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Publication Date
2009-08-19

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ABSTRACT

When fractures in otherwise hard rocks are filled with fluids (oil, gas, water, CO_2), the type and physical state of the fluid (liquid or gas) can make a large difference in the wave speeds and attenuation properties of seismic waves. The present work summarizes methods of deconstructing theses effects of fractures, together with any fluids contained within them, on wave propagation as observed in reflection seismic data. Additional studies of waves in fluid-saturated granular media show that the behavior can be quite different from that for fractured media, since these materials are typically much softer mechanically than are the fractured rocks (i.e., having a very small drained moduli). Important fluid effects in such media are often governed as much by fluid viscosity as by fluid bulk modulus.

INTRODUCTION

Detection and resource management of fluid reservoirs are commonly performed using seismic reflection surveys. When reservoirs contain fractures or cracks, these sources of high permeability and fluid-saturated porosity have a strong impact on the seismic wave analysis. Of special significance to seismic waves is the fact that aligned fractures result in seismic wave anisotropy [1]. We can also understand very directly the sources of the anisotropy due to fractures by considering a method introduced by Sayers and Kachanov [2]. Elastic constants, and therefore the Thomsen [3] parameters, can be conveniently expressed in terms of the Sayers and Kachanov [2] formalism. Furthermore, in the low crack density limit (which is also consistent with the weak anisotropy approach of Thomsen [3]), we obtain direct links between the Thomsen parameters and the fracture properties. These links suggest a method of inverting for fracture density from wave speed data.

THOMSEN'S SEISMIC WEAK ANISOTROPY FORMALISM

Thomsen's weak anisotropy formalism [3], being an approximation designed specifically for use in short offset velocity analysis for exploration geophysics, is clearly not exact. Approximations incorporated into the formulas become most apparent for angles θ greater than 15 degrees from the vertical, especially for compressional velocities $v_p(\theta)$ and vertically polarized shear velocities $v_{sv}(\theta)$. Angle θ in seismic exploration is typically measured from the spatial z-vector pointing directly down into the earth.

Exact velocity formulas for *P*, *SV*, and *SH* seismic waves at all angles in a *VTI* (vertically transverse isotropic) elastic medium are known and available in many places [4,5], so they will not be listed here. Expressions for phase velocities in Thomsen's weak anisotropy limit can also be found in many places, including [3] and [4]. The pertinent expressions for phase velocities in *VTI* media as a function of angle θ are:

$$v_p(\theta) \simeq v_p(0)(1 + \delta\, sin^2\,\theta\, cos^2\,\theta + \epsilon\, sin^4\,\theta), \tag{1}$$

$$v_{sv}(\theta) \simeq v_s(0)\left(1 + \left[\frac{v_p^2(0)}{v_s^2(0)}\right](\epsilon - \delta)\, sin^2\theta\, cos^2\theta\right), \tag{2}$$

and

$$v_{sh}(\theta) \simeq v_s(0)(1 + \gamma sin^2\theta). \tag{3}$$

In our present context, $v_s(0) = \sqrt{\frac{c_{44}}{\rho_0}}$, and $v_p(0) = \sqrt{\frac{c_{33}}{\rho_0}}$, where c_{33}, c_{44}, and ρ_0 are two stiffnesses of the cracked medium and the inertial density of the isotropic host elastic medium. We assume that the cracks have insufficient volume to affect the overall mass density significantly. In each formula, Thomsen's approximations have included a step that removes the square on the left-hand side of the exact equation, by then expanding a square root on the right hand side. This step introduces a factor of $\frac{1}{2}$ multiplying the $sin^2\theta$ terms on the right hand side, and --- for example --- immediately explains how equation (3) is obtained from the exact result. The other two equations for $v_p(\theta)$ and $v_{sv}(\theta)$, *i.e.*, (1) and (2), involve additional approximations we will not attempt to explain in the space available.

The three Thomsen [3] seismic parameters for weak anisotropy with *VTI* symmetry are $\gamma = \frac{c_{66}-c_{44}}{2c_{44}}$, $\epsilon = \frac{c_{11}-c_{33}}{2c_{33}}$, and δ, which is determined by $2c_{33}(c_{33} - c_{44})\delta = [(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2]$. All three of these parameters can play important roles in the velocities given by (1)-(3) when the crack densities are high enough. If crack densities are very low, then the *SV* shear wave will actually have no dependence on angle of wave propagation. Note that the so-called anellipticity parameter $A = \epsilon - \delta$, vanishes when $\epsilon \equiv \delta$, which *does happen as a rule* for very low crack/fracture densities.

HORIZONTAL FRACTURES, CRACK-INFLUENCE PARAMETERS, AND VTI SYMMETRY

To illustrate the Sayers and Kachanov [2] crack-influence parameter method, consider the situation in which all the cracks in the system have the same vertical (or *z*-)axis of symmetry. (We use 1,2,3 and x, y, z notation interchangeably for the axes.) Then, the cracked/fractured system is not isotropic, and we have the first-order compliance correction matrix for horizontal fractures, which – in Voigt 6x6 matrix notation -- is:

$$\Delta S_{ij}^{(1)} = \rho_c \begin{pmatrix} 0 & 0 & \eta_1 & & & \\ 0 & 0 & \eta_1 & & & \\ \eta_1 & \eta_1 & 2(\eta_1 + \eta_2) & & & \\ & & & 2\eta_2 & & \\ & & & & 2\eta_2 & \\ & & & & & 0 \end{pmatrix}, \tag{4}$$

where i,j = 1,2,3. The two lowest order crack-influence parameters from the Sayers and Kachanov [2] approach are η_1 and η_2. The scalar crack density parameter is defined -- for penny-shaped cracks having number density $n = \frac{N}{V}$ and radius in the plane of the crack equal to a -- to be $\rho_c = 4\pi n a^3/3$. The aspect ratio of these cracks is $\alpha = \frac{b}{a}$. Considering orientational averages of (4) provides a direct connection to the isotropic case, which is of great practical importance, because it permits us to estimate the parameters η_1 and η_2 by studying isotropic cracked/fractured systems, using well-understood effective medium theories [6, 7].

Now consider horizontal fractures, as just illustrated by the correction matrix $\Delta S_{ij}^{(1)}$. The axis of fracture symmetry is uniformly vertical, and so such a reservoir would exhibit *VTI* symmetry. The resulting expressions for the Thomsen parameters in terms of the Sayers and Kachanov parameters η_1 and η_2 are given by

$$\gamma_h = \frac{c_{66}-c_{44}}{2c_{44}} = \rho_c \eta_2 G_0, \tag{5}$$

and

$$\epsilon_h = \frac{c_{11}-c_{33}}{2c_{33}} = \rho_c[(1+\nu_0)\eta_1 + \eta_2]\left(\frac{E_0}{1-\nu_0^2}\right) \simeq \left(\frac{2\rho_c \eta_2 G_0}{1-\nu_0}\right). \tag{6}$$

Background shear modulus is G_0, with corresponding Poisson ratio is ν_0. Young's modulus is $E_0 = 2(1+\nu_0)G_0$. Also $\delta = \epsilon$ to the lowest order in crack density parameters. We chose to neglect the term η_1 in the final expression of (6), as this is on the order of a 1% correction to the term retained. Values of η_1 and η_2 can be determined from simulations and/or effective medium theories [6, 7]. They depend on the elastic constants of the background medium, and on the shape of the cracks (assumed to be penny-shaped in these examples). General formulas are available for computing the coefficients for any model of the crack microstructure, but the most common model is penny-shaped cracks. Formulas defined in terms of the results of particular models or measurements of E and η at finite crack density are:

$$\eta_2 = \frac{1}{2\rho_c}\left[\frac{1+\nu}{E} - \frac{1+\nu_0}{E_0}\right], \quad \eta_1 = -\frac{1}{2\rho_c}\left[\frac{\nu}{E} - \frac{\nu_0}{E_0}\right], \tag{7}$$

as can be found for example in [9].

INCORPORATING FLUID EFFECTS

Recall that Gassmann's formula [10-12] for fluid-substitution in an isotropic porous medium can be written in the form:

$$\frac{1}{K_d} - \frac{1}{K_u} = \frac{\beta}{K_d}[1 + (\varphi K_d/\beta K_f)(1 - K_f/K_m)]^{-1} = \frac{\beta B}{K_d} \tag{8}$$

where K_u is the undrained modulus, K_d is the drained modulus, K_m is the mineral or solid modulus, K_f is the bulk modulus of the pore fluid, φ is the porosity, and $\beta = 1 - K_d/K_m$ is the Biot-Willis or effective-stress coefficient [11]. For an isotropic distribution of cracks, the Sayers-Kachanov correction matrix for compliance is

$$\Delta S_{ij}^{(1)} = \rho_c \begin{pmatrix} (\eta_1+2\eta_2) & \eta_1 & \eta_1 & & & \\ \eta_1 & (\eta_1+2\eta_2) & \eta_1 & & & \\ \eta_1 & \eta_1 & (\eta_1+2\eta_2) & & & \\ & & & \eta_2 & & \\ & & & & \eta_2 & \\ & & & & & \eta_2 \end{pmatrix}, \quad (9)$$

where the influence factors η_1 and η_2 are fixed numbers (having units of compliance, since the crack density is dimensionless) for a particular type of crack shape (say penny-shaped, for example) and isotropic background elastic medium. Then, it is not hard to show that these combinations are also related to Sayers-Kachanov crack-influence parameters according to

$$\frac{1}{K_d} - \frac{1}{K_u} = \frac{\beta B}{K_d} = 2(3\eta_1 + \eta_2)\rho_c B. \quad (10)$$

This equation shows explicitly how the undrained modulus deviates from the drained modulus as the Skempton coefficient B [whose definition is given implicitly by (8)] varies from zero to unity (that is to say, K_u approaches K_m). Another useful form of the equation presenting the same information is

$$\frac{1}{K_u} - \frac{1}{K_m} = 2(3\eta_1 + \eta_2)\rho_c(1-B). \quad (11)$$

This alternative equation shows how the undrained modulus varies for very high pore-fluid bulk modulus K_f, as it approaches the bulk modulus value K_m for the surrounding rock. Equations (8) and (10) show that all appearances of the Sayers-Kachanov parameters in the upper left corner of the matrix in (9) should be modified by multiplying them by the factor $(1-B)$, while the parameters in the lower right corner remain unaffected by the presence of the fluid. This procedure gives a prescription for making the crack-influence parameter approach rigorously consistent with the results of both Biot and Gassmann [10-12].

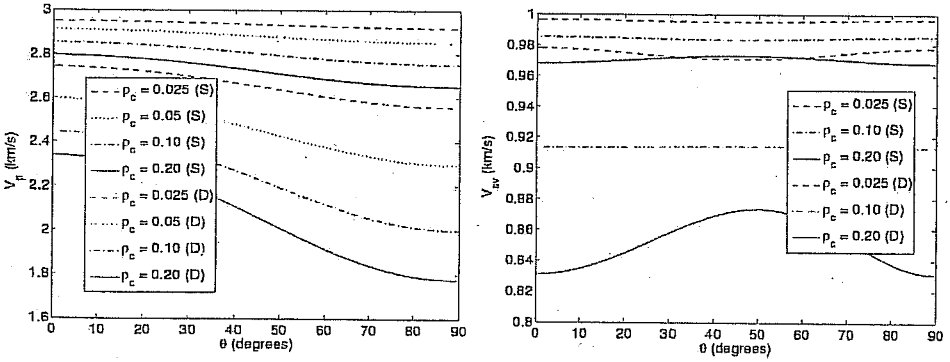

Figure 1: For aligned horizontal cracks, examples of anisotropic compressional (P) wave speed and shear (SV) wave speed for host medium (before addition of cracks) having Poisson's ratio: $\nu_0 = 0.4375$. Velocities near the top are those for liquid saturated cracks (S) for which Skempton's coefficient is B = 0.85; those below having the same type of line marking are for B = 0.00 (D = dry cracks). Calculations are based on the exact VTI formulas for anisotropic velocity (not for the Thomsen weak anisotropy approximation).

EXAMPLES

Figure 1 illustrates the results obtained from the theory described previously. A system of aligned horizontal cracks will display vertical transverse isotropy (VTI), just as in the case of randomly oriented vertical cracks. But the effects of the cracks on the velocities for the different geometries are not the same. Also, because the horizontally polarized shear wave (SH) depends only on the stiffnesses c_{44}, and c_{66}, and furthermore since these coefficients depend on the crack density but not at all on fluid content, we will not consider the SH waves further in our examples. For the quasi-compressional (qP) waves and the quasi-shear (qSV) waves, the behavior does depend the crack density, both when the cracks are dry and when they are liquid saturated. Liquid saturation always stiffens the medium, both in compression and in shear (except for the SH shear waves already excluded from consideration because then there is no mechanical effect, only a uniform increase in density and therefore a uniform, but small, decrease in these velocities). So we expect to see media having dry cracks with lower wave speeds in both quasi-P and quasi-SV waves. This is exactly what can be observed in the two examples in Figure 1. For the dry case, we use B = 0.0, and for the liquid-saturated case we use B = 0.85, for our purposes of illustration. For both types of waves, we see that the effects of the liquid on the wave speeds are substantial. Furthermore, since the SH shear waves do not depend on the fluid content, there will be a very easy to detect difference in the observed behavior due to the shear wave birefringence in these anisotropic cracked media.

To implement the methods described here, we have made use of earlier results of Berryman and Grechka [7] for the values of crack-influence parameters η_1 and η_2 (as well as some higher order corrections), for a background medium having Poisson's ratio equal to 0.4375 (which is reasonably consistent with some types of sandstone reservoirs). The assumed background values of the wave speeds are V_p = 3.0 km/s, V_s = 1.0 km/s. Inertial density is $\rho_0 = 2200.0$ kg/m^3. Results obtained, based on earlier computational work of Grechka, were $\eta_1 = -0.0192$, $\eta_2 = 0.3994$, and the higher order crack-influence parameters needed for crack densities greater than $\rho_c = 0.05$ (dimensionless) were found to be η_3 = -1.3750 and η_5 = 0.5500. All the crack-influence parameters have units of compliance.

CONCLUSIONS

The Sayers and Kachanov [2] crack-influence parameters are ideally suited to analyzing mechanics in reservoirs having aligned fractures and exhibiting *VTI* or *HTI* symmetry. Detailed discussion of results obtained for the higher crack density examples presented in Figure 1 will be provided in the oral presentation, but the main ideas are all contained in references [7] and [8]. Additional modeling [13] shows that Thomsen's weak anisotropy formalism [3] is valid for crack densities up to about $\rho_c \simeq 0.05$, but should be replaced by more exact calculations if the crack density is $\rho_c \simeq 0.1$ or higher.

A very similar analysis can be carried through for fluid-saturated granular media [14], but space limitations preclude us from discussing the details of this work any further here.

ACKNOWLEDGMENTS

The author thanks V. Grechka, M. Kachanov, S. R. Pride, M. Schoenberg, L. Thomsen, and I. Tsvankin for helpful collaborations and conversations. Work performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Berkeley National Laboratory under Contract No. DE-AC02-05CH11231 and supported specifically by the Geosciences Research Program of the DOE, Office of Basic Energy Sciences, Division of Chemical Sciences, Geosciences and Biosciences.

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