Title

Preliminary Analysis and Simulation Results of Microwave Transmission Through an Electron Cloud

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Preliminary analysis and simulation results of microwave transmission through an electron cloud [*]

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Abstract

The electromagnetic particle-in-cell (PIC) code VORPAL is being used to simulate the interaction of microwave radiation through an electron cloud. The results so far show good agreement with theory for simple cases. The study has been motivated by previous experimental work on this problem at the CERN SPS [1], experiments at the PEP-II Low Energy Ring (LER) at SLAC [4], and proposed experiments at the Fermilab Main Injector (MI). With experimental observation of quantities such as amplitude, phase and spectrum of the output microwave radiation and with support from simulations for different cloud densities and applied magnetic fields, this technique can prove to be a useful probe for assessing the presence as well as the density of electron clouds.

INTRODUCTION

This paper shows some preliminary results of work in progress for a planned exhaustive study on the physical processes involved with the transmission of microwaves through a beam channel containing electron clouds. Successful experimental observation of the physical phenomena would serve as a very useful probe for detecting electron clouds and their properties at different locations of a beam channel. Given the complexity of the problem, and in order to be able to correlate the properties of the electron distribution with the expected experimental observations, a good understanding of the problem through simulations and theoretical analysis is necessary.

Experimental results conducted at the SPS have been reported [1], and some preliminary experimental investigation has also been conducted at the PEP II low energy ring [4]. Further experiments need to be conducted in order to measure an expected "phase shift" caused by the electron clouds on the microwave after it has propagated through the beam channel. At the same time more features need to be added into the simulation for a better representation of the experimental conditions. While it is clear that more progress is required in the experimental and simulation efforts, we can already report that results obtained so far at PEP II do not violate any theoretical result that is based on the analysis in this paper. Further, simulation results presented here, which are done for idealized conditions show good agreement with analytic results.

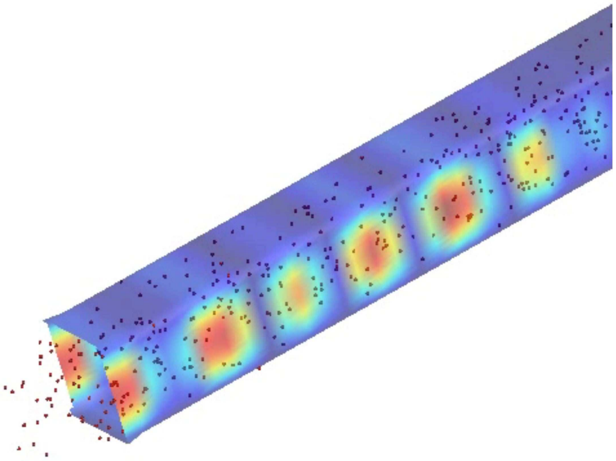

Figure 1: A schematic of the simulation process. The microwave source is placed a small length away from the left boundary.

To simulate the physical process, we use the electromagnetic particle-in-cell code VORPAL [2]. For the sake of simplicity, and as an initial part of the study, we use a channel with a square cross-section. The transverse boundaries are conducting and boundaries along the longitudinal direction consist of phase matched layers (PML) which cause the wave to decay and be absorbed, thus preventing any reflection in that direction. Figure 1 is a snapshot of the simulation process.

DISCUSSION OF THE DISPERSION RELATION AND THE RESULTING PHASE SHIFT

The method for deriving the wave dispersion relationship has been described in Ref. [3]. The derivation involves a perturbation about an equilibrium configuration of the Maxwell and fluid equations and where all higher order perturbation terms are neglected. As a result, the electron equation of motion is linear. In this model we do not expect any attenuation in the amplitude of the wave and the

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frequency spectrum remains invariant.

For a cold, homogeneous electron distribution that completely fills the beam channel and with no external magnetic field, the wave dispersion relationship for a TE mode is relatively simple and given by

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2} - \frac{\omega_c^2}{c^2} \tag{1}$$

Over here, k is the wave number, c the speed of light, ω is the wave frequency, ω_p is the plasma frequency and ω_c is the cutoff frequency of the waveguide. The plasma frequency ω_p is given by

$$\omega_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \tag{2}$$

where n_e is the electron number density, e is the electron charge, ϵ_0 is the free space permittivity and m_e is the electron mass.

If we assume the wave to propagate over a distance ΔL, then the phase advance of the wave is given by $k\Delta L$. It is clear from Eq. 1 that the phase advance of the wave depends upon the plasma frequency, a property of the electron distribution. If we linearize Eq. 1 about small ω_p and compute the difference between the phase advance with and without electrons, respectively, over a length ΔL, we obtain

$$\frac{\Delta\Phi}{\Delta L} = \frac{\omega_p^2}{2c(\omega^2 - \omega_c^2)^{1/2}} \tag{3}$$

This is the so called “phase shift” per unit length.

Figure 2 shows plots of the phase shift per unit length against frequency for different electron densities. The electron densities used here are $n_e = 1 \times 10^{12}$, 2×10^{12}, 4×10^{12} and 7×10^{12} and the corresponding plasma frequencies are 8.9, 12.7, 18.0 and 23.8 MHz respectively. The cutoff frequency of the waveguide is 2 GHz. It should be noted that Eq. 1 is linearized about small ω_p, so one must take higher order terms when ω approaches a value very close to ω_c. The plots give an indication of what an optimum value of the microwave frequency should be in order to determine the electron density. Higher frequencies lead to a lower variation in phase shift with density. At the same time, one cannot approach very close to the cutoff while performing the experiment and yet expect the signal to be detectable. In this case, we expect that a microwave frequency of 3 GHz is reasonable.

The analytic results in this section provide a good understanding of the problem, but one must take into account that they are valid under certain idealized conditions. For example, the formulation above assumes a cold, homogeneous electron distribution with no external magnetic field. Further, the electron equation of motion is linearized in a quasi-equilibrium regime. One must also take into account the periodic passage of the beam and a spatial and temporal variation in the electron density. In particle-in-cell simulation, these approximations may be relaxed. In the next section we shall show through simulations that real accelerator conditions fall into a regime where many of the above assumptions hold, and so the analytic result is a good starting point in understanding the problem.

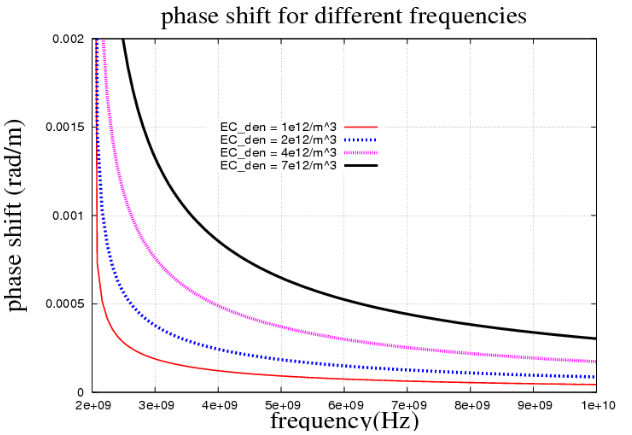

Figure 2: Plots of phase shift per unit length for different electron densities with $\omega_c = 2$ GHz.

Table 1: Simulation parameters

dimensions, $x \times y \times z$	$7.5 \times 7.5 \times 50$ cm
no. of grid cells, $x \times y \times z$	$16 \times 16 \times 32$
time of propagation	7×10^{-8} s
number of time steps	7560
TE$_{01}$ mode cutoff frequency	2 GHz
microwave frequency	3 GHz

SIMULATION RESULTS AND CALCULATION OF PHASE SHIFT

In this section, we present the simulation results obtained from the electromagnetic particle-in-cell code VORPAL. The system consists of a channel with a square cross section. The electron distribution fills up the whole channel and is initially uniform and cold. Other parameters used in the simulation are given in Table 1.

Figure 3 shows that there is no attenuation in the amplitude of the wave that was propagated. Although the length of propagation in the simulation (0.5 m) is much less than what it would be in an experiment (20–30 m), a careful analysis of the data still shows that one would expect no attenuation even if the simulation was carried out for a more realistic length. The electron density was $n_e = 2 \times 10^{12}$ m^{-3}. This is higher that what one would expect in a real accelerator. Thus, we see that the linear approximation, which become more valid with decreasing electron densities, still holds.

Figure 4 shows the logarithm of the frequency spectrum at the beginning, middle and end of the simulation length. It is clear that there is no shift or spread in the wave frequency due to the electrons. The electron density was still

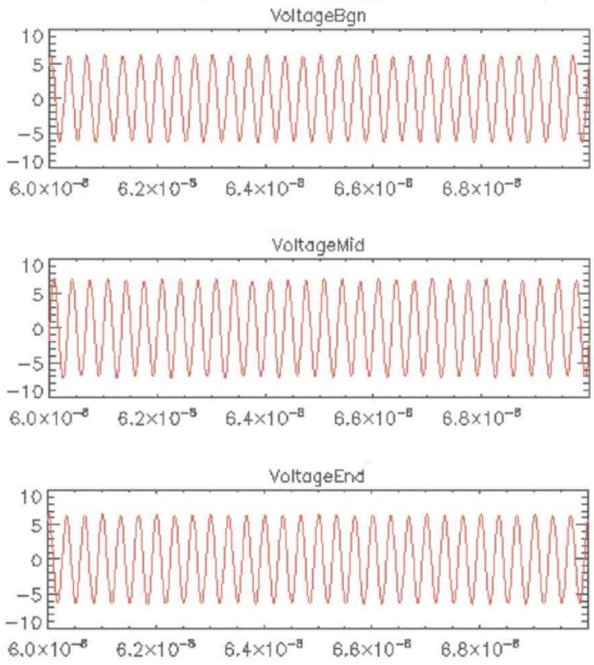

Figure 3: Wave amplitude vs. time at the beginning, middle and end of simulation length.

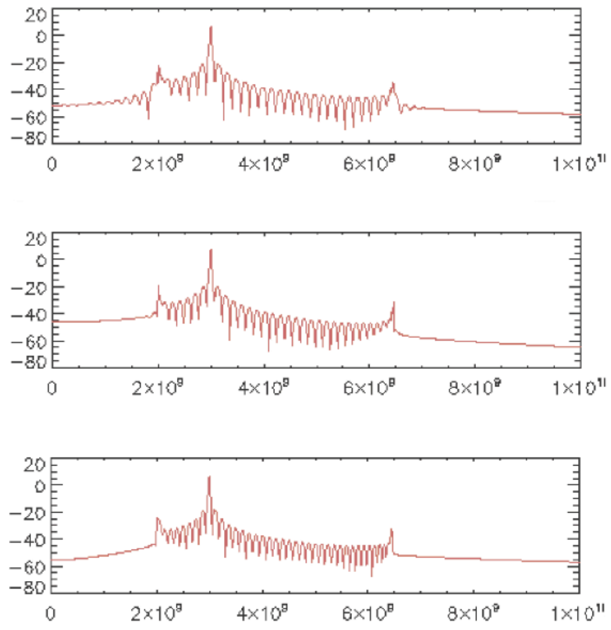

Figure 4: Logarithm of frequency spectrum at the beginning, middle and end of simulation length.

$n_e = 2 \times 10^{12}$ m^{-3}. The absence of any amplitude attenuation or frequency broadening due to the electrons are consistent with observations from experiments carried out at the PEP II LER. These observations are to be finalized and reported in a future publication [5].

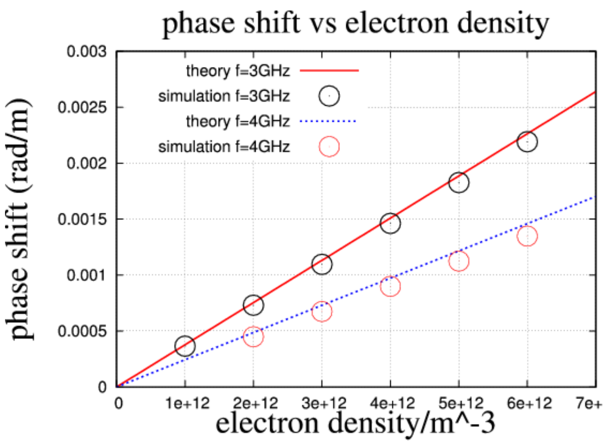

Figure 5: A comparison of phase shift obtained from computation and analytically.

We now proceed to show that despite the absence of any change in amplitude and frequency, there is an observable phase shift that varies with electron density. In order to compute the phase shift in the simulations, the wave is propagated through the channel in the absence of any electrons. This is then repeated for a known density of electrons. The oscillation of the electric field at the end of the channel is recorded for both cases and their amplitudes normalized. The difference of the two normalized oscillations with differing initial phases, ϕ_1 and ϕ_2 respectively, gives us

$$\begin{aligned}
&\sin(\omega t + \phi_1) - \sin(\omega t + \phi_2) \\
&\quad = 2 \sin\left[\frac{\phi_1 - \phi_2}{2}\right] \cos\left[\omega t + \frac{\phi_1 - \phi_2}{2}\right] \\
&\quad \simeq (\phi_1 - \phi_2) \cos\left[\omega t + \frac{\phi_1 - \phi_2}{2}\right] \qquad (4)
\end{aligned}$$

For very small values of phase shift, which is indeed the case here, we obtain a difference wave with an amplitude equal to the phase shift.

Figure 5 shows the theoretical as well as computed phase shifts for two values of frequencies. It may be noted that the approximations made in deriving the analytic result become more valid with decreasing electron density. Most values of electron densities used in the computation are well above those occurring in real machines. Despite this, the computational results compare very well with the analytic ones and also show that the two results get closer with decreased electron density. The results are shown for two values of microwave frequencies and the cutoff frequency was 2 GHz for both the cases.

SUMMARY

We have been able to perform simulations of the propagation of microwaves through electron clouds. A formulation that predicts the phase shift due to the electrons confined within the finite boundaries of a beam pipe has been presented. Calculations have shown that the simulation and analytic results agree very well with each other. These calculations included a regime that exceeded realistic values of electron cloud densities, thus showing that real machine conditions fall when within the domain in which linear theory is valid. This is an update on work in progress and we intent to include more physical features into the simulation and the analytic models in the future. Notable among them would be the effect of the periodic passage of a beam. Given the relativistic mass of the beam particles, it is clear that the beam would not contribute to the dispersion relation in the form of a plasma. However, the beam provides a time varying electric field and the extent of such an effect on the dispersion relation still needs to be estimated. The geometry of the cross section will become important in the presence of a beam and so we need to move to more realistic geometries. The influence of external magnetic fields also needs to be investigated. A dipole magnetic field provides an anisotropic medium. This would lead to differing dispersion relations in the horizontal and vertical components of the wave. As a result of this, one could in principle detect the properties of electron clouds within dipoles by studying the rotation of the plane of polarization of the microwave caused by this anisotropic medium. It should also be noted that electron cloud densities will have a spatial as well as a temporal variation and the effect of including this into the calculation needs to be studied. The PIC code VORPAL is well equipped to include all the above mentioned modifications. By conducting simulations that more closely represent parameters specific to existing machines we intend to provide a guidance to the experimental efforts in the study of this proposed electron cloud detection method.

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