 shows the temperature sensor array on each layer, with the central region delineated.

MODEL VALIDATION

In order to validate the wall model presented earlier, one should be able to (1) adequately predict the fluxes from measured temperatures and (2) derive physically correct thermal wall parameters. Furthermore, the measured temperatures and fluxes must have enough different frequencies to insure that the process of fitting the data and finding the thermal parameters will be valid for any temperature history. For this reason, the best driving strategies should contain all frequency components typically encountered (i.e., a "white-noise" spectrum).

To test ETTU and to validate the model, a section of a wall was built in the authors' laboratory. The wall (from side one towards side two) was made of (3/4 in. (19 mm)) plywood, (2 in. (51 mm)) high-density rigid board insulation, and (1/2 in. (13 mm)) gypsum board. The driving heat fluxes used by ETTU for this wall consisted of three sections of twelve hours duration each. The first and third sections were symmetric white noise, and the middle section was white noise with a DC offset. As can be seen from the charging behavior of the measured temperatures and fluxes (Fig. 4), the first several hours of data are dominated by the initial conditions; because this warm-up effect is undesirable in this frequency-based fitting procedure, the first ten hours of data in the analysis, were eliminated reducing the data to 26 hours. A third-order model (eight wall parameters) was used to process the data. The resulting STPs are shown below:

$U[W/m^2\text{-}K]$	τ[hr]	a_1	b_1	a_2	b_2	a_3	b_3
0.64	1.69	2.06	0.26	-2.99	-7.49	7.58	-22.67

The U-value shown is to be compared with the U-value of 0.60 calculated from thermal properties data listed in the ASHRAE Handbook—1977 of fundamentals volume. The comparison between the measured and predicted heat fluxes is shown in Figs. 5 and 6. Notice, again, the comparatively good tracking ability both for the relatively steady period in the left half and for the highly variable periods at the beginning and end.

One-sided Model

The model development and validation has concentrated on so-called "two-sided" walls-that is, walls for which the heat flux is measured on both sides. In many experiments and for many applications, heat flux data is neither recorded nor required for both sides of the wall. (Normally, the "outside" flux is the one missing.)

If the measurement is single-sided, only the storage factors for the measured side can be determined. The two most important parameters (U and τ), however, will still be determined by the same procedure — albeit with less accuracy than for a two-sided wall. Accordingly, there will be n_o+2 STPs in a one-sided analysis.

As an example, consider the data set shown in fig. 7. The measurements were made by a cement association's in Skokie, IL., using their dynamic hot-box.[12] The walls consisted of 13 mm exterior stucco, hollow-core concrete block, 19 mm furring strips, and 13 mm foil-backed gypsum board. The Simplified Thermal Parameters for this block wall are:

$U[W/m^2-K]$	τ[hr]	a_1	a_2
1.50	2.72	6.54	-25.09

The U-value determined by the model is to be compared with the U-value of 1.2 W/m^2-K reported by the laboratory.

As shown in Fig. 7, the correspondence between predicted and measured heat fluxes is quite good using a second-order fit ($n_m=2$). The discrepancy between the U-value calculated from the data using the model and the other may be due to the fact that the measured fluxes used in the model calculation came from a fluxmeter attached directly to the surface of the test wall; the lab data use an overall hot-box heat balance to calculate heat flow through the wall.

SUMMARY

This analytic technique, in conjunction with ETTU, can be used to evaluate the dynamic thermal characteristics of walls in-situ. Clearly, the applicability of the model is not restricted to field measurements, nor is the data acquisition system restricted to ETTU. Data measured using heat-flowmeter arrays or hot-boxes (both portable and laboratory-based) can be readily analyzed to derive the STPs of a wall, or even of a roof or a floor section.

In the future, ETTU will be used on a representative sample of walls to compile a catalog of STPs that can be compared to their theoretically calculated counterparts. In addition, field measurements will be continued in order to shed some light on the effect of different kinds of insulation retrofits and the age of the wall on its thermal performance, since either may cause measured and theoretical performance to differ markedly. Such measurements should shed some light on the effectiveness of different kinds of insulation retrofits and on the effect of age on walls.

APPENDIX

Theoretical Derivations

HOMOGENEOUS WALLS

In deriving the equations used for analyzing the thermal performance of walls, the diffusion equation will be presented that describes the thermal transport of energy through material — in this case a homogeneous slab (i.e., a wall slab made up of a single layer of a particular material):

$$\frac{dT(x,t)}{dt} = \alpha \frac{d^2T(x,t)}{dx^2} \tag{A1}$$

where: T is the temperature as a function of time and position [^oC]
x,t are the spatial and temporal coordinates, respectively
α is the thermal diffusivity [m^2/s]

In general, diffusivity can be a function of temperature, position, and time; for this application it is assumed to be constant. Furthermore, only a rectangular slab with one-dimensional heat flow will be considered. Carslaw and Jaeger [13] show the solution of this boundary value problem in terms of the temperature:

$$T(x,t) = T_o(x) + \sum_{n=1}^{\infty} \frac{2n}{\pi\tau} e^{-\frac{t}{\tau}n^2} \sin\left(\frac{n\pi x}{L}\right) \int_0^t e^{\frac{t'}{\tau}n^2} \left[T^1(t') - (-1)^n T^2(t') \right] dt' \tag{A2}$$

where: L is the thickness of the slab [m]
T^1, T^2 are the temperatures at the two surfaces [K]
τ is the fundamental time constant [s]
$T_o(x)$ is calculated from initial conditions below [K]

$$\tau = \frac{L^2}{\alpha \pi^2} \tag{A3.1}$$

$$T_o(x) = \frac{2}{L} \sum_{n=1}^{\infty} e^{-\frac{t'}{\tau}n^2} \sin\left(\frac{n\pi x}{L}\right) \int_0^L T(x',0) \sin\left(\frac{n\pi x'}{L}\right) dx' \tag{A3.2}$$

The initial condition of the temperature can be removed by including the past history of the surface temperatures (i.e., extend the integral to minus infinity). This allows a minor simplification of the expression above:

$$T(x,t) = \sum_{n=1}^{\infty} \frac{2n}{\pi\tau} \sin\left(\frac{n\pi x}{L}\right) \int_0^{\infty} e^{-\frac{t'}{\tau}n^2} \left[T^1(t-t') - (-1)^n \, T^2(t-t') \right] dt' \tag{A4}$$

This expression allows the temperature to be calculated at any position and at any time from the past history of the surface temperatures. The goal, however, is to calculate the heat flux, which is related to the gradient of the temperature, at the two surfaces:

$$J(x,t) = -\,U\,L\,\frac{dT(x,t)}{dx} \tag{A5}$$

where: J is the heat flux [watts/m^2]
U is the conductance [watts/m^2-K]

Because the evaluation of the gradient contains an infinite sum, one cannot take the derivative before summing. Thus, one cannot, in complete generality, simplify the problem any further; one can, however, introduce a reasonable, simplifying assumption that will allow the derivation to be continued.

The infinite sum indicated in the above equations is a sum over time constants, (τ/n) that begin at the fundamental time constant τ (i.e., n=1), and approach zero as the summation index (n) gets larger. To be perfectly general all of these time constants must be included in the analysis, but — as in any real experiment — there will be some minimum time constant below which all time constants are no longer important;* this minimum time constant implies a finite maximum limit to the summation, n_o:

$$n_o^2 \simeq \frac{\tau}{\tau_o} \tag{A6}$$

where: n_o is the maximum limit of the summation
τ_o is the minimum time constant [s]

While it is true that for sufficiently large n, each integral becomes negligible, those terms cannot be ignored given that there are an infinite number of them. However, because the temperature will not have changed appreciably until the exponential has become negligible, one can treat the temperature as being constant for those terms:

$$\int_0^{\infty} e^{-\frac{t'}{\tau}n^2} T^{(1,2)}(t-t')\,dt' \simeq \int_0^{\infty} e^{-\frac{t'}{\tau}n^2} T^{(1,2)}(t)\,dt' \tag{A7.1}$$

$$= \frac{\tau}{n^2} T(x,t) \qquad \text{for } n \gg n_o \tag{A7.2}$$

The infinite sum can be broken up into two parts at n_o; leaving the first n_o terms unchanged and

* The presence of a maximum frequency component (as is always the case for discrete data) implies a minimum time constant in the analysis.

substituting the relation above for all other terms:

$$T(x,t) = \sum_{n=1}^{n_o} \frac{2n}{\pi \tau} \sin(\frac{n\pi x}{L}) \int_o^{\infty} e^{-\frac{t'}{\tau}n^2} \Big[T^1(t-t') - (-1)^n T^2(t-t') \Big] \, dt' \tag{A8}$$

$$+ \sum_{n=n_o+1}^{\infty} \frac{2}{\pi n} \sin(\frac{n\pi x}{L}) \Big[T^1(t) - (-1)^n T^2(t) \Big]$$

The second sum can be simplified by using the following two trigonometric series:

$$\sum_{n=1}^{\infty} \frac{\sin n\theta}{n} = \frac{\pi}{2} - \frac{\theta}{2} \tag{A9.1}$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\theta}{n} = - \frac{\theta}{2} \tag{A9.2}$$

Therefore,

$$\sum_{n=n_o+1}^{\infty} \frac{2}{\pi n} \sin(\frac{n\pi x}{L}) = 1 - \frac{x}{L} - \sum_{n=1}^{n_o} \frac{2}{\pi n} \sin(\frac{n\pi x}{L}) \tag{A10.1}$$

$$\sum_{n=n_o+1}^{\infty} (-1)^n \frac{2}{\pi n} \sin(\frac{n\pi x}{L}) = - \frac{x}{L} - \sum_{n=1}^{n_o} (-1)^n \frac{2}{\pi n} \sin(\frac{n\pi x}{L}) \tag{A10.2}$$

These two identities can be used to eliminate all summation terms above the cutoff:

$$T(x,t) = T^1(t) - \frac{x}{L} (T^1(t) - T^2(t)) + \sum_{n=1}^{n_o} \sin(\frac{n\pi x}{L}) \frac{2L}{\pi n} \tag{A11}$$

$$\left[\frac{n^2}{\tau} \int_o^{\infty} e^{-\frac{t'}{\tau}n^2} [T^1(t-t') - (-1)^n T^2(t-t')] \, dt' - [T^1(t) - (-1)^n T^2(t-t')] \right]$$

Since this form of the equation does not contain any infinite sums, one can differentiate this expression and evaluate the derivatives at the two limits without having to explicitly evaluate the sums.

$$J^1(t) = J(0,t) = - \lim_{x\to 0} U \, L \, \nabla T(x,t) \tag{A12.1}$$

$$J^2(t) = - J(L,t) = \lim_{x\to L} -U \, L \, \nabla T(x,t) \tag{A12.2}$$

where: J^1, J^2 are the fluxes into surfaces 1 and 2
T^1, T^2 are the temperatures on surfaces 1 and 2

Note that in the definition of these two terms the surface fluxes have been defined as positive if they flow into the wall. Thus, the two surface fluxes may be expressed in terms of the history of the surface temperatures and the thermal parameters:

$$J^1(t) = U\ (T^1(t) - T^2(t)\ ) + 2U \sum_{n=1}^{n_o} F_n^1(t) - (-1)^n\ F_n^2(t) \qquad \text{(A13.1)}$$

$$J^2(t) = U\ (T^2(t) - T^1(t)\ ) + 2U \sum_{n=1}^{n_o} F_n^2(t) - (-1)^n\ F_n^1(t) \qquad \text{(A13.2)}$$

where: F^1, F^2 are the normalized temperature filters [K]

$$F_n^{(1,2)}(t) = \frac{n^2}{\tau} \int_0^{\infty} e^{-\frac{t'}{\tau}n^2}\ (\ T^{(1,2)}\ (t) - T^{(1,2)}(t-t')\ )\ dt' \qquad \text{(A14)}$$

The Fs are called filters because they are equivalent to low-pass filter functions of time constant τ/n^2 for the past history of the temperature. Note that the first term in each of the two equations above is the steady-state heat flux, ($U*\Delta T$). The second term represents corrections to the steady-state heat flux arising from thermal storage — for massless walls, it disappears.

Since no upper limits were put on the value of n_o in the preceding discussion, one can allow n_o to become arbitrarily large so as to increase the precision of this approximation; in the limit of $n_o \rightarrow \infty$, the expressions become analytically exact. However, for most walls $n_o<5$ is usually sufficient for approximating thermal performance under actual conditions.

FREQUENCY REPRESENTATION

All of the formulae derived above describe the thermal flux in terms of the change over time of the surface temperatures. For some purposes (such as predicting fluxes from temperature histories) this is the ideal representation, but for other purposes (such as calculating thermal parameters from a set of fluxes and temperatures), an analysis in the frequency domain is better suited, which can be done by Fourier-transforming Eq 13 relating temperature to flux:

$$J^1(w) = U\ (T^1(w) - T^2(w)\ ) + 2U \sum_{n=1}^{n_o} F_n^1(w) - (-1)^n\ F_n^2(w) \qquad \text{(A15.1)}$$

$$J^2(w) = U\ (T^2(w) - T^1(w)\ ) + 2U \sum_{n=1}^{n_o} F_n^2(w) - (-1)^n\ F_n^1(w) \qquad \text{(A15.2)}$$

where: w is the angular frequency [rad/s]
$J(w)$ is the amplitude of the flux at that frequency
$F(w)$ is the amplitude of the filter at that frequency
$T(w)$ is the amplitude of the temperature at that frequency

The frequency components are related to their temporal counterparts as follows:

$$J^{(1,2)}(w) = \int_{-\infty}^{\infty} e^{iwt} J^{(1,2)}(t)\ dt \qquad \text{(A16.1)}$$

$$F_n^{(1,2)}(w) = \int_{-\infty}^{\infty} e^{iwt} F_n^{(1,2)}(t)\, dt \qquad (A16.2)$$

$$T^{(1,2)}(w) = \int_{-\infty}^{\infty} e^{iwt} T^{(1,2)}(t)\, dt \qquad (A16.3)$$

where: $i = \sqrt{-1}$

The equation for the filters can be simplified by using the defining relation for the filters:

$$F_n^{(1,2)}(w) = \left[1 - \frac{n^2}{\tau} \int_0^{\infty} e^{-\frac{t'}{\tau} n^2} e^{iwt'}\, dt' \right] \int_{-\infty}^{\infty} e^{iwt}\, T^{(1,2)}(t)\, dt \qquad (A17)$$

The second term can be recognized as the Fourier transform of the temperature and the first term can be reduced by simple integration.

$$F_n^{(1,2)}(w) = \left[\frac{-iw\tau}{n^2 - iw\tau} \right] T^{(1,2)}(w) \qquad (A18)$$

Thus, in the frequency domain, these filters are simply proportional to the temperatures, greatly facilitating the determination of τ.

In any frequency analysis of a system there a set of transfer functions relate each of the inputs (T^1 and T^2) to each of the outputs (J^1 and J^2) and completely specify the system:

$$J^1(w) = H^1(w)T^1(w) - H^0(w)T^2(w) \qquad (A19.1)$$

$$J^2(w) = H^2(w)T^2(w) - H^0(w)T^1(w) \qquad (A19.2)$$

where: H^1 is the transfer function for side 1
H^2 is the transfer function for side 2
H^0 is the transfer function across the wall

These transfer functions can be found from the Fourier inversion:

$$H^0(w) = U + 2U \sum_{n=1}^{n_o} (-1)^n \frac{-iw\tau}{n^2 - iw\tau} \qquad (A20.1)$$

$$H^1(w) = U + 2U \sum_{n=1}^{n_o} \frac{-iw\tau}{n^2 - iw\tau} \qquad (A20.2)$$

$$H^2(w) = U + 2U \sum_{n=1}^{n_o} \frac{-iw\tau}{n^2 - iw\tau} \qquad (A20.3)$$

letting $n_o \rightarrow \infty$ and performing the infinite sum in closed form:

$$H^0 = U \frac{\sqrt{-iwt}}{\sinh(\sqrt{-iwt})} \qquad \text{(A21.1)}$$

$$H^2 = H^1 = U \frac{\sqrt{-iwt}}{\tanh(\sqrt{-iwt})} \qquad \text{(A21.2)}$$

INHOMOGENEOUS WALLS

Thus far, the calculations have been for one-dimensional homogeneous walls; however, because few real walls can be described as homogenous, the model must be corrected accordingly. This has been done by applying correction terms to the lowest order filters of the homogeneous model. There is no a priori reason for this generalization to be exact, and yet it works sufficiently well to use it as an approximate description of real walls:

$$J^1(t) = \underline{J}^1(t) + 2U \sum_{n=1}^{n_m} a_n F_n^1(t) \qquad \text{(A22.1)}$$

$$J^2(t) = \underline{J}^2(t) + 2U \sum_{n=1}^{n_m} b_n F_n^2(t) \qquad \text{(A22.2)}$$

where: J^1, J^2 are predicted inhomogeneous fluxes (W/m^2) at surfaces 1 and 2
$\underline{J}^1$, $\underline{J}^2$ are homogenous (uncorrected) fluxes (W/m^2) at surfaces 1 and 2
n_m is the number of correction factors

The homogeneous fluxes, $\underline{J}^1$, $\underline{J}^2$, are defined by Eq. A15. Note that henceforth the notation $\underline{X}$ indicates that a quantity is from the homogeneous solution, rather than the general solution.

In terms of the transfer functions,

$$H^0(w) = \underline{H}^0(w) \qquad \text{(A23.1)}$$

$$H^1(w) = \underline{H}^1(w) + 2U \sum_{n=1}^{n_m} a_n \frac{-iw\tau}{n^2 - iw\tau} \qquad \text{(A23.2)}$$

$$H^2(w) = \underline{H}^2(w) + 2U \sum_{n=1}^{n_m} b_n \frac{-iw\tau}{n^2 - iw\tau} \qquad \text{(A23.3)}$$

where: $\underline{H}(w)$ are the homogenous transfer functions
$H(w)$ are the corrected transfer functions

Again, the homogeneous transfer functions, $\underline{H}^0$, $\underline{H}^1$, $\underline{H}^2$, are defined by Eq. A21. The correction terms can be interpreted as surface storage factors that indicate the relative amount of storage that occurs on the surfaces of the wall compared to the interior.

The number of inhomogeneous terms can be estimated from the time constant and some knowledge of the highest frequency of interest (i.e., the highest frequency one is interested in duplicating accurately or, equivalently, the highest frequency expected in the data):

$$n_m = \sqrt{2w_{max}\,\tau} \tag{A24}$$

where: w_{max} is the maximum frequency of interest

COMBINING LAYERS

Transfer functions of a many-layered wall can be calculated from the transfer functions of its individual layers by conceptually using the flux out of one layer as the flux into the next layer. Mathematically, this chaining process is a matrix multiplication of the appropriate combinations of the transfer functions. The general relations for calculating the combined transfer function from two individual transfer functions is given below.

$$H_o = \frac{H'_o H''_o}{H'_2 + H''_1} \tag{A25.1}$$

$$H_1 = H'_1 - H_o \frac{H'_o}{H''_o} \tag{A25.2}$$

$$H_2 = H'_2 - H_o \frac{H''_o}{H'_o} \tag{A25.3}$$

where: H are the combined transfer functions
H' are the first layer transfer functions
H'' are the second layer transfer functions

Note that H'_1 and H''_2 represent the exposed surfaces, while H'_2 and H''_1 represent the surfaces internal to the combined wall.

This combinatorial rule can be used in two ways. It can be used to calculate the exact transfer function when the true thermal properties of all the component layers are known, and it can be used to calculate the approximate thermal properties when two composite walls are being combined.

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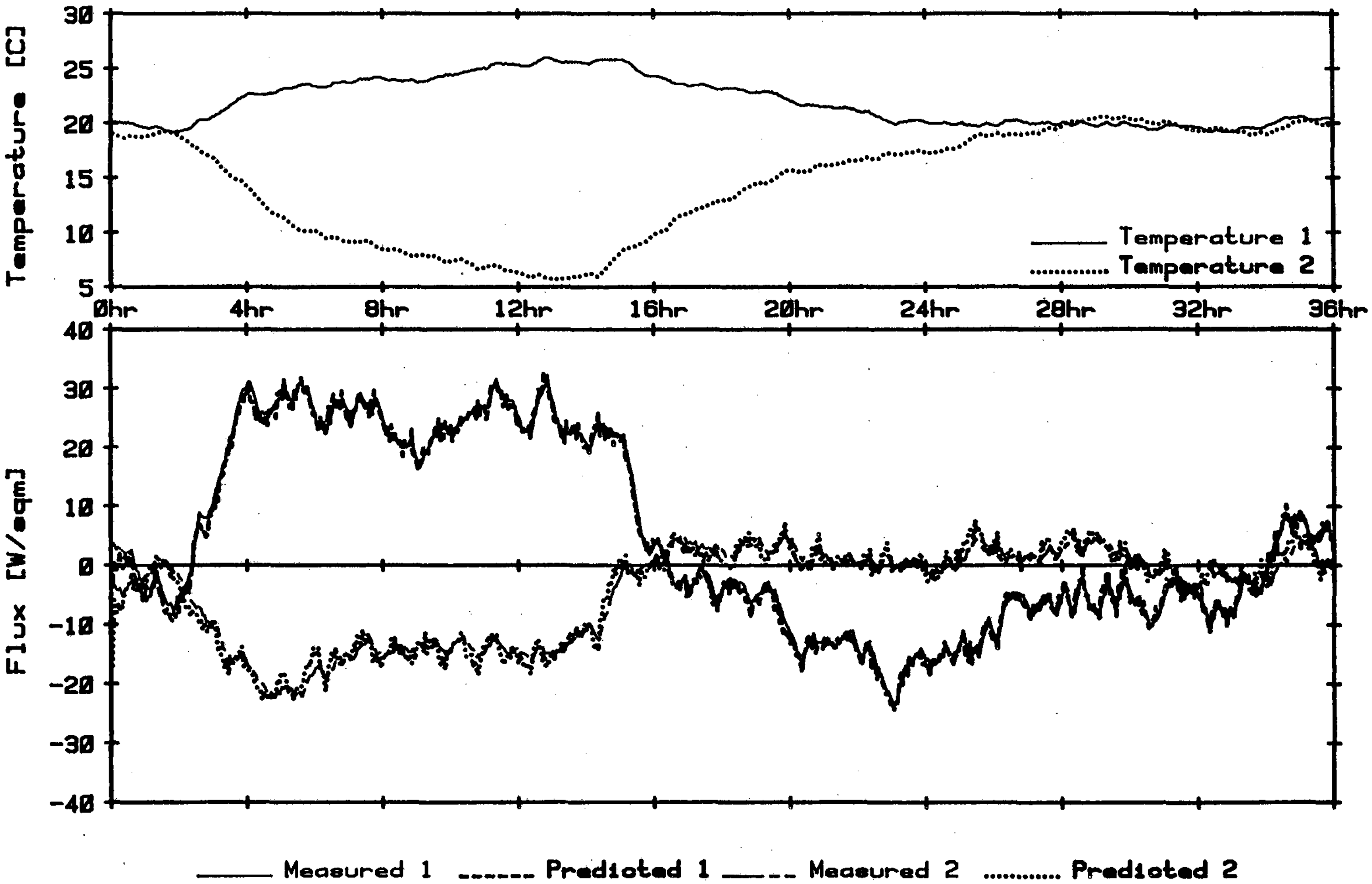


Figure 1. Measured and predicted data for illustrative wall.

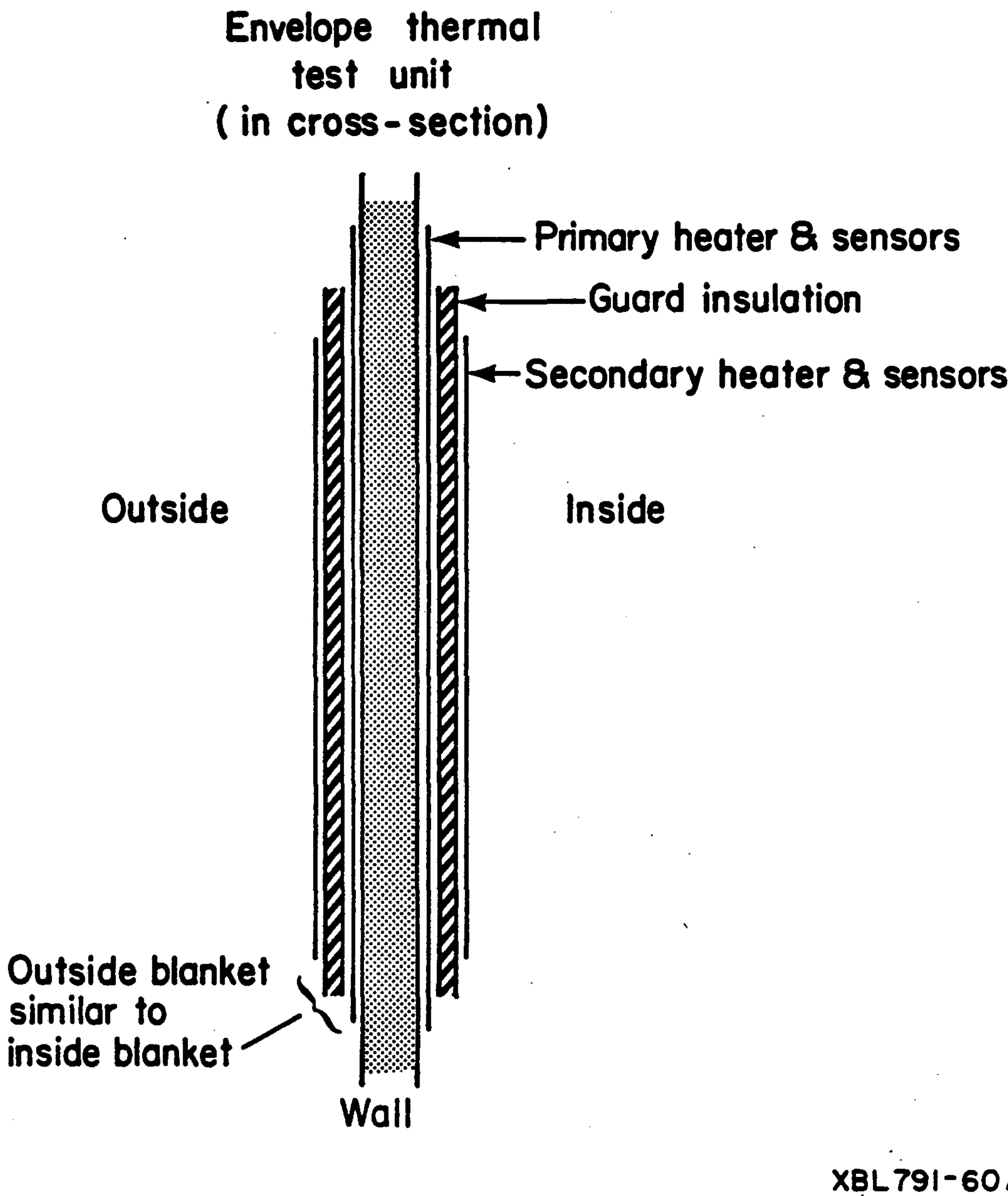
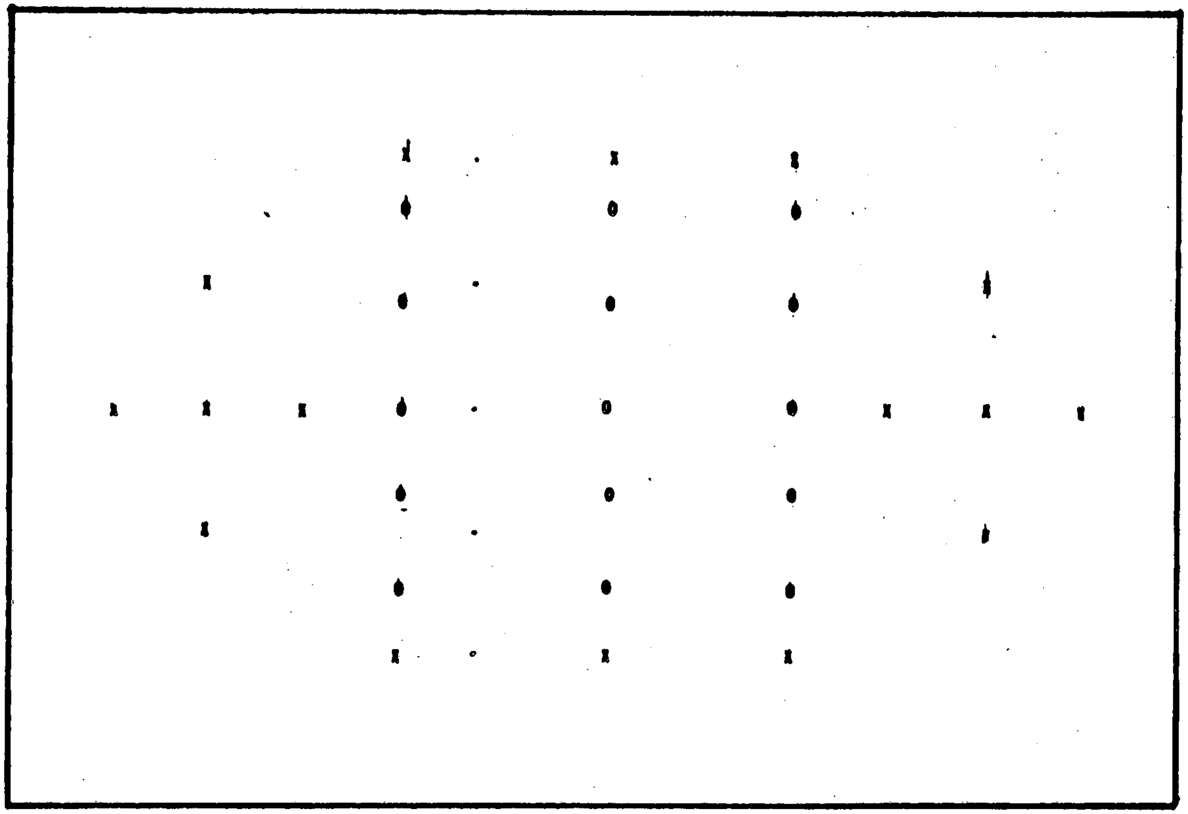


Figure 2. Schematic of Envelope Thermal Test Unit (cross section).



x: edge sensors

o: center sensors

Figure 3. ETTU temperature sensor array.

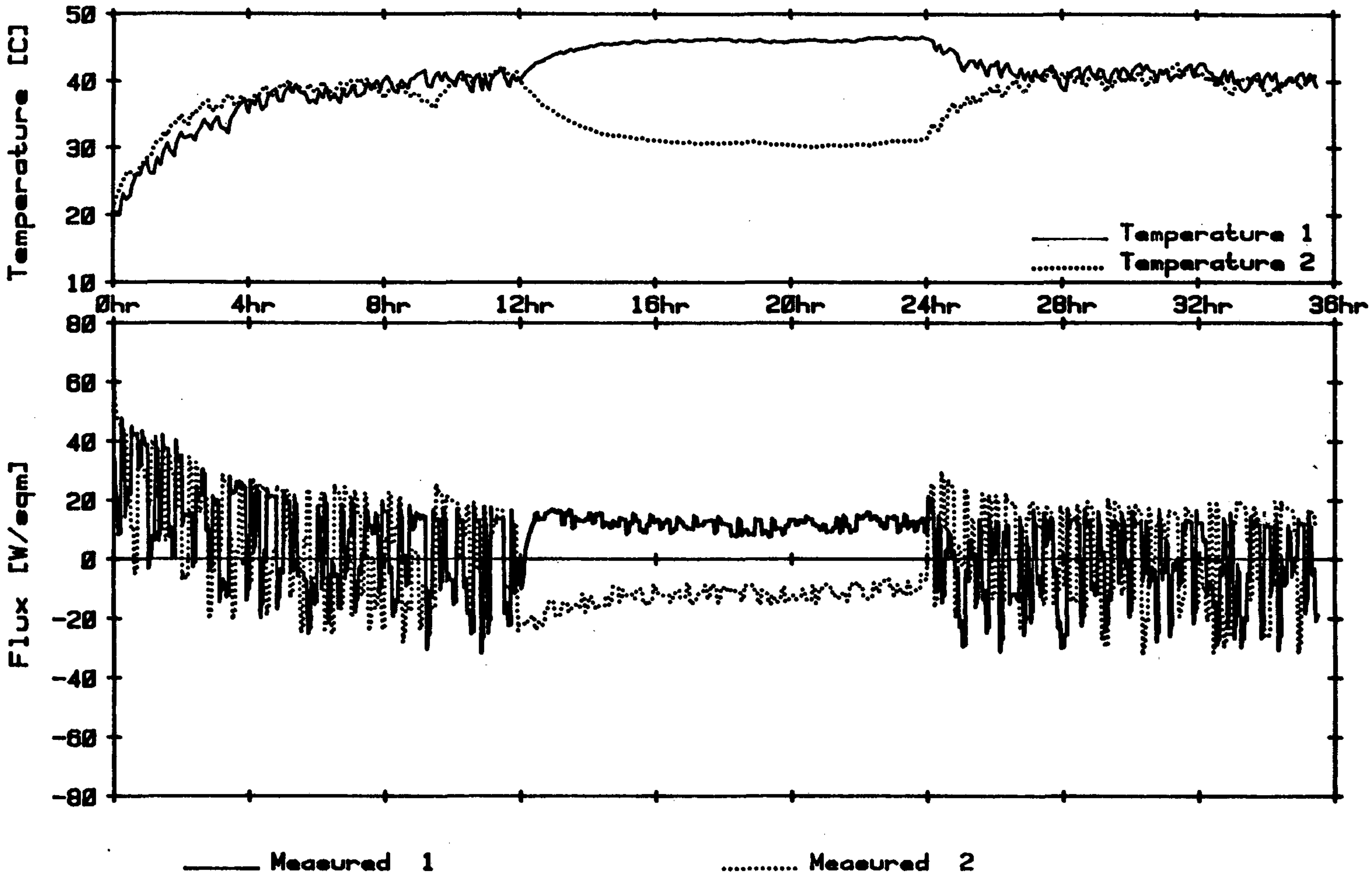


Figure 4. Measured heat flux and temperature data for laboratory wall.

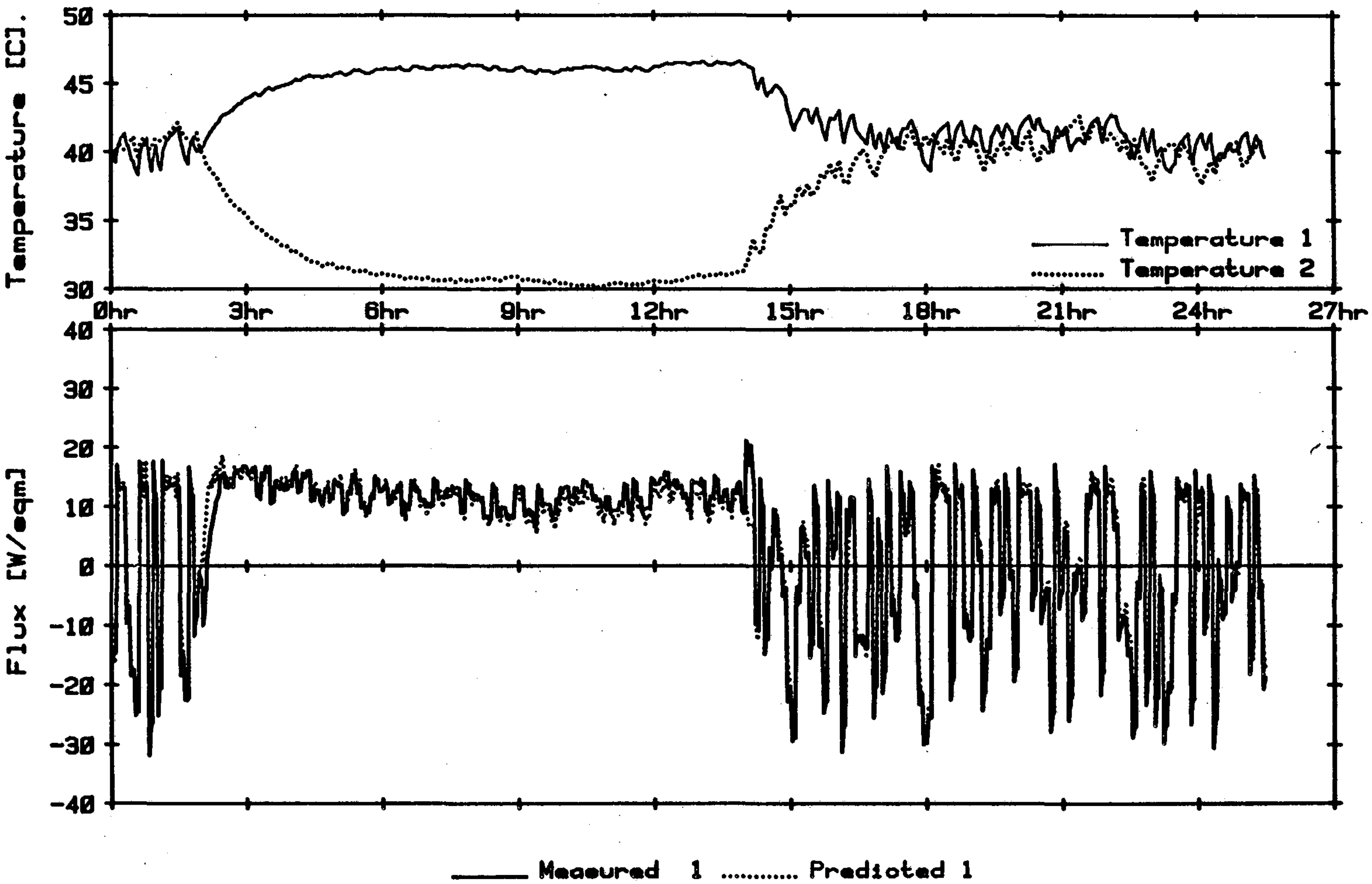


Figure 5. Measured and predicted data for side 1 of laboratory wall.

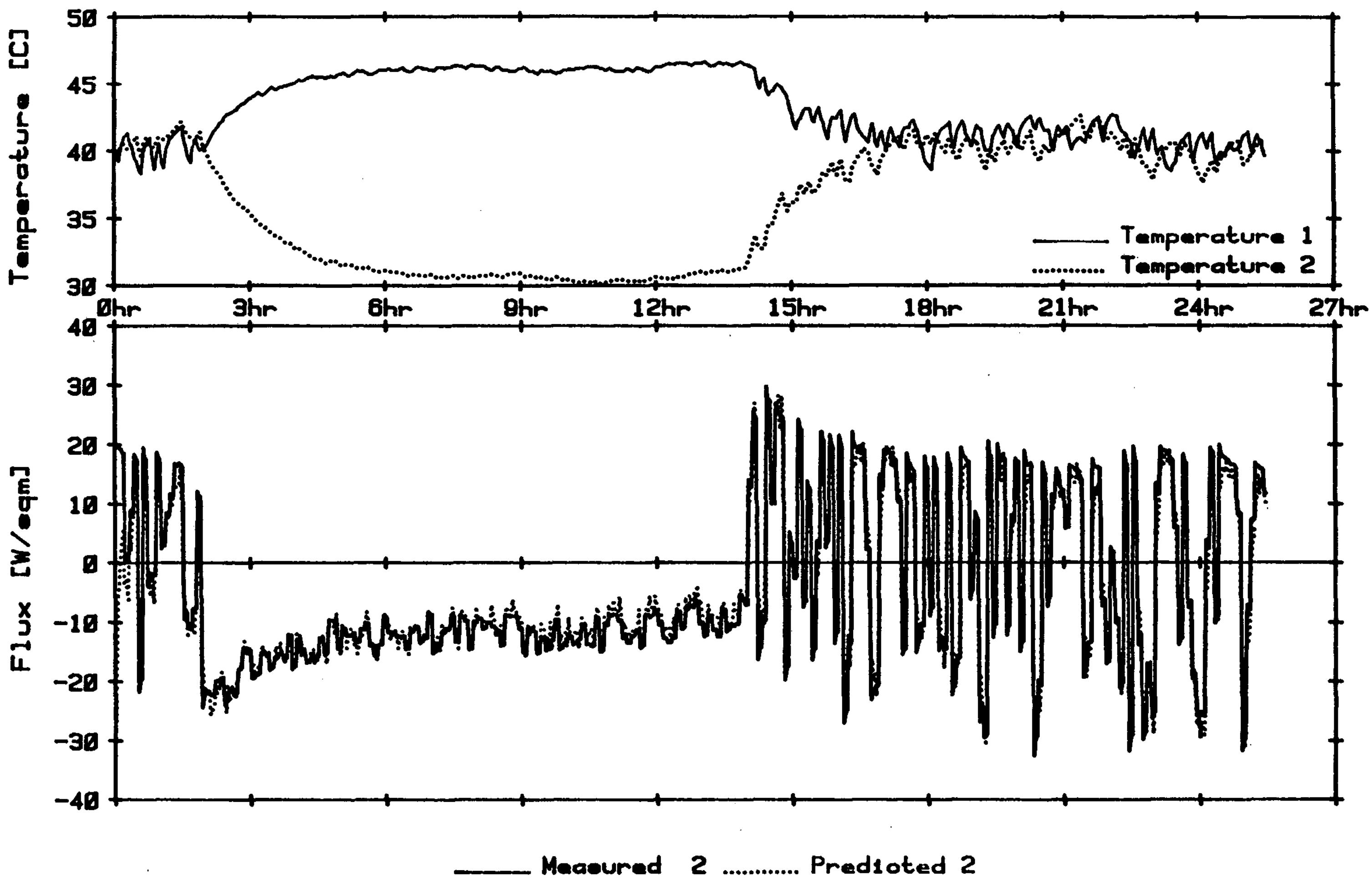


Figure 6. Measured and predicted data for side 2 of laboratory wall.

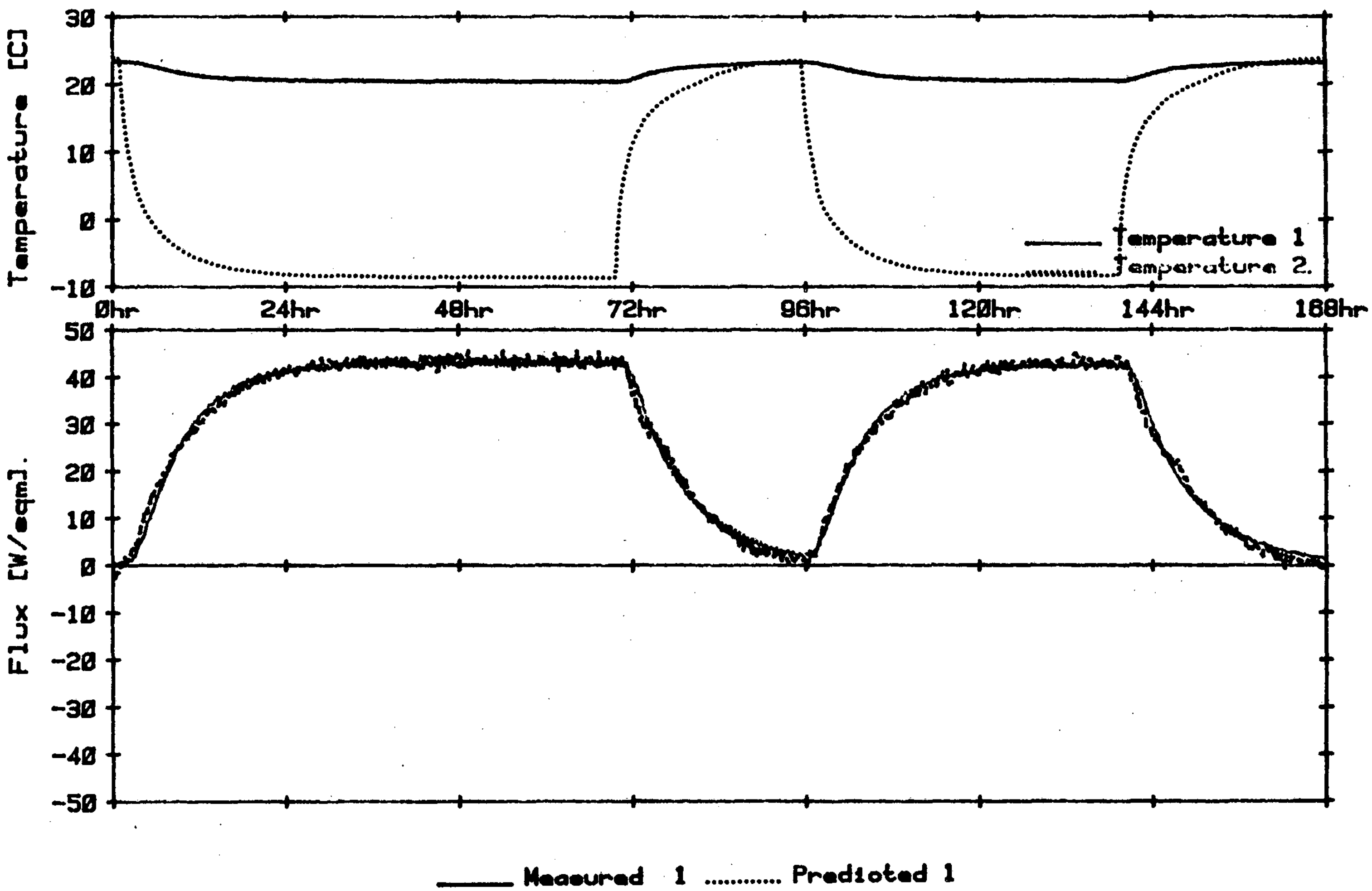


Figure 7. Measured and predicted data for cement association laboratory wall.

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