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G. Lambertson

April 1990

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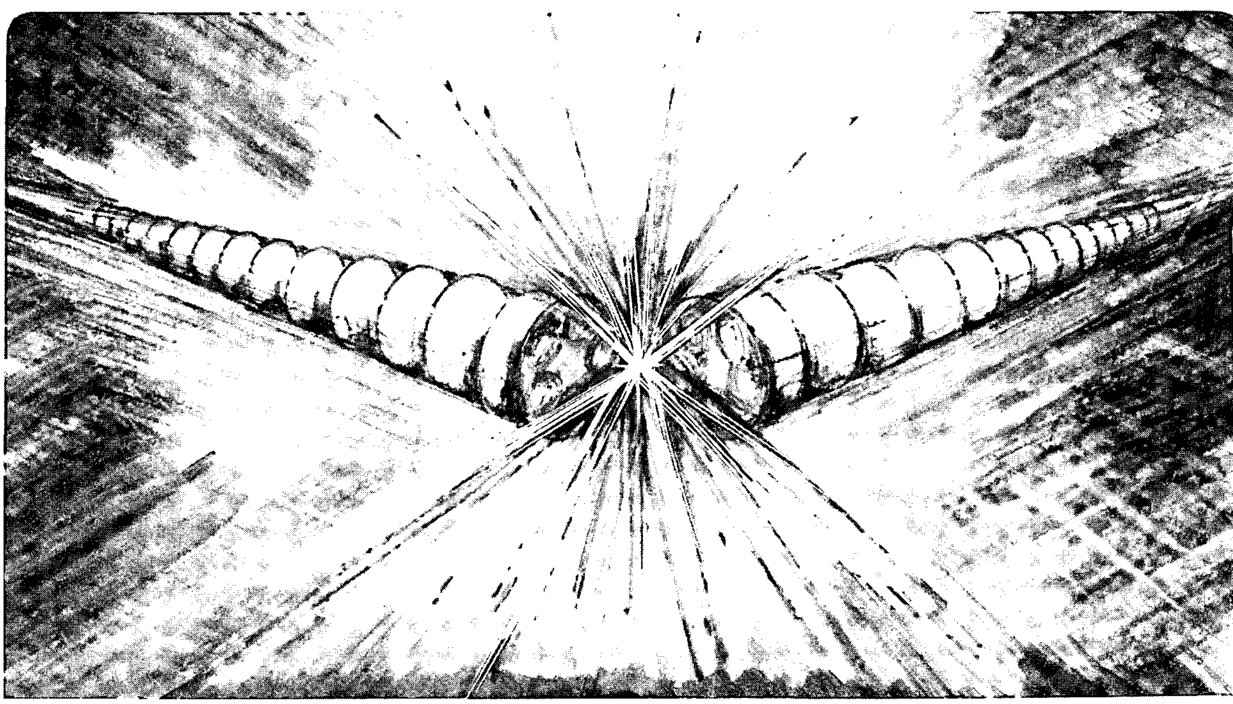

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BECON-80

BEAM IMPEDANCE OF A SPLIT CYLINDER

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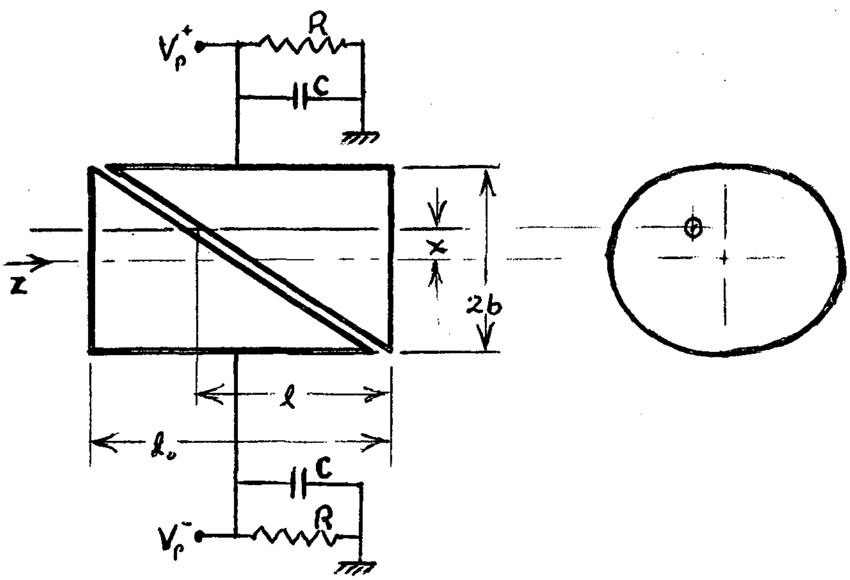
Beam Impedance of a Split Cylinder

A common geometry for position electrodes at moderately low frequencies is the capacitive pickup consisting of a diagonally-divided cylinder that encloses the beam trajectory.



For the simplified system sketched here, a relatively direct approach will give the longitudinal and transverse beam impedances ($Z_{||}$ and $Z_{\perp}$) at low frequencies.

We recall the definitions of the impedances:

$$Z_{||} = \frac{-\Delta E}{eI} = \frac{-V_{||}}{I} \text{ with } V_{||} = \int_z E_z e^{j\omega z/\beta c}\, dz \qquad (1)$$

$$Z_\perp = j\frac{1}{I}\frac{d}{dx}\left(\frac{\Delta p_\perp c}{e}\right) = j\frac{1}{\beta I x}V_\perp(x) \tag{2}$$

$$\text{with } V_\perp(x) = \int_z (E+vxB)_\perp e^{j\omega z/\beta c}\,dz \quad \text{(at position x)} \tag{3}$$

Here ΔE and $\Delta p_\perp$ are the perturbations in the energy or momentum of a single charge caused by passage through the electrode when it is excited by

$$(I \text{ or } I_x)\, e^{j(\omega t-\beta cz)}.$$

Each half of the electrode pair is driven by the induced current, which is given by the rate of change of the induced charge. The voltage developed in each half is thus

$$V_p = jI\frac{\omega \ell}{\beta c}\frac{R_L}{1+j\omega CR_L} \tag{4}$$

$$\text{with } \ell = \frac{\ell_o}{2}\left(1 \pm \frac{x}{b}\right).$$

To find $V_{||}$ when x = 0, note that axial fields E_z are concentrated at the ends of the electrode which the particle encounters at $Z = \mp\frac{\ell_o}{2}$. The application of Eq. 1 gives, for $\ell_o << 2\beta$ ƛ.

$$V_\parallel = V_p 2j\sin(\omega \ell_o/2\beta c) \approx V_p j\frac{\omega \ell_o}{\beta c}$$

$$V_\parallel = -I\frac{1}{2}\left(\frac{\omega \ell_o}{\beta c}\right)^2 \frac{R_L}{1+j\omega C R_L} \tag{5}$$

and we find

$$Z_\parallel = \frac{1}{2}\left(\frac{\omega \ell_o}{\beta c}\right)^2 \frac{R_L}{1+j\omega C R_L} \tag{6}$$

For the estimation of $Z_\perp$, we assume that the transverse electric field averaged over length ℓ_o is:

$$\overline{E_\parallel} = \frac{-1}{2b}\left(V_p(x) - V_p(-x)\right)$$

$$= -j\frac{x}{2b^2}\, I\frac{\omega \ell_0^2}{\beta c}\,\frac{R_L}{1+j\omega C R_L}\ . \tag{7}$$

Insert $V_\perp = \overline{E_\perp}\ell_o$ in Eq. 2 to obtain for the x-direction.

$$Z_\perp = \frac{1}{2b^2}\,\frac{\omega \ell_o^2}{\beta^2 c}\,\frac{R_L}{1+j\omega C R_L}\ . \tag{8}$$

The transverse impedance is often related to $Z_\parallel/n$, where $n = R\omega/c$ at orbit radius R. In these terms, the $Z_\perp$ found above becomes

$$Z_\perp = \frac{R}{b^2}\,\frac{Z_\parallel}{n}\ . \tag{9}$$

As an alternative to estimating the transverse field to find $Z_\perp$, one may calculate $V_{||}$ as a function of x and find $\Delta p_\perp$ using the Panofsky-Wenzel theorem which states

$$\frac{\partial p_\perp}{\partial t} = -\frac{\partial E}{\partial x} \quad \text{or}$$

$$j\omega \Delta p_\perp = -\frac{\partial E}{\partial x} \tag{10}$$

In using this theorem it is important to note that we have a partial derivative of the energy gain E, with the current held at position x. In contrast, the derivative in Eq. 2 is the total derivative that gives the effect of a change in the position of the beam current.

For this purpose, $\Delta E(x)$ is found as the sum of the effects of the separate halves at their appropriate voltages $V_p(\pm x)$ and lengths $\frac{\ell_o}{2}\left(1 \pm \frac{x}{b}\right)$.

$$\frac{\Delta E}{e} = V_{||}(x) = -V_p(x)\, j\frac{\omega \ell_o}{2\beta c}\left(1+\frac{x}{b}\right) - V_p(-x) j\frac{\omega \ell_o}{2\beta c}\left(1-\frac{x}{b}\right) \tag{11}$$

To apply Eq. 10, we note that the spatial variation of ΔE is contained in $V_p(\pm x)$ only. Hence, for $\partial E/\partial x$ we differentiate only Vp.

$$\frac{\partial V_p(\pm x)}{\partial x} = \pm \frac{1}{b} V_p(o) \tag{12}$$

to find

$$\frac{\Delta p_\perp c}{e} = j\frac{c}{w}\frac{\partial V_\parallel}{\partial x} = \frac{\ell_o}{2\beta b}\left[\left(1+\frac{x}{b}\right)V_p(o)-\left(1-\frac{x}{b}\right)V_p(o)\right] \tag{13}$$

$$\frac{\Delta p_\perp c}{c} = jI\frac{x}{2b^2}\frac{\omega \ell_o^2}{\beta^2 c}\frac{R_L}{1+j\omega CR_L}$$

Use this in Eq. 2 to obtain the result found before in Eq. 8.

Example, the BNL AGS Booster pickups:

24 radial, 24 vertical sets

$R_L = 300\ \Omega$	$b = 0.0765$ m
$C = 50$ pF	$\beta = 0.55$ at injection
$\ell_o = 0.2$ m	$R = 32.1$ m

Calculate $Z_\parallel/n$ and $Z_\perp$ with $n = \omega/\omega_\infty$.

$$\omega_\infty = \frac{c}{R} = 2\pi\ 1.49 \text{ MHz.}$$

$$\frac{Z_\parallel}{n} = n\frac{R_L}{2}\left(\frac{\ell_o}{\beta R}\right)^2 \frac{1}{1+j(ncCR_L/R}$$

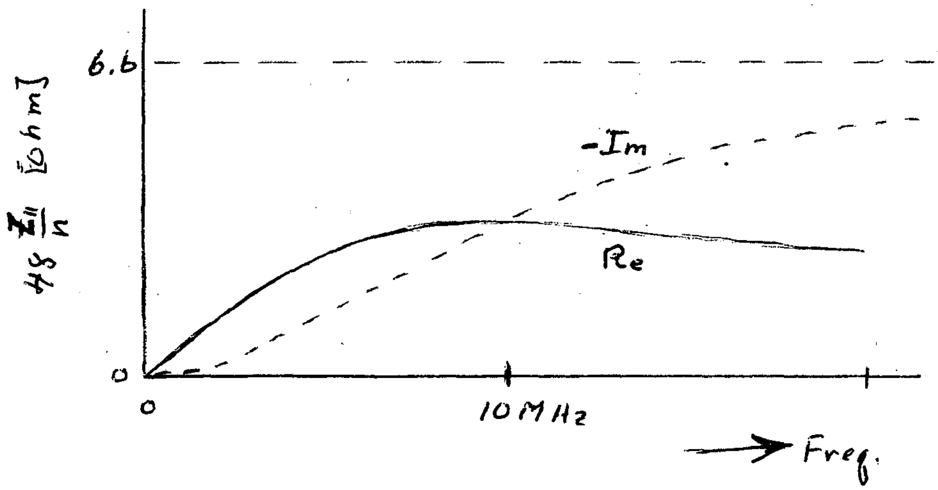
$$\frac{Z_\parallel}{n} = 0.1375\frac{0.14\ n}{1+j\ 0.14\ n} \text{ for one set}$$

For 48 sets,

$$48\frac{Z_\parallel}{n} = 6.6\frac{0.14\ n}{1+j\ 0.14\ n} \text{ ohm.}$$

In each transverse direction there are 24 sets, giving

$$Z_{\parallel} = 24\,\frac{R}{b^2}\,\frac{Z_{\parallel}}{n} = 1.81 \times 10^4\,\frac{0.14\,n}{1 + j\,0.14\,n}\ \text{ohm/m.}$$



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