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MODE FLATTENING IN LONG CAVITIES

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Authors

Birge, Robert W.
Ehlers, Kenneth
Carothers, James.

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MODE FLATTENING IN LONG CAVITIES

Robert W. Birge, Kenneth Ehlers, and James Carothers

September 27, 1951

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MODE FLATTENING IN LONG CAVITIES

Robert W. Birge, Kenneth Ehlers, and James Carothers

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

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A. INTRODUCTION AND THEORY

The operation of a long linear accelerator presents two problems in mode excitation. First, the desired mode must be established or excited in the cavity. Second, after the establishment of the proper mode, the electric and magnetic field distributions must be brought to, and maintained at their desired configurations.

The prospect of the construction of a very large and electrically long accelerator has made it advisable to investigate the problems set forth above.

Theories dealing with the two problems have been developed. The problem of exciting the zeroeth transverse magnetic mode has been treated by Luis W. Alvarez (Engineering Notes - file M-24). This mode (TM_{010}) is the operational mode of a long proton linear accelerator. In this mode, the axial electric field is in phase at all points along the length of the cavity, and the magnetic field has a transverse component only (Fig. 1). The theory treats the case of a cylindrical cavity of slowly varying radius. This simple case is then extended to that of an accelerator whose "cells" vary slightly in natural frequency as a function of axial position. Briefly, the Alvarez theory is that it will always be possible to set up some TM mode in a cavity. Inability to construct a large machine to minute tolerances will probably mean that the mode attainable will be a distorted, higher order mode. Measurement of the field along the length of the cavity will show the number and

position of the nulls, and the cavity distortion will then be shown by the
fact that these nulls are not located at the theoretically correct positions.
Proper tuning of the cavity will move the null points to their correct positions
and when this tuning has been completed, the next lower mode can be excited.
This procedure is repeated until the first mode can be excited. After this
null has been correctly positioned the zeroeth order TM mode can be obtained.

It is not possible to assume that a large machine can be constructed with
the stringent tolerances required to allow it to operate in the zeroeth mode
without some form of tuning. The UCRL 40 foot linear accelerator, for example,
was brought into the proper mode by inserting diaphragms into the cavity, and
then tuning the individual cells to the desired frequency. On a machine of
much larger dimensions, the diaphragm method could prove extremely cumbersome
and costly. The system of tuning or "shifting nulls" to the proper position
would be much more convenient and economical should this method be possible.

The relation, given by the theory, existing between the fractional
radius errors of the cavity wall and the null displacement is:

$$\frac{\Delta a}{a} = - \frac{K^2}{2N^2} \frac{\Delta z}{L} \tag{1}$$

where ΔZ is the null displacement
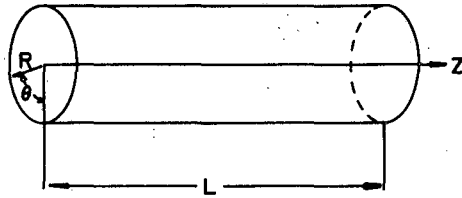
L is the length of the cavity

N is the electrical length of the cavity or L/λ air ($\lambda = 2.61$ a

where a is the radius of the unloaded cavity or, in the
loaded case $\lambda = C/F$

K is the number of the mode being considered.

With this relation, the frequency corrections needed to properly place the
null can be calculated.

CYLINDRICAL COORDINATES ARE θ, R, & Z

ZEROTH (TM_{010}) MODE

Ist (TM_{011}) MODE

+ ·   LINES OF H

—   LINES OF E

NOTE –   IN ZERO MODE BOTH H & E IN PHASE ALONG Z
       –   ONE NULL IN FIRST MODE (ENDS OUT OF
           PHASE)

TI 2663

Fig. 1

After the TM_{010} mode is once established, Panofsky's theory can be introduced. This theory deals with a procedure to be followed to reduce the field configuration to the desired form. (Engineering Notes - file #M-18, also UCRL-236.) It is essentially a first order perturbation theory that relates changes in the values of the electric and magnetic fields to changes in the resonant frequency of a cavity, by the relation:

$$\frac{\Delta F (Z)}{F} = \frac{n^2}{8N^2} \cdot \frac{\Delta H (Z)}{H} \tag{2}$$

Here

$\Delta F (Z)$ is the frequency shift that would be present in any given section of the cavity due to improper wall size.

F is the resonant frequency (ν of TM_{010}).

N is the electrical length of the cavity in units of λ_{air} $(N = L/\lambda a)$.

H is the field in the cavity, measured at some convenient spot.

$\Delta H (Z)$ is the deviation of this field due to variations in the cavity wall size. (Deviations of the cavity radius from the nominal radius.)

n is the number of the Fourier component of the field which is being considered.

To apply the method to a cavity, the following steps are taken:

(1) The magnetic field is measured along the length (Z) of the cavity. This given us H (Z).

(2) A Fourier analysis of H (Z) is carried out to as many terms as is desired. The coefficients are then converted to fractional amplitudes by dividing by the average field value. This will permit the calculation of a fractional frequency change later.

(3) Each of the fractional harmonic amplitudes $\dfrac{\Delta H (Z)}{H}$ is then multiplied by $n^2/8N^2$ (Equation 2). It is to be noted that the frequency corrections thus obtained are greater by the factor n^2 for the higher harmonic perturbations. Conversely, small random variations in the cavity dimensions are more apt to introduce only the lowest harmonic distortions in the field. The denominator $(8N^2)$ indicates the relative frequency corrections for a long cavity (large N) will be smaller, but it also shows that small deviations from the nominal radius of such a cavity will induce much greater field perturbations than would similar deviations in a small N cavity.

(4) After the values $a_1/8N^2$, $4a_2/8N^2$, $9a_3/8N^2$, etc, have been found, a synthesis is performed to give the net frequency change required at each tuning point along Z.

$$\frac{\Delta Fi}{F} \sum_{n=1}^{m} \frac{n^2 a_n}{8N^2} \quad \frac{\cos \pi n Z_i}{L} \tag{3}$$

where:

　　　m is the number of harmonics considered

　　　Z_i is the distance from the end of the cavity to the

　　　　i th tuning point.

(5) When the ΔFi have been found, it is necessary to convert these frequency changes to tuning adjustments on the cavity, using experimentally found values of ΔF versus tuning adjustment.

It was found that the simplest method for making a Fourier analysis of this type, was to use a calculating machine and a table of cosines giving the values for $\dfrac{n \pi Z}{L}$. It was the fastest, and the most accurate method.

A perturbation analysis of the type described was made on the UCRL 40

foot linear (N = 8.3) accelerator after it had been brought into a zeroeth

mode. Corrections were successfully applied by shimming the length of the

individual drift tubes. No attempt was made to find out whether large errors

could be corrected this way.

The purpose of the work to be described was to apply these two theories

to very long cavities to see if they would offer a practical means of ob-

taining a TM_{010} mode and of giving a field which is of constant amplitude

(flat) along the length of the cavity.

B. EQUIPMENT

1. Electronics

The intent of the program, to investigate the field problems encountered

in electrically long resonant cavities, necessitated the eventual construction

of a model test cavity with an electrical length (N) of approximately 20. To

perform this function in a practical and economic manner, the physical length

of the model should not be excessive. To keep this length within reason, it

was obvious that the use of micro-wave resonant frequencies would be required.*

It must be remembered however, that as the electrical length for a given

physical length increases with a decreasing wavelength, the theoretical Q and

the mode spacing also decrease, and tolerances become more stringent. This

fact was sufficient to eliminate the use of frequencies at the high end of

the micro-wave region.

In line with the above considerations, a frequency in the region of

3000 megacycles was selected as the driving frequency.

---

* An unloaded model cavity operating with a frequency of 500 mc, and an N
of 18, would be 35.4 feet long.

This choice allowed the use of standard 3 in. inner diameter stock brass pipe for cavity construction. Unloaded, a cavity constructed of this stock with an electrical length of 18, would be approximately 6 feet long. These were considered to be convenient dimensions.

To begin the program, it was felt that little test equipment would be required and the majority of that which was needed was already available to the project. The following is a brief description of the more important equipment used.

a. Signal Generator.

A Navy type TS-403/U (Hewlett-Packard Model 616A) signal generator was used to excite the cavity. This generator covers a frequency range from 1800 mc (16.7 cm) to 4000 mc (7.5 cm) with a single frequency control. The generator uses a 2K28 reflex klystron as the oscillator and has a power output of approximately one milliwatt. The single frequency control on the front of the panel changes both the electrical length of the oscillator's resonant chamber and automatically tracks the repeller voltage. It was found a bit difficult to adjust the frequency of the oscillator to the resonant frequency of the test cavity, since a rather large frequency range is covered by the one control in the $7\frac{1}{2}$ turns.

It was therefore necessary to modify the generator slightly to provide for a frequency vernier control. A potentiometer was substituted for one of the fixed resistors in the repeller voltage tracking circuit. This control allowed a frequency change of $\frac{1}{2}$ megacycle, and made precise tuning of the unit a simple matter.

The signal generator can be pulse modulated by an internal pulse generator circuit and was also modified to allow frequency modulation of the oscillator by an external sawtooth voltage.

b. Spectrum Analyzer.

Perhaps the most useful of the associated test equipment was the TSS-4SE spectrum analyzer. This 10 cm band analyzer of M.I.T. Radiation Laboratory design covers the band from 2300 to 3600 megacycles.

Basically, the spectrum analyzer is a narrow band super-heterodyne receiver. The local oscillator, a 707B reflex klystron, is electronically tuned in frequency by applying a linear modulating voltage. This same sawtooth voltage is applied to the horizontal deflecting plates which produces a plot on the scope face indicating power vertically and frequency horizontally.

A frequency marker circuit is incorporated on the analyzer chassis, to place one megacycle pips on the trace. With these pips, difference frequency measurements can be made with ease. The marker pips are produced by a tunable high frequency oscillator with a range from 75 to 150 mc, heavily modulated by a second oscillator operating at one megacycle. The output signal from the modulated oscillator is applied to a crystal multiplier to produce a 1 megacycle modulated harmonic in the tuning range of the analyzer.

This marker frequency provided a rapid method of determining Δf for Q measurements, and also permitted an accurate measurement of the frequency difference between the lower transverse magnetic modes. The spectrum analyzer also was used as a resonance indicator while tuning the signal generator to the resonant frequency of the cavity.

c. Frequency Meter

The need for accurately determining resonance frequencies was limited although difference frequency readings were quite important. A Signal Corps type TS-117/GP frequency meter with a range of 2400 to 3600 mc served as the basic frequency standard. The unit is a resonant cavity type wave meter, but is not sufficiently well calibrated for small difference frequency measurements.

d. Detection.

An AN/APR-4 radar receiver was used to detect and indicate the magnitude of the magnetic field along the Z axis of the test cavity. This receiver was equipped with a TN-54/APR-4 tuning unit which tunes the range of 2150 to 4000 megacycles.

A 4 in. 200 microamp Weston meter was used in place of the small field strength meter which is a standard part of the receiver.

The receiver has characteristics which make it a useful tool for radio-frequency measurements.

Noise reading was less than 2 percent on most scales. The meter is linear over 75 percent of the range to better than 1 percent of full scale. The receiver used had a full scale sensitivity on the lower scale of approximately 40 micro volts.

The wide band width of the receiver eliminated the need for precision tuning as $\frac{1}{2}$ megacycle change in input frequency resulted in a drop of less than 5 percent in meter reading.

In use the receiver was set on its more sensitive scales to permit the use of a small reading loop. Meter readings were used directly for plotting the amplitude of H versus Z as the reading loop was moved from point to point along the cavity.

Figure 2 is a photograph of the signal generator, spectrum analyzer, and receiver alongside a test cavity with an unloaded electrical length of 22.8.

2. Cavity Construction

a. Short Cavities.

The first cavity constructed was intended primarily as a familiarization device to present a real picture of the problems to be encountered. This
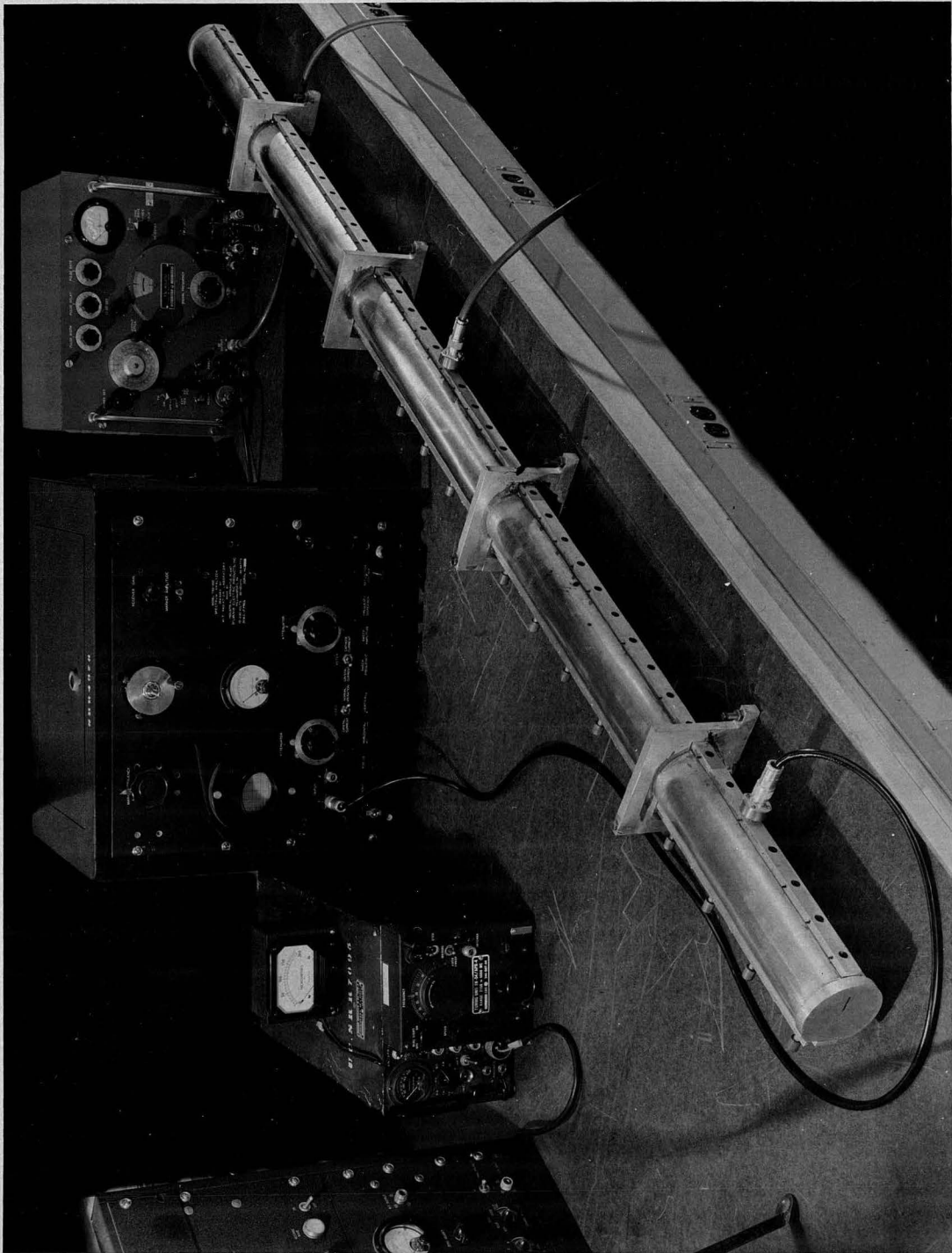
FIG. 2

model, constructed from a 22 inch piece of 3 inch brass pipe, resonated at a

frequency of 3009 mc and had an electrical length of 5.6.

Excitation of the cavity revealed that many different modes could be

excited within the frequency range of the signal generator. Thus it was

necessary to be able to recognize the type of mode generated. The simplist

expedient for discerning whether a mode was a TE or TM, was to rotate the

loop through 90°. If the loop reading indicator read a maximum with the loop

oriented parallel to the axis of the cavity, and dropped to zero when the

loop was rotated 90°, a TM mode was indicated. If the action was not similar

to that described, it indicated a transverse electric mode. When a doubt

existed, a second check was made, using an E probe which gave an indication

only when a TE mode existed.

Magnetic field measurements were first made with the use of a loop, crystal

diode, and galvanometer. This system provided a simple way to determine re-

sonance. Later, when more accuracy was desired, the field measurements were

made with an APR-4 receiver. It was still convenient to use the galvanometer

as a resonance indicator, before attempting to tune the receiver.

According to the theories presented in the Introduction the problems of

mode shifting and mode flattening can be solved by tuning various portions of

the cavity to compensate for the errors of construction. This tuning could

be accomplished in several possible ways and the first system tried was that

of using a series of small flat paddles mounted along the inside wall of the

cavity. These paddles could be rotated to intercept more or less magnetic

field lines as desired. The effect of the paddle was to make an apparent

decrease in the size of the cavity at the point where it was located, thus

increasing the resonant frequency.

The paddle operating in the H field was able to produce a frequency change,

but this change was quite small. When projected to a comparable change in the proposed machine, the paddle became a rather large and cumbersome piece of machinery.

It was noted, however, that should the paddle be inserted to the center of the cavity a sizable tuning effect was available. This effect is illustrated in Fig. 3. This chart shows the change in frequency produced by pulling a 0.250 in. brass ball through the 3 inch dimension of the small cavity. The "BB" was suspended on nylon line and its direction of travel was perpendicular to the axial electric field. By intercepting the magnetic lines near the skin of the cavity the BB produces an increase in resonant frequency, but as it is moved closer to the axis, or region of maximum E, the effect is that of increasing the capacity of the parallel resonant circuit represented by the cavity, and the resonant frequency drops.

It was thus concluded that to bring the eventual tuning mechanism down to a practical size it would be wiser to make adjustments in the region of maximum E. As the drift tubes in a linear accelerator are located in this region, it was logical to assume that changes in their physical size could be made to perform the tuning functions.

End Caps. The cavity ends are removable to allow access to the inside of the cavity for the insertion of loading elements. The end caps were equipped with phosphor-bronze spring finger ends that grip against the inside of the cavity to make rf contact. The spring fingers were silver plated to reduce the contact losses to a minimum.

While this cap seemed to be an effective seal it did have an effect upon the field configuration of the unloaded cavity. When a measurement of H versus Z was taken at 12 points along the wall of the cavity, a curve was obtained

X (SCALE IN INCHES)

Y (SCALE IN MEGACYCLES)
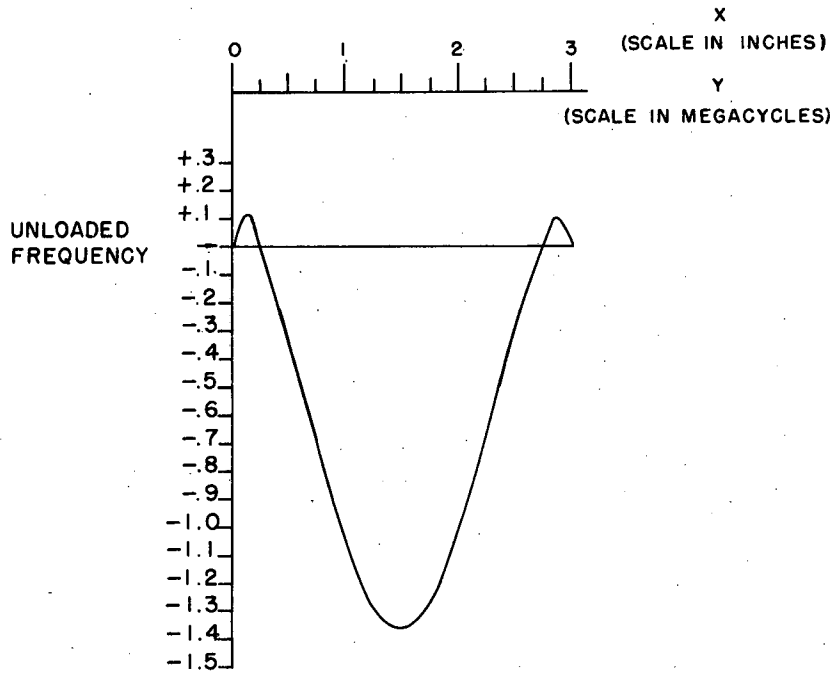
BB = .250" DIAMETER SUSPENDED BY 14 MIL NYLON

CAVITY = D = 3" − Z = 22" −
        f_o = 3009 MC UNLOADED λ_{AIR} = 9.97 CM = 3.925"
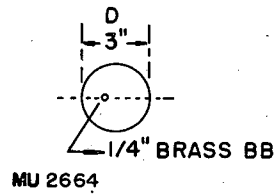   N = 5.6

MU 2664

Fig. 3

that dipped at both ends as shown in Fig. 4-a.

Both end caps were drilled and tapped at their centers for an 8-32 machine screw. When these screws were inserted, the E field of the cavity at the ends could be raised producing a relatively flat, unloaded cavity as shown in Fig. 4-b.

A logical explanation is that the spring fingers, because of their finite size, reduce the effective size of the end sections. This in turn makes the end sections resonate at a slightly higher frequency than the remainder of the cavity. Insertion of the end tuners adds capacity to these sections by decreasing the volume of E. This added capacity retunes the end sections to the normal resonant frequency.

In an attempt to eliminate the necessity for tuning out the effects of the end caps as described, a second cap was constructed. This cap used the coil spring type rf contact as illustrated in Fig. 4-c. These phosphor-bronze coil springs were also silver plated for minimum contact losses.

A chart of H versus Z for this type end cap showed a distinct raise in H at the ends. Pushing through end tuners of the type used previously only served to make the H near the end increase. As shown in Fig. 4-c the rf contact with the cavity wall for this end cap is made slightly behind the actual physical end of the cap. The protruding portion of the end cap could thus be thought of as a large end tuner inherently established in the system. This means the ends will be tuned lower in frequency than the remainder of the cavity. To flatten the curve illustrated, one must decrease the cavity wall size. Small strips of copper were laid along the wall in the ends of the cavity to decrease the apparent radius and a relatively flat field configuration was obtained thereby.

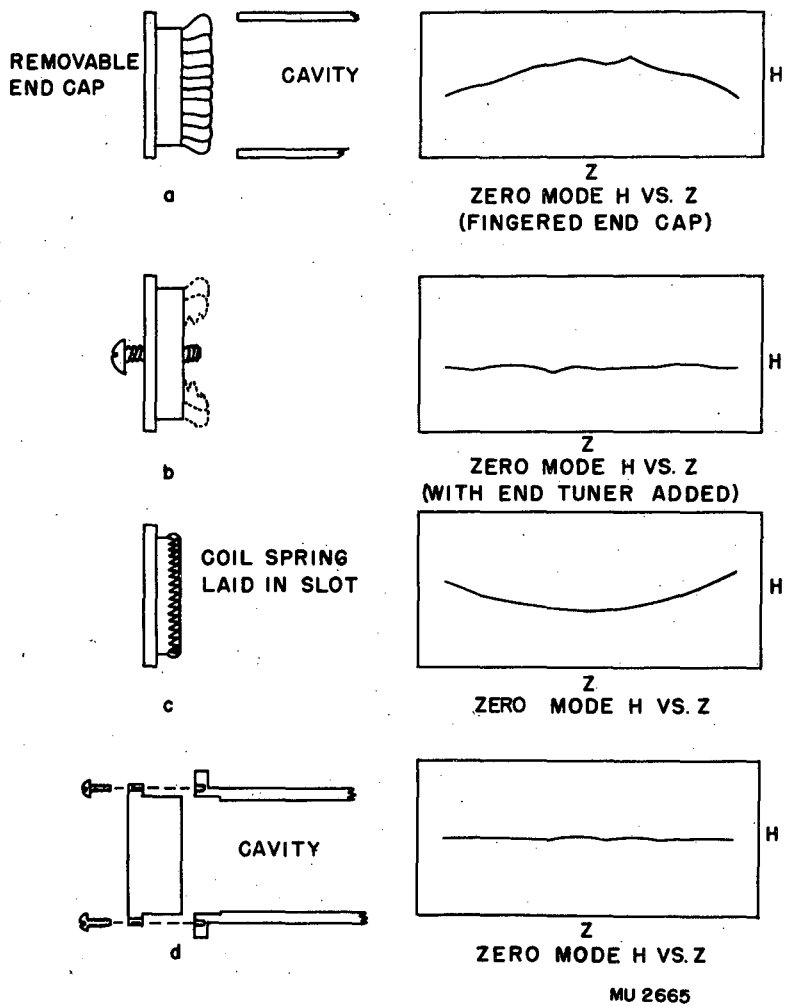A third type of end cap was constructed. This cap was essentially a

Fig. 4

plug which formed a butt joint with the cavity wall. The rf contact was made by clamping the plug to the cavity with four $\frac{1}{4}$-20 machine screws (Fig. 4-d).

This end cap proved to be the most successful and required no added tuning to allow the ends to resonate at the same frequency as the remainder of the cavity. The chart of H versus Z obtained with this end cap was essentailly flat.

Loaded Cavity. A second test cavity was constructed of 3 in. brass stock pipe. It was 44 in. in length. This model was equipped with removable end caps of the plug type.

For loading, ten drift tubes were constructed, four of which were adjustable in length. These drift tubes were $\frac{1}{2}$ in. in diameter and 1.5 in. long. The length of the adjustable drift tubes could be varied 30 mils either side of the 1.5 in. dimension. The drift tube faces were not rounded nor was any attempt made to design the tubes to follow any predetermined specifications.

Loaded, this cavity resonated at 2320 mc which corresponded to an N of 8.64. Electrically, this was slightly longer than the UCRL linear accelerator which has an N of 8.3.

Although the presence of the drift tube stems should discourage the excitation of T.E. modes, many were present in this cavity. Between 2320 megacycles and 2940 megacycles, 29 separate TM and TE modes could be excited. This provided an excellent opportunity to become familiar with the behavior of the various modes.

Attempts to flatten the zeroeth mode were quite successful. However, the method consisted mainly of trial and error adjustments and it was not obvious that any sort of consistent procedure could be developed.

b. Long Cavity

Considerable experience was gained from the work with the short cavities. The next step was to construct a long cavity.

A design was chosen on the basis of "one cell" tests. The cavity was 90 in. long, 3 in. in diameter, and was loaded with 45 drift tubes of uniform size, spaced evenly with 2 in. between centers. Half of the drift tubes were adjustable in length from the outside of the cavity. The cavity and all the drift tubes were constructed of brass and then silver plated to reduce losses as shown in Fig. 5, a photograph of the completed cavity.

Drift Tube and Cavity Design. The drift tubes used in the long cavity were designed (Fig. 6) to meet the following general specifications:

$$\beta = L/\lambda \quad \text{between 0.25 and 0.5}$$

$$g/L \quad \text{between 0.25 and 0.5}$$

$$D/d \quad \text{between 5.2 and 6}$$

The operating resonant frequency was arbitrarily chosen to be approximately 2400 megacycles. This frequency was chosen to allow the drift tubes to be of a convenient size; however, it was not possible to use a much lower frequency without being outside the band covered by the majority of our test equipment.

Using the standard 3 in. diameter brass pipe for the construction of the cavity, D was fixed at 3 in. This choice implies a d of approximately $\frac{1}{2}$ in. A 2 in. cell length L was chosen.

In order to find the resonant frequency (hence β) associated with certain values of t and g it was necessary to perform actual tests with the one cell model.

A one cell cavity was constructed (Fig. 7) and resonated with various length drift tubes. The data from this cavity led to the final cell dimensions
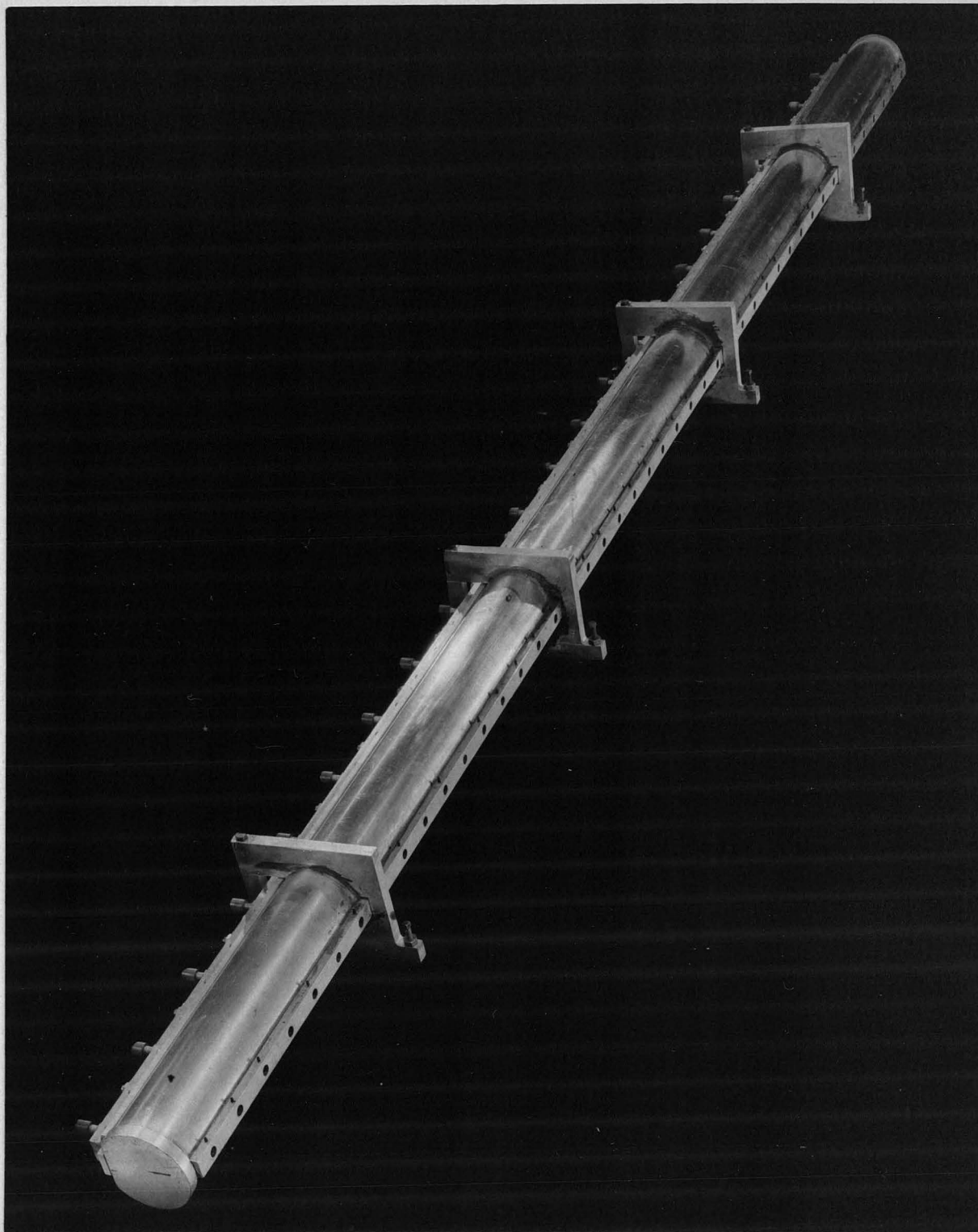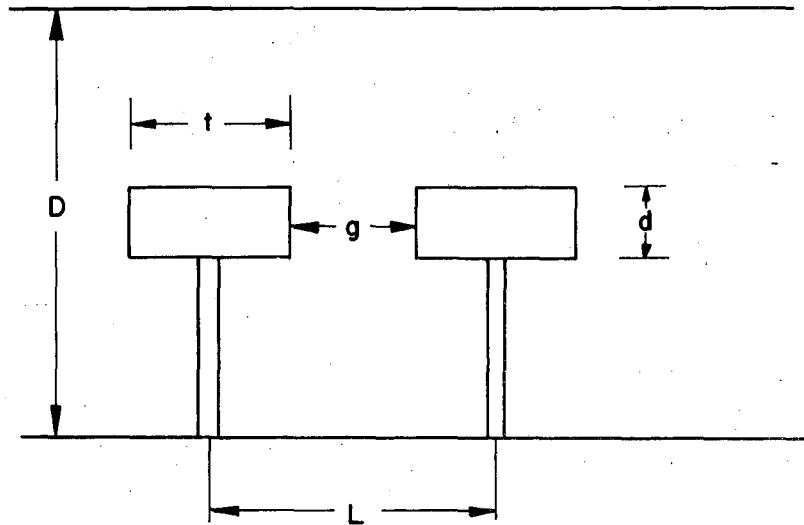
FIG. 5

D = INNER DIAMETER OF CAVITY
d = DIAMETER OF DRIFT TUBE
f = LENGTH OF DRIFT TUBE
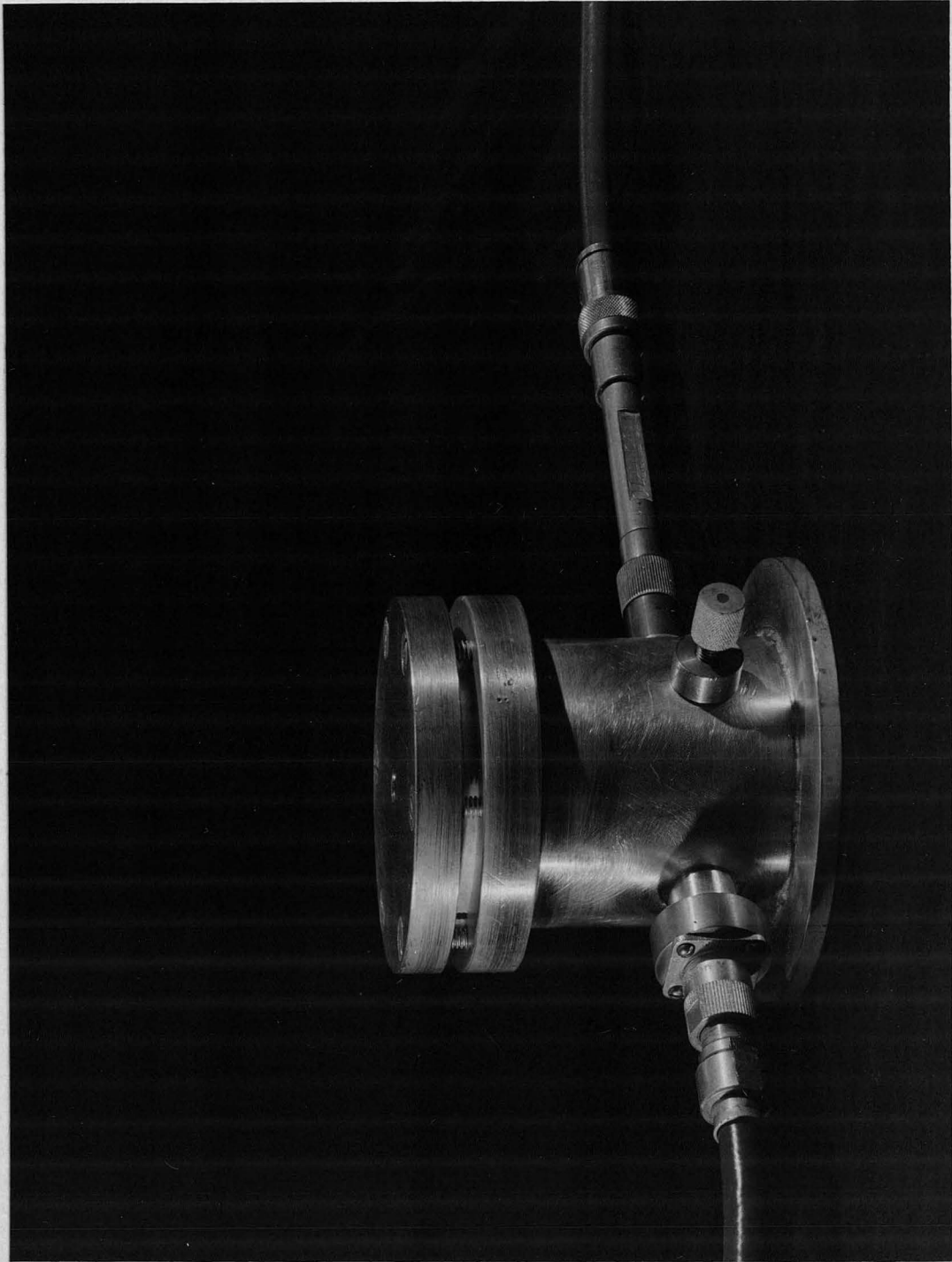g = GAP BETWEEN ADJACENT TUBES
L = CELL LENGTH

MU 2666

Fig. 6

FIG. 7

given below which were consistent with the desired specifications.

$D = 3$ in. $L/\lambda = 0.41 = \beta$

$d = 0.5$ in. $D/d = 6$

$t = 1.175$ in. $g/L = 0.41$

$L = 2$ in. $N = L/\lambda$ (for $N = 20$, $L = 98$ in.)

$\lambda = 12.5$ cm

$g = 0.825$ in.

The length of the cavity (90 in.) was selected to allow a convenient alignment of drift tubes with an adjustable drift tube at each end as well as in the middle of the cavity. Considerable thought had been given to the use of a 30 section electronic switching tube to act as the basis for an electronic Fourier analyzer.

To perform this function, it was desirable that the cavity be constructed to accomodate 15 evenly spaced probes. The 90 in. dimension would thus allow a probe to be located at each third drift tube.

Because of its length, the interior of the cavity was not accessible through removable end caps. To eliminate this problem, the cavity wall was slit down the Z axis to accommodate the placement of a drift tube support bar (Fig. 8). With this arrangement, drift tubes could be conveniently mounted, aligned, and the entire assembly then secured into position.

A 10 mil (0.010 in) copper foil connected the drift tube support bar to the cavity end walls, and provided a low loss current path. Mechanically the support bar was secured to the cavity by 4/40 machine screws located at 6 in. spacings on both sides of the bar (see Fig. 9).

As the end plates did not need to be removed, they were hard soldered in position. Four brackets were also soldered to the cavity outer wall to prevent the support bar slit from closing or becoming deformed due to any strain
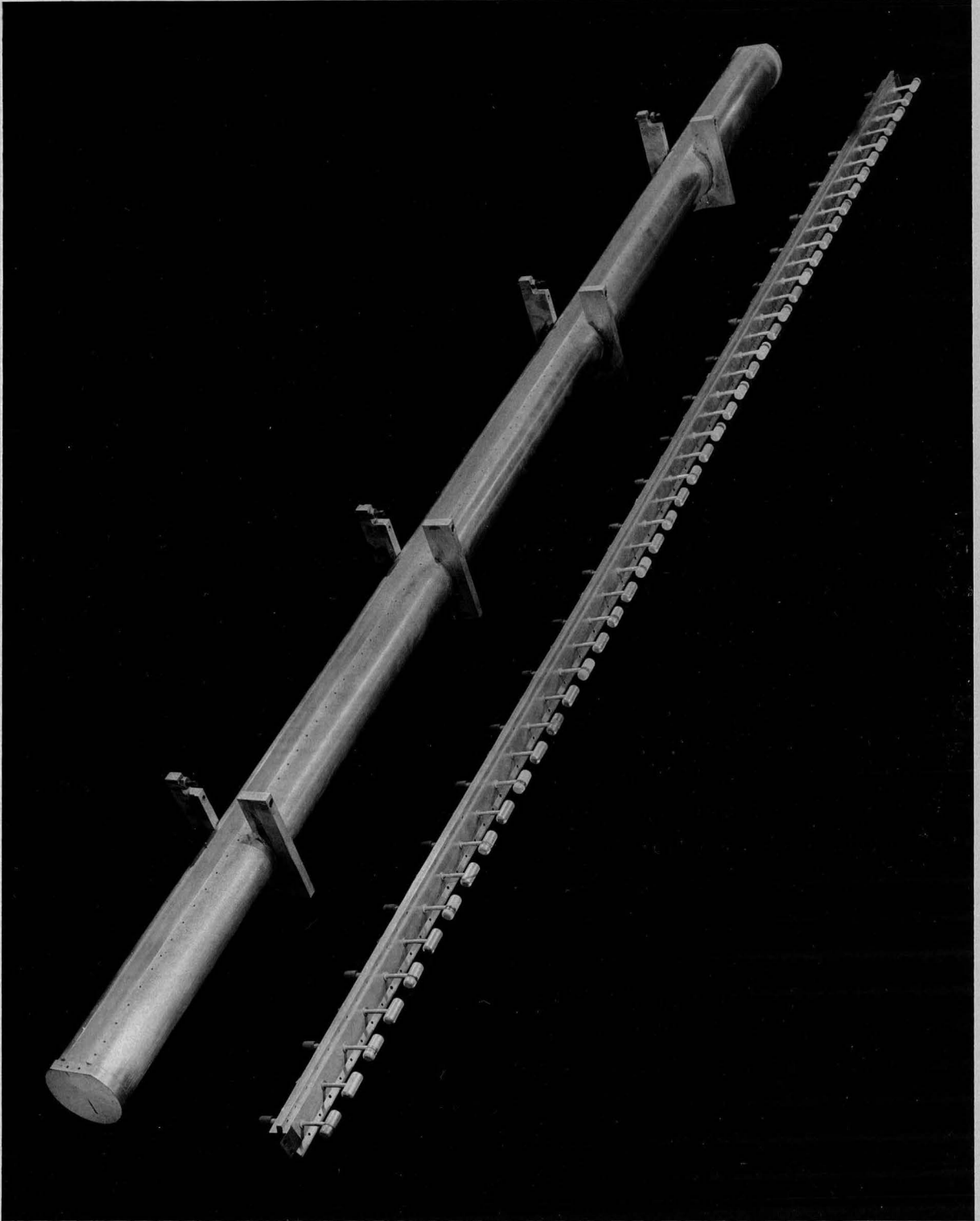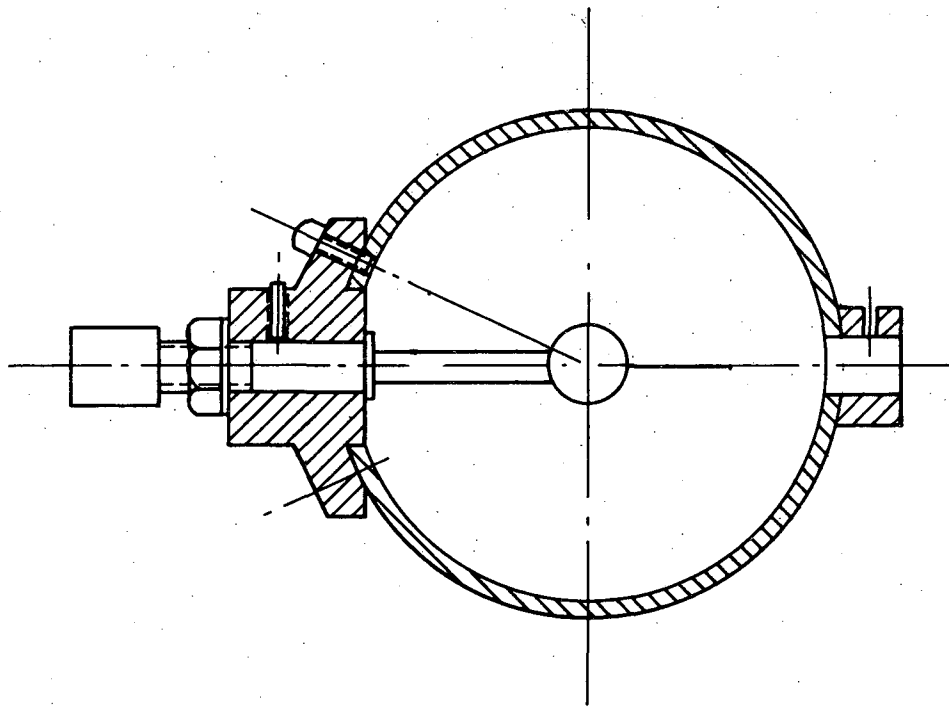
FIG. 8

MU 2667

Fig. 9

in the extruded pipe.

Of the 45 drift tubes to be mounted in the cavity, 23 were adjustable (Fig. 10) and 22 were solid and non-adjustable (Fig. 11).

The adjustable drift tubes were constructed to have one movable end providing ± 45 mils adjustment. Initially no provision was made for electrical contact other than that provided by the sliding surface between the body and movable end. The drift tubes were attached to the support bar with 5/16-32 hex nuts.

Figure 12 shows the general construction. Unloaded, this cavity resonated at 3008 mc, the same as the two smaller model cavities, which corresponds to an electrical length of $N = 22.8$. When fully loaded the resonant frequency was 2400 mc or $N = 18.3$.

C. DIFFICULTIES

1. Overlap of Modes due to Improper Mode Spacing.

When the cavity was first excited, numerous modes were detected in the region predicted by the one cell cavity test. It was soon noted that the zereoth mode could not be excited, and that the lowest TM mode detectable was the first mode (TM_{011}). It was possible by appropriate tuning (to be discussed later) to shift the null of this mode to the center of the cavity, but the zero mode was still difficult to excite. Thus the next step was to correct the position of the nulls in the second mode. It was then possible to excite the zero mode.

Due to close spacing of the zeroeth and first modes, there seemed to be an overlap of the two. This condition was enhanced by the low Q of the loaded cavity. The mode spacing for the lower TM series is predicted by the relation
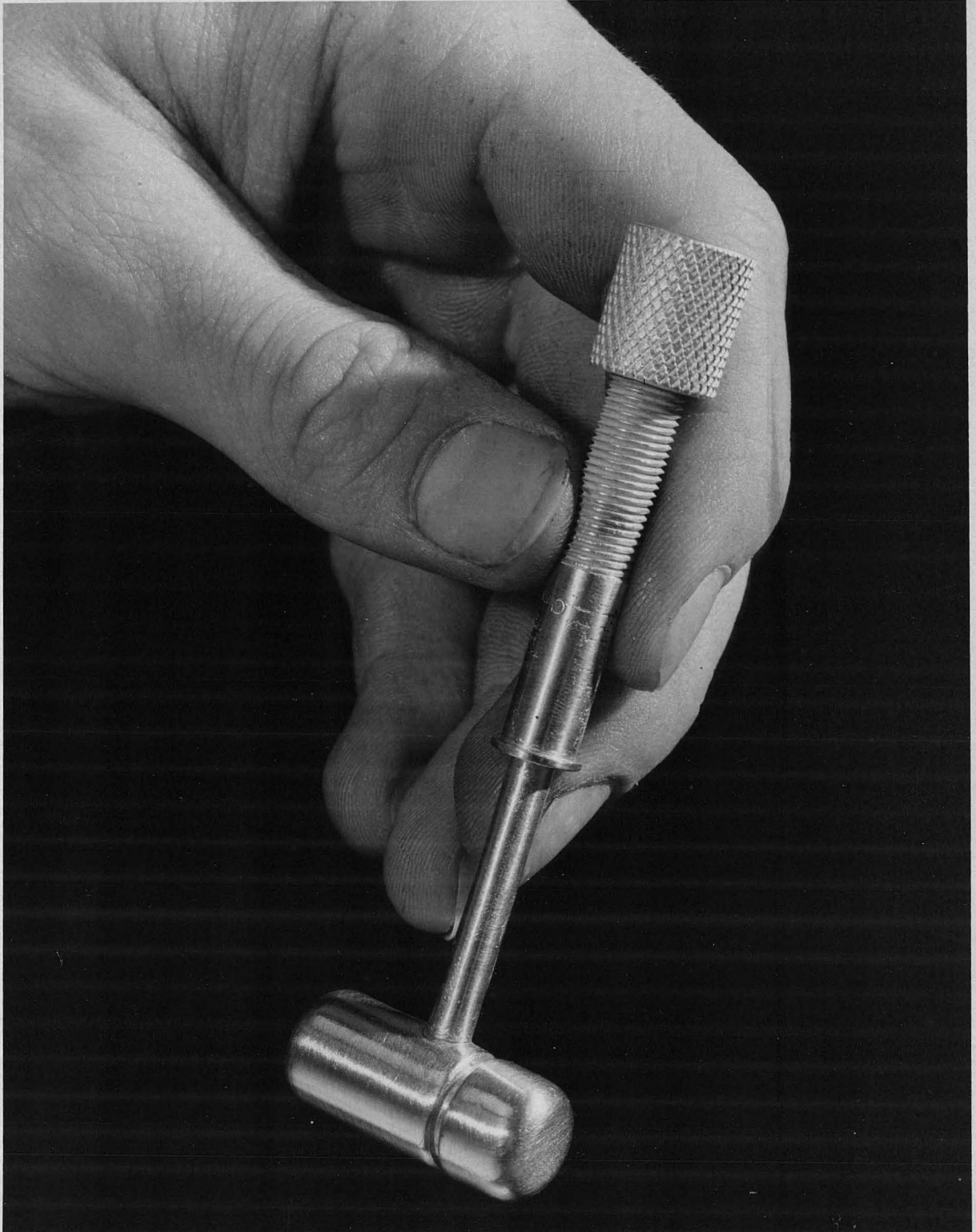
$$\Delta F = F_K - F_o = K^2 F_o / 8N^2 \tag{3}$$
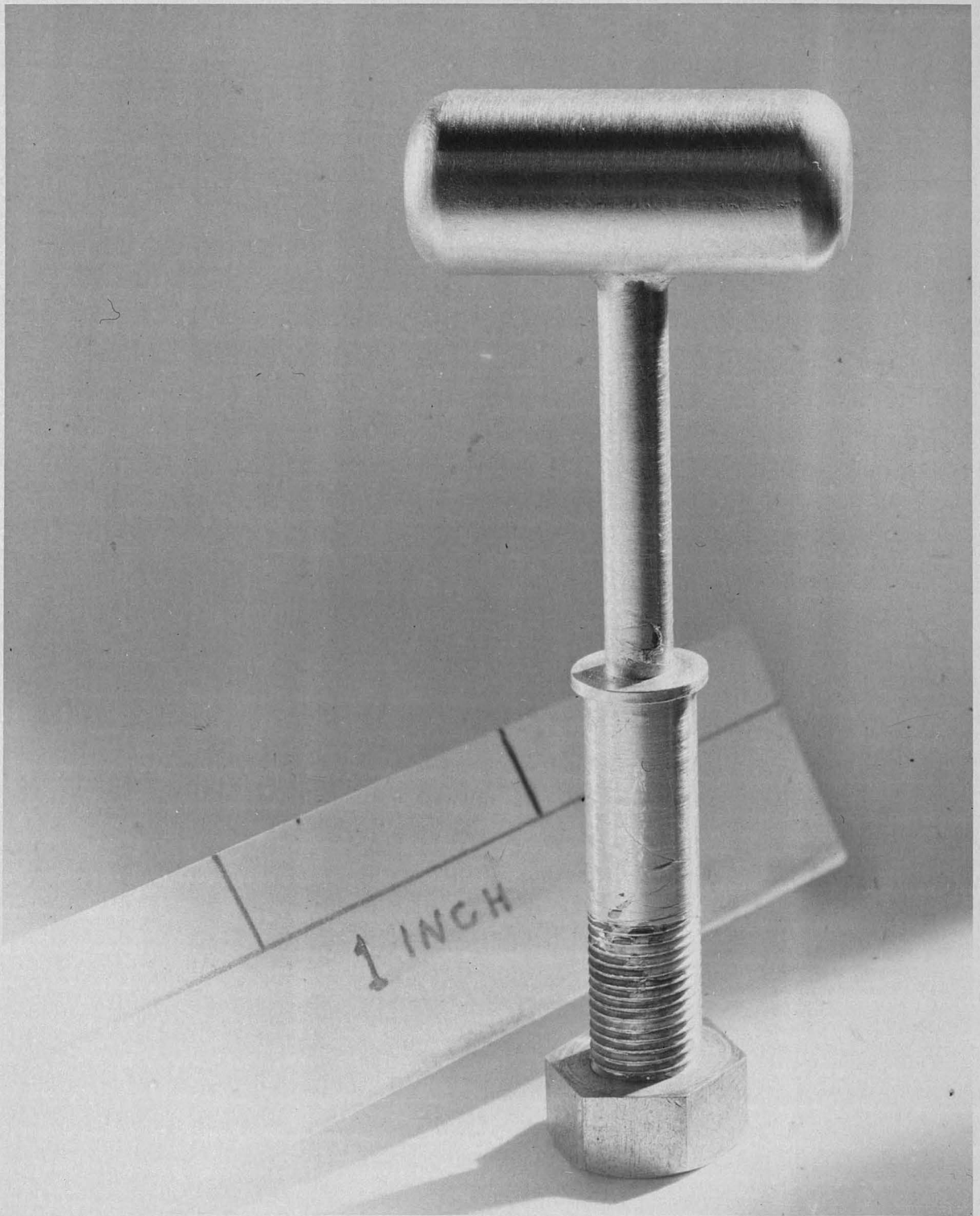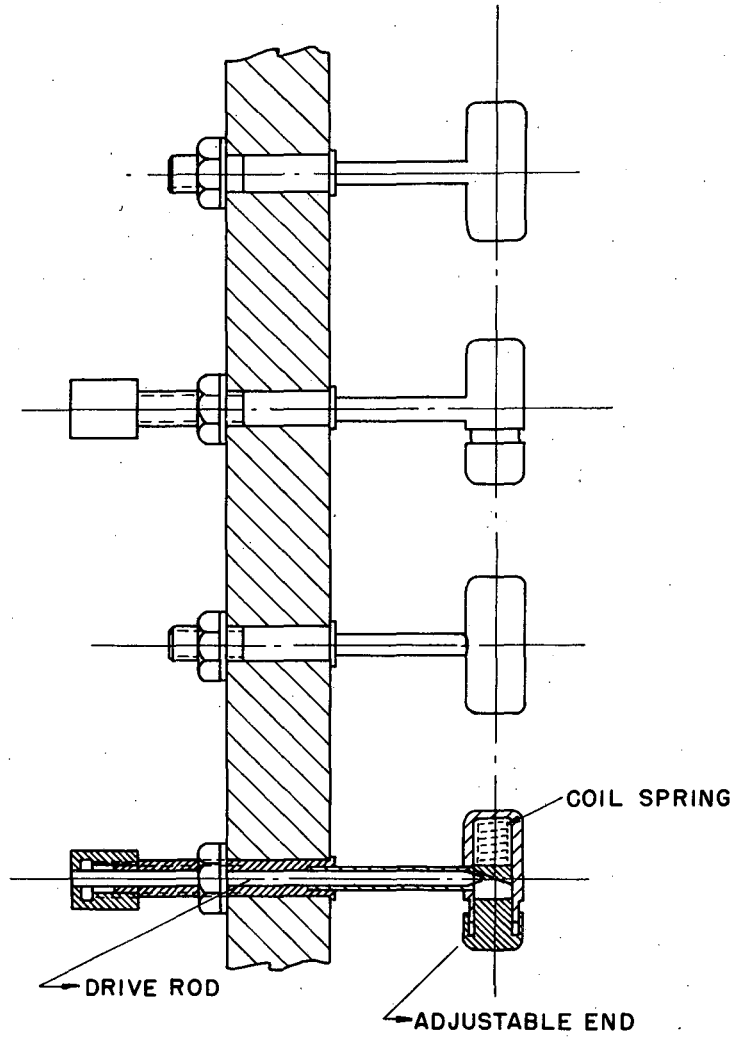
FIG. 10

FIG. 11

COIL SPRING

DRIVE ROD

ADJUSTABLE END

MU 2668

Fig. 12

For the 90 in. cavity this would indicate a spacing from the TM_{010} to the TM_{011} of 0.9 megacycles (K= 1). By using the Spectrum Analyzer as an indicator, and shifting the excitation frequency this mode spacing could be determined. Often the spacing was less than 0.9 mc as predicted by equation (3). The Q of the loaded cavity was measured and found to be 6000, or a $\frac{1}{2}$ power width of $\Delta F = 0.4$ mc. With these conditions, it was difficult, even by feeding at the null of the first, to prevent the modes from overlapping.

It was found that the spacing of the modes could be changed at will in a very simple manner. If changes in the drift tube lengths are made in the center of the cavity, where the first mode has a null, there will be very little or no effect on the frequency of the first mode (or other odd modes). The resonant frequency of the zereoth (and higher even modes) however, will be changed appreciably. Changing the length of the drift tubes near the cavity end where a field exists in both the zero and first modes, will change the frequency of both modes. However, as the E field of the first mode is much higher in these regions, than is the field of the zereoth, the frequency of the first will be changed to a greater degree than the frequency of the zereoth. By a combination of such adjustment, it was found possible to increase the mode spacing to several magacycles. Conversely, the mode spacing could be decreased to the point where the two modes completely overlapped.

2. Drift Tube Adjustments

Establishment of a zero mode in the 90 in. model was now quite possible. However, attempts to produce a uniform field by the use of the harmonic analysis procedures repeatedly failed. Many instrumental difficulties had to be overcome before reliable results could be obtained.

Changes in drift tube length necessary to correct a given field configuration were generally so small that they could not accurately be inserted by means

of the external adjustments (one rotation of the drive screw = 11 mils). See
Figs. 10 and 12. Rather than construct elaborate tuning indicators, the first
solution was to remove the entire drift tube support bar (Fig. 8) and measure
each individual adjustable drift tube with a micrometer to put in the desired
change. However, the possible misalignment when the bar was replaced, coupled
with the poor contact problem to be discussed later, made this procedure seem
unsatisfactory at the time.

a) External Micrometer.

The result of the Fourier analysis is given in mils of increase or de-
crease in length of the various adjustable drift tubes. Thus the correction
is not directly concerned with the true physical length of each drift tube,
but only in the amount of effective change to be made. This fact made it
possible to install changes in drift tube lengths by a method which did not
require the complete disassembly of the cavity. The equipment consisted of
a rod pivoted at the cavity wall (Fig. 13) with one end of the rod resting
on the adjustable end of the drift tube to be corrected. The other end ex-
tended outside the cavity far enough to give a 4 to 1 multiplication in
movement. The external tip was then observed with a traveling microscope
(Fig. 14).

With this arrangement, one-tenth mil (0.0001 in.) change in the length
of the drift tube could be observed.

To eliminate the necessity for drilling additional holes through the
cavity wall, the system was designed to use the holes already used for reading
the field configuration.

Aside from the original purpose of measuring the drift tube adjustments
without disassembling the cavity, the external micrometer provided a much
more rapid system, and a complete set of adjustments (23 tubes) could be

R-2 = 4 × R-1

TRAVELING MICROSCOPE
TO OBSERVE Δ L OF DRIFT TUBE

R-2

TENSION WEIGHT

R-1

DRIFT TUBE SUPPORT BAR
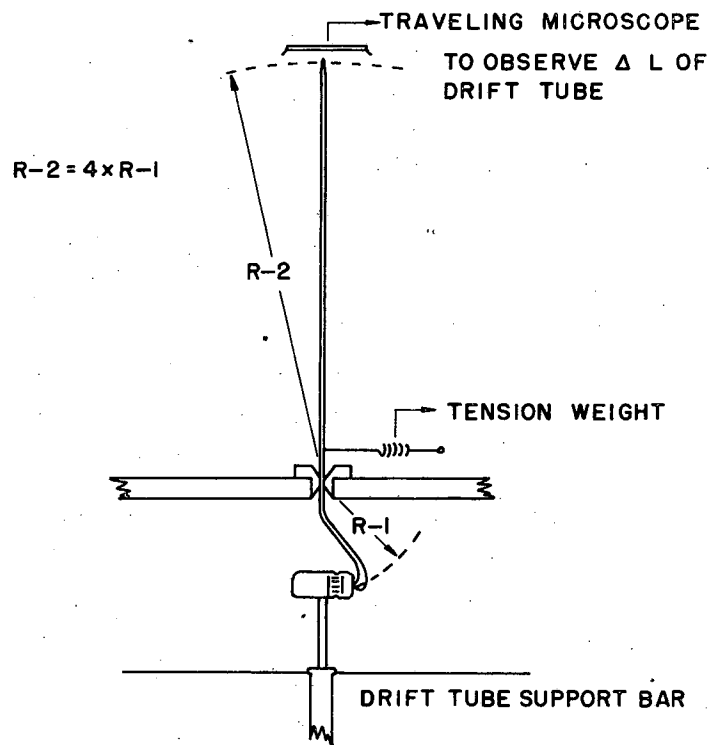
MU 2669

Fig. 13

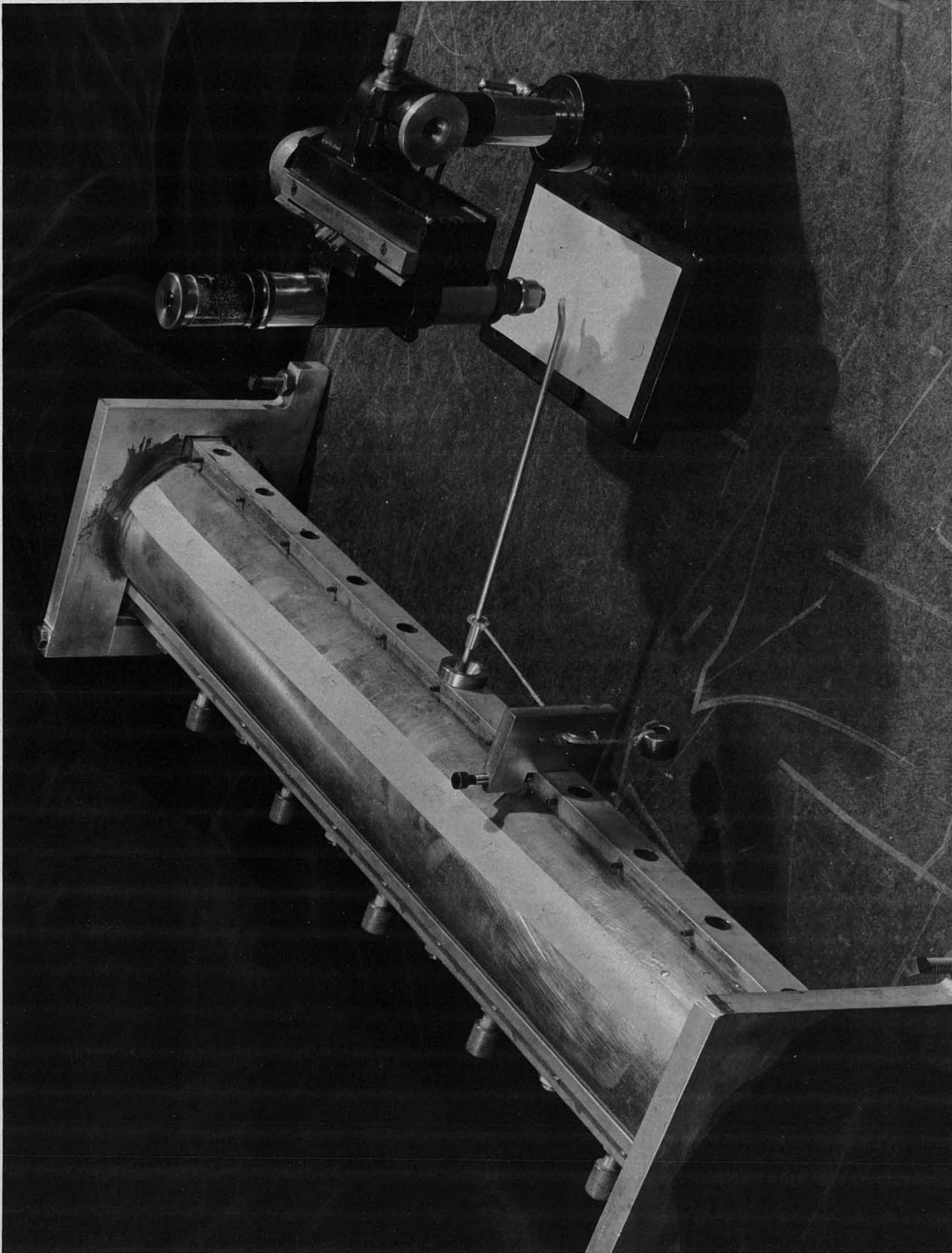FIG. 14

made in half the time originally required.

3. Low Cavity Q

Another major problem was associated with the poor contact that existed at the sliding joint of the adjustable drift tubes. This manifested itself in many ways. First, adjusting the drift tubes changed the contact resistance, resulting in varying power losses and discontinuous changes in field configuration. Hence, readings taken before and after a given harmonic change did not necessarily have any correlation with the correction. Secondly, when the zero mode was excited, it was possible to obtain field plots which produced a maximum reading at the point of feed (see Fig. 15). This result was explained in terms of the attenuation caused by the high resistance joints. Thirdly, it was necessary to feed the cavity at the null point of the first mode to prevent first mode excitation since the wide resonance curves made the modes overlap even when the spacing was correct. However, as adjustment of the drift tubes were made, this null tended to shift and the alternatives were either to follow the null with the feed loop, or to leave the feed loop in the initial position. The first possibility was found completely infeasible, due to the dependence of the field on the feed point. Corrections made for one feed position had no applicability for the field resulting from another feed point. The procedure of leaving the feed point fixed had little more success due to mode overlap. As soon as corrections were made, the null of the first mode was no longer at the feed point, and excitation of this mode became possible. The field in the cavity was then a mixture of partially corrected zero mode and uncorrected first mode. The only solution to these problems seemed to be to effect a definite increase in cavity Q.

It is to be noted also that in the perturbation derivation given by Panofsky, a term involting 1/Q is dropped with respect to terms involving
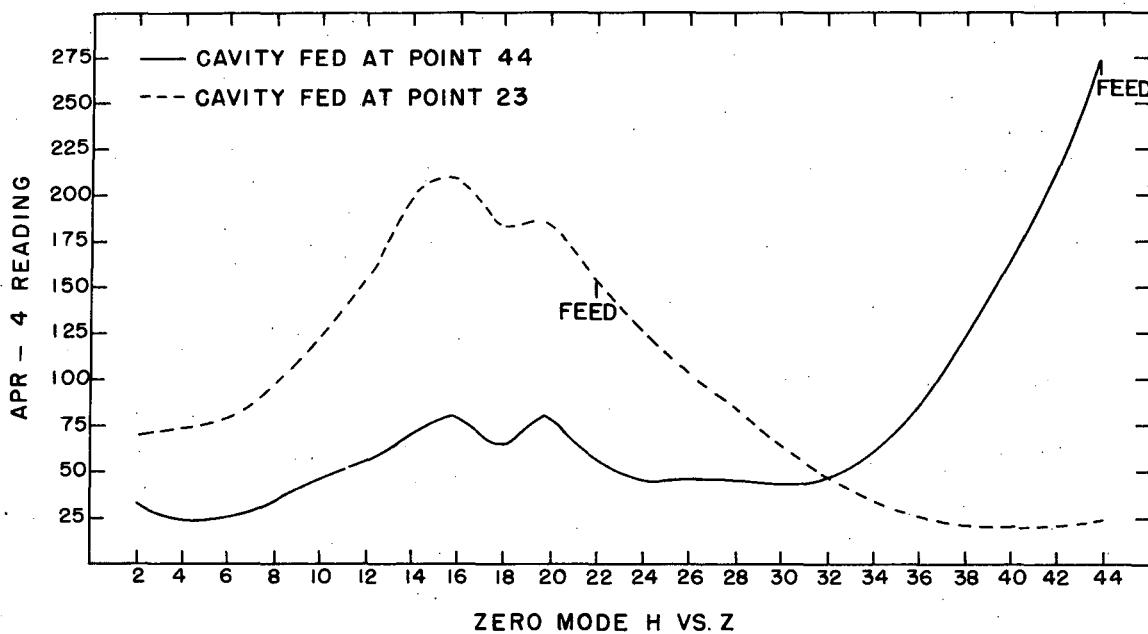
Fig. 15

the mode separation. Thus there is no guarantee that this method of flattening the zero mode should work unless the Q is sufficiently high to eliminate overlap of modes. Therefore a program to eliminate the losses in the cavity was instituted.

To insure a permanent physical contact between the two parts of the adjustable drift tubes, a split flexible foil was soldered to both parts. This section was constructed of one mil copper foil and extreme caution was required to prevent damage to the foil during the soldering process (Fig. 16). The presence of the foil reduced the adjustment possible to ± 0.025 in. but this was far greater than was ever required. The only difficulty encountered with the joint was that the copper foil was capable of only a small number of complete flexures before breaking. However, the very small adjustments which were required in actual practice caused less severe strain and a reasonable number of adjustments was possible.

The Q of each modified drift tube was separately checked in the one cell cavity. The measured Q of the unloaded one cell cavity was 15,000 to be compared with a theoretical Q of 17,500. The method used for measuring Q was to drop the voltage reading of a loop coupled to the APR-4 receiver to the 0.707 point by changing the signal generator frequency, and then noting this frequency change on the spectrum analyzer. This method was rapid and sufficiently accurate for the purpose.

Before modifying the adjustable drift tubes, they were checked in the one cell cavity and the resulting Q measurements ranged from 2500 to 6000. After the modification, Q obtained from the same test ranged from 12,000 to 14,000. These improved readings compared very favorably with the Q measured with a solid drift tube.
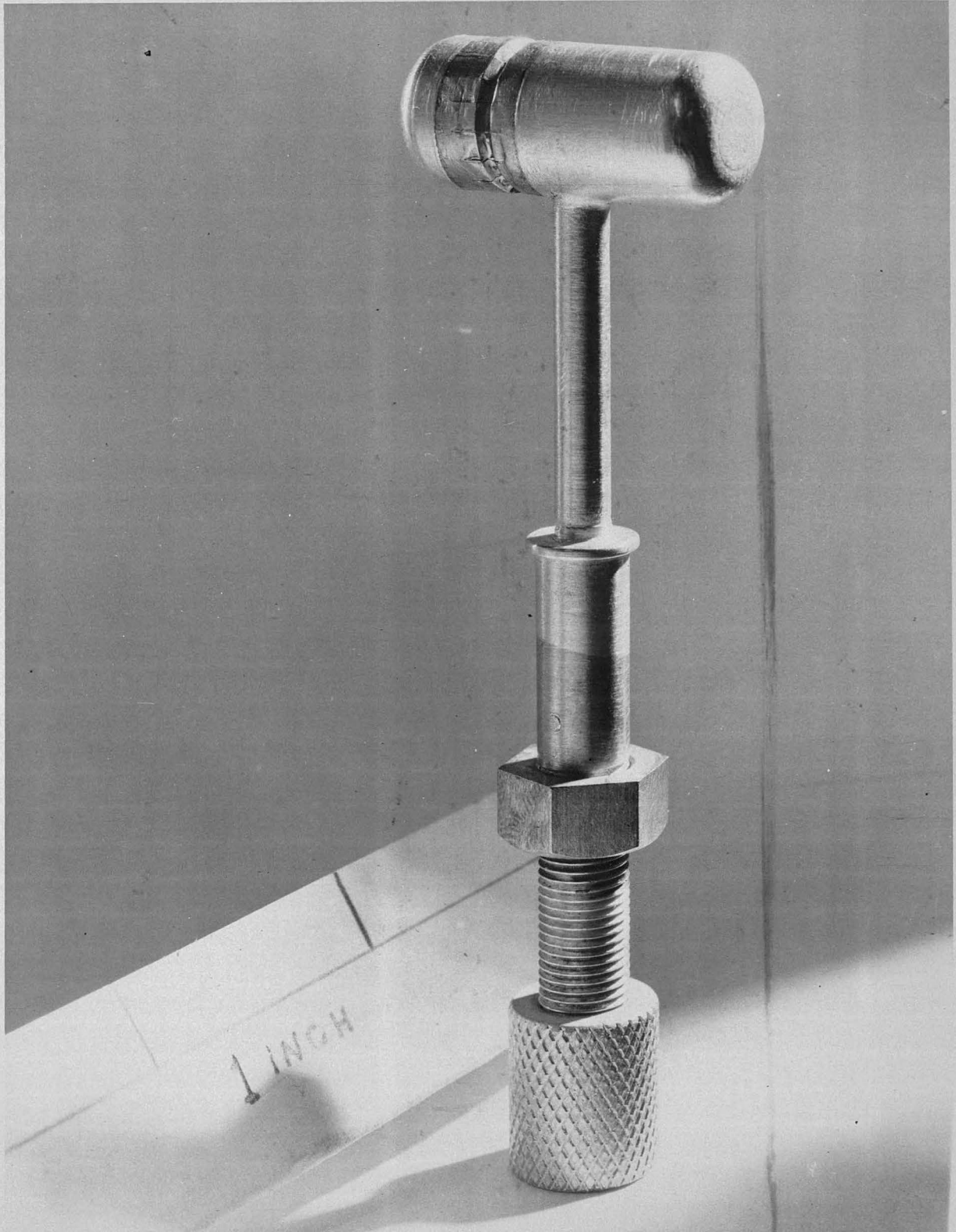
FIG. 16

The Q of the unloaded 90 in. cavity was measured at 27,000 and had a theoretical Q of 34,000. When loaded with the modified drift tubes the measured Q was 17,500. This resulted in a half power resonance width of approximately 0.13 mc. and with the correct mode spacing, overlap of modes no longer existed. It was now possible to make consistent and reproducible changes in the cavity. Also eliminated was the effect on the field of the feed point. Fig. 17 shows the plot of the zero mode obtained by feeding at opposite ends of the cavity. The two charts are essentailly the same.

D. NODE SHIFTING

After initially exciting the cavity, a search for the lowest TM mode was made. The second mode and first modes were visible; however, the zereoth was not readily detectable. A chart of these lower two modes was made as shown in Fig. 18b, the 2nd mode, and Fig. 18a, the 1st mode.

From the charts of these two modes, it was possible to obtain some information as to what tuning would be required to allow the excitation of the zero mode.

1. Measurement of Phase and Amplitude

As plotted in Fig 18 a and b, phase is not shown. It is to be remembered that opposite sides of each null are 180° out of phase. It was quite important in determining the mode number to check this phase as the curve produced by plotting loop readings are sometimes deceptive, particularly if the null should fall at a point that can not be measured. A phase check was always performed. This was accomplished by placing a coaxial "T" at the input of the receiver and using two reading loops connected to the "T" by equal lengths of RG-9/U coax cable. The loops were inserted one at a time on either side of a suspected null and individual readings noted. The loops were then inserted
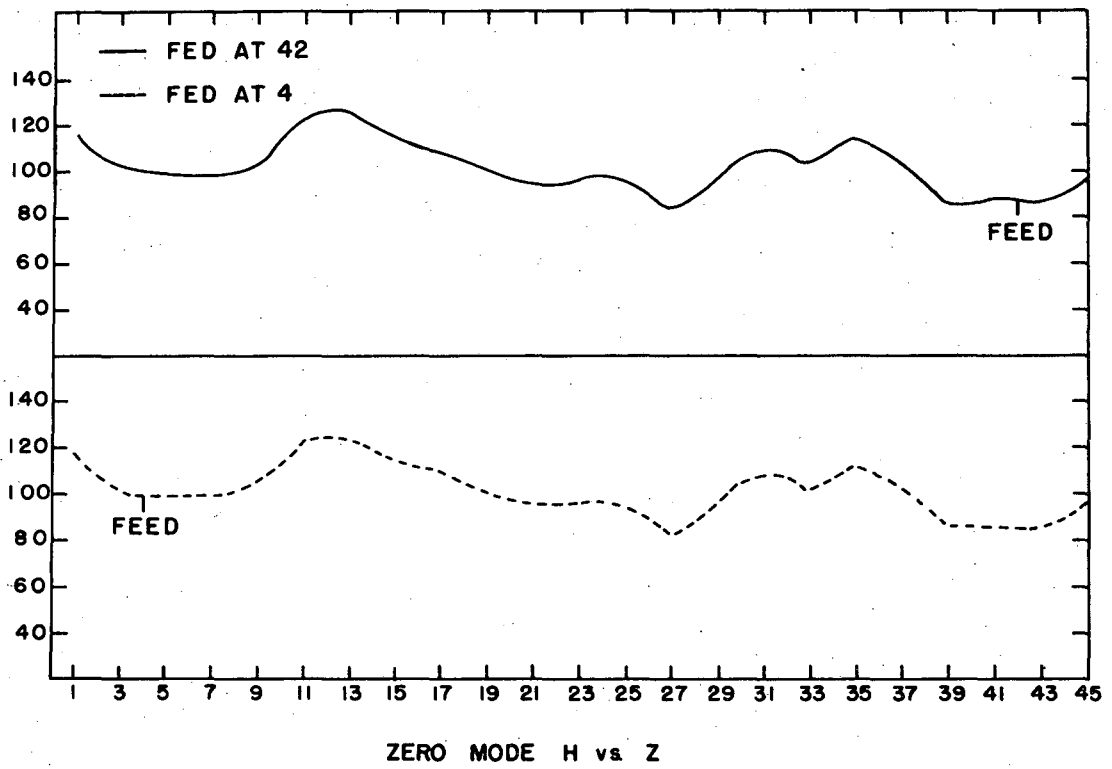
Fig. 17
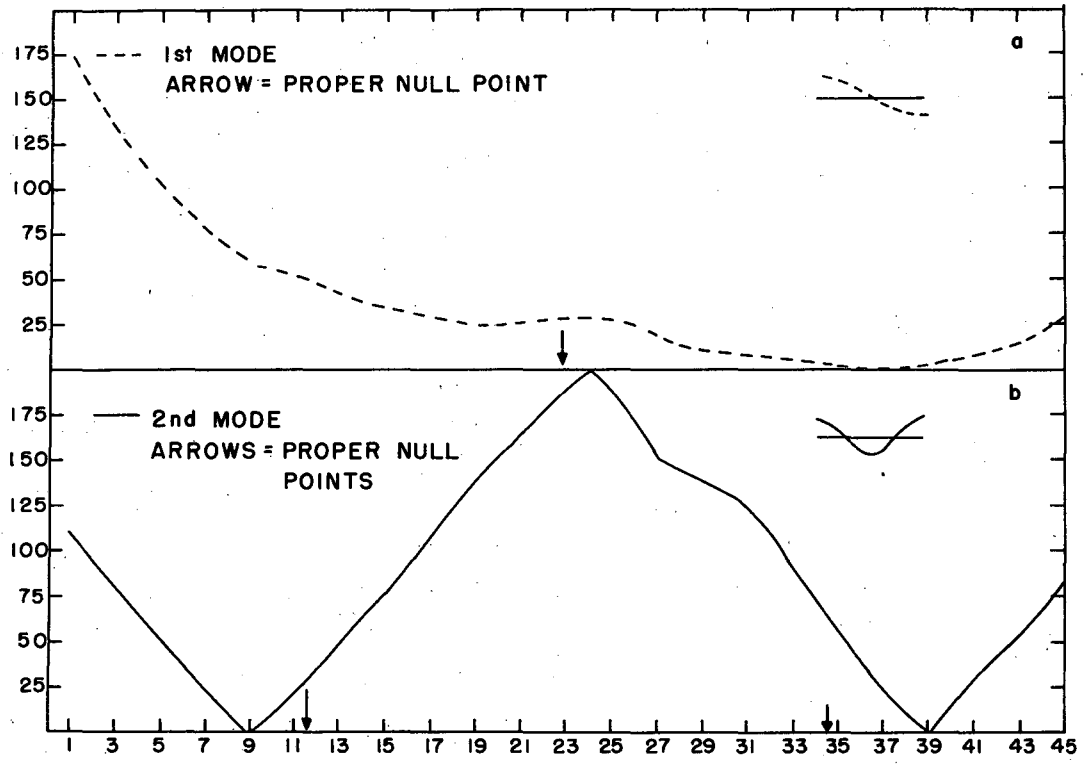
H vs. Z

MU 2672

Fig. 18

simultaneously and if the resultant field strength meter reading was the sum
of the two, the points checked were in phase. A difference reading, however,
indicated that the two points were of opposite polarity and a null existed.

Considering the shape of the 1st mode curve only, we could roughly esti-
mate the relative radius of the two ends of the cavity as indicated in Fig. 19.

#1   #45
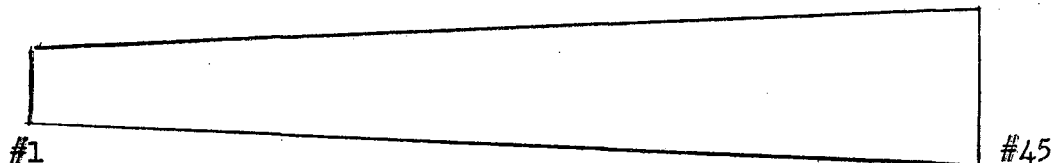
Fig. 19

In this configuration, the right end of the cavity is shown to be tuned to a
lower frequency than the left end. From Fig. 18b a relation can be obtained
of the middle section of the cavity to the two ends as shown in Fig. 20.
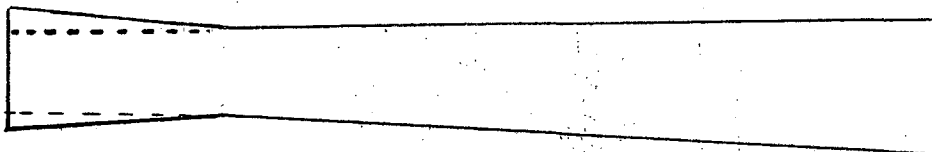
Fig. 20

Thus to move our null, the ends should be tuned to a higher frequency by
shortening the lengths of the drift tubes in the end regions. The drift
tubes at the right end, however, should be shortened more than those at the
left end.

A relation between the percent change in frequency and the percent change
in drift tube length $\Delta F/\Delta L$ was obtained from the one cell model cavity. The
figure so obtained agreed fairly well with numbers obtained from the drift
tube design group. It was found empirically, however, early in the tests,
that when the fields in a cavity were not uniform certain changes in one part

of the cavity were not equivalent to the same change made elsewhere. Also it was not evident that the calculated amount of field change resulted from a given drift tube adjustment.

The frequency shift resulting from a given change in drift tube length is not only a function of the amount of change but is also dependent upon the magnitude of the fields present at the drift tube relative to the magnitude of the fields elsewhere. The theory of the BB method of measuring electric fields shows that the frequency shift caused by the cutting out of a given volume of electric field is proportional to the square of the field strength. Thus when the field is non-uniform, larger changes in frequency occur where the fields are high than where the fields are low. This factor made the exact amount of correction required to shift the node a bit difficult to determine, and overshoot in the corrections was quite common.

The use of the radial beam system however, made this problem quite simple.

2. Radial Beam Tube

The radial beam system is the direct result of the desire to have a continuous visual presentation of the field configuration down the length of the experimental cavity. The method of using a single reading loop to measure the magnitude of the magnetic field at numerous points along the cavity, is perhaps more accurate. However, it is a lengthy task as these readings must be plotted before one can obtain a visual indication of the field.

Thought was given to the possibility of presenting the readings from a series of loops all located along the Z axis, on a cathode ray sweep. Essentially all that was required was a fast mechanical switch that would switch from probe to probe in sequence and in synchronism with the sweep of the oscilloscope. The radial beam system serves this function, acting as a

fast electronic switch.

Fifteen loops are coupled to the cavity at an even spacing and the output of each loop is then used to represent the average amplitude of a 6 in. section.

These signals are fed to a National Union RBE30G-3 electrostatically focused and deflected tube. This tube is basically 30 triode amplifiers with a common anode. As just 15 loops have been used, only 15 of the 30 sections of the tube are used. The grids of the tube, arranged in a circle about the cathode, are brought out separately. A common plate circles the grid structures with one single output lead.*

The beam of electrons from the encircled cathode is electrostatically swept from grid to grid by a 6 phase AC voltage applied to 6 screens or deflection plates. Small slits in these screen structures allow electrons to pass to the plate at a grid controlled rate, when the beam is in line with the slit and grid structure.

To facilitate construction, the sweep rate used is 60 cps. However, sweep rates in the order of one megacycle/sec can be used with the inclusion of proper 6 phase circuits.

A schematic of the basic sweep system is shown in Fig. 21. Six phase 60 cps AC voltage is provided by the phase shifting network. A variac T-1 allows control of the voltage level. A 10K potentiometer P-1 provides an adjustable negative voltage to the deflection plates to provide a focusing of the cathode beam.

---

* Actually the plate of one stage is separate and is brought out separately, which is useful for synchronization, but it is not used in this application.

T-I- O TO 135V A.C. VARIAC
M-I- O TO 400 V A.C. METER

PHASE-SHIFTING NETWORK (6 PHASE)

1/2a   T-1
1100 C
1/2 a

50K   50K   .05   .02
60K   50K   60K   50K
.05   .02   50K   50K

+ 375V

OUTPUT
AMPLIFIER

300 TO 500   8µ   100   8µ   20µ
D.C.   K
VARIABLE

TO SCOPE
.1µf

O TO 10 K
P-I

10 K

12 TRIODES ARE SHOWN IN RADIAL
BEAM TUBE, ACTUALLY 30 EXIST.

.1µ

20 K   500K   50K   8µf
                        60C7
                        450 Ω
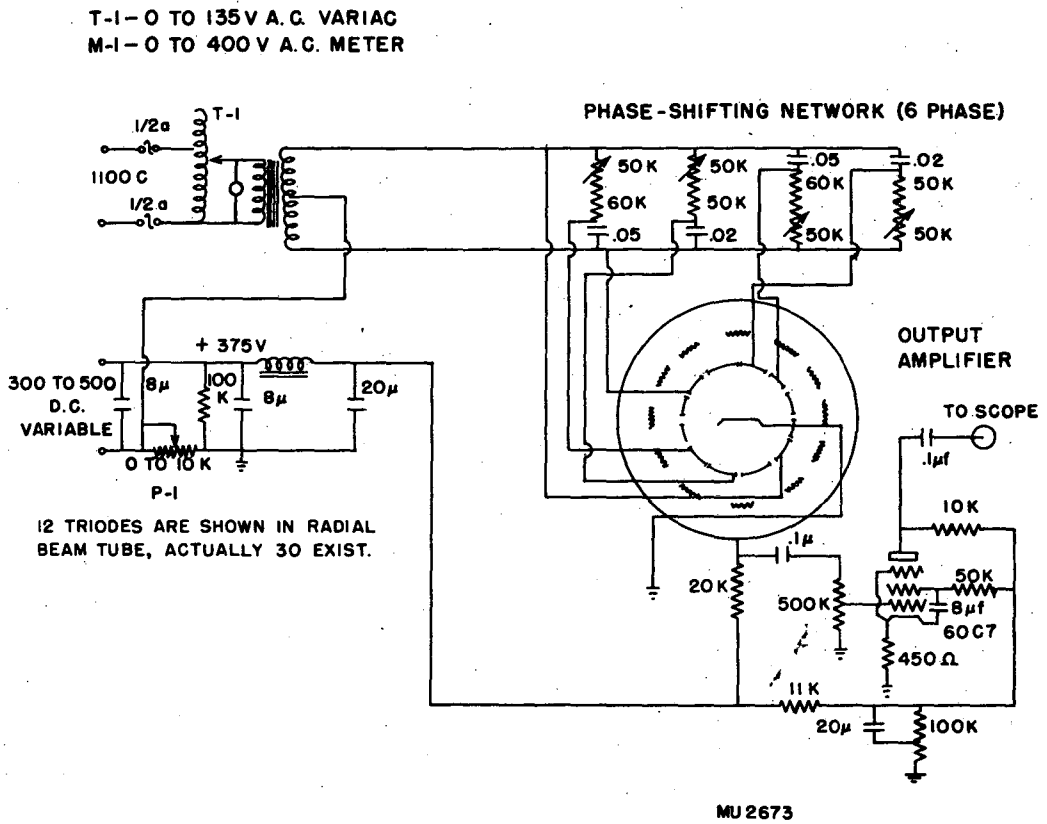11 K
20µ   100K

MU 2673

Fig. 21

In the common plate circuit is a 6AC7 amplifier which feeds the vertical
deflection plates of the oscilloscope on which the pattern is presented.
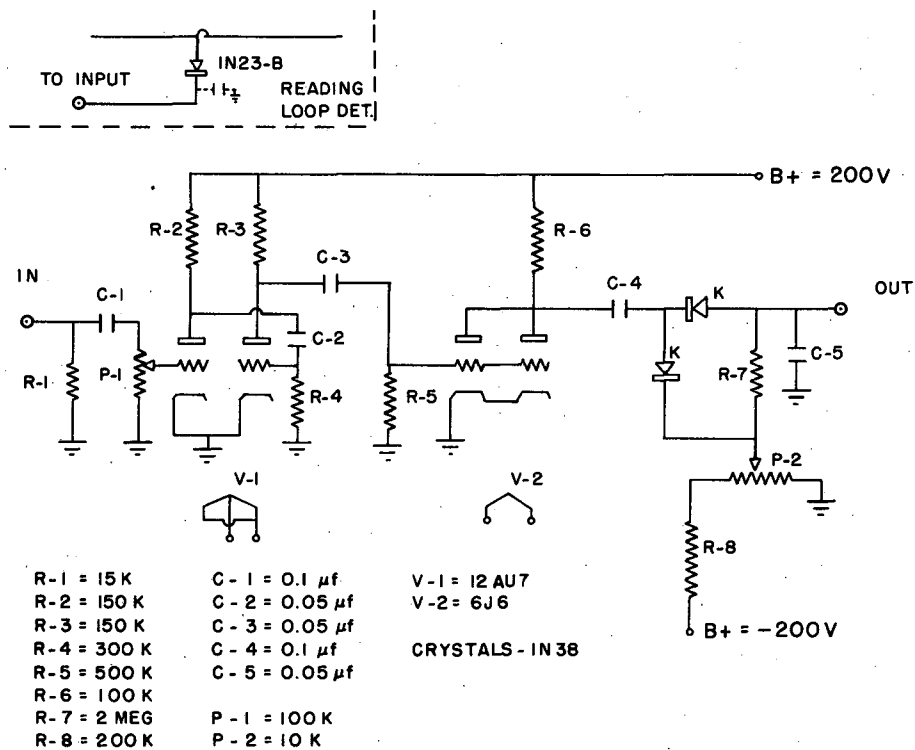
3. Radial Beam Amplifier.

In order to maintain as high a Q in the cavity as possible during radial
beam operation, it was necessary to couple a minimum of power from the cavity.
All power consumed by the crystal rectifiers while providing signal voltage
from the radial beam grids increases the value of the denominator in the formula,
$Q = 2\pi$ energy stored/energy lost per cycle. Hence, the presence of 15 power con-
suming loops can drastically effect the cavity Q. The Q requirement thus meant
that the loop coupling to the cavity should be very loose. Such coupling, how-
ever would require that the crystal rectifier outputs be amplified before be-
ing presented to the radial beam tube.

Figure 22 is a schematic of the 3 stage amplifier which followed each
pick-up loop. To eliminate the need for DC amplifiers, the signal generator
was operated in a pulsed rf manner. The pulse recurrence frequency was set
arbitrarily at 4000 cps and the pulse width at approximately 10 μ seconds,
which were the maximum settings of the "pulse rate" and "pulse width" controls
of the TS-403/U signal generator.

This type of cavity excitation allowed the use of conventional amplifier
stages, as an AC signal is now presented to the grid of the 1st stage.

The gain control for the amplifier is located in the grid circuit of
the input stage. The plate of the final tube, a 6J6, feeds a peak reading
rectifier circuit which rectifies the amplified signal and allows a straight
DC voltage to be fed to the grids of the radial beam tube.

Fifteen amplifier channels were constructed on a common chassis and they
amplify the signals from 15 cavity reading loops. The total gain of each

R-1 = 15 K
R-2 = 150 K
R-3 = 150 K
R-4 = 300 K
R-5 = 500 K
R-6 = 100 K
R-7 = 2 MEG
R-8 = 200 K

C - 1 = 0.1 µf
C - 2 = 0.05 µf
C - 3 = 0.05 µf
C - 4 = 0.1 µf
C - 5 = 0.05 µf

P - 1 = 100 K
P - 2 = 10 K

V -1 = 12 AU7
V -2 = 6J6

CRYSTALS - IN 38

MU 2674

Fig. 22

amplifier channel is approximately 2000.

4. Bias Control.

The bias controls serve to smooth out the uneven tube characteristics
of the radial beam triodes. They are adjusted with no signal connected to the
amplifier inputs and are set so that the output pips from the radial beam tube
are even and as small as possible. Then the various input channels are ad-
justed so that the overall gain of the channels, including the crystal recti-
fiers, is the same.

The graphic picture produced by the radial beam system allows sensitive
adjustments to be made while viewing the null position. Fig. 23 is a photo-
graph of the radial beam indicator, and shows the shift in the null of the
first mode caused by a 1.5 mil change in length of the 45th drift tube.

As this 1.5 mil adjustment was made in the region of maximum E, it pro-
duced the radical change pictured. From the theroy, this much shifting of the
null would have required a 1 mil motion in 10 drift tubes.

The situation is not so extreme as presented here because in calculations
from theory, one should allow for the fact that the fields are more concen-
trated between the nulls in the higher modes. In the first mode this should
amount to a factor of $\pi/2$ increase in effectiveness of the end drift tubes
in relation to the average but not the factor of 10 found experimentally.
However, the use of the radial beam tube display makes it relatively easy
to account for the different tuning functions empirically, thus allowing
for the correct changes to be made. Fig. 24 shows a 14 point radial beam
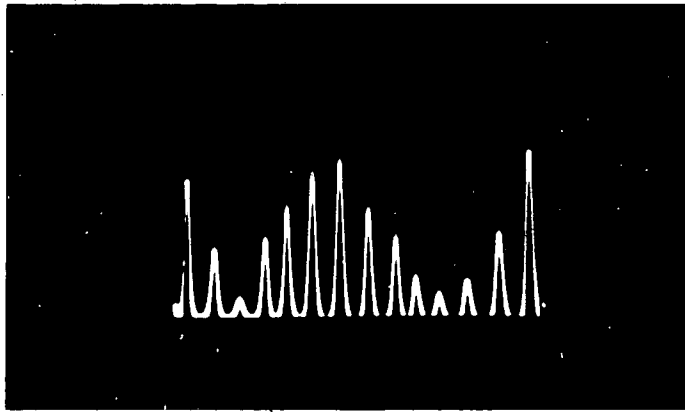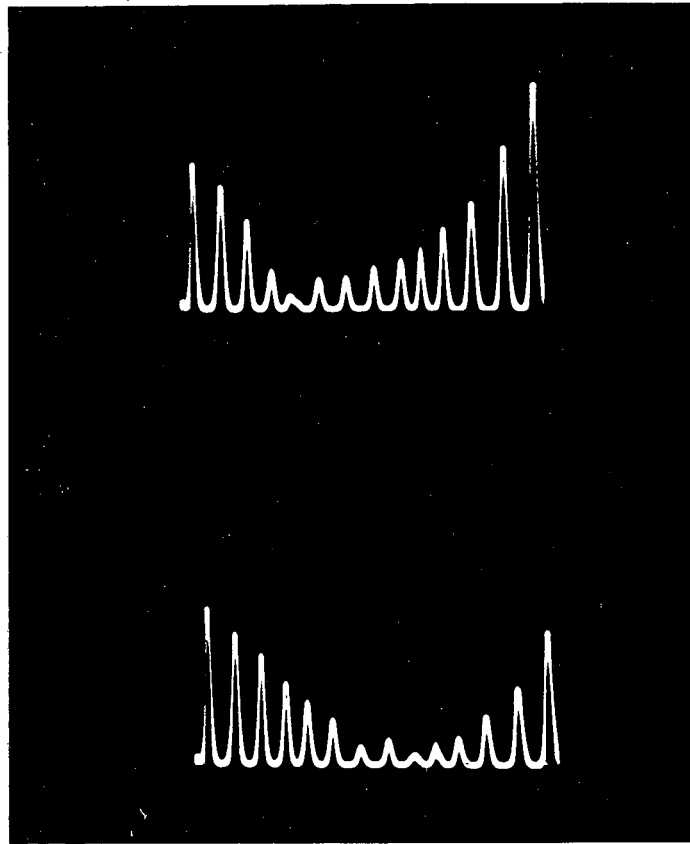presentation of the 2nd mode.

FIG. 23

FIG. 24

E. MODE FLATTENING

When the zeroeth mode was excited, after improving the cavity Q, a series of corrections were made according to the harmonic analysis. To insure that the corrections would not overshoot as a result of the non-uniform fields, only $\frac{1}{2}$ of the correction factor was used. Also, only the first two harmonics were initially analyzed. Fig. 25 consists of Graph I, initial distribution, and Graph II, after correcting 1st and 2nd harmonics, and Graph III and IV as described below.

A 4th harmonic correction was then made which lowered the content of this harmonic from 32 percent to 9 percent. This was also made on the $\frac{1}{2}$ correction basis of ΔF versus $\Delta L = 1$ mc/mil. It is to be noted that this correction also produced a change in the content of the 1st and 2nd harmonics. (Fig. 25 - Graph III.)

Finally all 5 harmonics were corrected, this time using the full correction factor of $\Delta F/\Delta L = 0.5$ mc/mil. (Graph IV - Fig. 25.) The resultant harmonic percentages in Graph IV are shown in Table I.

Table I

1st	$=$	2.2%
2nd	$=$	-4.3%
3rd	$=$	-0.06%
4th	$=$	0.6%
5th	$=$	-1.4%

These percentages would amount to the corrections per drift tube multiplied by the appropriate cosine shown in Table II.
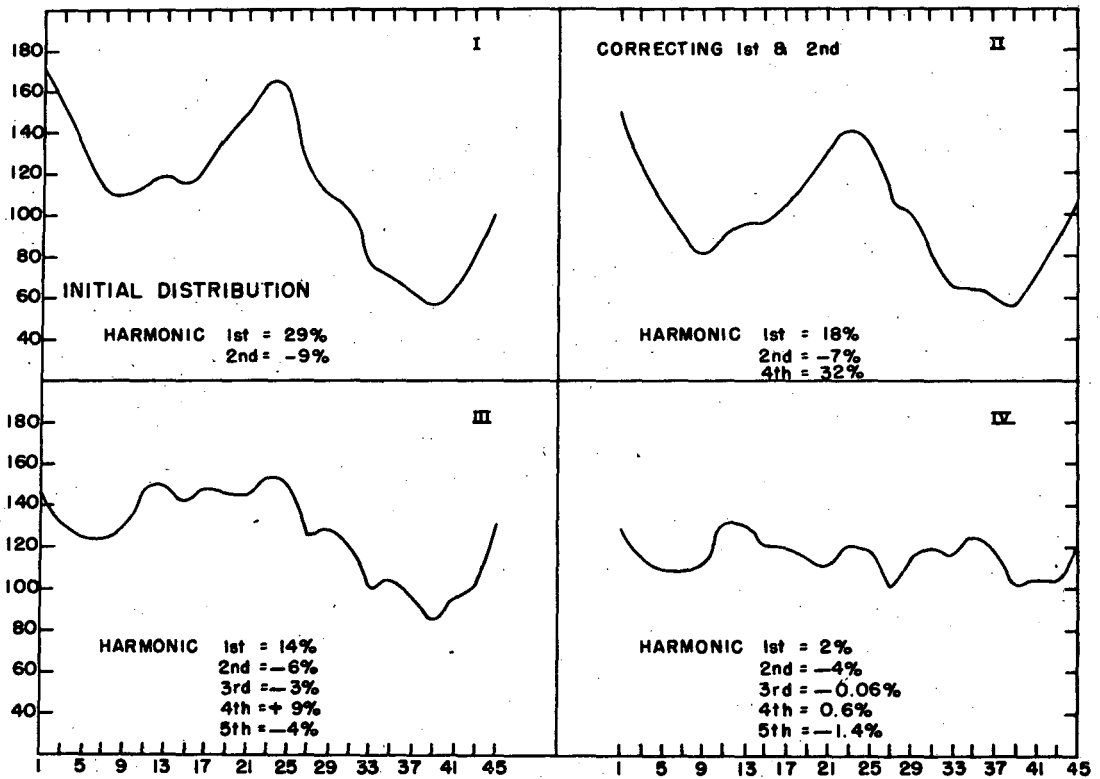
MU 2675

Fig. 25

Table II

1st	=	0.04 mils (0.00004 in)
2nd	=	0.31 mils
3rd	=	0.008 mils
4th	=	0.16 mils
5th	=	0.60 mils

Graph IV of Fig 25 is relatively flat in so far as contributions from the first five harmonics, however it is quite obvious that higher harmonic perturbations exist.

In connection with the overshoot mentioned previously it is interesting to note that while using $\frac{1}{2}$ of the correction factor the 1st and 2nd harmonic corrections gave 1/3 the required amount and the fourth gave nearly 2/3 the required amount.

Just how much of this variation can be related to errors in installing the correction is not known. However, the evidence is convincing that when higher harmonics are corrected, lower harmonic perturbations are made. That is, when the fields are not uniform initially, the cosine functions are not orthogonal and pure harmonics can not be put into the field distribution. However, the correction process seems to be convergent if done in the following manner: The low harmonics are roughly corrected first. Then the high harmonics are corrected and finally the low harmonics are again trimmed up.

This correction sequence was used to flatten the 90 in. cavity to the percentages for eight harmonics (Fig. 26a) listed in Table III

Table III

1st	=	-5.6%	5th	=	0
2nd	=	-0.9%	6th	=	+1 %
3rd	=	0	7th	=	+1.2%
4th	=	-3.6%	8th	=	-0.1%
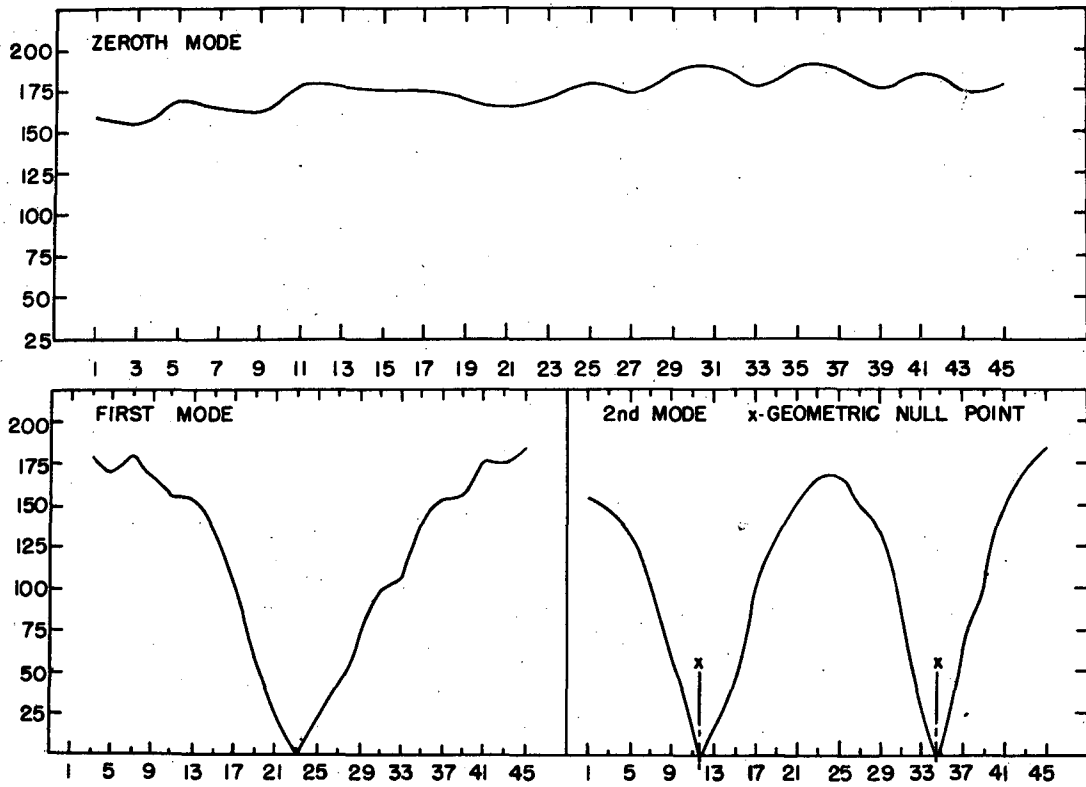
Fig. 26

The -5.6 percent of 1st harmonic remaining would have required a maximum
change of 0.1 mil (0.0001 in) and this was too small to correct with the in-
struments available.

Fig. 26 b and c are plots of the 1st and 2nd modes with the same cavity
configuration. It can be noted that the high harmonic perturbations which exist
in the zero mode are also apparent to a degree in the other two modes pictured.

The nulls of the first and second modes are located at the geometrically
correct position. The difference frequency between the zeroeth and first
modes was measured at 0.8 megacycles and between the first and second at 2.4 mc.
Both are close to the calculated spacings as predicted by Equation (3).

F. AUTOMATIC DEVICES.

To perform an adequate harmonic analysis of the field configuration in a
"long" cavity, it will be necessary to provide the following minimum information:

1. Indicate (on meters or otherwise) the presence of individual har-
monics. These indications should be continuous for each harmonic,
and not affected by the duty cycle of pulsed C. W. operation.

2. When the presence of any harmonic is indicated, the polarity of
the harmonic must also be shown.

3. The analysis should include sufficient harmonics, $N = (L/\lambda)$ harmonics
has been suggested (L. W. Alvarez, Engineering Notes, File M-24),
but this may be more than required after cold alignment is complete.

Many additions to the above requirements may be added, but it is felt
that should the above information be made available to an operator, he could,
with the addition of proper correction equipment, keep the field configuration
at its designed shape.

An additional requirement that may be needed (particularly if a non-uniform

field shape is to be used) would be the indication of the percentage of the harmonic present in all cases.

It is quite important also that the word "accuracy" be added to the list of requirements.

When one envisages the dimensions of the proposed machine, it is not difficult to understand that even the simplest of analysis and correction equipment will be quite complex. Thus, simplicity of equipment should be strived for, but not at the expense of accuracy. As cost is also a factor which must be considered, thought should be given to circuits which can provide accuracy with the use of less expensive components.

Several methods for making automatic harmonic analyses have been pointed out. In particular are two methods suggested by Dr. Alvarez in Engineering Notes M-24. One method measures the output of a series of audio filters into which are put the readings of each loop successively. The radial beam tube could be used to augment this system. The other method involves rotating the individual loops according to the appropriate cosine function and adding the signals algebraically at rf level.

A system suggested originally jointly by W. Panofsky and W. Brobeck consists of resistor chains from which $V \cos \frac{N \pi i}{L}$ can be read directly when voltage V is applied across a given chain. These readings can be added and read directly for each harmonic and the reading transferred to another resistor chain that does the synthesis; that is, the percent harmonic (multiplied by $n^2/8N^2$) is now applied across a resistor chain and various outputs taken off corresponding to the cosine of the positions of the various adjustors. One may obtain these singly or as a result of adding all harmonics simultaneously.

Each of these systems should be quite suitable on a full scale system. In all cases one must be careful to preserve phase so that corrections are applied in the right direction. Also it seems advisable that harmonic corrections be made individually, with a human element to transfer information so that instability and hunting will not result.

The systems proposed so far are perhaps more complex than need be to fulfill the basic requirements previously mentioned. It would thus seem desirable to investigate in more detail a basic simplified system. This system, in addition to providing simplicity, would be accurate and readily flexible for the inclusion of further automatic controls should such be deemed advisable.

G. SUMMARY OF BASIC SYSTEM

A number of small loops mounted along the Z axis, sample the relative strength of the magnetic field. The voltages induced in these loops are rectified and fed to resistor voltage dividers which comprise the R of the individual RC filter circuits. By proper tap points and wiring, (Fig. 27) these circuits can be made to read the presence of a harmonic plus the harmonic polarity. To provide for the continuous indication of several harmonics, several rows of loops would be needed, one for each harmonic to be indicated. It is recommended that the loops be mounted in the same relative position in each cavity cell for each harmonic. Thus as cell length increases, loop spacing will also increase. As several harmonics will be analyzed, additional loops must be added radially at each position. Corrections must then be made for the non-uniform spacing.

The total number of loops required will be large, and to provide uniformity and ease of construction they should be mass produced from common jigs. The resistors could be identical.

Figure 27 shows an arrangement for detecting the presence of a first harmonic. This circuit will indicate on a zero center, no load vacuum tube voltmeter the following relation:

$$V_1 \cos \frac{\pi Z_1}{L} + V_2 \cos \frac{\pi Z_2}{L} + V_3 \cos \frac{\pi Z_3}{L} = V_5 \cos \frac{\pi Z_5}{L}$$

$$- V_6 \cos \frac{\pi Z_6}{L} - V_7 \cos \frac{\pi Z_7}{L}$$

As Loop 4 of Fig. 27 falls at a point where the cos is zero, it is not connected and would not be installed.

By tapping the resistors at a different voltage point and rearranging the wiring slightly, the system can be changed to indicate a 2nd harmonic as illustrated in Fig. 28. The relation indicated is:

$$V_1 \cos \frac{2\pi Z_1}{L} + V_2 \cos \frac{2\pi Z_2}{L} - V_3 \cos \frac{2\pi Z_3}{L} - V_4 \cos \frac{2\pi Z_4}{L} - V_5 \cos \frac{2\pi Z_5}{L}$$

$$+ V_6 \cos \frac{2\pi Z_6}{L} + V_7 \cos \frac{2\pi Z_7}{L} .$$

Note - The sum of all cosines for any harmonic should be zero. If this is not true as in Fig. 28, loop positions must be altered or steps taken to balance out the reading error.

If it should be necessary to position a loop in a cell at a point that is not the same relative position occupied by the remaining loops an error in the field analysis can result. This error will be due to the fact that the amplitude of the magnetic field is not constant along the length of the loaded cell. Fig. 29b is a chart of the magnitude of the magnetic field vs L as measured along the wall of a model cavity. Fig. 29a is a diagram of the approximate geometry of the test cell in which $\beta = 0.6$. The variation pictured in Fig. 29b is more extreme than the variations that will be encountered. In all cases however, loop positioning can be made flexible by compensating for
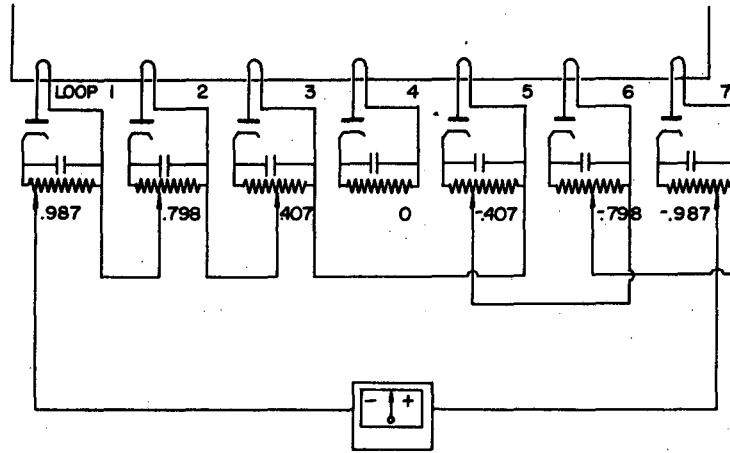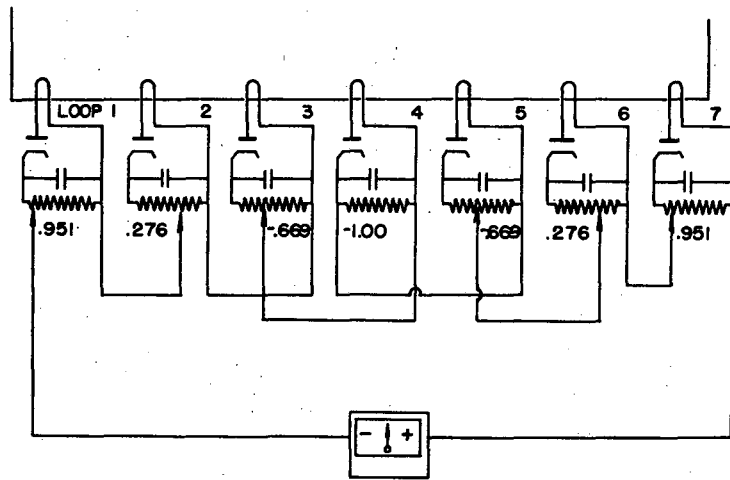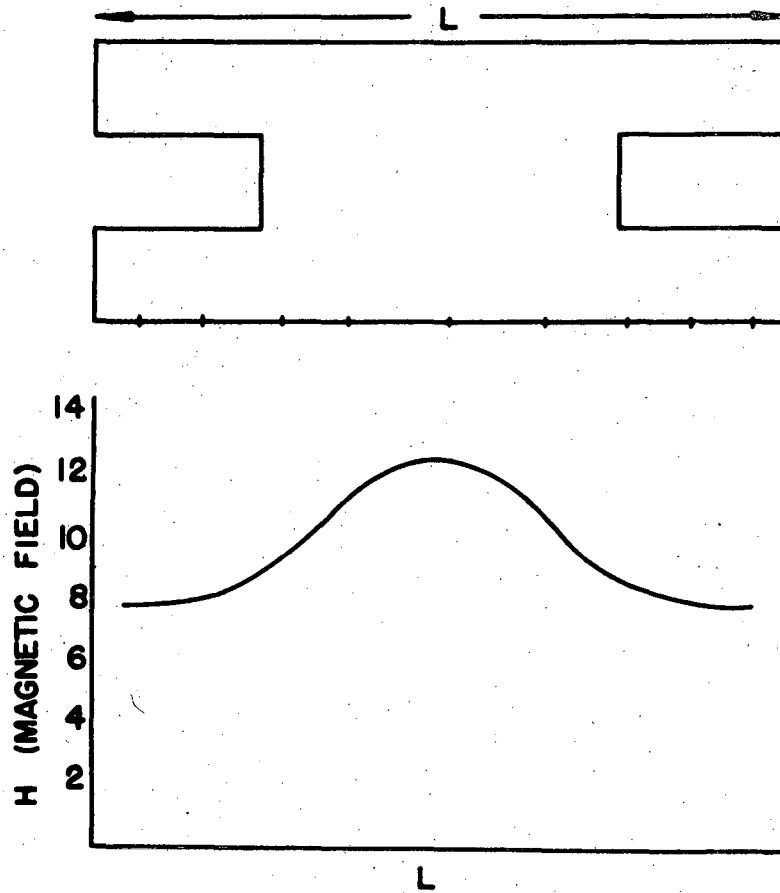
FIG. 27

FIG. 28

MU 2677

Fig. 29

such field changes. This compensation could be made by altering the tap point of the resistor strip involved.
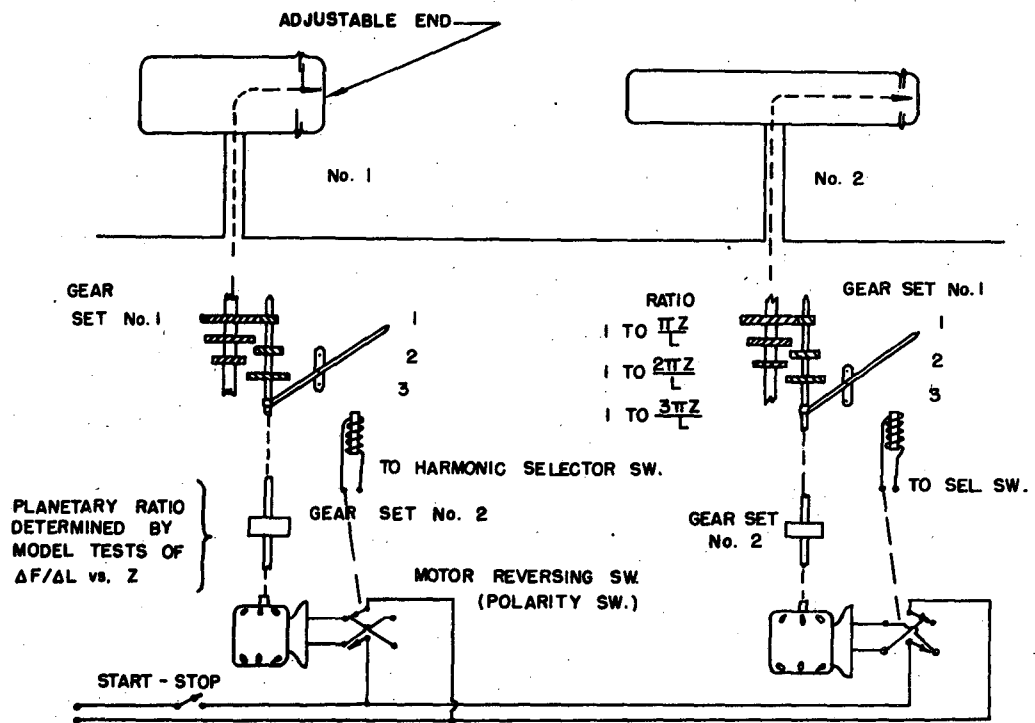
A number of systems similar to those shown in Figs. 27 and 28 can be installed to indicate the presence and polarity of any desired number of harmonics. A mechanism must now be considered, utilizing this information, to remove the indicated perturbations.

A basic mechanical system for synthesis would consist of the following arrangement: Each adjustable element will be controlled by two sets of gears. One set would have a gear ratio of 1 to $\cos \frac{n \pi Z}{L}$ and as several harmonics are to be individually corrected, a switchable gear ratio must be provided.

The cavity will be composed of numerous cells, all of which resonate at the same frequency. But because of design characteristics, they are not of the same geometry. Hence a certain change in one cell (drift tube length) will not produce the same frequency shift as a similar change in a different cell of the same cavity. To provide a constant frequency shift per cell a second set of gears is required which would have a gear ratio determined by $\Delta F/\Delta L$ vs Z. This set must be designed from data obtained from model tests.

Fig. 30 shows two drift tubes which can be assumed to be in opposite ends of a cavity. Of the three harmonic gear ratios shown (in Set #1) the first harmonic is engaged. This gear ratio will be changed as the different harmonics are switched in for correction. The harmonic selector switch and motor reversing switch must be coordinated to provide the proper direction of motor rotation with changes of sign of the harmonic and of $\cos \frac{n \pi Z}{L}$.

Gear Set #1 can be directly determined from the position of the adjustable element along the Z axis of the cavity. Gear Set #2 ($\Delta F/\Delta L$ vs Z) can be fixed and need not be switched once its ratio has been determined.

ADJUSTABLE END

No. 1   No. 2

GEAR
SET No. 1   RATIO   GEAR SET No. 1

1   1 TO $\frac{\pi Z}{L}$ 1

2   1 TO $\frac{2\pi Z}{L}$ 2
3   3
                     1 TO $\frac{3\pi Z}{L}$

TO HARMONIC SELECTOR SW.   TO SEL. SW.

PLANETARY RATIO
DETERMINED   BY   GEAR SET No. 2   GEAR SET
MODEL TESTS OF   No. 2
$\Delta F/\Delta L$ vs. Z

                     MOTOR REVERSING SW.
                     (POLARITY SW.)

START - STOP

MU 2679

Fig. 30

H. OPERATIONAL PROCEDURE

The operator notes on his meters which harmonic is being indicated. He sets his selector switch to this harmonic and also sets the polarity switch to the harmonic polarity indicated. The proper gear ratio and direction of motor rotation have thus been determined. He then pushed the "start" button which energizes the adjusting motors and holds the switch down until the meter has returned to zero. It is not important that he know that for a given harmonic percentage, the eighthharmonic would require 64 times the correction needed by a first harmonic. He would notice, however, that it was necessary to hold the switch on for a longer time, before the meter returned to zero. By correcting harmonics individually, the basic system eliminates the need for $\frac{n^2}{8N^2}$ information.

As mentioned previously, this system is considered as a basic one, and should it actually be used, considerable refinements should be included.