LBNL-41236
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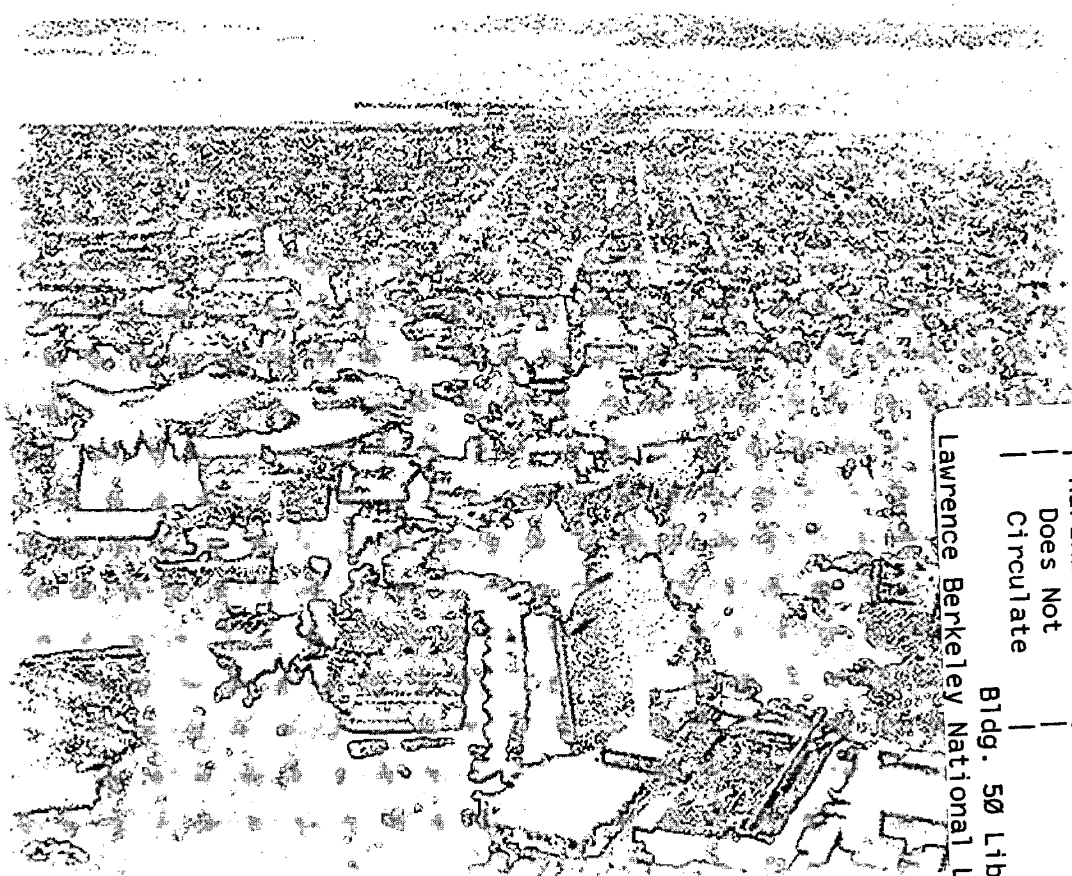
December 1997

Submitted to
Groundwater



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This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

SOMETHING TO THINK ABOUT....DARCY-BUCKINGHAM LAW

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Edgar Buckingham (1867-1940), a physicist, was with the Physical Laboratory of the Bureau of Soils, U.S. Bureau of Agriculture at the turn of the century. His theoretical and experimental studies on the dynamic movement of soil gases and soil moisture led to a major contribution (Studies on the movement of soil moisture, U.S. Department of Agriculture, Bull. No. 38, 61p., 1907) which is part of the foundation of soil physics. Based on the works of Fourier and Ohm, Buckingham rigorously defined the concept of capillary potential and proposed that the steady flux of moisture through an unsaturated soil is directly proportional to the gradient of potential, the constant of proportionality being a property of the soil. The mathematical form of this statement was much the same as that of Darcy's Law, except that the parameter of proportionality was recognized by Buckingham to be a function of capillary potential. In essence, the physics of the unsaturated soil was shown to be much more complicated than that of the saturated sand with which Darcy was concerned half a century earlier. In a single stroke, Buckingham provided a paradigm for soil physics and unified the flow processes in the saturated and the unsaturated zone. Some soil physicists persuasively argue that we should use the phrase "Darcy-Buckingham" Law in place of Darcy's Law.

In modern notation, Darcy-Buckingham Law can be expressed as,

(1) $$Q_x = -K(\psi)\left(\frac{d}{dx}(z+\psi)\right) A(x) \quad ,$$

where Q_x is volumetric flux in the x direction, K is hydraulic conductivity which is a function of gauge pressure head ψ, z is elevation and A is area of cross section which is a function of position. This equation is equally valid for the unsaturated zone ($\psi < 0$) and the saturated zone ($\psi > 0$). We rely heavily on this equation whether we do curiosity-driven research or applied work. It is worthwhile to be thoughtful about the intrinsic nature of this equation.

If we know the groundwater potential at two locations (points) in a saturated medium and the hydraulic conductivity of the material, we think nothing of calculating groundwater velocity ($q = Q/A$) between the points by a simple calculation, $q = -K(\Delta\phi/L)$, where $\Delta\phi$ is the difference in potentiometric head ($\phi = z + \psi$) and L is the distance between the points. In doing so we implicitly assume that the flow is unidimensional (as through a pipe of constant cross section) between the points. A consequence of this assumption is that the gradient of ϕ is constant anywhere between the two points. However, the gradient will vary along x if the pattern involves convergent or divergent flow paths.

Suppose ϕ is known at two points in an unsaturated soil. Here, even if we assume constant cross section, we cannot calculate q as easily. This is so because, in an unsaturated soil the gradient of potential varies between the two points of interest even if the cross sectional area is constant. Therefore, the calculation of q in this case is a much more involved task. For illustration, consider a vertical column of unsaturated soil of uniform cross sectional area, A. By starting with (1) and some mathematical manipulations, one can show that,

(2)
$$z_2 - z_1 = \int_{\psi_2}^{\psi_1} \frac{K(\psi)}{K(\psi) + q} .$$

Because q occurs within the integral, q can only be iteratively evaluated even if the functional form of $K(\psi)$ is known *a priori*. Only in the special case when K is exponentially related to ψ can q be evaluated explicitly. Even in this case, if the cross sectional area is variable, q can be evaluated only iteratively because the integral will take on an even more involved form. Conversely, if we conduct an experiment with a vertical column bounded by ψ_1 and ψ_2 and we measure $q = Q/A$, then we must assume some functional form for the dependence of K on ψ before we can meaningfully interpret the information in terms of hydraulic conductivity. In essence, the relation between flux and potential drop along the flow path in the unsaturated soil is an *implicit* one whereas the relation is *explicit* in the case of a homogeneous saturated soil (Darcy's Law).

The practical implication is that if we measure gauge pressure heads at different locations in an unsaturated soil, we cannot directly calculate flux using (1) because we cannot measure gradients at points. To simplify the situation, even if we assume one dimensional flow between adjoining points, we can only evaluate flux iteratively even if the functional dependence of K on ψ is known.

The task becomes considerably more involved when one has to deal with convergent or divergent flow patterns.

Another important implication of the nature of integral in (2) is that the inverse problem of estimating K(ψ) from experimental data is in general quite difficult. There is one special experimental situation in which K can be unambiguously measured at a specific value of ψ. This pertains to a vertical column of a homogeneous soil in which the gauge pressure is maintained at a given desired value throughout the column. Here, water moves down purely under gravity and one may explicitly apply Darcy's Law to estimate K at the particular value of ψ. This possibility affirms the credibility of Darcy-Buckingham Law. In principle, one could carry out a series of these experiments to establish the K (ψ) relation for a soil. However, such unambiguous evaluation is time-consuming in the laboratory and the method is difficult to apply in the field.

Finally, we note that Darcy-Buckingham Law pertains to the liquid phase in an unsaturated soil and hence, the potential of relevance is the potential in the liquid phase. A physical implication is that at a given magnitude of the gauge pressure head at the outlet, there exist a finite number of *continuous* water pathways between the inlet and the outlet. These pathways will be laterally separated by other air-filled pathways of larger interconnected pores. If the unsaturated zone is thick, then, Buckingham's work requires that we visualize fine, micron-size (or even sub-micron size) pathways which extend continuously over long distances from one end of the column to the other, conducting water.

Darcy-Buckingham Law is the primary foundation which we have to quantitatively understand the moisture migration in unsaturated soils. Extensive experimentation in the field and in the laboratory confirm that this empirical law is valuable in explaining many observations pertaining to moisture migration in soils. Yet, the implicit nature of relation between flux and potential difference, the difficulties inherent in unambiguously measuring the dependence of K on ψ and the channelization of flow along long, slender pathways suggest that significant limitations exist in regard to our ability to precisely quantify the flow of water in unsaturated soils.

Under the circumstances, fundamental questions confront us as earth scientists: how best may we make use of a reasonable mechanistic idea whose applicability to a natural system is constrained severely by limitations of measurements and observation? How should we exercise quantitative judgment when the natural system of interest conforms to physical principles only in a crude way?

This work was supported partly by the Director, Office of Energy Research, Office of Basic Energy Sciences of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 through the Earth Sciences Division of the Ernest Orlando Lawrence Berkeley National Laboratory.

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Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098