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BUILDING A COMPUTER MODEL OF LEARNING CLASSICAL MECHANICS *

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ABSTRACT

A computational model of learning in a complex domain is described and its implementation is discussed. The model supports knowledge-based acquisition of problem-solving concepts from observed examples, in the domain of physics problem solving. The system currently learns about momentum conservation, in a psychologically plausible fashion, from a background knowledge of Newton's laws and the calculus. The background knowledge is consistent with the mathematical abilities of a college student who has been exposed to calculus. In its contribution to machine learning, this research is important for artificial intelligence. From a psychological perspective it demonstrates the computational consistency of a mechanism that may underlie human learning in a complex domain. This work also has implications for computer-aided instruction, in that it advances a learning model for a complicated domain involving both symbolic and numerical reasoning.

INTRODUCTION

In complex domains like physics, people seem to understand general rules best if they are accompanied by illustrative examples. A large part of most physics texts is taken up by examples and exercises. Indeed, there is psychological evidence that a person who discovers a rule from examples learns it better than one who is taught the rule directly [Egan74] and that illustrative examples provide an important reinforcement to general rules [Bransford76, Gick83].

Solving physics problems requires complicated and diverse reasoning. Simply knowing all of the formulae is insufficient. A student must understand the physics behind each formula - knowing how, when, and why it applies. Furthermore, a skilled physicist is able to make quick judgements concerning which aspects of a situation are irrelevant or have negligible effect. We are investigating the process by which a mathematically-sophisticated student acquires these skills.

Physics problem-solving was selected as a domain for several reasons. First, it is concerned with adult learning and so does not suffer the confounding influence of maturation. Second, it forces us to address the very important but neglected problem of combining symbolic and numeric reasoning. Finally, one of us (Shavlik) has had extensive experience as a teaching assistant observing human students struggling through their first physics class.

In its initial state the implementation of our model is capable of performing many of the mathematical manipulations expected of a college freshman who has encountered the calculus. Through tutoring with examples, the system acquires concepts taught in a first semester college physics course; hence the name of the system, **Physics 101**. Newton's laws - which are provided to the system - suffice to solve all problems in classical mechanics (see page 10-1 of [Feynman63]), but the general principles that are consequences of Newton's laws are interesting for their elegance as well as their ability to greatly simplify the solution process. Since the system's physical knowledge rests on the strong foundation of Newton's laws, only its mathematical abilities will limit the physical concepts it can acquire.

Explanation-based learning [DeJong81, DeJong83], is a computer-based knowledge acquisition method that utilizes sophisticated domain representations. In this type of learning a computer generalizes a problem solution into a form that can be later used to solve conceptually similar problems. The generalization process is driven by the *explanation* of why the solution worked. It is the deep knowledge about the domain that allows the explanation to be developed and then extended. We are applying this paradigm to the learning of classical physics. This approach to learning has much in common with [Mitchell85, Silver83, Winston83]. See [DeJong85] for a full discussion.

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DESCRIPTION OF THE MODEL

Our learning model is inspired by our intuitions concerning the importance of concrete experiences when acquiring abstract concepts. We are implementing this psychologically plausible model as a experiment to test if it is computationally consistent.

Figure 1 contains the model. After a physical situation is described and a problem is posed, the student attempts to solve the problem. We are interested in the process of learning during a successful solution; our attention is currently focussed on learning from a teacher's example. When the student cannot solve a problem, he requests a solution from his instructor. The solution provided must then be verified; additional details are requested when steps in the teacher's solution cannot be understood. We divide the process by which the student understands an example into two phases. First, using his current knowledge about mathematics and physics, the student verifies that the solution is valid. At the end of this phase the student knows that his instructor's solution solves the current problem, but he does not have any understanding of *why* his teacher chose these steps to solve the problem. During the second phase of understanding, the student determines a reason for the structure of each expression in the teacher's solution. Especially important is understanding new formulae encountered in the solution. After this phase the student has a firm understanding of how and why this solution solved the problem at hand. At this point he is able to profitably generalize any new principles that were used in the solution process, thereby increasing his knowledge of classical physics.

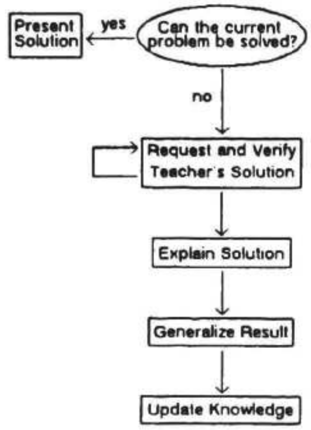

Figure 1. A Model of Learning Classical Mechanics

In our model, physical situations are represented as *worlds*, which comprise a number of *objects*. Figure 2 presents the representation of the generic world, **worldN**, which is used to instantiate specific physical situations. Objects have four measurable attributes; mass, position, velocity, and acceleration. The relationships among these attributes, with respect to symbolic differentiation and integration, are known, although they are not shown in this figure. Only the algebraic relationships are shown.

Newton's second and third laws also appear in figure 2. (Newton's first law is a special case of his second law.) The second law says that the net force on an object equals its mass times its acceleration. The net force is decomposed into two components; the external forces and the internal forces between objects in the world. An external force results from any external fields that act upon an object. Object *i*'s internal force is the sum of the forces the other objects exert on object *i*. These *inter-object* forces are constrained by Newton's third law, which says that every action has an equal and opposite reaction.

The current implementation of the model learns the physical concept of momentum conservation by analyzing its teacher's solution to a simple collision problem. A fuller description of the learning process can be found in [Shavlik85]. The sample problem is shown in figure 3. In this one-dimensional problem there are two objects moving in free space, without the influence of any external forces. (Nothing is known about the forces between the two objects. Besides their

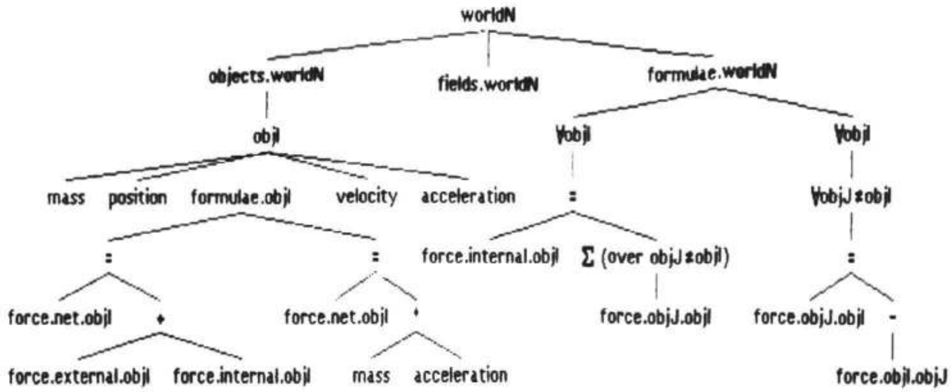

Figure 2. The Generic Representation of a Physical Situation

mutual gravitational attraction, there could be, for example, a long-range electrical interaction and a very complicated interaction during the collision.) In the initial state (**state A**) the first object is moving toward the second, which is stationary. Some time later (**state B**) the first object is recoiling from the resulting collision. The task is to determine the velocity of the second object after the collision.

First, **Physics 101** unsuccessfully attempts to solve the problem using its initial physical knowledge. The system cannot solve this problem, though, as the force exerted on object two by object one is not known. At this point the system requests a solution from its teacher. The solution provided can be seen in figure 4. Without explicitly stating it, the instructor takes advantage of the principle of conservation of momentum, as the momentum (*mass* × *velocity*) of the world at two different times is equated. After that, various algebraic manipulations lead to the answer. In order to accept the answer, **Physics 101** has to verify each of the steps in this solution.

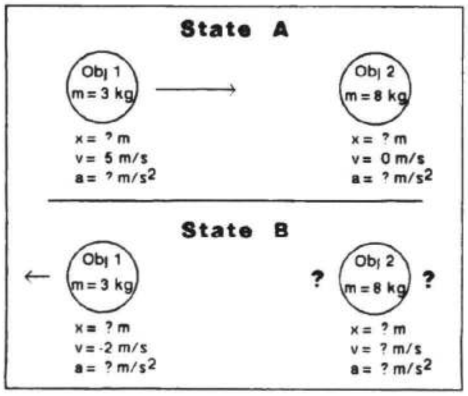

Figure 3. A Two-Body Collision Problem

$$\text{mass}_{obj1,stateA}\,\text{velocity}_{obj1,stateA,X} + \text{mass}_{obj2,stateA}\,\text{velocity}_{obj2,stateA,X}$$
$$= \text{mass}_{obj1,stateB}\,\text{velocity}_{obj1,stateB,X} + \text{mass}_{obj2,stateB}\,\text{velocity}_{obj2,stateB,X}$$
$$3_{kg}\,5_{m/s} = 3_{kg}\,{-2}_{m/s} + 8_{kg}\,\text{velocity}_{obj2,stateB,X}$$
$$15_{kg\,m/s} = -6_{kg\,m/s} + 8_{kg}\,\text{velocity}_{obj2,stateB,X}$$
$$\text{velocity}_{obj2,stateB,X} = 2.63_{m/s}$$

Figure 4. The Teacher's Solution

Four possible classifications of a teacher's solution steps have been identified. Besides being mathematically correct, the instructor's calculations must be physically consistent.

(1) A known formula could have been used: *force* = *mass* × *acceleration* is of this type.

(2) New variables can be defined in order to shorten later expressions. A formula such as *momentum* = *mass* × *velocity* would fall in this category.

(3) Equations can be algebraic variants of previous steps. The replacement of variables by their values also falls into this category.

(4) The teacher can specify an equation that states a relationship among known variables, yet the system knows of no algebraically equivalent formula. These steps require full justification, which the system does by using its abilities to symbolically reason with calculus. Only the equations falling in this category are candidates for generalization.

The last three steps in figure 4 are easily verified in our model, as they are simple algebraic manipulations. The hard part is determining a physical justification for the first equation in the teacher's solution. Since the two sides of this initial equation only differ as to the state in which they are evaluated, an attempt is made to determine a time-dependent expression describing the general form of one side of the equation. Using its physical and mathematical knowledge, **Physics 101** determines that

$$mass_1\ velocity_{1,X}(t) + mass_2\ velocity_{2,X}(t) = constant_1 \tag{1}$$

This result validates the first equation in the teacher's solution, as the left-hand side of this equation can be equated for any two times.

At this point the system has ascertained that the teacher's solution does indeed solve the collision problem. In the next step, it tries to understand why the newly-experienced formula is structured the way it is. This formula has been validated - that is, **Physics 101** knows it is mathematically and physically correct but the system must determine *why* the instructor used this equation.

In the initial equation of figure 4, the teacher used four variables to determine the value of object two's velocity. The system analyzes its teacher's solution and detects that summing the two objects' momenta eliminates from the calculation the force each object exerts upon the other, regardless of the details of these forces. (This is a consequence of Newton's third law.) Since each object in a physical situation potentially exerts a force on every other object, in the general case cancelling the net inter-object force upon an object requires summing the momenta of *all* the objects.

Equation 2 presents the result **Physics 101** obtains by extending its instructor's solution technique to a world with an arbitrary number of objects.

$$\sum_{i=1}^{N} mass_i\ velocity_i = \int \sum_{i=1}^{N} force_{external,i}\ dt \tag{2}$$

This formula says: *The total momentum of a collection of objects is determined by the integral of the sum of the external forces on those objects.* A second problem, which involves three bodies under the influence of an external force, has been solved by **Physics 101** using this generalized result.

Much research on learning involves relaxing constraints on the entities in a situation, rather than generalizing the number of entities themselves. Nonetheless, many important concepts require generalizing number. Explanation-based learning provides a solution to a major problem, namely, how do you know when it is valid and proper to generalize the number of entities? For example, compare the concept of *tripod* with *bicycle wheel*. Both concepts contain a number of repeated components. Suppose a three-legged tripod and a 25-spoked wheel are observed. An explanation-based system can build a general concept for each. The general tripod concept will contain precisely three legs, as any other number of legs is unstable. The general wheel concept, however, will allow a variable number of spokes. The explanation of a component's functionality dictates when it is valid and proper to generalize its number.

CONCLUSION

By analyzing a worked example, the current implementation of **Physics 101** is able to derive a formula describing the temporal evolution of the momentum of any arbitrary collection of objects. This formula can be used to solve a collection of complicated collision problems. Other physical concepts to be learned by the system include work, friction, conservation of energy, simple harmonic motion, and conservation of angular momentum. As these additional concepts are learned, previously-learned concepts will have to be refined. This work will also investigate how the system can learn to estimate which features of a problem can be *ignored* when solving the problem, an important trait possessed by experts.

We have developed a model for learning in a complex domain requiring both symbolic and numeric reasoning. Our approach is knowledge-based: the system requires and applies detailed knowledge about the calculus and Newton's laws. Once a new concept is learned, it is added to the system's knowledge base and is thereby available to help solve future problems and as a stepping stone toward acquiring more difficult concepts. This research contributions to machine learning and psychological modeling. It also has implications for intelligent computer-aided instruction in rich domains where sophisticated learning models are necessary.

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