

Lawrence Berkeley National Laboratory

Recent Work

Title

HIGH FIELD MAGNET DEVELOPMENT ANALYSIS ""MECHANICS OF THIN RINGS WITH DIPOLE-LIKE LOADS

Permalink

<https://escholarship.org/uc/item/5bx5n8dj>

Author

Meuser, Robert B.

Publication Date

1979-09-01

LBID-107 c.1

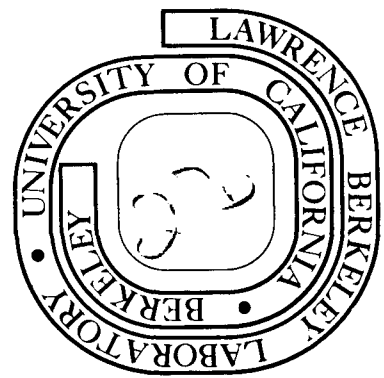
RECEIVED
LAWRENCE
BERKELEY LABORATORY

OCT 31 1979

LIBRARY AND
DOCUMENTS SECTION

For Reference

Not to be taken from this room



LBID - 107 c.1

DISCLAIMER

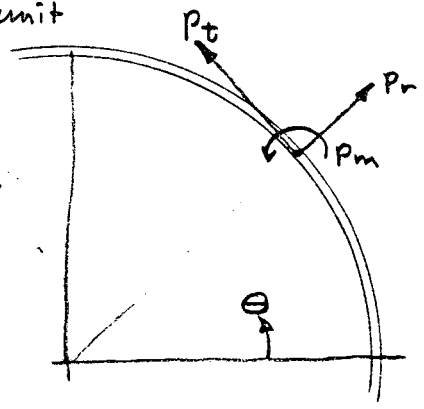
This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

LAWRENCE BERKELEY LABORATORY - UNIVERSITY OF CALIFORNIA		CODE	SERIAL	PAGE
ENGINEERING NOTE		MD 1111	M5399	1 of 6
AUTHOR	DEPARTMENT	LOCATION	DATE	
R. MEUSER	MECH. ENG.	BERK	SEP 17 1979	
PROGRAM - PROJECT - JOB				
HIGH-FIELD MAGNET DEVELOPMENT				
ANALYSIS				
TITLE				
MECHANICS OF THIN RINGS WITH DIPOLE-LIKE LOADS				

This is a summary (believe it or not); no derivations are incl. Usual thin ring/shell assumptions apply: deflections due to extension of middle surface and shear are ignored.

We consider the following distributed unit loads on a circular thin ring.

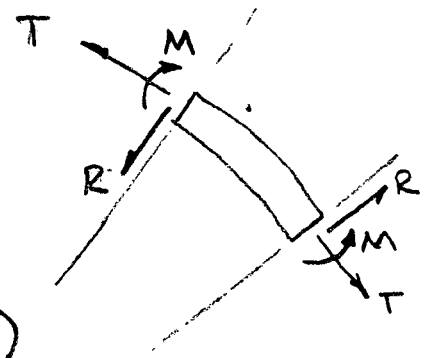
$$\begin{aligned}
 p_r &= C_r \cos^2 \theta && \text{for units,} \\
 p_t &= C_t \sin \theta \cos \theta && \text{see page 6} \\
 p_m &= C_m a \sin \theta \cos \theta
 \end{aligned}$$



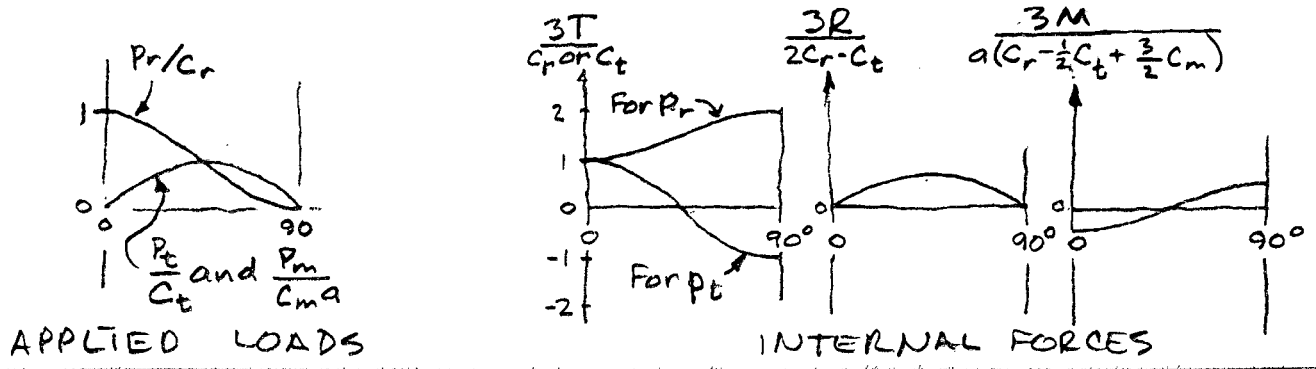
These are the sorts of loads that can be produced by a circular-cross-section bending magnet; the quantitative correspondence will be covered later.

These loads result in the following internal forces in the ring:

$$\begin{aligned}
 T &= \frac{1}{3} [C_r (1 + \sin^2 \theta) + C_t (1 - 2 \sin^2 \theta)] \\
 R &= \frac{1}{3} (2 C_r - C_t) \sin \theta \cos \theta \\
 M &= \frac{1}{3} a (C_r - \frac{1}{2} C_t + \frac{3}{2} C_m) (\sin^2 \theta - \frac{1}{2})
 \end{aligned}$$



as illustrated below



ENGINEERING NOTE

MD 1111

M5399

2 OF 6

AUTHOR

DEPARTMENT

LOCATION

DATE

R. MEUSER

MECH

BERK

SEP 17 1979

The maximum and minimum values are as follows:

$$\left. \begin{aligned} T_{\theta=0} &= \frac{1}{3} C_r \\ T_{\theta=90} &= \frac{2}{3} C_r \end{aligned} \right\} \text{for } p_r \text{ only}$$

$$T_{\theta=|\infty|} = \pm \frac{1}{3} C_t \quad \text{for } p_t \text{ only}$$

$$R_{\theta=45} = \frac{1}{6} (2C_r - C_t)$$

$$M_{\theta=|\infty|} = \mp \frac{1}{6} (C_r - \frac{1}{2} C_t + \frac{3}{2} C_m) a$$

The body forces on an element of a thin winding having a cosine- θ distribution of currents are

$$dF_r = 2m \frac{B_B}{B_A} \cos^2 \theta d\theta$$

$$dF_t = -2m \left(1 + \frac{B_B}{B_A}\right) \cos \theta \sin \theta d\theta$$

$$\text{where } m \equiv \frac{B_A^2 \hat{a}}{4\pi} \quad (\text{cgs em units})$$

$$\equiv \frac{B_A^2 \hat{a}}{\mu_0} \quad (\text{S.I. mks units})$$

B_A = central field contributed by the winding.

B_B = " " " " " " iron yoke.

\hat{a} = coil mean radius

(For a thick winding, see MS256, MEUSER, NOV 14, 1978)

B_A and B_B are expressed in terms of the current density and geometry as follows

$$B_A = \frac{\mu_0 J}{2}, \quad B_B = B_A (a/b)^2$$

where $J = J_0 \cos \theta$, J_0 = peak current/circumference

LAWRENCE BERKELEY LABORATORY - UNIVERSITY OF CALIFORNIA		CODE	SERIAL	PAGE
ENGINEERING NOTE		MD1111	M5399	3 OF 6
AUTHOR	DEPARTMENT	LOCATION	DATE	
R. MEUSER	MECH	BERK	SEP 17 1979	

The resulting loads on the surrounding rings are as follows:

For a coil that sticks to the rings, with no force transmitted tangentially within the coil, the body forces are transmitted directly to the rings:

$$P_r = \frac{dF_r}{d\theta} = 2m \frac{B_B}{B_A} \cos^2 \theta$$

$$P_t = \frac{dF_t}{d\theta} = -2m \left(1 + \frac{B_B}{B_A}\right) \cos \theta \sin \theta$$

$$p_m = -\frac{dF_t}{d\theta} (a - \hat{a}) = 2m \left(1 + \frac{B_B}{B_A}\right) (a - \hat{a}) \cos \theta \sin \theta$$

or

$$C_r = 2m \frac{B_B}{B_A}$$

$$C_t = -2m \left(1 + \frac{B_B}{B_A}\right)$$

$$C_m = 2m \left(1 + \frac{B_B}{B_A}\right) \frac{a - \hat{a}}{a}$$

(For applied bending moments, p_m , the philosophy is that the ring has a small but finite thickness, so shear stresses applied at the inside surface result in applied moments: $p_m = \uparrow (a - \hat{a})$.)

ENGINEERING NOTE

MD1111

M5399

4 of 6

AUTHOR

DEPARTMENT

LOCATION

DATE

R. MEUSER

MECH

BERK

SEP 17 1979

For a "slippery" coil, no shear is transmitted to the ring, but tangential forces are transmitted from element to element within the coil. The resulting radial loads are

$$P_r = m \left(1 + 3 \frac{B_B}{B_A} \right) \cos^2 \theta$$

or

$$C_r = m \left(1 + 3 \frac{B_B}{B_A} \right), \quad C_t = 0, \quad C_m = 0$$

The internal ring forces expressed in terms of the aperture field components for a thin cosine- θ winding are:

for a sticky coil:

$$T_{\theta=0} = -\frac{2}{3} m$$

$$T_{\theta=90} = \frac{2}{3} m \left(1 + 3 \frac{B_B}{B_A} \right)$$

$$R_{\theta=45} = \frac{1}{3} m \left(1 + 3 \frac{B_B}{B_A} \right)$$

$$M_{\theta=|0|} = \mp ma \left[\left(1 + 3 \frac{B_B}{B_A} \right) + 3 \left(1 + \frac{B_B}{B_A} \right) \frac{a-\hat{a}}{a} \right]$$

for a slippery coil:

$$T_{\theta=0} = \frac{1}{3} m \left(1 + 3 \frac{B_B}{B_A} \right)$$

$$T_{\theta=90} = \frac{2}{3} m \left(1 + 3 \frac{B_B}{B_A} \right)$$

$$R_{\theta=45} = \frac{1}{3} m \left(1 + 3 \frac{B_B}{B_A} \right)$$

$$M_{\theta=|0|} = \mp \frac{1}{6} ma \left(1 + 3 \frac{B_B}{B_A} \right)$$

ENGINEERING NOTE

MD 1111

M5399

5 OF 6

AUTHOR

DEPARTMENT

LOCATION

DATE

R. MEUSER

MELU

BERK

SEP 17 1979

The local deflections are:

$$\delta_x = \frac{1}{3} \frac{M_0 a^2}{EI} \cos^3 \theta$$

$$\delta_y = -\frac{1}{3} \frac{M_0 a^2}{EI} \sin^3 \theta$$

$$\delta_r = \frac{1}{3} \frac{M_0 a^2}{EI} (\cos^2 \theta - \sin^2 \theta) = \frac{1}{3} \frac{M_0 a^2}{EI} \cos 2\theta$$

$$\delta_t = -\frac{1}{3} \frac{M_0 a^2}{EI} \sin \theta \cos \theta = -\frac{1}{3} \frac{M_0 a^2}{EI} \cdot \frac{1}{2} \sin 2\theta$$

$$\delta_m = -\frac{M_0 a}{EI} \sin \theta \cos \theta = -\frac{M_0 a}{EI} \cdot \frac{1}{2} \sin 2\theta$$

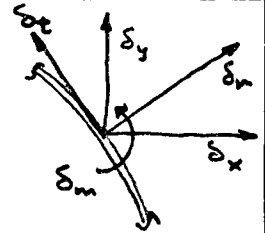
where $-M_0$ is the value of the bending moment at $\theta = 0$, or $+M_0$ is its value at $\theta = 90^\circ$.

The maximum and minimum values are as follows

$$\delta_x \theta=0 = \delta_r \theta=0 = -\delta_y \theta=90 = -\delta_r \theta=90 = \frac{1}{3} \frac{M_0 a^2}{EI}$$

$$\delta_t \theta=45^\circ = -\frac{1}{6} \frac{M_0 a^2}{EI}$$

$$\delta_m \theta=45^\circ = -\frac{1}{2} \frac{M_0 a}{EI}$$



ENGINEERING NOTE

MD1111

M5399

6 OF 6

AUTHOR

DEPARTMENT

LOCATION

DATE

R. MEUSER

MECH

BERK

SEP 17 1979

Nomenclature

P_r, P_t, P_m	Applied loads, see sketch, p. 1
C_r, C_t, C_m	See equations, top of p. 1
a	Radius of ring
\hat{a}	Coil avg. radius
T, R, M	Internal forces, see sketch, p. 1
dF_r, dF_t	Element body forces, radial and tangential
B_A	Field in aperture for no iron
B_B	Field in aperture from iron
b	Iron inside radius
m	see lower-middle, p. 2
$\delta_x, \delta_y, \delta_r, \delta_t, \delta_m$	See sketch, p. 5
E	Elastic modulus
I	Section moment of inertia

Units, dimensions: Any consistent units are OK except m is defined only in terms of SI and cgs-emu units

The loads, internal forces, and section modulus can be either for the whole ring or for unit axial length
In English units the dimensions are as follows:

	Whole ring	Unit length
P_{rt}, C_{rt}, R, T	lb/in	lb/in ²
P_m, C_m	in-lb/in	in-lb/in ²
M	in-lb	in-lb/in
dF_r, dF_t	lb/in	lb/in ²
m	lb/in	lb/in ²

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720