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COULOMB DISSOCIATION OF RELATIVISTIC DEUTERONS

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ABSTRACT

Using the Weiszacker-Williams method of virtual quanta and a non-relativistic perturbation theory calculation of the deuteron photo-dissociation cross section, the double differential cross section for coulomb dissociation of relativistic deuterons is calculated. Numerical results are presented and the application of the results to the subject of tagged neutron beams is discussed.

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I. SUMMARY

When a high energy deuteron beam collides with a target, some fraction of the deuterons dissociate into neutron-proton pairs with low relative velocity, thus producing a neutron "tagged" by a proton of about the same energy. In a recent Bevatron experiment[1] which examined this process to provide information for the development of a tagged neutron beam, it was observed that tagging occurred for heavy targets (U^{238}) but not for light targets (Be^9). Since this behavior conflicts with the $A^{1/3}$ dependence expected of a peripheral nuclear process, it has been suggested[1] that tagging may be produced electromagnetically. In this paper we derive the cross-section for deuteron dissociation using the Weizsacker-Williams method of virtual quanta and present numerical results for the double differential cross-section[2]. In section II we review the method and derive the expression for the cross-section. Readers primarily interested in the results should proceed directly to section III in which the numerical results are presented and discussed.

IIa. THE WEIZACKER-WILLIAMS METHOD[3]

The electromagnetic field of a relativistic particle of charge q=Ze passing the observation point at impact parameter b is approximately composed of a transverse electric field and magnetic field given by:

$$\vec{E}(t) = Ze \frac{\gamma b}{\left(b^2 + \gamma^2 v^2 t^2\right)^{3/2}} \hat{x}$$

$$\vec{B}(t) = \beta E(t)\hat{y}. \qquad (1)$$

As $\beta \to 1$ $\vec{B}$ and $\vec{E}$ become perpendicular and equal, and thus become similar to free electromagnetic radiation. To exploit this equivalence C. F. Weizsacker and E. J. Williams independently developed the following method in which the fields are Fourier decomposed into an equivalent photon distribution.

The Fourier transform $E(\omega)$ of the above electric field gives the intensity distribution[4]:

$$I\ (\omega,b) = \frac{c}{2\pi}\ |\ E(\omega)|^2 = \frac{1}{\pi^2}\ \frac{q^2}{c}\ \frac{1}{\beta^2 b^2}\left(\frac{\omega}{\gamma}\frac{b}{v}\right)^2 K_1^2\left(\frac{\omega b}{\gamma v}\right) \qquad (2)$$

Integrating the above formula over impact parameters and using a low frequency approximation to the resulting K functions we find:

$$I(\omega) = \frac{2}{\pi}\frac{q^2}{c}\frac{1}{\beta^2}\left[\ln\left(\frac{1.123\gamma v}{\omega b_{min}}\right) - \frac{\beta^2}{2}\right] \qquad (3)$$

where b_{min} is the nuclear radius.
The equivalent photon distribution can now be found by using the relation:

$$I(\omega)\ d\omega = \hbar\ \omega\ \frac{dN(\hbar\ \omega)}{d(\hbar\ \omega)}\ d\ (\hbar\ \omega); \qquad (4)$$

thus

$$\frac{dN(\hbar\ \omega)}{d(\hbar\omega)} = \frac{2Z^2}{\pi}\ \frac{e^2}{\hbar c}\ \frac{1}{\beta^2}\ \frac{1}{\hbar\omega}\left[\ln\left(\frac{1.123\ \gamma v}{\omega b_{min}}\right) - \frac{\beta^2}{2}\right]. \qquad (5)$$

Given the deuteron photo-dissociation cross-section for a single real incident photon $\frac{d\sigma}{d\Omega}$, the cross-section for coulomb dissociation would be:

$$\frac{d^2\sigma}{d(p)d\Omega} = \frac{dN(\hbar\omega)}{d(\hbar\omega)}\ \frac{d(\hbar\omega)}{dp}\ \frac{d\sigma(\hbar\omega)}{d\Omega}\ . \qquad (6)$$

It is important to remember that this calculation is done in the deuteron rest frame and so it is the field of the target nucleus which is Fourier decomposed.

IIb. DEUTERON PHOTO-DISSOCIATION CROSS SECTION

We now derive the deuteron photo-dissociation cross section using the non-relativistic quantum theory of radiation. The Hamiltonian is:

$$H = H_0 + \mathscr{H} + \mathscr{H}' \tag{7}$$

where

$$H_0 = \sum_{\vec{k},\alpha} \hbar\omega_k \, a^{\dagger}_{\vec{k},\alpha} a_{\vec{k},\alpha} + H_d \tag{8a}$$

$$\mathscr{H} = \frac{-e}{\mu c} \, \vec{p} \cdot \vec{A}(\vec{r}) = -ie\sqrt{2\pi\hbar} \sum_{\vec{k},\alpha} \sqrt{\omega_k} \, [a_{\vec{k}} e^{i\vec{k}\cdot\vec{r}} + a^{\dagger}_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}}] \, \vec{r}\cdot\varepsilon_{\vec{k},\alpha} \tag{8b}$$

$$\mathscr{H}' = \frac{e^2}{2\mu c^2} \, A^2(\vec{r}). \tag{8c}$$

In the above, the creation and annihilation operators $a^{\dagger}_{\vec{k},\alpha}$ and $a_{\vec{k},\alpha}$ refer to plane-wave photon states of wave number $\vec{k}$ and polarization α. H_d is the deuteron Hamiltonian and μ, r, and p refer to the relative neutron-proton motion. Here and in the following all quantization volume factors will be systematically dropped.

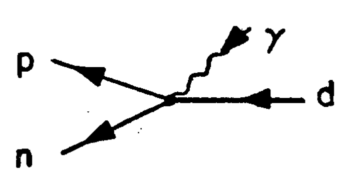
The initial state will be the deuteron ground state plus one photon of wave number $\vec{k}$ and polarization α:

$$|i\rangle = |H_d\rangle \; |\vec{k},\alpha\rangle \tag{9}$$

and the final state a free-particle plane wave and no photon:

$$|f\rangle = |\vec{k}'\rangle \; . \tag{10}$$

We will use $\mathscr{H}$ as the perturbation mixing the above two eigenstates of H_0. The first non-vanishing term in the perturbation expansion for the transition matrix is of the form:



giving a differential cross section

$$d\sigma = \frac{2\pi}{\hbar c} |\langle \vec{k}'| \frac{-e}{\mu c} \vec{p}\cdot\vec{A}(\vec{r})| H_d, \vec{k}, \alpha\rangle|^2 \rho(E')$$
$$= \frac{2\pi}{\hbar c} |\langle \vec{k}'| -ie\sqrt{2\pi\hbar\omega_k}\, e^{i\vec{k}\cdot\vec{r}}\, \vec{r}\cdot\varepsilon_{\hat{k},\alpha}|H_d\rangle|^2 \rho(E'). \tag{11}$$

The density of final states $\rho(E')$ is given by

$$\rho(E') = \frac{dN}{dE'} = \frac{\mu k'}{(2\pi)^3\hbar^2}\, d\Omega \tag{12}$$

for dissociation into the interval of solid angle $d\Omega$, giving:

$$\frac{d\sigma}{d\Omega} = \frac{\mu k' e^2 \omega_k}{2\pi\hbar^2 c} |\langle \vec{k}'|\ \vec{r}\cdot\varepsilon_{\hat{k},\alpha}|H_d\rangle|^2 \tag{13}$$

where we have made use of the fact that since $k \ll 1/r_0$ for photons in the range of interest with r_0 being the characteristic "size" of the deuteron wave function, we have $e^{i\vec{k}\cdot\vec{r}} \cong 1$ (electric dipole approximation).

To simplify the matrix element we insert the unit operator

$$|\ell,m\rangle\langle\ell,m| \tag{14}$$

(implied sum over ℓ, m) into (13) yielding:

$$\frac{d\sigma}{d\Omega} = \frac{\mu k' e^2 \omega_k}{2\pi\hbar^2 c} |\langle \vec{k}'|\ell,m\rangle\langle\ell,m|\vec{r}\cdot\hat{\varepsilon}_{\hat{k},\alpha}|H_d\rangle|^2 . \tag{15}$$

If m is choosen to be measured along the direction of $\hat{\varepsilon}_{\hat{k},\alpha}$ we see that since $r\cdot\hat{\varepsilon}_{\hat{k},\alpha}$ is the zeroth component of a spherical tensor operator of rank one, the Wigner-Eckart theorem insures that the only non-vanishing matrix element will be between $|H_d\rangle$ (which we assume is s-wave) and $\langle 1, 0_\alpha|$;

$$\frac{d\sigma}{d\Omega} = \frac{\mu k' e^2 \omega_k}{2\pi\hbar^2 c} |\langle k'|1,0_\alpha\rangle\langle 1,0_\alpha|\vec{r}\cdot\hat{\varepsilon}_{\hat{k},\alpha}|H_d\rangle|^2$$

$$= \frac{\mu k' e^2 \omega_k}{2\pi\hbar^2 c} \left| 4\pi i \; {}^0Y_1^{\alpha}(\hat{k}')^* \langle 1, 0_\alpha | \vec{r}\cdot\hat{\varepsilon}_{\hat{k},\alpha} | H_d \rangle \right|^2$$

$$= \frac{8\pi\mu k' e^2 \omega_k}{\hbar^2 c} \left| {}^0Y_1^{\alpha}(\hat{k}') \right|^2 \left| \int j_1(k'r)\vec{r}\cdot\hat{\varepsilon}_{\hat{k},\alpha}\phi_d(r)\, {}^0Y_1^{\alpha *}(\hat{r})dV \right|^2 \tag{16}$$

where

$$\phi_d(r) = \langle \vec{r} | H_d \rangle . \tag{17}$$

Doing the angular integral and utilizing Blatt and Weisskophf's[5] calculation of the radial integral, we get:

$$\frac{d\sigma}{d\Omega} = \frac{16\pi}{3} \frac{e^2}{\hbar c} \frac{|{}^0Y_1^{\alpha}(\hat{k}')|^2}{\gamma_d^2(1-\gamma_d r_{ot})} \left(\frac{k'\gamma_d}{k'^2+\gamma_d^2} \right)^3 \tag{18}$$

where γ_d is the decay constant in the deuteron asymptotic wave function $e^{-\gamma r}$ and r_{ot} is a neutron-proton scattering length. We thus have the differential cross section for proton (or equivalently neutron) production by a linearly polarized photon beam in terms of the angle of the emitted proton (neutron) relative to the polarization direction of the photon. For unpolarized radiation we average over two orthagonal polarizations:

$$\tfrac{1}{2}\sum_\alpha \frac{d\sigma}{d\Omega} = \frac{8\pi}{3} \frac{e^2}{\hbar c} \frac{1}{\gamma_d^2(1-\gamma_d r_{ot})} \left(\frac{k'\gamma_d}{k'^2+\gamma_d^2} \right)^3 \left(\frac{3}{4\pi} \right) (\cos^2\theta_1 + \cos^2\theta_2) \tag{19}$$

where θ_α refers to the angle between $\hat{\varepsilon}_{\hat{k},\alpha}$ and k'. Since $\hat{\varepsilon}_{\hat{k},\alpha}$, $\hat{\varepsilon}_{\hat{k},\alpha}$, and $\hat{k}$ form a right-handed coordinate system by construction we have:

$$\cos^2\theta_1 + \cos^2\theta_2 = 1 - \cos^2\theta_{\hat{k}} = \sin^2\theta_{\hat{k}} \tag{20}$$

so we can write the unpolarized cross section

$$\frac{d\sigma}{d\Omega} = \frac{8\pi}{3} \frac{e^2}{\hbar c} \frac{1}{\gamma^2(1-\gamma_d r_{ot})} \left(\frac{k'\gamma_d}{k'^2+\gamma_d^2} \right)^3 \left(\frac{3}{4\pi} \sin^2\theta_{\hat{k}} \right). \tag{21}$$

The momenta of the outgoing proton and neutron are approximately equal in the deuteron rest frame, yielding the relations:

$$E_n = 1/2\ (\hbar\ \ - B). \quad \text{(in MeV)}$$
$$k'^2 = 4.824 \times 10^{24}\ (\hbar\omega - B). \tag{22}$$

Using these and plugging in eq. (5) for the photon distribution, we get

$$\frac{d^2\sigma}{dEd\Omega} = \frac{32\ e^2/\hbar c}{3\gamma_d^{\ 2}\ (1-\gamma_d r_{ot})} \left(\frac{Z}{\beta}\right)^2 \frac{1}{h\omega} \left(\frac{k'\gamma_d}{k^2 + \gamma_d^{\ 2}}\right)^3$$
$$\times \left[\ln \left(\frac{1.123\gamma v}{\omega b_{min}}\right) - \frac{\beta^2}{2} \right] \tag{23}$$

$$= \frac{64\ e^2/\hbar c}{3\gamma_d^{\ 2}(1-\gamma_d\ r_{ot})}\left(\frac{Z}{\beta}\right)^2 \frac{\mu}{\hbar^2\ (k'^2+\gamma_d^{\ 2})} \left(\frac{k'\gamma_d}{k'^2+\gamma_d^{\ 2}}\right)^3$$
$$\times \left[\ln\left(\frac{2.246\ \gamma\ v\ \mu}{\hbar\ b_{min}(k'^2+\gamma_d^{\ 2})}\right) - \frac{\beta^2}{2}\right]\left(\frac{3}{4\pi}\ \sin^2\theta\right) \tag{24}$$

III. NUMERICAL RESULTS

The deuteron rest frame cross-section in eq. (24) is transformed to the lab using the familiar relations between lab and beam frame quanties:

$$\left.\frac{d^2\sigma}{dp\,d\Omega}\right|_{lab} = \left.\frac{d^2\sigma}{dE\,d\Omega}\right|_{beam} \frac{p_{lab}}{E_{lab}} \frac{\sin\Theta_{beam}}{\sin\Theta_{lab}}$$

$$\tan\Theta_{lab} = \frac{1}{\gamma}\,\frac{1}{\cos\Theta_{beam} - (\beta/\beta_{lab})} \qquad (25)$$

$$p_{\perp\,lab} = p_{\perp\,beam}$$

$$p_{\parallel\,lab} = \beta\gamma E_{beam} + \gamma p_{\parallel\,beam}$$

In Fig. 1-3 the numerical results for the cross sections in the lab at $p_{incident} = 5.8$ GeV/c $\equiv 2p_{nom}$ are plotted. Fig. 1 gives the double differential cross-section for a Uranium target as a function of neutron lab angle for various lab momenta. The cross-section is peaked at $\theta = 0.5^o$ with a maximum value of 1.6 barns/sterad-MeV/c and is almost entirely contained in the interval 0^o-2^o. Fig. 2 gives the cross-section plotted vs lab momentum for various lab angles; it exhibits a peak at one half the incident deuteron momentum and width of about 140 MeV/c. The cross-section integrated over the momentum spectrum is plotted in Fig. 3 for various targets. The form is approximately that of Fig. 1 and for U^{238} the maximum is 245 barns/sterad.

IV. TAGGED NEUTRON BEAMS

To produce a tagged neutron beam an apparatus of the sort illustrated in Fig. 4 would be used. The neutrons would be selected by a slit and proceed to the experimental area while the protons would be momentum analyzed. Coincidences between the experimental apparatus and the proton momentum analyzer would reveal which neutrons are tagged. Since the proton and neutron momenta are equal and opposite in the center of mass, selecting a particular neutron angle and proton momentum would yield a neutron with a specific momentum. Thus the momentum resolution of the beam is only limited by the resolution of the proton momentum analysis, the angular width of the neutron slit, and multiple coulomb (elastic) scattering in the target.

An important experimental consideration is the possiblity of producing a polarized neutron beam from the dissociation of a polarized deuteron beam. Since in the electric dipole approximation the photon does not couple to the neutron or proton spin, their intitial polarization will be conserved by coulomb dissociation. The protons produced will have the same polarization as the neutrons and thus provide a convienent check on the neutron beam polarization.

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Footnotes and References

1. P. Bowles, C. Leemann, W. Mehlhop, H. Grunder, O. Piccioni, R. Thomas, D. Scipione, R. Garland, J. Sebek, Bull. Am. Phys. Soc. 17, 1188 (1972).

2. Previous results on this subject may be found in the following articles: G. Faldt, Phys. Rev. D2, 846 (1970 and S. M. Dancoff, Phys. Rev. 72, 1017 (1947).

3. A more complete derivation may be found in J. D. Jackson, Classical Electrodynamics (Wiley, New York, 1962) pp 520 - 524.

4. K_1 is the first order modified Bessel funtion. v, c, β, and γ refer to the velocity of the particle which produces this field.

5. J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (Wiley, New York, 1952) pp. 609 - 613.

Figure Captions

Fig. 1 A plot of the double differential cross-section for the production of neutrons in the lab momentum interval dp and lab solid angle interval $d\Omega$ by coulomb dissociation of 5.8 GeV/c deuterons colliding with a U^{238} target. This figure plots $\frac{d^2\sigma}{dpd\Omega}$ versus neutron lab angle for neutron lab momenta of 2900, 2960, and 3020 MeV/c. $P_{nom} \equiv P_{beam}/2$.

Fig. 2 The same cross-section as in Fig. 1 but here plotted versus neutron lab momentum for neutron lab angles $0{,}1^{o}$, 0.5^{o}, and 0.8^{o}.

Fig. 3 The cross-section in Figs. 1 and 2 integrated over neutron lab momenta 2.7 GeV/c < p < 3.1 GeV/c and plotted versus neutron lab angle. Also plotted are the corresponding cross-section for Pb^{207}, Sn^{119}, and Fe^{86} targets.

Fig. 4 A schematic illustration of a possible experimental configuration for the production of a tagged neutron beam.

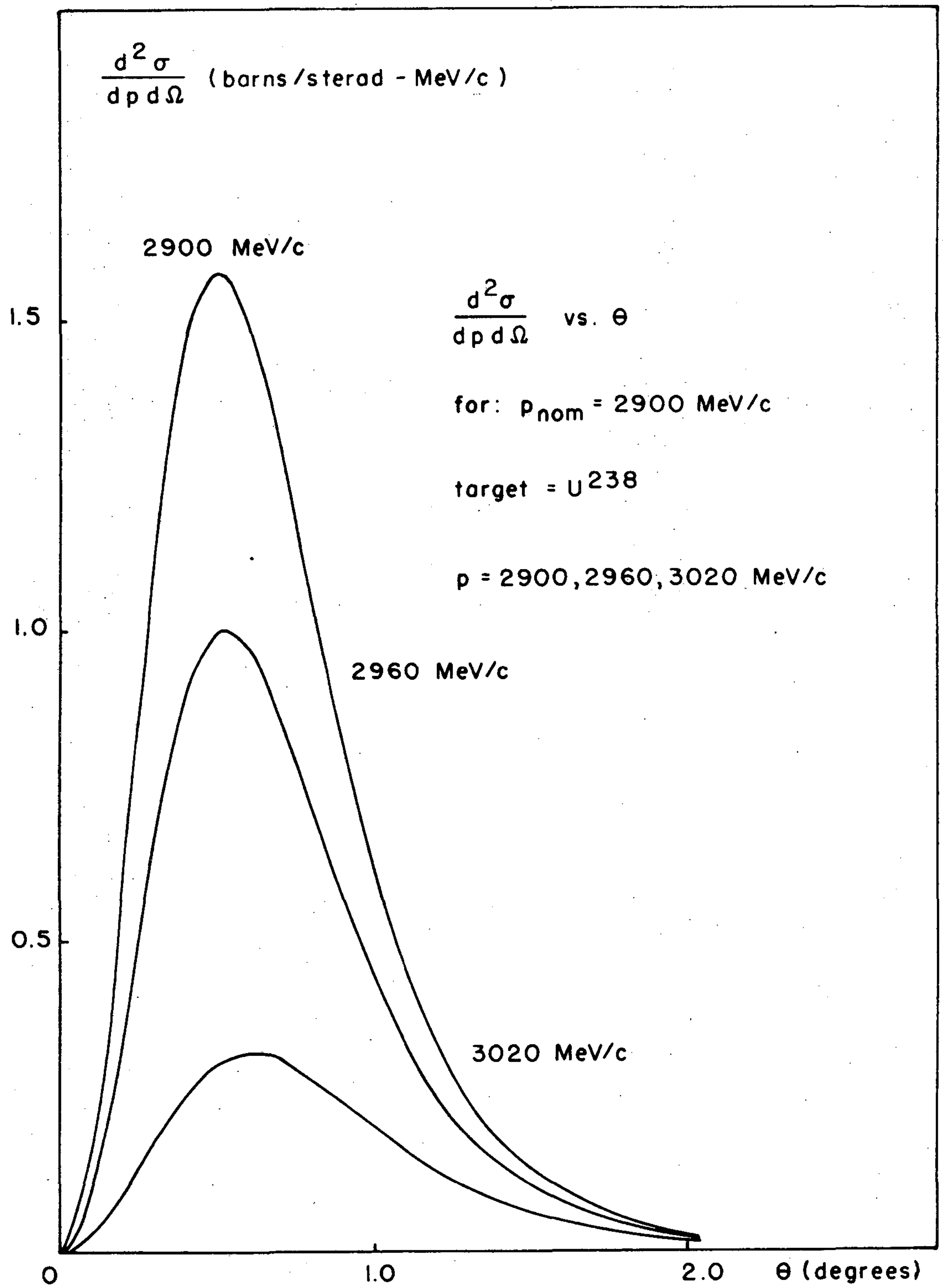

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Fig. 1

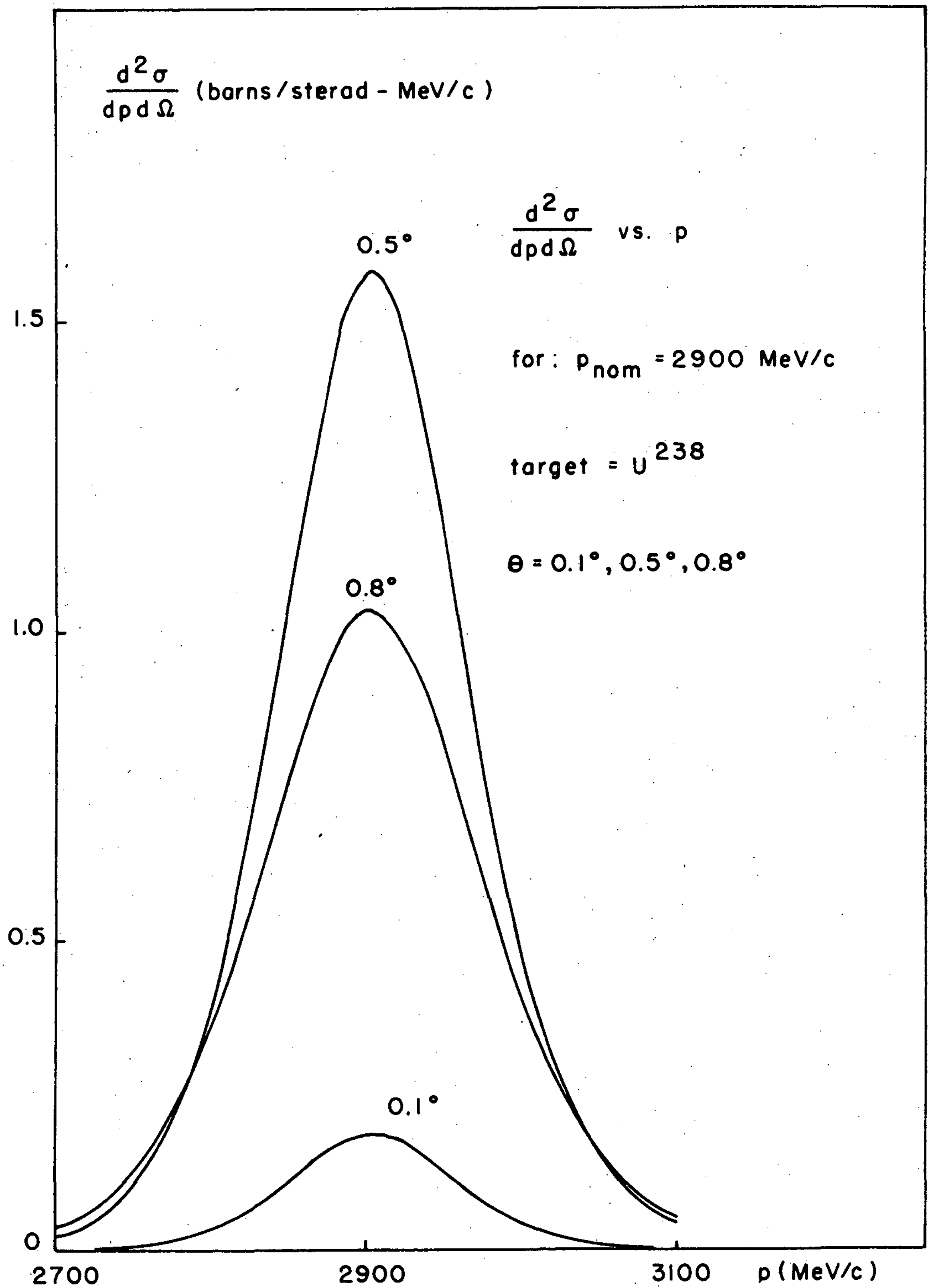

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Fig. 2

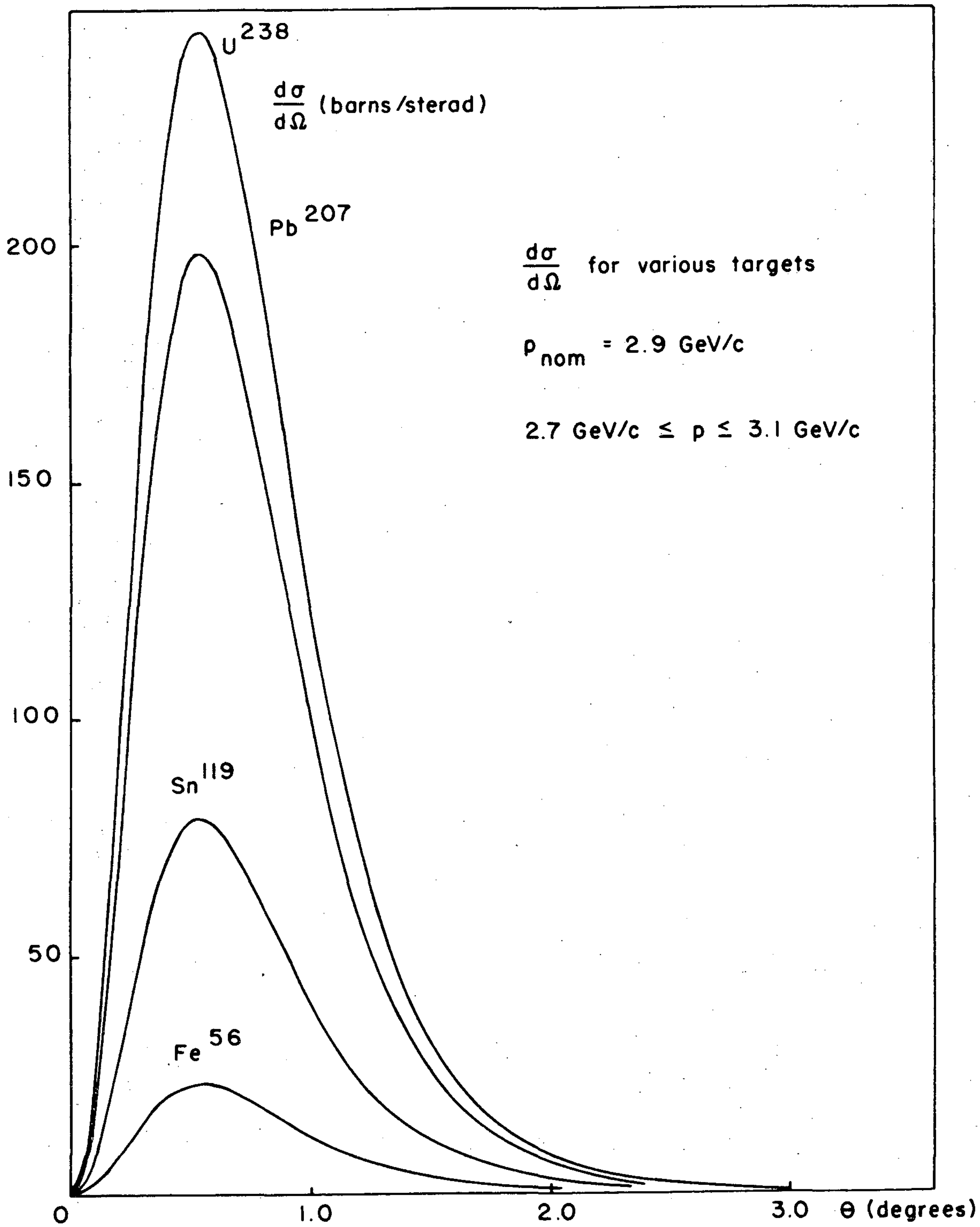

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Fig. 3

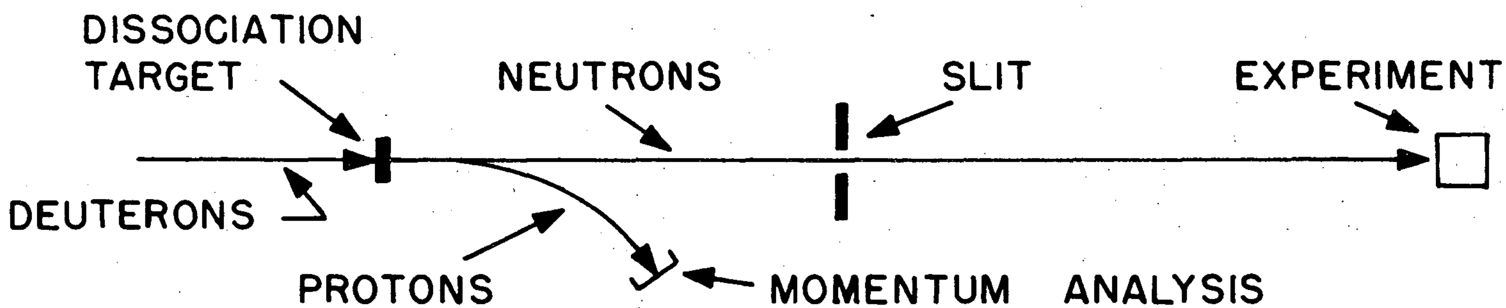

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Fig. 4

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