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Axion Isocurvature and Magnetic Monopoles

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Abstract

We propose a simple mechanism to suppress axion isocurvature fluctuations using hidden sector magnetic monopoles. This allows for the Peccei-Quinn scale to be of order the unification scale consistently with high scale inflation.

1 Introduction

Cosmic inflation not only provides a framework to address many puzzles of early universe cosmology [1, 2] but also incorporates a mechanism that seeds the formation of the structure in the universe [3]. An exciting aspect of the inflationary mechanism is that it also sources gravitational waves. If inflation occurs at a sufficiently high scale ($\sim 10^{15}$–10^{16} GeV), the amplitude of these gravitational waves is large enough to leave a measurable imprint on the polarization of the cosmic microwave background (CMB) [4]. A number of CMB polarization experiments are presently searching for this signal [5]. A positive signal in such an experiment would have interesting implications for particle physics, especially for ultra-light bosonic fields. Bosonic fields with masses lighter than the inflationary Hubble scale are efficiently produced by inflation and can cause isocurvature perturbations in the CMB [6]. High scale inflation thus leads to interesting constraints on ultra-light bosons, including the QCD axion provided the axion decay constant f_a is greater than the inflationary scale.

It is widely regarded [7] that a discovery of inflationary gravitational waves would rule out the QCD axion with a decay constant $f_a \gtrsim 10^{16}$ GeV, a range that is favored by several theoretical considerations [8]. Experiments have also been proposed recently to search for the QCD axion in this parameter range [9], and it is of great interest to delineate the viable parameter space accessible to these efforts. For example, this bound disappears if the QCD axion acquires a large mass during inflation, damping the production of isocurvature modes. At the end of inflation, however, this mass has to nearly vanish for the QCD axion to solve the strong CP problem. While models achieving this do exist (see [10] for example), they face the difficulty that the mechanism responsible for generating a large axion mass during inflation has to violate the Peccei-Quinn symmetry while ensuring that this violation remains sufficiently sequestered from the axion after inflation. This task is made even more difficult by the fact that these dynamics must couple to the inflaton. Other proposals to alleviate the tension between high scale inflation and the QCD axion include a dynamically changing Peccei-Quinn breaking scale [11]. While reasonable, such models sacrifice some of the theoretical arguments underlying high f_a axions. There are also attempts that involve transfer of the axion isocurvature from one species to another [12], but these typically deplete the dark matter abundance of the axion, eliminating one of the promising ways to search for them. It might also be possible to relax these constraints by dumping entropy into the universe around the QCD phase transition [13], but these channels are constrained [7].

In this paper, we investigate an alternative possibility: what if the QCD axion acquires a large mass *after* inflation, which subsequently disappears *before* the QCD phase transition? If this mass is larger than the Hubble scale during a large interval, somewhere between the reheating and QCD scales, then the axion field oscillates earlier and the fluctuations in the field will be damped, relaxing into the minimum of the potential generating this large mass. When this mass (and potential)

subsequently disappears, the average axion field takes a value corresponding to this minimum. Since this minimum is in general displaced from the QCD minimum, the misalignment between these two points regenerate a cosmic abundance of the QCD axion when the axion reacquires a mass during the QCD phase transition, enabling it to be dark matter. The isocurvature perturbations, however, will be small since the initial evolution of the field causes the perturbations to coalesce around the initial minimum, while the subsequent dark matter abundance is generated by the homogeneous misalignment between the QCD minimum and the initial minimum.

How can we give such a large initial mass that then disappears almost completely? We accomplish this by coupling the QCD axion to a new $U(1)'$ gauge group. If the reheating of the universe produces magnetic monopoles under this $U(1)'$, the monopole density generates a mass for the axion [14]. This is because topological terms like $F\tilde{F}$ become physical in the presence of magnetic monopoles due to the Witten effect [15]. Specifically, it gives a free energy density that depends on a background axion field value, thus creating an effective mass for the axion. This mass is sufficient to damp isocurvature perturbations in the axion field. After the perturbations have been damped, the monopole density can be efficiently eliminated by breaking the $U(1)'$ symmetry, resulting in confinement and subsequent annihilation of the monopoles. The monopole density forces the axion field to relax into θ', a point on the potential chosen by CP phases in the $U(1)'$ sector. Since this phase need not be aligned with the QCD minimum at θ_{QCD}, the axion generally acquires a homogeneous cosmic abundance during the QCD phase transition, with suppressed isocurvature perturbations. For large $f_a \gg 10^{12}$ GeV, this misalignment needs to be small, $|\theta' - \theta_{\mathrm{QCD}}| \ll 1$, but this can be environmentally selected [16]. We show that there is sufficient time for the damping of axion isocurvature fluctuations so that axion dark matter with a unification scale f_a is consistent with high scale inflation giving an observable size of the gravitational wave polarization signal.

The organization of this paper is as follows. In Section 2, we review the required amount of damping of axion isocurvature fluctuations consistent with current observations. In Section 3 we introduce our basic mechanism, and in Section 4 we present a minimal model realizing it. We show that the model can consistently accommodate unification scale axion dark matter with high scale (unification scale) inflation. In Section 5, we discuss monopole annihilations due to $U(1)'$ breaking in detail, showing that they efficiently eliminate monopoles. In Section 6, we discuss extensions/modifications of the minimal model in which the issue of radiative stability of the $U(1)'$ sector existing in the minimal model does not arise. We conclude in Section 7.

2   Required Damping of Isocurvature Perturbations

Inflation generally induces quantum fluctuations of order $H_{\mathrm{inf}}/2\pi$ for any massless field, where H_{inf} is the Hubble parameter during inflation. This implies that if $U(1)_{\mathrm{PQ}}$ is broken before or during

inflation, then the angle $\theta = a/f_a$ of the axion field a has fluctuations

$$\delta\theta(T_{\text{R}}) \approx \frac{H_{\text{inf}}}{2\pi f_a}, \tag{1}$$

at temperature T_{R}, when the radiation dominated era starts due to reheating.[1] Since the axion potential is flat during inflation, these fluctuations are converted to isocurvature density perturbations upon the generation of the axion mass.

There is a tight constraint on the amount of allowed isocurvature perturbations from the Planck data [17], which can be written as (see, e.g., [18])

$$\frac{\Omega_a}{\Omega_{\text{DM}}} \frac{\delta\theta(T_{\text{QCD}})}{\theta_{\text{mis}}} \lesssim 4.8 \times 10^{-6}, \tag{2}$$

where θ_{mis} is the average axion misalignment angle, while $\delta\theta(T_{\text{QCD}})$ is the angle fluctuation of the axion field at temperature $T_{\text{QCD}} \sim 1$ GeV. Here, Ω_a and $\Omega_{\text{DM}} \simeq 0.24$ represent the axion and total dark matter abundances, respectively, and we assume $\theta_{\text{mis}} > \delta\theta(T)$ throughout.[2] Using the expression for the axion relic density

$$\frac{\Omega_a}{\Omega_{DM}} \approx 1.0 \times 10^5 \, \theta_{\text{mis}}^2 \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{1.19}, \tag{3}$$

(which requires $\theta_{\text{mis}} \lesssim 0.003$ for $f_a \simeq 10^{16}$ GeV, possibly realized through environmental selection effects [16]), we may rewrite Eq. (2) as

$$\delta\theta(T_{\text{QCD}}) \lesssim 1.5 \times 10^{-8} \sqrt{\frac{\Omega_{\text{DM}}}{\Omega_a}} \left(\frac{10^{16} \text{ GeV}}{f_a} \right)^{0.6}. \tag{4}$$

Assuming the standard cosmological history after inflation, $\delta\theta(T_{\text{QCD}}) \approx \delta\theta(T_{\text{R}})$, so that we find

$$H_{\text{inf}} \lesssim 9.4 \times 10^8 \text{ GeV} \sqrt{\frac{\Omega_{\text{DM}}}{\Omega_a}} \left(\frac{f_a}{10^{16} \text{ GeV}} \right)^{0.4}. \tag{5}$$

This severely constrains inflationary models in the presence of a unification scale axion [7]. In particular, unification scale axion dark matter—$\Omega_a = \Omega_{\text{DM}}$ and $f_a \sim 10^{16}$ GeV—is inconsistent with unification scale inflation—$E_{\text{inf}} \equiv V_{\text{inf}}^{1/4} \sim 10^{16}$ GeV, which leads to $H_{\text{inf}} = E_{\text{inf}}^2/\sqrt{3}\bar{M}_{\text{Pl}} \sim 10^{13}$ GeV, where $\bar{M}_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale.

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[1]In this paper we adopt the instant reheating approximation for simplicity, so that the universe is radiation dominated right after inflation. An extension of our analysis to more general cases (including a matter dominated era before reheating) is straightforward.

[2]This condition requires $H_{\text{inf}} \lesssim 2 \times 10^{14}$ GeV $\sqrt{\Omega_a/\Omega_{\text{DM}}}(f_a/10^{16}$ GeV$)^{0.4}$; for comparison, see Eq. (5) and an estimate below it for unification scale inflation.

Below, we discuss a scenario in which axion isocurvature fluctuations are damped due to dynamics after inflation. Defining the (inverse) damping factor Δ by

$$\Delta = \frac{\delta\theta(T_{\mathrm{QCD}})}{\delta\theta(T_{\mathrm{R}})}, \tag{6}$$

Eq. (4) yields

$$\Delta \lesssim 1 \times 10^{-4} \sqrt{\frac{\Omega_{\mathrm{DM}}}{\Omega_a}} \left(\frac{f_a}{10^{16}\ \mathrm{GeV}}\right)^{0.4} \left(\frac{10^{13}\ \mathrm{GeV}}{H_{\mathrm{inf}}}\right). \tag{7}$$

Here, we have normalized f_a and H_{inf} by the values corresponding to unification scale axion and inflation, respectively. This gives the required amount of damping.

3   Basic Mechanism

Our basic idea of suppressing axion isocurvature fluctuations is that the axion mass obtains extra contributions beyond that from QCD in the early universe so that it is larger than the Hubble parameter in some period. In this period, axion isocurvature perturbations are reduced because of the damped oscillations of the axion field, giving $\Delta < 1$.

We do this by introducing a coupling of the axion to a hidden $U(1)'$ gauge group

$$\mathcal{L} \sim \frac{1}{f_a}\, a F'^{\mu\nu} \tilde{F}'_{\mu\nu}. \tag{8}$$

We assume that at some temperature T_M after inflation ($T_M \lesssim T_{\mathrm{R}} \approx E_{\mathrm{inf}}$), monopoles of $U(1)'$ are created. This can happen, for example, if a hidden sector $SU(2)'$ gauge group is broken to $U(1)'$ at that scale.[3] In the presence of magnetic monopoles, the coupling in Eq. (8) induces an effective mass for the axion [14]:

$$m_a^2(T) = \gamma \frac{n_M(T)}{f_a}, \tag{9}$$

where γ is determined by the structure of the $U(1)'$ sector, such as the gauge coupling and matter content. (γ may in general depend on temperature, although it is not the case in the explicit model considered below.) $n_M(T)$ is the number density of the monopoles; assuming the abundance determined by the Kibble-Zurek mechanism [19], we find

$$n_M(T) \approx \alpha \left(\frac{T}{T_M}\right)^3 H(T_M)^3, \tag{10}$$

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[3]If the creation of monopoles is associated with $G \to G' \times U(1)'$ symmetry breaking in the hidden sector, where G and G' are non-Abelian gauge groups, then we would need to have two axion fields in the ultraviolet so that the QCD axion remains after G' gives a large mass to one linear combination of the two axion fields.

where $H(T)$ is the Hubble parameter at temperature T, and $\alpha \gtrsim 1$.[4] The contribution of Eq. (9) makes the axion mass effect dominates over the Hubble friction

$$m_a(T) \gtrsim 3H(T), \tag{11}$$

below some temperature T_i ($\leq T_M$), so that the axion field is subject to damped oscillations for $T \lesssim T_i$.

We assume that $U(1)'$ is spontaneously broken at some temperature T_f ($\ll T_i$), so that monopoles quickly disappear.[5] Axion isocurvature fluctuations are then damped efficiently between temperatures T_i and T_f. Suppose

$$m_a^2(T) \propto T^n, \tag{12}$$

($n = 3$ for a constant γ). Since the axion "number density" $m_a(T)\delta\theta(T)^2$ scales as T^3 while Eq. (11) is satisfied, we find

$$\delta\theta(T) \propto T^p, \qquad p \approx \frac{6-n}{4}, \tag{13}$$

in this period. The final damping factor is thus

$$\Delta \approx \left(\frac{T_f}{T_i}\right)^{\frac{6-n}{4}}, \tag{14}$$

which can be compared with the required amount of damping from observations, Eq. (7).

Note that the average axion field $\langle\theta\rangle = \langle a\rangle/f_a$ after the operation of this damping mechanism is determined by the structure of the hidden sector (the original hidden sector $\bar\theta$ parameter), which in general differs from the minimum of the late-time axion potential, θ_{QCD}. A homogeneous displacement of the axion field from θ_{QCD}, determining the late-time axion dark matter abundance, is not controlled by the present mechanism, unless we make an extra assumption. For $f_a \gg 10^{12}$ GeV, the value of this displacement must be small, but it can be environmentally selected to be consistent with $\Omega_a \leq \Omega_{\text{DM}}$ [16].

4   Minimal Model

We now consider the minimal model in which the $U(1)'$ sector below T_M contains only a charged scalar field φ, which breaks $U(1)'$ at scale T_f ($\ll T_M$). In this case, the factor γ in the expression for the induced axion mass, Eq. (9), is

$$\gamma \approx \tilde\gamma \frac{T_M}{f_a}, \tag{15}$$

---

[4]Note that α can be much larger than $O(1)$, depending on the dynamics of the phase transition; see e.g. [20]. In this case, monopole-antimonopole annihilations at $T \sim T_M$ may become important; see Section 6.1 for such a scenario.

[5]An alternative possibility will be discussed in Section 6.2.

where we have used $T_M \lesssim f_a$, and $\tilde{\gamma} \approx O(1)$ assuming that the $U(1)'$ gauge coupling is of order unity.[6] The axion mass just after the monopole production is then given by

$$\frac{m_a(T_M)}{3H(T_M)} \simeq 0.2\sqrt{\alpha\tilde{\gamma}}\, g_{*M}^{\frac{1}{4}}\sqrt{\frac{T_M^3}{f_a^2 \bar{M}_{\text{Pl}}}}, \tag{16}$$

where we have used $H(T_M) = \rho(T_M)^{1/2}/\sqrt{3}\bar{M}_{\text{Pl}}$ and $\rho(T_M) = (\pi^2/30)g_{*M}T_M^4$ with g_{*M} being the effective number of relativistic degrees of freedom at temperature T_M. Assuming that T_M is not much smaller than the unification scale, this number is roughly of order unity (and at least not too much smaller than of order unity). The axion field thus starts having damped oscillations at $T \sim T_i$, within a few orders of magnitude from T_M. Specifically

$$T_i \simeq 1 \times 10^{11} \text{ GeV}\, \alpha\tilde{\gamma}\sqrt{\frac{g_{*M}}{100}} \left(\frac{10^{16} \text{ GeV}}{f_a}\right)^2 \left(\frac{T_M}{3 \times 10^{15} \text{ GeV}}\right)^4. \tag{17}$$

Note that if T_i in this expression exceeds T_M, e.g. because of $\alpha \gg 1$, then T_i must be set to T_M.

At temperatures below T_i, axion isocurvature fluctuations are damped. Since Eq. (15) implies $n = 3$, so that $p \approx 3/4$ (see Eq. (13)),

$$\frac{\delta\theta(T)}{\delta\theta(T_i)} \approx \left(\frac{T}{T_i}\right)^{\frac{3}{4}}. \tag{18}$$

Therefore, to avoid the observational constraint of Eq. (7), we need

$$T_f \lesssim 2 \times 10^5 \text{ GeV}\, \alpha\tilde{\gamma}\sqrt{\frac{g_{*M}}{100}} \left(\frac{\Omega_{\text{DM}}}{\Omega_a}\right)^{\frac{2}{3}} \left(\frac{T_M/E_{\text{inf}}}{0.3}\right)^4 \left(\frac{10^{16} \text{ GeV}}{f_a}\right)^{1.5} \left(\frac{E_{\text{inf}}}{10^{16} \text{ GeV}}\right)^{\frac{4}{3}}, \tag{19}$$

where we have used $H_{\text{inf}} \approx E_{\text{inf}}^2/\sqrt{3}\bar{M}_{\text{Pl}}$. We here generate the required value of T_f simply by the Brout-Englert-Higgs mechanism associated with φ:

$$V_{\text{hid}} = \lambda' \left(|\varphi|^2 - v'^2\right)^2, \tag{20}$$

with $v' \approx T_f$. We find that unification scale axion dark matter with unification scale inflation can be made consistent by our mechanism.

Incidentally, ignoring $U(1)'$ breaking, we find that monopoles dominate the energy density of the universe at temperature

$$T_* \simeq 6 \times 10^6 \text{ GeV}\, \alpha\sqrt{\frac{g_{*M}}{100}} \left(\frac{T_M}{3 \times 10^{15} \text{ GeV}}\right)^3 \left(\frac{m_M}{3 \times 10^{15} \text{ GeV}}\right), \tag{21}$$

which is slightly below the upper bound in Eq. (19) in the relevant parameter region. Here, m_M is the monopole mass. This implies that the universe may be monopole dominated toward the end of the damped oscillation period, $T_f \lesssim T \lesssim T_i$.

---

[6]It is important here that the $U(1)'$ sector does not contain a light fermion charged under $U(1)'$. If it did, virtual fermions would partially screen the charge surrounding a monopole, allowing it to spread over a distance or order m_f^{-1}. Here, m_f is the fermion mass. This would suppress the induced mass of the axion so that $\gamma \approx m_f/f_a$ [14]. This will be relevant for models in Section 6.1.

5 Monopole Annihilations

Here we discuss annihilations of monopoles after $U(1)'$ is spontaneously broken at some temperature T_S ($\sim T_{\rm f}$). After $U(1)'$ is spontaneously broken, monopoles and antimonopoles become connected by strings. For monopole-antimonopole annihilations to occur, the string-monopole system must lose their energies, and there are several processes that can contribute to the energy loss.

We assume the existence of a renormalizable coupling between the $U(1)'$ and standard model sectors, e.g. a quartic coupling between the $U(1)'$ breaking and standard model Higgs fields or a kinetic mixing between $U(1)'$ and $U(1)$ hypercharge:

$$\mathcal{L} \sim \epsilon\, \varphi^\dagger \varphi h^\dagger h, \qquad \epsilon F'_{\mu\nu} F_Y^{\mu\nu}. \tag{22}$$

We will find that monopoles quickly disappear, well within a Hubble time, unless the coupling ϵ is significantly suppressed. Note that cosmic strings formed by $U(1)'$ breaking are harmless for $T_S \lesssim 10^{15}$ GeV [21].

5.1 Monopole friction

Suppose the correlation length of the $U(1)'$ breaking field, φ, is of order or larger than the average distance between monopoles at $T \sim T_S$:

$$d(T_S) \sim n_M(T_S)^{-\frac{1}{3}} \sim \frac{\bar{M}_{\rm Pl}}{\alpha^{1/3} T_S T_M}. \tag{23}$$

In this case, strings will connect monopoles through the shortest possible path, and the energy of a monopole-antimonopole pair to be dissipated is

$$E_0 \sim \eta\, d(T_S) \sim \frac{\bar{M}_{\rm Pl} T_S}{\alpha^{1/3} T_M}, \tag{24}$$

where we have estimated the string tension η to be of order T_S^2.

If the monopoles scatter with a thermal bath of temperature T_S through a coupling of strength ϵ, as in Eq. (22), then the energy loss rate due to friction is [22]:

$$\dot{E} \sim -\epsilon^2 T_S^2 v^2, \tag{25}$$

where v is the velocity of the monopoles, which is given by

$$v \sim \begin{cases} \left(\frac{T_S^2 d(T_S)}{m_M}\right)^{\frac{1}{2}} \sim \left(\frac{T_S \bar{M}_{\rm Pl}}{\alpha^{1/3} T_M^2}\right)^{\frac{1}{2}} & \text{for } T_S \ll \frac{\alpha^{1/3} T_M^2}{\bar{M}_{\rm Pl}}, \\ 1 & \text{for } T_S \gtrsim \frac{\alpha^{1/3} T_M^2}{\bar{M}_{\rm Pl}}, \end{cases} \tag{26}$$

where the former and latter cases correspond to nonrelativistic and relativistic monopoles, respectively. In each case, the annihilation timescale $\tau_{\mathrm{ann}} \sim |E_0/\dot{E}|$ is given by

$$
\tau_{\mathrm{ann}} \sim \begin{cases} \frac{T_M}{\epsilon^2 T_S^2} & \text{for } T_S \ll \frac{\alpha^{1/3} T_M^2}{\bar{M}_{\mathrm{Pl}}}, \\ \frac{\bar{M}_{\mathrm{Pl}}}{\epsilon^2 \alpha^{1/3} T_S T_M} & \text{for } T_S \gtrsim \frac{\alpha^{1/3} T_M^2}{\bar{M}_{\mathrm{Pl}}}. \end{cases} \tag{27}
$$

In both cases, this timescale is of order or shorter than the Hubble timescale, $t_S \sim \bar{M}_{\mathrm{Pl}}/T_S^2$, unless ϵ is much smaller than of order unity.

5.2 Particle production from strings

If the correlation length of φ is much smaller than the average monopole distance at T_S, then we expect that a string connecting a monopole-antimonopole pair to have a significant number of kinks (from a Brownian formation), and particle production from the string contributes significantly to the dissipation.

Based on the analysis in Ref. [22], we estimate that the power for a string of thickness δ and length L to radiate standard model particles is

$$
P \sim \frac{\epsilon^2}{\delta \, \xi(T_S)}, \tag{28}
$$

per a portion of a string of length $\xi(T_S)$, where $\xi(T_S)$ ($\ll d(T_S)$) is the correlation length of φ.[7] In the case of Brownian strings, the average string length is given by

$$
L \sim \frac{d(T_S)^2}{\xi(T_S)}, \tag{29}
$$

so that the total energy of the string-monopole system to be dissipated and the emission power from it are

$$
E_0 \sim \eta L \sim \frac{T_S^2 \, d(T_S)^2}{\xi(T_S)}, \tag{30}
$$

$$
\dot{E} \sim P \frac{L}{\xi(T_S)} \sim \frac{\epsilon^2 T_S \, d(T_S)^2}{\xi(T_S)^3}, \tag{31}
$$

where we have used $\eta \sim T_S^2$ and $\delta \sim 1/T_S$. The monopole-antimonopole annihilation timescale is thus

$$
\tau_{\mathrm{ann}} = \frac{E_0}{\dot{E}} \sim \frac{1}{\epsilon^2} T_S \, \xi(T_S)^2 \ll \frac{1}{\epsilon^2} T_S \, d(T_S)^2 \sim \frac{\bar{M}_{\mathrm{Pl}}^2}{\epsilon^2 \alpha^{2/3} T_S T_M^2}. \tag{32}
$$

Again, this is of order or shorter than the Hubble timescale, $t_S \sim \bar{M}_{\mathrm{Pl}}/T_S^2$, unless ϵ is much smaller than order unity.[8]

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[7]The process of energy dissipation may be much faster, $P \sim \epsilon^2 \eta(\delta/\xi(T_S))^{1/3}$, if cusps form efficiently [23]. Here we adopt a conservative estimate of Eq. (28), which is sufficient to eliminate the monopoles quickly.

[8]In the analysis in this subsection, we have ignored the effect of the increase of the relevant correlation length

6   Technical Naturalness of $U(1)'$

In Section 4, we have presented the minimal model in which $U(1)'$ breaking is achieved by a scalar field φ with the potential Eq. (20). As it stands, the scale appearing in this potential, v', is not radiatively stable. The radiative stability of this scale is qualitatively and quantitatively different from the problem of protecting the QCD axion from quantum corrections. The $U(1)'$-breaking field φ is a scalar much like the standard model Higgs field whose mass needs to be protected at scales above T_S, unlike the QCD axion whose mass needs to be protected to the level of $\sim 10^{-5} m_a$. Existing ideas to address the hierarchy problem may thus be leveraged to solve this issue. In this section, we discuss extensions/modifications of the minimal model in which the issue of radiative stability does not arise.

6.1   Supersymmetric $U(1)'$ sector

One way to construct a technically natural model is to make the $U(1)'$ sector supersymmetric. This requires promoting the $U(1)'$-breaking field φ to chiral superfields $\Phi(+1)$ and $\bar{\Phi}(-1)$. The complication arises because the induced axion mass is suppressed in the presence of light fermions charged under $U(1)'$, as mentioned in footnote 6. To obtain a significant contribution to the axion mass, we need to have a supersymmetric mass for Φ and $\bar{\Phi}$:

$$W = M_\Phi \Phi \bar{\Phi}. \tag{33}$$

The breaking of $U(1)'$ is then caused by supersymmetry-breaking squared masses for Φ and $\bar{\Phi}$ of order $\tilde{m}^2 \sim T_S^2$. To maximize the axion mass, we also take $M_\Phi \sim T_S$.[9] The coupling between the $U(1)'$ and the standard model sectors needed for monopole annihilations can be taken as a kinetic mixing between $U(1)'$ and $U(1)$ hypercharge: $\mathcal{L} \sim \epsilon \left[\mathcal{W}'^\alpha \mathcal{W}_{Y\alpha} \right]_{\theta^2}$ (see Section 5). This implies that the standard model is also supersymmetric above the scale $\sim (4\pi/\epsilon)\tilde{m}$.

With this setup, the induced axion mass is given by Eq. (9) with

$$\gamma \approx \frac{M_\Phi}{f_a} \sim \frac{T_S}{f_a}. \tag{34}$$

Plugging this into Eq. (19) with $T_S \sim T_{\rm f}$, we find that α must be much larger than 1 for the model to work. We thus suppose that the dynamics of the phase transition producing monopoles is such

---

due to interactions of the strings with the thermal bath, which may become important for $T_S \lesssim T_M^2/\bar{M}_{\rm Pl}$. In this case, however, the analysis in the previous subsection applies, which also says that monopoles quickly disappear after $U(1)'$ symmetry breaking.

[9]The coincidence of the scales $\tilde{m}$ and M_Φ is analogous to the μ problem in the minimal supersymmetric standard model, which can be addressed, e.g., as in Ref. [24].

that $\alpha \gg 1$. The largest possible abundance of monopoles obtained in this case is determined by the freezeout abundance (instead of Eq. (10)), which is given by [25]

$$n_M(T) \approx \left(\frac{T}{T_M}\right)^3 \frac{\sqrt{g_{*M}}T_M^4}{\bar{M}_{\mathrm{Pl}}}, \tag{35}$$

where we have assumed an $O(1)$ $U(1)'$ gauge coupling. The axion mass at $T \sim T_M$ is then

$$\frac{m_a(T_M)}{3H(T_M)} \sim \frac{\sqrt{T_S \bar{M}_{\mathrm{Pl}}}}{g_{*M}^{1/4} f_a}, \tag{36}$$

so that the axion field starts damped oscillations at

$$T_{\mathrm{i}} \sim \frac{T_S T_M \bar{M}_{\mathrm{Pl}}}{\sqrt{g_{*M}} f_a^2}. \tag{37}$$

This gives the damping factor of

$$\Delta \approx \left(\frac{T_{\mathrm{f}}}{T_{\mathrm{i}}}\right)^{\frac{3}{4}} \sim \left(\frac{f_a^2}{T_M \bar{M}_{\mathrm{Pl}}}\right)^{\frac{3}{4}}. \tag{38}$$

We find that the mechanism is not as strong as in the minimal model, but it can still save the scenario with f_a, T_M, E_{inf} as large as $\sim 10^{15}$ GeV.

6.2 Possibility of unbroken $U(1)'$

We finally mention an alternative (and very different) possibility that $U(1)'$ monopoles may be efficiently eliminated without breaking $U(1)'$. This may happen if the monopole under consideration is in fact a dyon that also carries a charge under a hidden non-Abelian gauge group G' (to which the axion field does not couple). In this case, if G' confines at a scale Λ', then dyons can be subjected to extra strong annihilation processes.

Suppose the G' sector contains light particles that are electrically charged under G'. When G' confines at $T \sim \Lambda'$, dyons pick up these light particles, becoming G' hadrons. At this point, the dyon-antidyon annihilation cross section is expected to become large $\sim 1/\Lambda'^2$, as in the analogous situation for a heavy stable colored particle [26]. This will efficiently eliminate dyons if the confinement scale is sufficiently low $\Lambda' \lesssim 100$ TeV, giving $T_{\mathrm{f}} \sim \Lambda'$. Since this scenario does not require breaking of the $U(1)'$ symmetry, the $U(1)'$ sector need not have a light charged scalar or fermion, which would, respectively, lead to the issue of radiative stability and axion mass suppression. Further studies of this possibility, including a detailed analysis of whether dyon annihilation is indeed strong enough, are warranted.

10

7 Conclusions

In this paper we have presented a mechanism that suppresses axion isocurvature fluctuations due to the dynamics of a hidden $U(1)'$ sector coupled to the axion field. In particular, this sector produces $U(1)'$ monopoles at $T \sim T_M$, which disappear at $T \sim T_{\rm f}$ ($\ll T_M$). For temperatures between $T_{\rm i}$ ($\gg T_{\rm f}$) and $T_{\rm f}$, the effective axion mass induced by the monopoles makes the axion heavier than the Hubble parameter, so that the isocurvature fluctuations are damped. Since the average value of the axion field after the damping is not necessarily at the minimum of the zero-temperature potential determined by QCD, homogeneous coherent oscillations after the QCD phase transition may still produce axion dark matter [16].

We have presented a minimal model in which this mechanism successfully operates. This model accommodates a large enough time interval in which the axion isocurvature fluctuations are damped, so that axion dark matter with a unification scale decay constant can be consistent with unification scale inflation. We have also discussed extensions/modifications of the minimal model in which the issue of radiative stability does not arise.

Since the axion provides a leading solution to the strong CP problem, it is important to fully study its consistency. If a future CMB experiment discovers inflationary gravitational wave signals, it would exclude naive axion models with the Peccei-Quinn symmetry broken before the end of inflation. Our mechanism makes the QCD axion alive even in such a case, without requiring the Peccei-Quinn symmetry breaking scale to be below the inflationary scale. This is particularly important for a string axion, which has a virtue that explicit breaking of the Peccei-Quinn symmetry (which needs to be extremely small to solve the strong CP problem [27]) is generated only at a nonperturbative level [8]. Our mechanism allows for a string axion to be a consistent solution to the strong CP problem even if inflationary gravitational wave signals are discovered, and it would also keep open the possibility that axion dark matter may be discovered by high precision experiments such as those proposed in Ref. [9].

Note added:
While completing this paper, we received Ref. [28] which discusses a similar idea.

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