Title
HIGH FIELD MAGNET DEVELOPMENT ANALYSIS ""MECHANICS OF THIN RINGS WITH DIPOLE-LIKE LOADS

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PROGRAM — PROJECT — JOB

HIGH-FIELD MAGNET DEVELOPMENT

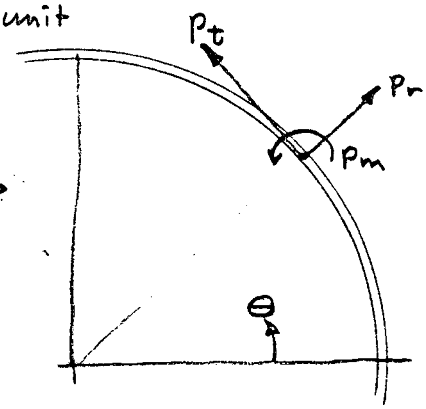
ANALYSIS

TITLE

MECHANICS OF THIN RINGS WITH DIPOLE-LIKE LOADS

This is a summary (believe it or not); no derivations are incl. Usual thin ring/shell assumptions apply: deflections due to extension of middle surface and shear are ignored.

We consider the following distributed unit loads on a circular thin ring.



$p_r = C_r \cos^2\theta$

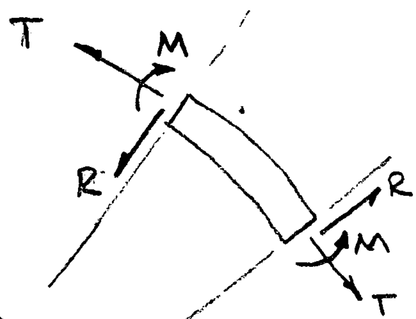
$p_t = C_t \sin\theta \cos\theta$

$p_m = C_m a \sin\theta \cos\theta$

for units, see page 6

These are the sorts of loads that can be produced by a circular-cross-section bending magnet; the quantitative correspondence will be covered later.

These loads result in the following internal forces in the ring:

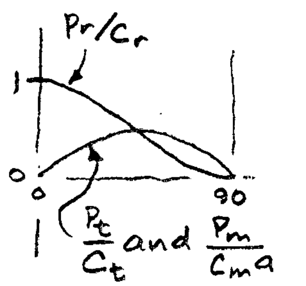


$$T = \tfrac{1}{3}\left[C_r(1+\sin^2\theta) + C_t(1-2\sin^2\theta)\right]$$

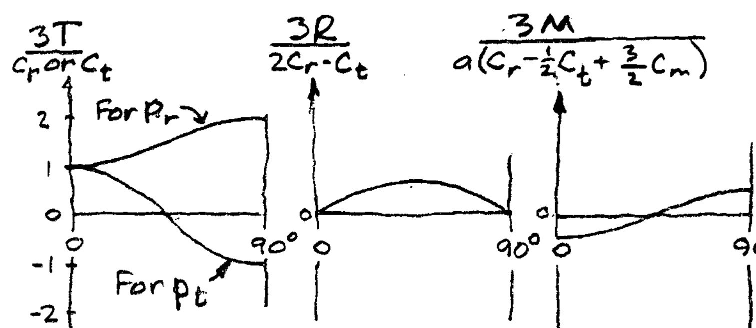
$$R = \tfrac{1}{3}(2C_r - C_t)\sin\theta\cos\theta$$

$$M = \tfrac{1}{3}a\left(C_r - \tfrac{1}{2}C_t + \tfrac{3}{2}C_m\right)\left(\sin^2\theta - \tfrac{1}{2}\right)$$

as illustrated below



APPLIED LOADS



INTERNAL FORCES

The maximum and minimum values are as follows:

$$T_{\theta=0} = \frac{1}{3} C_r \quad T_{\theta=90} = \frac{2}{3} C_r \quad \Bigg\} \text{ for } p_r \text{ only}$$

$$T_{\theta=\left|{0 \atop 90}\right|} = \pm \frac{1}{3} C_t \quad \text{for } p_t \text{ only}$$

$$R_{\theta=45} = \frac{1}{6}(2C_r - C_t)$$

$$M_{\theta=\left|{0 \atop 90}\right|} = \mp \frac{1}{6}\left(C_r - \frac{1}{2} C_t + \frac{3}{2} C_m\right) a$$

The body forces on an element of a thin winding having a cosine-θ distribution of currents are

$$dF_r = 2m \frac{B_B}{B_A} \cos^2\theta \, d\theta$$

$$dF_t = -2m\left(1 + \frac{B_B}{B_A}\right) \cos\theta \sin\theta \, d\theta$$

where $m \equiv \frac{B_A^2 \hat{a}}{4\pi}$ (cgs em units)

$\equiv \frac{B_A^2 \hat{a}}{\mu_0}$ (S.I. mks units)

B_A = central field contributed by the winding.
B_B = " " " " " iron yoke.

$\hat{a}$ = coil mean radius

(For a thick winding, see M5256, MEUSER, NOV 14, 1978

B_A and B_B are expressed in terms of the current density and geometry as follows

$$B_A = \frac{\mu_0 J_0}{2}, \quad B_B = B_A (a/b)^2$$

where $J = J_0 \cos\theta$, J_0 = peak current/circumference

The resulting loads on the surrounding rings are as follows:

For a coil that sticks to the rings, with no force transmitted tangentially within the coil, the body forces are transmitted directly to the rings:

$$p_r = \frac{dF_r}{d\theta} = 2m \frac{B_B}{B_A} \cos^2\theta$$

$$p_t = \frac{dF_t}{d\theta} = -2m\left(1 + \frac{B_B}{B_A}\right)\cos\theta \sin\theta$$

$$p_m = -\frac{dF_t}{d\theta}(a-\hat{a}) = 2m\left(1 + \frac{B_B}{B_A}\right)(a-\hat{a})\cos\theta \sin\theta$$

or

$$C_r = 2m\frac{B_B}{B_A}$$

$$C_t = -2m\left(1 + \frac{B_B}{B_A}\right)$$

$$C_m = 2m\left(1 + \frac{B_B}{B_A}\right)\frac{a-\hat{a}}{a}$$

(For applied bending moments, p_m, the philosophy is that the ring has a small but finite thickness, so shear stresses applied at the inside surface result in applied moments: $p_m = \tau(a-\hat{a})$.)

For a "slippery" coil, no shear is transmitted to the ring, but tangential forces are transmitted from element to element within the coil. The resulting radial loads are

$$p_r = m\left(1 + 3\frac{B_B}{B_A}\right)\cos^2\theta$$

or

$$C_r = m\left(1+3\frac{B_B}{B_A}\right), \quad C_t = 0, \quad C_m = 0$$

The internal ring forces expressed in terms of the aperture field components for a thin cosine-θ winding are:

For a sticky coil:

$$T_{\theta=0} = -\frac{2}{3}m$$

$$T_{\theta=90} = \frac{2}{3}m\left(1+3\frac{B_B}{B_A}\right)$$

$$R_{\theta=45} = \frac{1}{3}m\left(1+3\cdot\frac{B_B}{B_A}\right)$$

$$M_{\theta=\left|{0 \atop 90}\right|} = \mp ma\left[\left(1+3\frac{B_B}{B_A}\right)+3\left(1+\frac{B_B}{B_A}\right)\frac{a-\hat{a}}{a}\right]$$

For a slippery coil:

$$T_{\theta=0} = \frac{1}{3}m\left(1+3\frac{B_B}{B_A}\right)$$

$$T_{\theta=90} = \frac{2}{3}m\left(1+3\frac{B_B}{B_A}\right)$$

$$R_{\theta=45} = \frac{1}{3}m\left(1+3\frac{B_B}{B_A}\right)$$

$$M_{\theta=\left|{0 \atop 90}\right|} = \mp\frac{1}{6}ma\left(1+3\frac{B_B}{B_A}\right)$$

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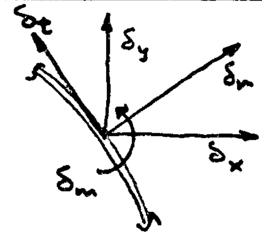

The local deflections are:

$$\delta_x = \frac{1}{3}\frac{M_0 a^2}{EI}\cos^3\theta$$

$$\delta_y = -\frac{1}{3}\frac{M_0 a^2}{EI}\sin^3\theta$$

$$\delta_r = \frac{1}{3}\frac{M_0 a^2}{EI}(\cos^2\theta - \sin^2\theta) = \frac{1}{3}\frac{M_0 a^2}{EI}\cos 2\theta$$

$$\delta_t = -\frac{1}{3}\frac{M_0 a^2}{EI}\sin\theta\cos\theta = -\frac{1}{3}\frac{M_0 a^2}{EI}\cdot\frac{1}{2}\sin 2\theta$$

$$\delta_m = -\frac{M_0 a}{EI}\sin\theta\cos\theta = -\frac{M_0 a}{EI}\cdot\frac{1}{2}\sin 2\theta$$

where $-M_0$ is the value of the bending moment at $\theta = 0$, or $+M_0$ is its value at $\theta = 90°$.

The maximum and minimum values are as follows

$$\delta_{x_{\theta=0}} = \delta_{r_{\theta=0}} = -\delta_{y_{\theta=90}} = -\delta_{r_{\theta=90}} = \frac{1}{3}\frac{M_0 a^2}{EI}$$

$$\delta_{t_{\theta=45°}} = -\frac{1}{6}\frac{M_0 a^2}{EI}$$

$$\delta_{m_{\theta=45°}} = -\frac{1}{2}\frac{M_0 a}{EI}$$

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Nomenclature

P_r, P_t, P_m	Applied loads, see sketch, p.1
C_r, C_t, C_m	See equations, top of p.1
a	Radius of ring
$\hat{a}$	Coil avg. radius
T, R, M	Internal forces, see sketch, p.1
dF_r, dF_t	Element body forces, radial and tangential
B_A	Field in aperture for no iron
B_B	Field in aperture from iron
b	Iron inside radius
m	see lower-middle, p.2
$\delta_x, \delta_y, \delta_r, \delta_t, \delta_m$	See sketch, p.5
E	Elastic modulus
I	Section moment of inertia

Units, dimensions: Any consistent units are OK except m is defined only in terms of SI and cgs-emu units

The loads, internal forces, and section modulus can be either for the whole ring or for unit axial length
In English units the dimensions are as follows:

	Whole ring	Unit length
$P_{r,t}$, $C_{r,t}$, R, T	lb/in	lb/in^2
P_m, C_m	in-lb/in	in-lb/in^2
M	in-lb	in-lb/in
dF_r, dF_t	lb/in	lb/in^2
m	lb/in	lb/in^2

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