Pressure diffusion waves in porous media

Dmitry Silin and Valeri Korneev, Lawrence Berkeley National Laboratory,*
Gennady Goloshubin, University of Houston

Summary

Pressure diffusion wave in porous rocks are under consideration. The pressure diffusion mechanism can provide an explanation of the high attenuation of low-frequency signals in fluid-saturated rocks. Both single and dual porosity models are considered. In either case, the attenuation coefficient is a function of the frequency.

Introduction

Theories describing wave propagation in fluid-bearing porous media are usually derived from Biot's theory of poroelasticity (Biot 1956ab, 1962). However, the observed high attenuation of low-frequency waves (Goloshubin and Korneev, 2000) is not well predicted by this theory.

One of possible reasons for difficulties in detecting Biot waves in real rocks is in the limitations imposed by the assumptions underlying Biot's equations. Biot (1956ab, 1962) derived his main equations characterizing the mechanical motion of elastic porous fluid-saturated rock from the Hamiltonian Principle of Least Action. However, using the Hamiltonian Principle for describing fluid flow in porous media imposes certain restrictions. These restrictions are related to the nature of Darcy's law. Darcy fluid velocity or, equivalently, superficial velocity is defined as the fluid flux through an elementary surface in the bulk volume. In real rocks, the fluid flows through an extremely complex system of numerous geometrically irregular pore channels with different orientations and cross-sectional areas, see Figure 1. In addition, the pore walls are rough surfaces. Due to this complex geometry of the pore space, individual fluid particles accelerate and slow down all the time and the dispersion of the Lagrangian velocities between can be very large. King Hubbert (1956) stated: "… deductions concerning Darcy-type flow made from the simpler Poiseuille flow are likely to be seriously misleading". At the same time, the equations describing elastic displacement of the solid skeleton are formulated at a microscopic scale, where characteristic length is comparable with the size of an individual grain and, therefore, that of an individual pore. In case of a parallel bundle of capillary tubes or other simplified model of porous medium, the velocity dispersion is smaller than in natural rocks. Therefore this scale mismatch may be not as well pronounced.

To narrow the above-mentioned gap between the length scales of Darcy's law and elastic equations, we propose to use the model of slightly compressible fluid flow in an elastic porous medium. Such a model results in a parabolic pressure diffusion equation. Its validity has been confirmed and "canonized", for instance, in transient pressure well test analysis, where it is used as the main tool since 1930th, see e.g. Earlougher (1977) and Barenblatt *et. al.*, (1990). The basic assumptions of this model make it applicable specifically in the low-frequency range of pressure fluctuations.

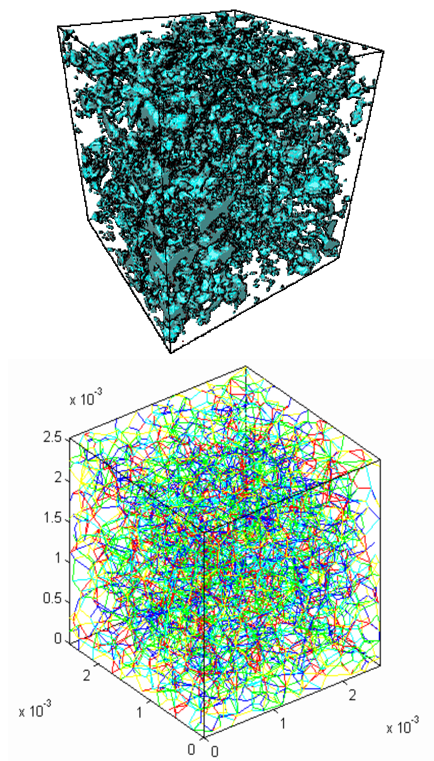

Figure 1: A 5 μm resolution image of the pore space of 2.5×2.5×2.5 mm^3 piece of Fontaineblaux sandstone and schematic representation of the connections between the pores (courtesy of Statoil).

In the next sections, we discuss the harmonic wave solutions to pressure diffusion equation in the contexts of single and dual porosity models. Since diffusion and heat conductance processes are usually described by parabolic equations, such waves are called diffusion or thermal waves (Tikhonov and Samarskii, 1963; Mandelis, 2001). As early as in 1851, Stokes (1851) used diffusion waves mechanism for measuring fluid shear viscosity.

Pressure diffusion waves

Diffusion waves have high frequency-dependent attenuation and slow velocity of propagation. To illustrate this fact, we consider two prototype problems: pressure diffusion waves in a classical porous medium and pressure diffusion waves in a dual-porosity medium.

Pressure diffusion waves in a classical porous medium

The pressure diffusion equation in a homogeneous porous medium characterized by porosity ϕ and permeability κ, has the following form:

$$\beta\phi \frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{\kappa}{\eta} \nabla p \right) \quad (1)$$

Here p and η are the pressure and viscosity of the fluid and β is the compressibility coefficient characterizing both the fluid and the solid skeleton simultaneously. Detailed derivation of Equation (1) is presented, *e.g.*, in Barenblatt *et. al.* (1990).

It is worthwhile to notice that the compressibility of the skeleton in Eq. (1) is expressed through the variation of porosity caused by variation of the fluid pressure. The definition of porosity requires an elementary volume much larger than that of an individual pore or grain. Darcy's velocity definition requires an elementary surface whose linear dimensions also are much larger than and individual pore or grain. Therefore, unlike in the classical Biot's theory, the elastic and hydraulic components of Equation (1) are defined in the same length scale.

If we consider a situation where the pressure depends only on one coordinate, say, if the pressure variations are due to an excitation source in a fracture or at the top of the reservoir, then Equation (1) is simplified:

$$\beta\phi \frac{\partial p}{\partial t} = \frac{\kappa}{\eta} \frac{\partial^2 p}{\partial x^2} \quad (2)$$

The latter equations admits a harmonic wave solution

$$p\left(t, z\right) = p_0 \exp(-\alpha x) \exp\left(ikx - i\omega t\right) \quad (3)$$

with

$$k = \alpha = \sqrt{\frac{\omega\phi\eta\beta}{2\kappa}} \quad (4)$$

The wavelength λ and velocity v are equal, respectively, to

$$\lambda = 2\pi \sqrt{\frac{2\kappa}{\omega\phi\eta\beta}} \quad \text{and} \quad v = \sqrt{\frac{2\kappa\omega}{\phi\eta\beta}} \quad (5)$$

For example, if we assume porosity of 20%, permeability of 500 mDarcy, fluid viscosity of 1 cp and compressibility coefficient of the order of 10^{-9} Pa^{-1}, then at a frequency of 20 Hz we obtain the wavelength approximately equal to 2 m and the phase velocity of 18 m/s. Such a small wavelength makes possible imaging of thin fluid-saturated permeable layers. Note that both the wavelength and the velocity of the elastic incident wave are significantly larger.

The high attenuation of pressure diffusion waves is a consequence of energy losses in the fluid flow in the pores. By virtue of equations (5), the quality factor does not depend on the frequency and is equal to $Q = 0.5$. It is also worthwhile to remark that if the permeability of the rock is determined by a system of fractures but the matrix is practically impermeable, then the pressure diffusion equation will have the same form (1), but with an anisotropic permeability tensor (Romm, 1966). However, in many cases the equation can be reduced to one-dimensional form (2) and the analysis presented above can be applied. Normally, a system of fractures has a very small volume but a relatively high permeability. Therefore, for a fractured rock, the ratio κ / ϕ defining both the wavelength and the velocity is much larger than for a "classical" porous medium. At the same time, the attenuation coefficient in a fractured medium is smaller than in matrix.

Pressure diffusion wave in a dual-porosity medium

The important role of the fractures in fluid transport in natural rocks is commonly recognized (Barenblatt *et. al.*, 1960; Romm, 1966). The system of fractures carries most of the fluid flow, whereas the matrix provides the storage for the fluid.

A theory of fluid flow accounting for the interaction between the fractures and the matrix blocks was proposed by Barenblatt *et. al.* (1960), see also Barenblatt *et. al.* (1990). The basic assumption of this theory is that at each point, the medium can be considered as a tangled combination of two media: the classical porous matrix and the one where the permeability is exclusively due to the system of fractures. For the sake of simplicity, we assume that both media are isotropic so that the permeability tensor is scalar. A representative volume of such media is much larger than the size of the individual porous blocks, but it is much smaller than the wavelength and the size of the reservoir under consideration. Pressure diffusion equation of the flow of slightly compressible fluid in a dual-porosity medium has the following form:

Pressure diffusion waves

$$\frac{\partial p_f}{\partial t} - \frac{\partial}{\partial t} \nabla \cdot \left(A \nabla p_f \right) = \nabla \cdot \left(B \nabla p_f \right) \quad (6)$$

where

$$A = \frac{L^2 \kappa_f}{a \kappa_m} \; \frac{\beta + \beta_{mm}}{\beta + \beta_{mm} - \beta_{mf}} \quad (7)$$

and

$$B = \frac{\kappa_f}{\phi_m \mu \left(\beta + \beta_{mm} - \beta_{mf} \right)} \quad (8)$$

A subscript f denotes the quantities related to the fractures and a subscript m stands for parameters related to the matrix. The compressibility coefficients have the following sense. The compressibility coefficients β_{mm} and β_{fm} characterize the increment of the matrix porosity due to the increment of the fluid pressure in the matrix blocks and in the surrounding fractures:

$$\frac{d\phi_m}{\phi_m} = -\beta_{mf} dp_f + \beta_{mm} dp_m \quad (9)$$

and the coefficient β is the compressibility of the fluid. The characteristic length L and the dimensionless factor a characterize the size and the geometry of the porous blocks and define the rate of the cross-flow between the matrix and the fractures at given p_f and p_m. Equation (6) is written down for the fracture pressures p_f. A similar equation can be obtained for the matrix pressure p_m and for the difference $p_m - p_f$.

A harmonic wave solution (3) to a one-dimensional version of Equation (6) is defined by the following frequency-dependent wave number and attenuation coefficient:

$$k = \sqrt{\frac{A\omega^2 + \omega\sqrt{A^2\omega^2 + B^2}}{2\left(A^2\omega^2 + B^2 \right)}} \quad (10)$$

$$\alpha = \sqrt{\frac{-A\omega^2 + \omega\sqrt{A^2\omega^2 + B^2}}{2\left(A^2\omega^2 + B^2 \right)}} \quad (11)$$

In this case, the quality factor also is a function of frequency. For inverse quality factor, one obtains

$$\frac{1}{Q} = 2\sqrt{1 + \left(\frac{A}{B}\omega \right)^2} - 2\frac{A}{B}\omega \quad (12)$$

At the limit $\omega \to 0$, the value of the quality factor converges to one half, i.e., to the value obtained earlier for a single-porosity medium. The plot of inverse quality factor versus dimensionless frequency

$$\Xi = \frac{L^2 \phi_m \mu \left(\beta + \beta_{mm} \right)}{\alpha \kappa_m} \omega \quad (13)$$

in shown in Figure 2. Note a remarkable similarity with the plot of inverse quality factor versus frequency for the second compressional wave in Berea sandstone, Fig. 4 in Berryman and Wang (2000).

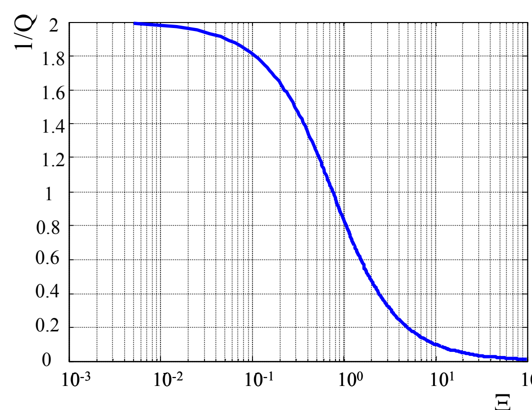

Figure 2: Inverse quality factor $1/Q$ versus dimensionless frequency Ξ: the asymptotic value at $\omega \to 0$ is equal to 2, regardless of the values of other parameters.

The attenuation attains its maximum at the frequency

$$\omega_{\max} = \frac{a\kappa_m}{\sqrt{3}L^2 \; \phi_m \eta \left(\beta + \beta_{mm} \right)} \quad (14)$$

The maximal attenuation coefficient itself is represented by the equation

$$\alpha_{\max} = \frac{\sqrt{\frac{a}{2}\frac{\kappa_m}{\kappa_f}\left(1 - \frac{\beta_{mf}}{\beta + \beta_{mm}} \right)}}{2L} \quad (15)$$

and the corresponding wave number is given by

$$k_{\max} = \frac{\sqrt{\frac{3a}{2}\frac{\kappa_m}{\kappa_f}\left(1-\frac{\beta_{mf}}{\beta+\beta_{mm}}\right)}}{2L} \qquad (16)$$

Thus, the respective quality factor is given by $Q = \sqrt{3}/2$, that does not depend on the properties of the medium and the fluid. For the wavelength and velocity, on obtains

$$\lambda_{\max} = \frac{4\pi L}{\sqrt{\frac{3a}{2}\frac{\kappa_m}{\kappa_f}\left(1-\frac{\beta_{mf}}{\beta+\beta_{mm}}\right)}} \qquad (17)$$

$$v_{\max} = \frac{2\sqrt{2}}{3}\frac{\sqrt{a\kappa_f\kappa_m}}{L\,\phi_m\eta\left(\beta+\beta_{mm}\right)\sqrt{1-\frac{\beta_{mf}}{\beta+\beta_{mm}}}} \qquad (18)$$

For example, if we assume fractures permeability $\kappa_f = 5$ Darcy, $L = 0.1$ m, $a = 1$, and take the matrix parameters from the previous section, at a frequency of 10 Hz we obtain $\lambda_{\max} \sim 15$ m, $\alpha_{\max} \sim 0.25$ m^{-1} and $v_{\max} \sim 33$ m/s.

Conclusions

The mechanism of pressure diffusion waves in fluid-saturated porous rocks has been investigated. This mechanism may play an important role in attenuation of low-frequency waves. Two prototype examples of diffusion wave model have been considered: elastic fluid flow in single and dual porosity media. Estimates show that the diffusion waves have relatively slow velocities and short wavelength. This circumstance partially explains the high-resolution seismic images of thin fluid-bearing layers observed in the field (Goloshubin and Korneev, 2000).

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