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ABSTRACT

General aspects of the problem of anomalous nuclear-structure-dependent contributions to the internal-conversion process are considered with the qualitative conclusion that the most likely cases for observation of anomalies will be in highly retarded electric or magnetic dipole transitions. Formulas for an elementary theory of anomalous internal conversion for E1 transitions are given. Selection rules for the relevant nuclear matrix elements are given in the quantum numbers appropriate to spheroidally deformed nuclei (K, N, n_z, Λ, Σ). Similar selection rules for M1 transitions are given on the basis of the anomalous operators previously derived by Church and Weneser.

The experimental data on dipole conversion coefficients of retarded transitions for odd-mass spheroidal nuclei are surveyed. It is noted that where retardation is ascribable to K forbiddenness (up to retardation from the single-proton rate by a factor of 10^9) no detectable anomalies are found, but where transitions are allowed by K-selection rules detectable conversion-coefficient anomalies may generally be found at retardations greater than 10^5 to 10^6 and are not found at lesser retardation. There are some exceptions to this general rule, though. From the present meager data the utility of selection rules in the asymptotic quantum numbers, N, n_z, Λ, and Σ, for anomalous-conversion matrix elements is open to question, although their utility in qualitatively explaining retardation of the radiative transitions is very evident.

The simple E1 theory is applied in an attempt to quantitatively explain the very anomalous 85-kev transition in Pa^{231}. Values of the two parameters in the simple theoretical expressions can be found to explain all three L-

subshell conversion coefficients. The magnitude of one parameter, the nuclear matrix element $\left\langle r^3 \, Y_1 \right\rangle$, is consistent with estimates from the single-particle model. However, the magnitude required of the other parameter is such as to suggest that there are important shortcomings in the theory.

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INTRODUCTION

The internal conversion process whereby a bound orbital electron is ejected during a nuclear electromagnetic transition generally occurs in parallel with photon emission. The ratio of conversion-electron ejection to photon emission is defined as the conversion coefficient, α, with α_K, α_{L_I}, etc., referring to conversion of K, L_I, or other electrons alone. Comparison of experimental absolute conversion coefficients or relative conversion coefficients (K/L ratios, L- or M-subshell ratios) with theoretical values constitutes the most generally useful means of determining gamma-transition multipolarities.

The overwhelming contribution to the normal internal-conversion process comes from regions outside the nuclear volume. The original calculations by Rose <u>et al.</u>[1] assuming a point nucleus represent therefore a good approximation in most cases, as the probability of the electrons penetrating the nucleus is small even for the heaviest nuclei. However, later calculations, by Sliv and Band[2], show some conversion coefficients (particularly M1) to be quite seriously affected when, instead of the point-nucleus model, they assume a nucleus of finite size but with all nuclear currents restricted to the surface. This correction is essentially a correction corresponding to improved electron wave functions. The intranuclear effects of the electron penetrating the nucleus are accounted for only in an average way by the model of Sliv <u>et al.</u>[2] restricting the currents to the surface of the nucleus. The correction is, however,

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of real importance, and recent experimental evidence on M1 conversion coefficients agrees better with the latter theoretical values.[3]

Church and Weneser[4] have further suggested that anomalous, model-dependent conversion coefficients may occur for retarded M1 transitions if one takes into account the distribution of currents throughout the nuclear volume. They have considered contributions to the internal conversion arising from integrals over the electron density within the nuclear volume, and they have shown that terms of this intranuclear contribution may obey certain selection rules in various approximate nuclear quantum numbers, which selection rules may be different or less restrictive than the selection rules governing both photon emission and the ordinary (electron outside the nucleus) internal-conversion contributions. Thus, if a transition is forbidden by the ordinary selection rules and is highly retarded but an intranuclear contribution to internal conversion is allowed, the conversion coefficient may be anomalous. There is an experimental case of an anomalous M1 conversion coefficient in Ta^{181}, which we shall refer to later.

It has been known for some time that L-subshell conversion ratios for the 60-kev transition to ground in Np^{237} were not in agreement with theoretical values.[5] More recently, evidence has been collected for other E1 transitions in the heavy region. (cf. Asaro, Stephens, Hollander and Perlman[6]) Some transitions exhibit $L_{I,II,III}$ conversion coefficients in agreement with the theoretical values of Rose, while others (notably the 85-kev transition to ground in Pa^{231}) exhibit anomalously large L_I and L_{II} conversion coefficients. Many of the low-energy E1 transitions of odd-A spheroidal nuclei have rates measurable by fast-coincidence techniques ($\tau \gtrsim 10^{-9}$ sec), and are thus greatly retarded from single-proton lifetime formula estimates.

From simple qualitative considerations one might suspect that intranuclear contributions to the internal conversion may be responsible for the anomalous E1 conversion coefficients for $s_{1/2}$ and $p_{1/2}$ electrons. In order for such special contributions to be at all competitive, a necessary condition is that neither the initial nor final electron wave function be vanishingly small within the nuclear volume. Terms involving both initial and final electron states with $j = 1/2$ (i.e., $s_{1/2}$ or $p_{1/2}$) would seem more likely to give anomalous intranuclear contributions than would terms involving j_i or j_f $> 1/2$.

Table I lists the continuum states available for internal conversion from various bound states for different multipolarities.

Table I

Allowed continuum states for internal conversion

Multipolarity / bound state	E1	M1	E2	M2
$s_{1/2}$	$p_{1/2}, p_{3/2}$	$s_{1/2}, d_{3/2}$	$d_{3/2}, d_{5/2}$	$p_{3/2}, f_{5/2}$
$p_{1/2}$	$s_{1/2}, d_{3/2}$	$p_{1/2}, p_{3/2}$	$p_{3/2}, f_{5/2}$	$d_{3/2}, d_{5/2}$
$p_{3/2}$	$s_{1/2}, d_{3/2}, d_{5/2}$	$p_{1/2}, p_{3/2}, f_{5/2}$	$p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2}$	$s_{1/2}, d_{3/2}, d_{5/2}, g_{7/2}$

The combinations where anomalous intranuclear terms might have the best chance of being significant are those with $j_i = j_f = 1/2$, namely, E1 and M1 conversion of $s_{1/2}(K,L_I)$ and $p_{1/2}(L_{II})$ electrons.

In the sections following there is given an elementary theory of the anomalous electric dipole conversion coefficients and an examination of its implications. Comparison with experimental data is made, and the strong points and shortcomings of the theory discussed.

THEORY OF THE ANOMALOUS TERMS IN THE E1 CONVERSION PROCESS

The probability for the ejection of an electron by the process of internal conversion is proportional to the matrix element $|U_{fi}|^2$ (in a perturbation approximation). We limit ourselves here to internal conversion accompanying E1 gamma radiation and write,[7,8] in direct analogy with the treatment given for M1 internal conversion coefficients (hereafter referred to as ICC) in Ref. 4,

$$U_{fi}(E1) \sim \sum_M \Big[\int_0^\infty d\tau_e\, \psi_f^*\, O_e(h_1)\, \psi_i \int_0^{r_e} d\tau_n\, \phi_f^*\, O_n(j_1)\, \phi_i + \int_0^\infty d\tau_n\, \phi_f^*\, O_n(h_1)\, \phi_i \int_0^{r_n} d\tau_e\, \psi_f^*\, O_e(j_1)\, \psi_i \Big] \quad (1)$$

Here ψ and ϕ, respectively, are the electron and nucleon wave functions. The integration over the nucleon coordinates $\int_0^{r_e} d\tau_n$ implies a complete angular integration over the angles of the nucleon position but an integration in the radial coordinates $\underline{r_n}$ only out to the radius of the electron $\underline{r_e}$. The first term thus accounts for the case when the electron is outside the nucleon. The second term represents the reversed situation in which the electron is inside the nucleon radius.

The nuclear operator is

$$O_n(j_1) = iW \frac{\partial}{\partial r} \left[r j_1(Wr) \right] Y_{1-M}, \tag{2}$$

where $j_1(Wr)$ is the regular spherical Bessel function and $\underline{W}$ the energy of the gamma ray (for a more complete account of the derivations leading up to Eq. (13) see Ref. 8.) The quantity $O_n(j_1)$ is rather independent[9] of the assumed interaction of the nucleon with the transverse photon field provided (a) that the assumed interaction is linear in the electromagnetic field, (b) that it is gauge-invariant, (c) that the long-wave length limit is approached (i.e., $Wr_n \ll 1$).

The electron operator is

$$O_e(j_1) = \left[-iW \frac{\partial}{\partial r} (r j_1) + \alpha_r W^2 r j_1 \right] Y_{1-M}, \tag{3}$$

where $\underline{j_1}$ as before is a function of $\underline{Wr}$. The matrix α_r is defined as

$$\alpha_r = \begin{pmatrix} 0 1 \\ 1 0 \end{pmatrix} \frac{\underset{\sim}{\sigma} \cdot \underset{\sim}{r}}{r}.$$

This expression (3) is derived on the basis that the interaction of the electron with the transverse photon field is

$$H_e = -e \underset{\sim}{\alpha_e} \cdot \underset{\sim}{A} (\underset{\sim}{r_e}), \tag{4}$$

where the matrix $\underset{\sim}{\alpha} = \begin{pmatrix} 0 1 \\ 1 0 \end{pmatrix} \underset{\sim}{\sigma}$.

The quantities $O_n(h_1)$ and $O_e(h_1)$ are obtained from Eq. (2) and (3) by everywhere replacing j_1 by h_1, the Hankel function of the first kind, corresponding to an outgoing wave.

Equation (1) may be conveniently rewritten as

$$U_{fi}(E1) \sim \sum_M \Big\{ \int_0^\infty d\tau_e \, \psi_f^* \, O_e(h_1)\psi_i \int_0^\infty d\tau_n \, \phi_f^* \, O_n(j_1)\phi_i$$

$$+ \int_0^\infty d\tau_n \, \phi_f^* \, O_n(h_1)\phi_i \int_0^{r_n} d\tau_e \, \psi_f^* \, O_e(j_1)\psi_i$$

$$- \int_0^\infty d\tau_n \, \phi_f^* \, O_n(j_1) \, \phi_i \int_0^{r_n} d\tau_e \, \psi_f^* \, O_e(h_1)\psi_i \Big\} \quad . \tag{5}$$

The first term in Eq. (5) now corresponds to the "point nucleus" case, i.e., it is the only surviving term if we let the nucleus shrink to a point. The other two terms represent "finite size" corrections to this limiting situation.

As the angular functions are identical in the corresponding operators for all three terms, it is easily shown that the "partial" conversion coefficient corresponding to ejection of a bound electron in state κ' into the free state κ is

$$\alpha_{\kappa\kappa'} \; (E1) = \overset{0}{\alpha}_{\kappa\kappa'} \; |1 + \lambda|^2, \tag{6}$$

where $\overset{0}{\alpha}_{\kappa\kappa'}$ is the partial conversion coefficient corresponding to a point nucleus, as far as nuclear matrix elements are concerned (and thus corresponds to the case in which only the first term in (5) is retained). Calculating $\overset{0}{\alpha}_{\kappa\kappa'}$ one should, strictly speaking, use electron wave functions adjusted for the finite extension of the central charge. In the qualitative considerations employed in the following, values of either Sliv or Rose are sufficiently accurate, even though neither strictly corresponds to our definition of $\overset{0}{\alpha}_{\kappa\kappa'}$. The term λ of Eq. (6) is defined as

$$\lambda = \frac{\langle I \| O_n(h_1) S(r, j_1) - O_n(j_1) S(r, h_1) \| I' \rangle}{\langle I \| \; O_n(j_1) \| \; I' \rangle \quad S(\infty, h_L)} \; , \tag{7}$$

where

$$S(r, j_1) = \int_0^r r^2 dr \quad \Big\{ W^2 r j_1 (f_\kappa g_{\kappa'} - g_\kappa f_{\kappa'}) + \Big[\frac{\partial}{\partial r} (r j_1) \Big] \quad (f_\kappa f_{\kappa'} + g_\kappa g_{\kappa'}) \Big\} \tag{8}$$

In Eq. (8) it is assumed that the expansion of $j_l(Wr)$ is employed, with only the leading term retained. Under this condition, $Wr \ll 1$, the second of the terms in Eq. (8) is dominant. In Eq. (7) the "double-bar" matrix element (reduced matrix element) is employed in the usual definition. (The quantities $\underline{I'}$ and $\underline{I}$ in the bra- and ket-vectors of Eq. (7) really denote all quantum numbers necessary to represent initial and final state, apart from the spacial projection $\underline{m_I}$ of the total angular momentum).

The expression $S(r,h_l)$ is obtained from Eq. (8) by employing h_l instead of j_l. Furthermore $S(\infty,h_l)$ corresponds to $S(r,h_l)_{r\to\infty}$.

Finally the total conversion coefficient $\alpha_{\kappa'}$ (i.e., respectively α_K, α_{L_I}, $\alpha_{L_{II}}$, etc.) is defined as

$$\alpha_{\kappa'} = \sum_{\kappa} \alpha_{\kappa\kappa'} . \tag{9}$$

The electron wave functions f_κ and g_κ ("small" and "large" components of the Dirac electron wave functions) have now to be estimated.

In the interior of the nucleus one may assume an electrostatic potential corresponding to a homogeneous charge distribution,[10,11]

$$v(r) = \frac{e^2 Z}{2R}\left[3 - \left(\frac{r}{R}\right)^2\right], \tag{10}$$

where $\underline{R}$ is the nuclear radius. In this potential, which is finite at the origin, one may find series expansions in $\underline{r}$ of f_κ and g_κ. The amplitude of the leading term is determined by matching at the nuclear boundary with the external solution. The leading terms are only weakly dependent on the particular shape of the interior potential assumed.

For L_I and L_{II} conversion one would expect the main contribution to the structure-dependent terms to originate from transitions $\kappa' = -1$ to $\kappa = 1$ (i.e., $s_{1/2}\to p_{1/2}$) and $\kappa' = 1$ to $\kappa = -1$ (i.e., $p_{1/2}\to s_{1/2}$), respectively. (The wave functions corresponding to these states have the largest amplitudes at the nuclear surface). Indeed, these are the leading contributions, although they are considerably weakened owing to a particular cancellation, discussed in the following paragraphs.

We employ the following internal expansions, treating as an example the case $s_{1/2} \rightarrow p_{1/2}$.

$$(\kappa' = -1;\ s_{1/2}) \quad f_{\kappa'} = \mathring{f}_{\kappa'}\left(\frac{r}{R}\right) + \ldots = -\frac{1}{3}\,[E - v'(0) - 1]\,\mathring{g}_{\kappa'}\, r + \ldots, \tag{11a}$$

$$g_{\kappa'} = \mathring{g}_{\kappa'} + \ldots, \tag{11b}$$

$$(\kappa = 1;\ p_{1/2}) \quad f_{\kappa} = \mathring{f}_{\kappa} + \ldots, \tag{11c}$$

$$g_{\kappa} = \mathring{g}_{\kappa}\left(\frac{r}{R}\right) + \ldots = \frac{1}{3}\,[E - v(0) + 1]\,\mathring{f}_{\kappa}\, r + \ldots, \tag{11d}$$

where use has been made of the Dirac equation to determine the relation between $\mathring{f}_{\kappa}$ and $\mathring{g}_{\kappa}$.

Retaining terms to leading order only and neglecting terms of order W (valid for transition energies much less than the electron rest mass), we obtain by substitution into Eq. (8) the expression

$$f_{\kappa} f_{\kappa'} + g_{\kappa} g_{\kappa'} \cong r\, \mathring{f}_{\kappa} \mathring{g}_{\kappa'}\, \frac{2}{3}\left[1 + \frac{v'(0) - v(0)}{2}\right] + \ldots\,, \tag{12}$$

where units $m = \hbar = c = 1$ are assumed throughout. Thus R is the nuclear radius in units of the electron Compton wave length, and $v'(0)$ and $v(0)$ are the initial and final values of the electrostatic potential at the origin. The two terms on the left side of (12) are opposite in sign and very nearly of equal magnitude unless there is a large change in effective potential at the origin during the transition. Assuming, e.g., a homogeneous charge distribution over a spheroid of constant volume but varying eccentricity, the potential at the origin depends on the eccentricity parameter δ (excess of major axis over minor axis) as

$$v(0) = \mathring{v}(0)\,(1 - \frac{4}{9}\,\delta^2). \tag{13}$$

If thus the nucleonic transition associated with the conversion process changes the equilibrium deformation from $\delta = 0.3$ to $\delta = 0.2$, $v(0)$ changes by approximately 2%, i.e., by ~1 Mev in the heavy-element region. It is quite possible that other effects may also tend to lift this cancellation.

Employing the expansions of Eqs. (11a-d) one arrives at the following simplified expression for the corrected conversion coefficient

$$\alpha_{\kappa\kappa'} = \overset{o}{\alpha}_{\kappa\kappa'} \left| 1 - i \frac{M_{\kappa\kappa'} \, C_{\kappa\kappa'} \, x}{W^{3/2} \, \overset{o}{\alpha}_{\kappa\kappa'}^{1/2} e^{i\delta}} \right|^2 \tag{14}$$

Here $\overset{o}{\alpha}_{\kappa\kappa'}$ is the normal "point nucleus" partial-conversion coefficient defined previously; $e^{i\delta}$ is the phase of the integral $S(\infty,h_L)$, which enters into the expression for $\overset{o}{\alpha}_{\kappa\kappa'}$ as the absolute value squared. These phases have not been published, but we estimate, for low-energy transitions, $\delta \cong \pm \frac{\pi}{2}$ (i.e., $S(\infty,h_L)$ almost purely imaginary) for E1 as well as for M1 conversion. The phase problem for the M1 case has been previously discussed by Church and Weneser.[4] Here $\underline{W}$ is the transition energy in units of mc^2; $C_{\kappa\kappa'}$ is a factor depending on the change of electrostatic potential at the origin brought about during the transition; $\underline{x}$ is a real quantity, the ratio of two nuclear matrix elements,

$$x = \frac{\langle I \| r^3 \, Y_1 \| I' \rangle}{\langle I \| r \, Y_1 \| I' \rangle} , \tag{15}$$

where $\underline{r}$ is expressed in units of $\sqrt{\frac{\hbar}{M\omega_o}}$ appropriate to the nucleon wave functions of Nilsson,[12] where the basic energy $\hbar\omega_o$ of the nuclear oscillator potentials employed in Ref. 12 is given as $80\ A^{-1/3}$ in units of mc^2. The correction term in Eq. (14) can thus be expected to be almost purely real for low-energy transitions. Furthermore, for nuclei in the heavy-element region and for gamma-ray energies less than ~100 kev the estimates of $M_{\kappa\kappa'}$ of Table II may be employed. The accuracy of the values of $M_{\kappa\kappa'}$ depend on the accuracy with which $\overset{o}{f}_\kappa$ and $\overset{o}{g}_\kappa$ may be estimated.

Values of these latter quantities may be obtained for bound states from the diagrams of Brysk and Rose[13] based on calculations that allow for screening and the finite size extension of the central nuclear charge. The tables of Reitz[14] have supplied values of the free-electron wave functions. A more detailed discussion on this point is found in Ref. 8.

It is clear that the calculated $M_{\kappa\kappa'}$ of Table II are not very exact but should be sufficiently accurate for semiquantitative estimates of the conversion anomalies.

Table II

Values of $M_{\kappa\kappa'}$ and $C_{\kappa\kappa'}$ for Z = 91 and low energies				
Shell	Initial	Final	$M_{\kappa\kappa'}\times 10^6$	$C_{\kappa\kappa'}$
K	$1s_{1/2}$	$p_{1/2}$	-5.3	$1 + \frac{v'(0)-v(0)}{2}$
		$p_{3/2}$	3.9	1
L_I	$2s_{1/2}$	$p_{1/2}$	-2.1	$1 + \frac{v'(0)-v(0)}{2}$
		$p_{3/2}$	1.6	1
L_{II}	$2p_{1/2}$	$s_{1/2}$	-1.9	$1 + \frac{v(0)-v'(0)}{2}$
		$d_{3/2}$	0.1	1
L_{III}	$2p_{3/2}$	$s_{1/2}$	1.4	1
		$d_{3/2}$*	--	-
		$d_{5/2}$*	--	-

*In these cases the leading anomalous conversion operator is of the type $r^5 Y_1$; the coefficient corresponding to $M_{\kappa\kappa'}$ is, however, so small that the anomalous contributions to these terms may be safely neglected.

Let us examine the implications of the simple theory, as expressed in Eqs. (14) and (15) and in Table II. The conversion coefficients for K, L_I, and L_{II} (i.e., $s_{1/2}$ and $p_{1/2}$) electrons in this formulation are functions of two parameters--the matrix element ratio $\underline{x}$, defined in Eq. (15), and the correction factor $C_{\kappa\kappa'}$, which depends on the electrostatic potential change. However, the L_{III} conversion coefficient is essentially a function of $\underline{x}$ only. For a typical heavy-element case (Z = 91, W = 0.17) an $\underline{x}$ value of about 400 to 600 should give rise to a second term in Eq. (13) of order 0.1, causing the partial-conversion coefficient for $p_{3/2} \rightarrow s_{1/2}$ transitions to increase or decrease by 20%, depending on the relative signs of $\underline{x}$ and $e^{i\delta}$. For $v'(0) = v(0)$ the theory predicts anomalies in the L_I and L_{II} subshells only slightly greater than that in the L_{III} subshell.

The correction factor $C_{\kappa\kappa'}$ theoretically will usually differ for $s_{1/2}$ and $p_{1/2}$ electrons. As $v'(0)-v(0)$ is increased from zero, $C_{\kappa\kappa'}$ increases from unity for $s_{1/2}$ and decreases from unity for $p_{1/2}$, and in the limit of very large potential change $C_{\kappa\kappa'}$ will be nearly of equal magnitude but of opposite sign for $s_{1/2}$ and $p_{1/2}$ electrons. In the next section we attempt some comparison with experiment.

The nuclei in which the anomalous cases occur lie in the region of spheroidal nuclei. Thus, one may expect selection rules in $\underline{K}$ and to a lesser extent in $\underline{N}$, $\underline{n_z}$, Λ, and Σ to be applicable to transition-matrix elements. The $\underline{K}$-selection rules as applied to beta and gamma transitions have been frequently discussed elsewhere. The $\underline{N}$, $\underline{n_z}$, Λ, Σ selection rules have also been applied successfully to beta[15,16] and gamma[16,17,18,19] transitions previously, although they are not generally as restrictive as the $\underline{K}$-selection rules.

As has been pointed out, the $\underline{N}$ quantum number should properly not be called an asymptotic quantum number as it is not dependent on the assumption of a very large deformation.[20] The evidence from Ref. 20 and from the studies by Hoffman and Dropesky[21] on the K-capture of Pu^{237} to Np^{237} may suggest that a breaking of the selection rule in $\underline{N}$ is associated with a quantitatively somewhat greater hindrance than in $\underline{n_z}$ and Λ.

Let us now consider the selection rules in $\underline{K}$ and in $\underline{N}$, $\underline{n_z}$, Λ, and Σ for the matrix elements (r^3Y_1) giving rise to anomalous E1 conversion contributions.

If the matrix element of rY_1 is weakened by $\underline{K}$-forbiddenness $(\Delta K>1)$, then this is also the case for r^3Y_1, which has the same $\underline{K}$-selection rule. However, the severe nucleonic selection rules that hinder E1 transitions, to some extent accounted for by the asymptotic selection rules in $\underline{N}$, $\underline{n_z}$, Λ, and Σ, are relaxed for r^3Y_1.

Tables III and IV list the appropriate selection rules. For $\Delta K = -1$ the selection rules are obtained by changing signs in the $\Delta\Lambda$ and $\Delta\Sigma$ columns.

Table III

Selection rules for radiative E1 transitions $(\sum_m rY_{1m})$

ΣK	Operator	ΔN	Δn_z	ΔΛ	ΔΣ
1	$x + iy$	±1	0	1	0
0	z	+1	+1	0	0
		-1	-1		

Table IV

Selection rules for anomalous E1 conversion $(\sum_m r^3Y_{1m})$

ΔK	Operator	ΔN	Δn_z	ΔΛ	ΔΣ
	$(x+iy)(x^2+y^2)$	±1,±3	0		
1	$(x+iy)\cdot z^2$	±1	0		
		+1,+3	+2	1	0
		-1,+3	-2		
	$z(x^2+y^2)$	±1,+3	+1		
		±1,-3	-1		
0				0	0
	z^3	+1	+1		
		+3	+3		
		-1	-1		
		-3	-3		

NUCLEAR-STRUCTURE CORRECTIONS FOR M1 CONVERSION COEFFICIENTS

The case of magnetic dipole K-shell ICC is treated in Ref. 4 by Church and Weneser. For the purpose of the survey of empirical data below we rewrite the final formulae of that reference. To preserve the analogy with the E1 case, Eq. (14), we express $\underline{r}$ in the units of nuclear dimensions $\sqrt{\frac{\hbar}{M\omega_o}}$ employed in Eq. (14). We obtain for the K-shell partial ICC (denoted $\beta_{\kappa\kappa'}$, and leading to the free $s_{1/2}$ state, i.e., $\kappa' = -1$, $\kappa = 1$).[4]

$$\beta_{-1\,-1} = \overset{o}{\beta}_{-1\,-1}\,(1 + xN_{-1\,-1})^2, \tag{16}$$

where

$$x = \frac{\langle\| r^2\,(\underset{\sim}{\ell} + 2\mu\underset{\sim}{\sigma}) - \mu\underset{\sim}{r}(\underset{\sim}{\sigma}\cdot\underset{\sim}{r})\|\rangle}{\langle\|\,\underset{\sim}{\ell} + \mu\underset{\sim}{\sigma}\,\|\rangle}, \tag{17}$$

and where the value of the constant $N_{-1\,-1}$ may be obtained from Eq. (6) of Ref. 4. Furthermore, Church and Weneser, on the basis of available M1 partial ICC's, $\overset{o}{\beta}_{\kappa\kappa'}$, rewrite the correction in terms of the total ICC. In the notation of the present paper (adopted for the use of the nucleonic wave functions of Ref. 12). Their result may be rewritten

$$\beta_K \approx \overset{o}{\beta}_K\,[1 + x\,N_K]^2. \tag{18}$$

The step between Eqs. (16) and (18) (or Eqs. (6) and (7) in Ref. 4) appears to invoke the approximating assumption of $N_K \ll 1$.

The constant N_K in Eq. (18) is given as

$$N_K = C(Z,W)\cdot R^{-2}. \tag{19}$$

The energy involved in the transition is denoted $\underline{W}$ as before ($\underline{k}$ in Ref. 4). Values of the constant $C(Z,W)$ are tabulated in Ref. 4. The nuclear radius $\underline{R}$ is to be expressed in the units above (for $A \approx 230$, $R \approx 3$). For $W \lesssim mc^2$ the factors N_K take on the values listed in Table V; for more accuracy Table I of Ref. 4 should be used for $C(Z,W)$. It should be noted that if Sliv's values are used for $\overset{o}{\beta}_K$ and $\overset{o}{\beta}_{-1\,-1}$ it is appropriate to replace $\underline{x}$ by $(x-R^2)$, as pointed out in Ref. 4. The same holds true for the E1 case. As large values of $\underline{x}$ are required for the transition to be detected as anomalous, this correction term to $\underline{x}$ is negligible, however.

Table V

Values of N_K for K conversion and $W < mc^2$.

The numbers in parentheses denote powers of 10.

Z	70	85	100
N_K	2 to 3(-3)	4(-3)	7 to 8(-3)

It is thus found that the quantity N_K, characterizing the anomalous corrections to the M1 ICC, is in general much larger than the corresponding quantity for E1 transitions $M_{\kappa\kappa'} W^{-3/2} \overset{o}{\alpha}_{\kappa\kappa'}^{1/2}$ (see Eq. (14); the correction there is, however, expressed in terms of the partial ICC). For example, in the experimentally interesting case of the 84-kev E1 transition in Pa^{231}, the latter quantity corresponding to the partial ICC $\alpha_{1\ -1}$ of L_I conversion (connecting the bound electron state $2s_{1/2}$ with the free state $p_{1/2}$) equals $\sim 2 \times 10^{-4}$. (For the purely hypothetical case of K conversion of the same energy we would have twice this value.) If the theoretically undetermined factor $C_{\kappa\kappa'}$ were of the order 1, the nuclear-structure deviations in ICC would be expected to be observed in E1 first for transitions that were 100 to 1000 times as hindered as M1 transitions showing anomalous conversion. However, there seems to be some experimental indication that $C_{\kappa\kappa'}$ indeed is of order 10, in which case the difference between E1 and M1 in this respect is less important.

Church and Weneser[4] gave three categories of hindered M1 transitions in which anomalies might be observable, and to them may be added the fourth category of transitions in strongly deformed nuclei, transitions for which there is hindrance in $\underline{N}$, n_z, Λ, or Σ.

In Table VI we give the "asymptotic" selection rules for M1 radiation and in Table VII the selection rules for the anomalous M1 conversion operator. As with Tables III and IV, the selection rules are only for $\Delta K = +1$ and 0, and the corresponding rules for $\Delta K = -1$ are obtained by changing signs in the $\Delta\Lambda$ and $\Delta\Sigma$ columns.

Table VI

Selection rules for radiative M1 transitions

$$\left[\sum_m \hat{e}_m \cdot (\underset{\sim}{L} + \mu\, \underset{\sim}{\sigma})\right]$$

ΔK	Operator	ΔN	Δn_z	ΔΛ	ΔΣ
1	σ_+	0	0	0	1
	L_+	0	±1	1	0
0	σ_z	0	0	0	0
	L_z	0,±2	0	0	0

Table VII

Selection rules for anomalous M1 conversion contribution

$$\sum_m \hat{e}_m \cdot \left[(\underset{\sim}{L} + 2\mu\underset{\sim}{\sigma})r^2 - \mu\, \underset{\sim}{r}\, (\underset{\sim}{\sigma}\cdot\underset{\sim}{r})\right]$$

ΔK	Operator	ΔN	Δn_z	ΔΛ	ΔΣ
1	$z^2\sigma_+$	+2 0 -2	+2 0 -2	0	1
	$(x^2+y^2)\sigma_+$	0,±2	0	0	1
	z^2L_+	+2 0 -2	+1,+3 ±1 -1,-3	1	0
	$(x^2+y^2)L_+$	0,±2	±1	1	0
	$(x+iy)^2\sigma_-$	0,±2	0	2	-1
	$(x+iy)z\sigma_z$	0,+2 0,-2	+1 -1	1	0
0	z^2L_z or $z^2\sigma_z$	+2 0 -2	+2 0 -2	0	0
	$(x^2+y^2)L_z$ or $(x^2+y^2)\sigma_z$	0,±2	0	0	0

Table VII (cont'd.)

ΔK	Operator	ΔN	Δn_z	ΔΛ	ΔΣ
	$(x+iy)z\sigma_-$	{0,+2; 0,-2}	{+1; -1}	1	-1
0	$(x-iy)z\sigma_+$	{0,+2; 0,-2}	{+1; -1}	-1	1
	$z^2\sigma_z$	{+2; 0; -2}	{+2; 0; -2}	0	0

QUALITATIVE SYSTEMATIZATION OF CONVERSION-COEFFICIENT ANOMALIES

The occurrence in the 60-kev E1 transition of Np^{237} of L-subshell ICC's in disagreement with Rose's theoretical values has been pointed out by Hollander et al.[5] L-subshell conversion coefficients have been studied for E1 transitions in neighboring isotopes, and some are found to be anomalous (notably 85-kev E1 in Pa^{231}), whereas others are normal. The experimental evidence in the heavy region is detailed in a forthcoming paper by Asaro, Stephens, Hollander, and Perlman.[6] Vartapetian[22,23] has reviewed lifetime and conversion-coefficient data for M1 and E1 transitions and has made the general observation for the heavy element E1's that the more delayed transitions usually exhibit conversion coefficients higher than theoretical for E1 and requiring more M2 admixture than is reasonable for explanation in many cases. Vartapetian suggests that the anomalies may be due to nuclear-structure effects not treated by the Rose or Sliv theoretical calculations.

We now wish to make a brief survey of experimental conversion-coefficient data for E1 and M1 transitions in the principal two regions of spheroidal nuclear deformation. It is beyond the scope of this paper to analyze exhaustively the experimental evidence. We confine our cases to those in which a lifetime or limit is known and exclude from consideration those cases for which only a single conversion coefficient is known and in which there is no independent evidence bearing on possible quadrupole admixture.

The simple theoretical treatment exhibited in the first section of this paper predicts (a) that significant anomalies should appear only in transitions highly hindered from the single-particle transition value, (b) that when such hindrance is attributable to violation of the K-selection rule, anomalies should not be appreciable, and (c) that anomalies should be favored for transitions in which the anomalous operator is allowed by "asymptotic" selection rules. The relevant experimental cases are summarized briefly in Tables VII through XI in inverse order of retardation from the single-particle transition rate. The cases are discussed individually in the Appendix. Separate tables are given for K-forbidden and K-allowed cases. Altogether there are four certain and four probable anomalous E1 cases and one certain M1 case.

Concerning the first of the three general theoretical predictions enumerated above we see, indeed, that all the clear cases of anomalies occur for transitions retarded from single-particle rates by factors of 1.5×10^4 or more.

Concerning the second, we see that except for the exceedingly retarded Hf^{180m} case, normal conversion coefficients are found in K-forbidden cases even though the retardation may be as large as 10^9 or more.

Concerning the third theoretical prediction, that asymptotic quantum-number selection rules for the anomalous operator are valid, there is some uncertainty. At the outset it should be borne in mind that violation of selection rules in n_z, Λ, or Σ is found to result, on the average, in retardation of only one order of magnitude in beta decay.[15] It has been suggested that cases such as those in Tables VIII and X owe their retardation to violation of these selection rules, although the especially high retardation of E1 transition associated with the removal of almost all the oscillator strength to the giant-resonance region of excitations seems to indicate some higher-order cancellation of matrix elements, in addition.

(Violation of the selection rules in Σ and Λ in E1 transitions is (for the normal cases $\Delta N = 1$) also associated with a violation $\underline{n_z}$. One may possibly make the distinction, however, as to the <u>order</u> of forbiddenness in $\underline{n_z}$ alone.) What the theory of anomalous ICC would lead us to seek is separate "threshold" values of retardation above which transitions would show anomalies (of greater

Table VIII

Survey of E1 transitions not K-forbidden

Nucleus	Gamma energy (kev)	Retardation factor	Probable state assignments I Kπ[$Nn_z\Lambda$]	Asymptotic classification for: Radiative trans. op. $\langle rY_1 \rangle$	Asymptotic classification for: Anomalous ICC_3 op. $\langle r^3Y_1 \rangle$	Conversion-coefficient observations
Np^{237}	267	5.5×10^8	$\frac{3}{2}\ \frac{3}{2}$-[521] → $\frac{5}{2}\ \frac{5}{2}$+[642]	h*	u**	α_K probably high by factor of 10.
Pa^{231}***	84	3×10^6	$\frac{5}{2}\ \frac{5}{2}$+[642] → $\frac{3}{2}\ \frac{3}{2}$-[521](?)	h(?)	u(?)	α_{L_I} high by 20. $\alpha_{L_{II}}$ high by 20. $\alpha_{L_{III}}$ high by 1.3±0.4.
Lu^{177}	146	5×10^6	$\frac{9}{2}\ \frac{9}{2}$-[514] → $\frac{7}{2}\ \frac{7}{2}$+[404]	h	h	α_K probably normal but uncertain.
Lu^{175}	282	1.4×10^6	$\frac{9}{2}\ \frac{9}{2}$-[514] → $\frac{7}{2}\ \frac{7}{2}$+[404]	h	h	α_K normal.
Pu^{239}	106	5×10^5	$\frac{7}{2}\ \frac{7}{2}$-[743] → $\frac{5}{2}\ \frac{5}{2}$+[622]	h	u	α_{L_I} high by 1.6±0.2. $\alpha_{L_{II}}$ high by 2.7±0.3. $\alpha_{L_{III}}$ not measurable.
Np^{237}	60	2.8×10^5	$\frac{5}{2}\ \frac{5}{2}$-[523] → $\frac{5}{2}\ \frac{5}{2}$+[642]	h	h	α_{L_I} high by 1.7. $\alpha_{L_{II}}$ high by 3.8. $\alpha_{L_{III}}$ normal. M-subshell pattern abnormal.
Np^{237}	26	3×10^5	$\frac{5}{2}\ \frac{5}{2}$-[523] → $\frac{7}{2}\ \frac{5}{2}$+[642]	h	h	α_L high by 2.
Tm^{169}	63	8×10^4	$\frac{7}{2}\ \frac{7}{2}$-[523] → $\frac{7}{2}\ \frac{7}{2}$+[404]	h	h	α_L normal.
Ac^{227}	27	6×10^4	State assignments unknown.			α_L ~normal but low. $\alpha_{M_{II}}$ low.

Table VIII (cont'd.)

Nucleus	Gamma energy (kev)	Retardation factor	Probable state assignments I Kπ[$Nn_z\Lambda$]	Asymptotic classification for: Radiative trans. op. $\langle rY_1 \rangle$	Asymptotic classification for: Anomalous ICC_3 op. $\langle r^3Y_1 \rangle$	Conversion-coefficient observations
Pa^{231}	26	~5 x 10^4	$\frac{5}{2}\ \frac{5}{2}+[642] \rightarrow \frac{5}{2}\ \frac{3}{2}-[521]$(?)	h(?)	u(?)	α_M-subshell ratios normal.
Pa^{234m}	29	1.5 x 10^4	? Note: odd-odd nucleus			α_L probably high (?).
Hf^{177}	208	<3 x 10^4	$\frac{9}{2}\ \frac{9}{2}+[624] \rightarrow \frac{9}{2}\ \frac{7}{2}-[514]$	h	h	α_K normal.
Am^{243}	85	<2 x 10^4	$\frac{5}{2}\ \frac{5}{2}+[642] \rightarrow \frac{5}{2}\ \frac{5}{2}-[523]$	h	h	L-subshell ratio and α_{total} normal.
Eu^{153}	98	<9 x 10^3	$\frac{5}{2}\ \frac{5}{2}-[532] \rightarrow \frac{5}{2}\ \frac{5}{2}+[413]$	h	h	α_K normal
W^{182}	152	?	? Note: even-even nucleus			α_K low, α_{L_I} high.

* h = hindered.

** u = unhindered.

*** Question marks indicate that the orbital assignments seem somewhat less certain then in the other cases listed.

Table IX

Survey of K-forbidden E1 transitions				
Nucleus	Gamma energy (kev)	Retardation factor	|ΔK|	Conversion-coefficient observations
Hf^{180m}	57.6	10^{15}	8 or 9	L-subshell pattern anomalous with L_I too high
Hf^{178m}	88.8	$2x10^{14}$	8 or 9	Total conversion coeff. normal for E1 within 20%.
Pu^{239}	316	$9.4x10^8$	3	α_K normal
	334	$8.4x10^8$	3	α_K normal
Re^{183}	382	$2x10^6$	2	α_K normal

Table X

Survey of M1 transitions not K-forbidden						
				Asymptotic classification for		
Nucleus	Gamma energy (kev)	Retardation factor	Probable state assignments	rad. trans. $\langle \ell + \mu g \rangle$	anomalous ICC op. $\langle r\ (g \cdot r) \rangle$	Conversion coefficient observations
Ta^{181}	482	$2.6x10^6$	$\frac{5}{2}\ \frac{5}{2}+[402] \rightarrow \frac{7}{2}\ \frac{7}{2}+[404]$	h	u	α_K high by factor of 2 to 10
Np^{237}	208	$1.3x10^4$	$\frac{3}{2}\ \frac{3}{2}-[521] \rightarrow \frac{5}{2}\ \frac{5}{2}-[523]$	h	u	α_K normal (95±10% of Sliv theo.)
Eu^{153}	102	$5x10^2$	$\frac{3}{2}\ \frac{3}{2}+[411] \rightarrow \frac{5}{2}\ \frac{5}{2}+[413]$	h	u	α_K normal (82±10% of Sliv theo.)

Table XI

Survey of K-forbidden M1 transitions				
Nucleus	Gamma energy (kev)	Retardation factor	|ΔK|	Conversion-coefficient observations
Tm^{169}	178	5×10^5	3	α_K normal (or slightly high)
	199	5×10^5	3	α_K normal
Pu^{239}	277.9	6.0×10^4	2	α_K normal
	228.2	4.7×10^4	2	α_K normal
	209.7	9×10^3	2	α_K normal (or low)

than, say, 50%): one threshold for transitions with unhindered anomalous operators and another higher threshold for transitions hindered in this operator. There is also a theoretical dependence of the anomalous ICC on atomic number. Numberical values exhibiting this dependence for M1 have been given by Church and Weneser,[4] and a similar dependence is expected for E1. (Our Table II gives only values of $M_{\chi\chi'}$ for Z = 91.)

Among the E1 cases in Table VIII all examples of transitions unhindered in r^3Y_1 are anomalous with the probable exception of the 26-kev in Pa^{231}, so we may presume the threshold retardation is about 10^5. It is somewhat surprising that the factor by which these examples are anomalous does not vary more, in view of the variation in retardation factor from 5×10^5 to 6×10^8. Perhaps in some cases of high retardation there is a change of configuration involving nucleons other than the odd one, and such rearrangement would decrease the anomalous-conversion matrix element classed as unhindered. Of the Table VIII heavy-element cases hindered in r^3Y_1, the 60- and 26-kev Np^{237} transitions at 3×10^5 retardation show anomalies while in Am^{243} at a retardation $< 2 \times 10^4$ (with presumably the same proton states as the Np^{237} 60-kev) the ICC's are normal. In the rare earth region the E1 cases of Table VIII are all hindered in r^3Y_1, and just one of them, Lu^{177} at 5×10^6 retardation, shows a possible anomaly. The other four cases with retardation ranging down from 1.4×10^6 to $< 9 \times 10^3$ are all normal. The threshold for anomalies in these "h" cases seems around 10^5 in the heavy region and 10^6 in the rare earth region. There is no evident difference in threshold for the cases hindered in r^3Y_1, and the unhindered cases. Probably the scatter in magnitudes of matrix elements is greater than the average separation of the "h" and "u" groups, but more experimental cases will be needed to establish the point.

All three M1 isomers of Table X are unhindered in the anomalous operator, and they indicate a threshold retardation somewhere between 2×10^4 and 10^6. The cases designated "normal" show α_K values somewhat lower than the Sliv theoretical values, but the unretarded M1 transitions probably generally exhibit such slightly lower values according to the analysis of Wapstra and Nijgh.[3]

MORE DETAILED COMPARISONS OF THEORY AND EXPERIMENT

The simple theory of Church and Weneser[4] for anomalous M1 conversion and the corresponding theory for E1 given in this paper, together with considerations of selection rules in the quantum numbers of deformed nuclei, have provided a basis for some systematization of the occurrence of anomalous conversion coefficients. It is next of interest to see if the simple theories are also capable of quantitative explanation of the anomalies.

In order to make really quantitative comparisons for E1 conversion it would be necessary to have theoretical partial-conversion coefficients (i.e., how much of E1 conversion of $s_{1/2}$ goes to $p_{1/2}$ and how much to $p_{3/2}$) and phases for the normal-conversion matrix elements, and these quantities have not been published. Nevertheless, in a particular case of exceedingly large deviations from the normal values of the ICC's we are relatively independent of a knowledge of these partial values. The experimental L_I, L_{II}, and L_{III} coefficients for the 85-kev transition in Pa^{231} are 1.32, 0.84, and 0.047, respectively, and Rose's theoretical values are 0.063, 0.042, and 0.037.

As is readily evident, the experimental ratios cannot be explained solely by M2 admixture* ($\alpha_{M2\ theo.}$ = 96, 8.5, 29.2) but some M2 admixture cannot at present be excluded.

One might attempt to test the theory (cf. Eqs. (14) and (15) and Table II) by examining first any anomaly of the L_{III} subshell conversion, which should depend only on x, the ratio of nuclear matrix elements. This procedure involves several difficulties: first, there is experimental uncertainty of at least 50% in the value; second, the relative partial-conversion coefficients to $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ final states are not known to us; third, it cannot be excluded that the entire small increase of experimental L_{III}-conversion coefficient over theory could be due to a small M2 admixture (0.03%). This admixture corresponds to an M2 half life of 4×10^{-4} sec. which is 10 times as long as the single-proton estimate, but still probably not long enough to be consistent with the fact that the M2 transition between the single-particle states assigned is classified as hindered. Depending upon which fraction of the total normal

* Provided M2 is not anomalous.

L_{III} internal conversion goes to final state $s_{1/2}$ (with which latter transition almost all the anomalous conversion is associated), the upper limit on x may be put between 5×10^3 (0%) and 2×10^4 (100%). (If we assume the fraction to be, e.g., 1/2, the upper limit on x from the experimental L_{III} ICC may be put at 1.5×10^4.) This upper limit corresponds to the hypothetical case (somewhat improbable in view of other empirical L_{III} cases) that the correction term is almost twice as large as the normal term but enters with opposite sign.

The experimental L_I and L_{II} ICC's depend both on the structure parameter x and on $C_{\kappa\kappa'}$. Because they are an order of magnitude (≈ 20) larger than the normal values, the analysis is rather independent of a knowledge of the partial ICC's, $\overset{\circ}{\alpha}_{\kappa\kappa'}$. We may rewrite Eq. (14) in the form

$$\alpha_{\kappa'} = \sum_{\kappa} \left| \sqrt{\overset{\circ}{\alpha}_{\kappa\kappa'}} - ie^{-i\delta} W^{-3/2} M_{\kappa\kappa'} C_{\kappa\kappa'} x \right|^2. \tag{20}$$

It is then apparent that in these particular cases $\overset{\circ}{\alpha}_{\kappa\kappa'}$ may be neglected, and from the empirical values of α_{L_I} and $\alpha_{L_{II}}$ for the Pa^{231} case considered, we obtain the relations

$$|x|^2 (0.37 + 0.63\, C^2_{1\,-1}) \approx 8.2 \times 10^8, \tag{21}$$

$$|x|^2\, C^2_{-1\,1} \approx 10 \times 10^8. \tag{22}$$

From the analysis of the L_{III} conversion we had

$$|x|^2 < 4.0 \times 10^8. \tag{23}$$

In the most elementary form of the theory [Eq. (14) and Table II] $C_{\kappa\kappa'}$ would be unity, corresponding to no change in electrostatic potential at the center of the nucleus and to a correspondingly high degree of cancellation of the two terms on the left of Eq. (12). With the uncertainty of our values of $M_{\kappa\kappa'}$ with both $C_{\kappa\kappa'}$ unity and, in addition, the uncertainty introduced by neglecting the contribution from the normal terms in Eq. (20), we cannot entirely exclude a solution $x \approx 3 \times 10^4$ and $C_{\kappa\kappa'} = 1$ to Eqs. (21) and (22). This x is somewhat larger than what is allowed by the inequality (23). The upper limit of this inequality corresponds in turn to a somewhat improbable case that the anomalous amplitude for L_{III} conversion enters with twice the size of the normal contribution, and with the opposite sign. The limit is furthermore lowered if part of the L_{III} conversion is due to M2 radiation.

Furthermore, we have calculated the single-particle value of the matrix element $< r^3Y_1>$ for the state assignments of Table VIII, using the wave functions of Ref. 12 in the so-called asymptotic approximation representing an approximate solution to the potential of Ref. 12 in the limit of large deformations. Using the empirical value on $|< rY_1>|$ from the gamma lifetime, we obtain $|x| \approx 1100$. The asymptotic approximation in particular and any single-particle wave function in general obtained from a simple model is more likely to overestimate than underestimate the value of $< r^3Y_1>$. However, a possible enhancement of x might result from a collective octupole deformation of the nucleus. An enhancement by a factor of 20 seems, however, excessive.

It seems more probable that the true physical situation is more nearly represented by a solution of relations (21)-(23) with an x of the order of the estimated single-particle value and large factors $C_{\kappa\kappa'}$. On a naive basis one may insert the expressions of Table II for $C_{\kappa\kappa'}$ and solve Eqs. (21) and (22) in terms of x and the quantity $[v'(0) - v(0)]$, i.e., the change of depth of the electrostatic potential. (One may notice that this quantity enters with a different sign in C_{1-1} and C_{-11}.) The quantity $[v'(0) - v(0)]$ is then given by the ratio between Eqs. (21) and (22), i.e., $[v'(0) - v(0)]$ is related essentially to the ratio L_I/L_{II}. Of the two solutions to the new relation so obtained, one corresponds to a very small value of $[v'(0) - v(0)]$ and requires the large x value already discussed and evaluated as improbable for other reasons. The other solution corresponding to a large value of $|v'(0) - v(0)|$, of the order of 10 Mev or more, is very sensitive to the exact ratio of the right sides of Eqs. (21) and (22). In view of the fact that the normal conversion amplitudes are neglected in comparison with the anomalous ones (the latter being only five times as large) and furthermore in view of the uncertainty of the estimate of $M_{\kappa\kappa'}$, the numbers on the right-hand sides of Eqs. (21) and (22) cannot be considered very accurate. The solution corresponding to $|v'(0) - v(0)| \gg 1$ is, however (in contrast to the other solution), very sensitive to the value of the ratio discussed. We can then mainly conclude that $C_{\kappa\kappa'}$ seems to be of order 10, and x of the order of the single-particle estimate or somewhat larger.

In summary, we can assign values to the two parameters $< r^3 Y_1 >$ and $v'(0) - v(0)$ to give a consistent explanation of the three L-subshell coefficients in the Pa^{231} case, and such that the actual value of the matrix element $< r^3 Y_1 >$ is not inconsistent with reasonable single-particle values. However, the magnitude of $v'(0) - v(0)$ required seems quite excessive. This shortcoming of the elementary theory clearly calls for refinements and consideration of effects in addition to the change in the electrostatic potential, which would tend to lift the cancellation in Eq. (12). The anomalous internal-conversion interaction takes place wholly within the nuclear volume (i.e., at short distances), and it would not be surprising if vacuum polarization or higher-order radiative corrections were significant. It is believed, for example, that such corrections are significant in calculation of x-ray fine-structure energy levels.[24] Another effect that might tend to remove the $s_{1/2} \leftrightarrow p_{1/2}$ cancellation in the "unperturbed" electron wave functions would result from a large change in the state of magnetization throughout the nuclear volume.

The probably anomalously converted 29-kev transition in Pa^{234} discussed by Vartapetian[22] (where possibly the same orbitals are associated with the transition as in Pa^{231}) presents a difficulty of a quantitative kind for this elementary theory, as this transition is only 1/100 as hindered as the 84-kev transition in Pa^{231}. In view of possible experimental difficulties in conversion coefficient measurements at such low energies, further work on the ICC and subshell ratios is important.

It is interesting to note the pattern of L-subshell anomalies for the E1 cases of Table VIII where such information is known (the 85-kev transition of Pa^{231}, the 60-kev and 26-kev transitions of Np^{237}, and the 106-kev of Pu^{239}). In no case is the L_{III} subshell definitely anomalous, although it may be about 30% too high in Pa^{231}. (This may be interpreted as strong support for the argument of $C_{\kappa\kappa'} \gg 1$). In Pa^{231}, the most striking case, the L_I and L_{II} subshells are equally enhanced by a factor of 20. In Np^{237} and Pu^{239} the enhancement is more modest, and is greater in the L_{II} subshell than in the L_I.

One may speculate on a possible effect on the K/L_I ratio associated with the existence of large anomalous matrix elements but more directly dependent on the phase of the normal-conversion amplitude. The K-as well as the

L_I-conversion involves initial $s_{1/2}$ states; the initial states, however, are characterized by different radial quantum numbers. The sign of $S(\infty, h_L)$ (cf. Eq. (8)) determines the phase of the "point charge" amplitude $\sqrt{\overset{\circ}{\alpha}_{\kappa\kappa'}}$. As we have not performed the calculation of $S(\infty, h_L)$ we have no way of quantitatively estimating the effect; it seems, however, conceivable that the phase of this quantity may be greatly different for the K-conversion and the L_I-conversion. It is thus possible that in anomalous conversion the K and L_I ICC's could deviate in different directions. This effect may be thought of as a possible explanation of the anomalous K/L_I ratio reported for an E1 transition in W^{182} (see discussion Appendix).

One might attempt a quantitative comparison also for some of the M1 transitions. The case that lends itself most readily for such a comparison is the 480-kev M1 transition in Ta^{181}. The M1 K-conversion coefficient obtained on the basis of the measured total ICC and angular correlation data, as discussed in the Appendix, is ≈ 0.5 as compared with Sliv's value $\alpha_K(M1) = 0.06$. Using Eq. (18) to fit the observed ICC, one may roughly estimate $|x|$

$$|x|^{expt} = 1000 - 2000.$$

Actually the approximation involved in the step between Eqs. (16) and (18) (i.e., between Eqs. (6) and (7) of Ref. 4) seems to require that $\underline{x}$ (or λ) not be too large. This introduces an additional error in the estimate of the order

$$\sqrt{\frac{\overset{\circ}{\beta}_{1-1} + \overset{\circ}{\beta}_{2-1}}{\overset{\circ}{\beta}_{1-1}}}$$

for large $\underline{x}$ values. Equation (18) thus underestimates $\underline{x}$.

A semitheoretical estimate of $\underline{x}$ is obtained by using the observed partial M1 lifetime to determine the absolute value of the normal M1 matrix element and the wave functions of Ref. 12 in the "asymptotic approximation" to determine the anomalous internal-conversion matrix element. This estimate of $\underline{x}$ gives

$$|x|^{theo} \approx 5000.$$

It is not disturbing to find the theoretical anomalous conversion matrix element as much as a factor of five too large. Indeed, most single particle theoretical matrix elements for gamma or beta transitions overestimate the experimental transition rates unless collective effects are important.

It is interesting to compare the Ta^{181} 480-kev case just treated with the case of the 208-kev M1 transition in Np^{237}. Both transitions are hindered for the radiative operator in the "asymptotic" selection rules and unhindered in the same rules for the anomalous internal-conversion operator, but they differ in their experimental hindrance factors. This latter transition is hindered by a factor $\approx 10^4$, compared with $\approx 10^6$ for the Ta^{181} transition considered.

The experimental value of α_K is 2.3, compared to Sliv's theoretical value of 2.4. This may be used to put a limit on $\underline{x}^{expt}$. With a deviation between theoretical and experimental α_K of, say, 10%, we have

$$|x|^{expt} < 10$$

(excluding the unlikely case that the anomalous contribution is of twice the magnitude but opposite in sign to the normal term.) The semitheoretical value of $\underline{x}$ obtained by taking the absolute value of the "normal" matrix element from the gamma lifetime and calculating the anomalous matrix element from the wave function of Ref. 12 in the "asymptotic approximation" is

$$|x|^{theo} \approx 50$$

Again the single-particle estimate of $\underline{x}$ seems too high, but as discussed with the Ta^{181} case, this overestimation is not particularly disturbing.

As a summary of the E1 and M1 cases treated quantitatively one may state that the anomalous terms introduced by Church and Weneser seem to account semi-quantitatively for the anomalous M1 case encountered and on the whole to be consistent with the cases of hindered M1 transitions in which no anomaly is found. As for the quantitative comparison of the anomalous E1 internal-conversion operator, the experimental effect seems somewhat greater than theoretically expected, and the large values of the correction factor $C_{\kappa\kappa'}$ obtained by the attempted fitting of experimental data may probably more appropriately be considered an indication of higher-order effects, neglected in the treatment presented here.

EFFECTS ON ANGULAR DISTRIBUTION

A few words may be in order regarding the effects of ICC anomalies on angular-distribution experiments involving conversion electrons. Church and Weneser have already discussed the M1 transitions. For E1 transitions the $s_{1/2}$ electrons (K,L_I) convert into $p_{1/2}$ and $p_{3/2}$ continuum states, and contributions to anisotropy arise from the $p_{3/2}$ part and--more importantly--from a $p_{1/2}$-$p_{3/2}$ interference. In the normal case $p_{1/2}$ and $p_{3/2}$ occur in comparable amount, whereas it is clear empirically from anomalous subshell ratios that the $p_{1/2}$ final state is generally most affected and $p_{3/2}$ very little. In a greatly enhanced K conversion, as is likely for the 267-kev transition in Np^{237}, the anisotropy in a conversion-line angular correlation would be depressed from normal, and the sign of the anisotropy might or might not be reversed from normal, depending on the relative phases of the normal and anomalous conversion to $p_{1/2}$ states. Angular-correlation experiments on conversion electrons could thus yield unique information on these relative phases.

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APPENDIX

The cases in Table VIII in the heavy-element region are thoroughly discussed in the paper of Asaro et al.,[6] but other cases in Tables VIII-XI that need special discussion are covered briefly here.

Table VIII Cases (K-Allowed E1)

Lu^{177}, 146 kev

The decay scheme of Yb^{177} to Lu^{177} has been studied by several groups in recent years.[25,26,27,28] A prominent feature of the photon spectrum is the 146-kev gamma ray depopulating a state measured as having 1.2×10^{-7} sec half life.[25,26] The conversion coefficient was reported by de Waard as $\alpha_K = 0.63 \pm 0.08$ and K/LM ≈ 3.5.[26] On this basis he designated the transition as 10% M2 – 90% E1, and Vartapetian[22] points out that this admixture requires M2 radiation of 1.3 times the single-particle rate. From this observation one might suspect the ICC for E1 to be slightly high. However, Mize et al.[28] re-measured α_K for the 146-kev gamma and set an upper limit $\alpha_K < 0.4$ and, using de Waard's K/LM ratio, they set a limit M2/E1 < 0.04. With this limit the M2 comparative lifetime is quite similar to that in the analogous transitions of 396 kev and 282 kev in neighboring Lu^{175}. For the 282-kev transition the angular correlation measurements fix the M2/E1 ratio and lead to the conclusion that α_K for the E1 is probably normal.[28] We must conclude from the present uncertain data that the 146-kev transition in Lu^{177} probably has a normal E1 conversion coefficient. Experimental work to resolve the disagreement on α_K and to establish L-subshell conversion ratios would be valuable.

Lu^{175}, 282 kev

Vartapetian[22,23] has measured the lifetime of the 396-kev level from which this E1 transition originates as $(3.4 \pm 0.3) \times 10^{-9}$ sec. Several studies[29-34] on Yb^{175} have helped to establish the decay scheme, and a theoretical interpretation of some features has been given by Chase and Wilets.[35]

From the angular correlation work of Åkerlind et al.[33] on the 282-113 cascade it is possible to determine the E1/M2 ratio of the 282-kev independently of its conversion coefficient. Assuming de Waard's value of E2/M1 = 0.3 for the 113-kev gamma (recent work of Hatch et al.[34] gives E2/M1

≅ 0.25, in reasonable agreement), one finds that the anisotropy measured by Åkerlind *et al*. supports 2% M2 – 98% E1 for the 282-kev transition. With this mixing ratio and Sliv's theoretical K-conversion coefficients (for E1, 0.0205, and for M2, 0.67) we calculate a theoretical normal ICC of 0.0334. Hatch *et al*. give an experimental α_K of 0.030 and Mize *et al*.[28] give 0.038, so we conclude that the E1 ICC here is normal. Vartapetian[22] has pointed out that the 2% M2 admixture requires an M2 transition rate half that of the single particle formula.

Tm^{169}, 63 kev

The Yb^{169} electron capture decay has been studied in several recent investigations.[27,34,36,37] Some features of the decay have been discussed also by Mottelson and Nilsson.[38]

The 63-kev transition proceeds from a level at 380 kev according to the decay scheme of Mihelich *et al*.,[37] and they measured a lifetime of 4.5×10^{-8} sec for the state. They determined an L-conversion coefficient of 0.19 ± .004 and Hatch *et al*.[34] give 0.15 for the same quantity. The corresponding Rose theoretical value for E1 is 0.15. Mihelich *et al*. measure relative L subshell ratios of 2.3/0.8/1.0, consistent with the theoretical ratios 2.2/0.8/1.0. We conclude that this transition has normal conversion coefficients.

Hf^{177}, 208 kev

The beta decay of Lu^{177} to Hf^{177} has received considerable study in recent years,[22,39-41] and the ground state spin of Hf^{177} has been determined as 7/2 by Speck and Jenkins.[42] Some information has also come from study of electron capture of Ta^{177},[43,44] and from Coulomb excitation.[45]

There appears to be a level at 321 kev populated both in beta decay and in electron capture. It decays by 321-kev and 208-kev gamma rays. For our study of the ICC problem the latter transition is of the greater interest, since there is information on the E1-M2 mixing from angular-correlation data.[22,40] The interpretation of the angular correlation requires independent knowledge of the M1-E2 ratio in the 113-kev cascade transition. From L-subshell ratios[39,43] one would calculate a small M1 admixture of 2 to 3%. The experimental anisotropy is consistent with M1/E2 = 0.03±.005 and a pure

E1 transition; this interpretation is proposed by Ofer.[41] However, the angular-correlation experiment does not fix the limit on M2 admixture in the 208-kev very well. Two percent M2 in the 208-kev is consistent with 2.8% or 3.2% M1/E2 possibilities in the 113-kev.

The experimental α_K of the 208-kev is 0.044,[39] to be compared with Sliv's theoretical α_K (E1) = 0.0446 and α_K(M2) = 2.05. We conclude in Table VIII that α_K is normal, although we cannot exclude the remote possibility that α_K(E1) is anomalously small and that there is M2 admixture.

A limit on the 321-kev state lifetime was set by Vartapetian[22,23] as $t_{1/2} < 4 \times 10^{-10}$ sec.

Eu^{153}, 98 kev

The level system of Eu^{153} has been studied by Coulomb excitation[46] and by beta decay[47-52] of Sm^{153} and electron capture[53-58] of Gd^{153}. Among the transitions is a 98-kev E1 transition to ground with a lifetime of $< 10^{-9}$ sec. Marty and Vergnes[56] report $\alpha_K = 0.3 \pm .1$, and Church[59] has given the α_K relative to that of the 103-kev M1 transition discussed later in this appendix. From his ratio we calculate $\alpha_K \approx 0.17$. The Sliv and Band theoretical α_K is 0.23. Hence, we conclude that α_K is normal within experimental uncertainty.

The initial- and final-state assignments are 5/2, 5/2 - [532] and 5/2, 5/2 + [413], respectively.[19]

W^{182}, 152 kev

Wapstra and Nijgh[3] have recently renormalized and discussed conversion coefficients in W^{182} and W^{183} from measurements by Murray et al.,[60] and they list conversion-coefficient comparisons for a few E1 transitions in W^{182}. The α_K of the 152-kev transition from 3, 2- (I,K π) to 2, 2+[61] is less than half the Rose theoretical values (which differ very little from Sliv and Band values, in this case). The 222-kev transition from 4, 4- to 3, 2+ exhibits a normal α_K for E1. It is interesting to note that the anomalous case is not K-forbidden, while the normal case is K-forbidden. With only one determination of these conversion coefficients we might maintain some reservation about labeling the 152-kev transition as anomalous. However, recent work of Gallagher on Re^{182} decay to W^{182} has clearly shown that there is an anomaly in the K/L_I ratio,[62] the experimental value being 2.8 and Rose's theoretical being 8.3. This K/L_I measurement and the α_K measurement together indicate an α_{L_I} higher than normal. In the main text of this paper we have discussed the possibility that the E1 anomaly might give a constructive contribution to L_I (or K) conversion and a destructive contribution to K(or L_I). Gallagher also finds probable L_I/K anomalies in other E1 transitions of W^{182}, but there is considerably more experimental uncertainty with them than with the 152-kev transition.

We do not have a lifetime determination for the 152-kev transition, but we may estimate the order of its retardation from that of the analogous 67-kev transition (2,2- → 2,2+). The 2,2- state has a half life of 1.03×10^{-9} sec according to measurements of Sunyar.[63] From this measurement and relative gamma intensity measurements[60] we calculate a retardation factor of 4.5×10^3 for the 67-kev E1.

It seems surprising that E1 anomalies should set in at such low retardation, but W^{182} is an even-even nucleus and is not really to be compared closely with odd-mass nuclei. Further experimental studies in this and similar even-even nuclei are certainly desirable.

Table IX Cases (K-Forbidden E1)

Hf^{180m}, 57.6 kev

This unusual 5.5-hr E1 isomeric transition[64,65] is the slowest E1 transition known; it is slower by a factor of 10^{15} than the single-particle formula predicts. The values $L_I:L_{II}:L_{III} \approx 5:0.5:1$ and $\alpha_L \approx 0.4$ have been reported,[65,66] from which one determines approximately that α_{L_I}, $\alpha_{L_{II}}$, and $\alpha_{L_{III}}$ are 0.31, 0.03, and 0.06 compared with theoretical values of 0.11, 0.051, and 0.058 for the three L subshells for E1. (Corresponding values for M2 are 67, 6.1, and 21). Clearly it is not proper to invoke M2 admixture, since $\alpha_{L_{III}}$ agrees for E1. The threefold enhancement of α_{L_I} and possible decrease of $\alpha_{L_{II}}$ is to be ascribed to anomalous E1 conversion contributions. The great retardation has been attributed to a high order of K-forbiddenness, $\Delta K = 8$ or 9. The appearance of anomalous internal conversion according to this interpretation indicates, however, that for the small components of the wave function by which the transition may proceed the anomalous matrix elements may greatly exceed those for the radiative transition.

Hf^{178m}, 88.8 kev

This E1 transition[43,67,68] seems analogous to the case of Hf^{180m} just discussed. With a lifetime of 3 sec, the retardation factor is 2×10^{14}. It is reported that the total conversion coefficient is 0.5[43] compared to the theoretical value of 0.46. Here again the retardation is presumably due mainly to K-forbiddenness, $\Delta K = 8$ or 9.

Pu^{239}, 316 kev and 334 kev

These weak E1 transitions have been seen arising from the 1.93×10^{-7} sec level at 391.8 kev.[69,70] From the branching ratios for transitions from this level we calculate that the 316-kev transition is retarded by a factor of 9.4×10^{8} and the 334 by 8.4×10^{8}.

Ewan and Knowles[70] measured the K-conversion coefficients of these gammas and found them in agreement with theoretical E1 values of Sliv or Rose, which are not very different from each other in this case.

The great slowness of these transitions is largely to be attributed to K-forbiddenness, since K_i is 7/2 (or possibly 5/2) and K_f is 1/2.[69,19]

Re^{183}, 382 kev

Newton and Shirley[71] have found a 382-kev E1 transition in Re^{183} following the decay of Os^{183}. The state at 496 kev from which this transition originates was measured to have a half life of 8×10^{-9} sec. According to Newton's interpretation the initial state is 9/2 - (K = 9/2) and the final state is 7/2 + (K = 5/2), being the first excited state of the ground rotational band. Thus, the transition is K-forbidden.

The experimental K-conversion coefficients is $(1.0 \pm 0.1) \times 10^{-2}$, to be compared with Sliv's theoretical value of 1.12×10^{-2}, and thus seems normal.)

Table X Cases (K-Allowed M1)

Ta^{181}, 482 kev

The levels and transitions of Ta^{181} have received much study through Coulomb excitation and beta decay of Hf^{181}. We shall not attempt to list references or give a detailed discussion, since the data have been thoroughly reviewed recently by Debrunner et al.[72] Suffice it to say here that the M1-E2 mixing ratio of the 482-kev transition has been carefully determined by gamma-gamma and electron-gamma angular correlations as 98% E2 + 2% M1. Its half life is 1.06×10^{-8} sec. Several determinations of α_K are listed by Debrunner <u>et al</u>., and they choose $\alpha_K = 0.026$ as a best value. Using Sliv's theoretical conversion coefficients one finds a significant discrepancy, which we choose to interpret as the M1 conversion coefficient being a factor of about 10 larger than normal and the unhindered E2 ICC's being normal. The experimental uncertainty on this enhancement factor is considerable, but there is clearly some enhancement outside limits of experimental error.

The initial- and final-state assignments 5/2, 5/2 + [402] → 7/2, 7/2 + [404] are supported by spin determinations as discussed by Debrunner <u>et al</u>.,[72] and the orbital assignments are based on a variety of evidence to be reviewed in a forthcoming publication.[19] The radiative M1 transition is allowed by K-selection rules but hindered in "asymptotic" quantum-number rules (see Table VI), probably explaining at least partially its great retardation (2.6×10^6). The anomalous M1 conversion operator is unhindered ($\Delta n_z = 0$, $\Delta\Lambda = 2$, $\Delta\Sigma = -1$) (see Table VII), a favorable situation for appearance of the ICC anomaly.

<u>Np^{237}, 208 kev</u>

The 208-kev M1 transition is a prominent feature of the beta decay of U^{237} (cf. Rasmussen, Canavan, and Hollander[73] and references listed therein). By L-subshell conversion ratios the E2/M1 mixing ratio is 0.5% or less. Within experimental error the α_K of 2.3 agrees with the Sliv theoretical M1 value of 2.4. The half life of the state at 267 kev from which the 208-kev transition originates was measured by Bunker, Mize, and Starner[74] as 5.4×10^{-9} sec.

The reasons supporting the state assignments associated with this transition exhibited in Table X are detailed in Ref. 73. With respect to the asymptotic quantum-number selection rules, the situation is the same as in Ta^{181} just discussed: $\Delta n_z = 0$, $\Delta\Lambda = 2$, $\Delta\Sigma = -1$. Hence, the radiative operator is hindered and the anomalous operator is unhindered.

Eu^{153}, 103 kev

The reader is referred to the paragraph on the 98-kev E1 transition in Eu^{153} for references on the Eu^{153} decay scheme. The 103-kev M1 transition is thought to proceed to ground from a state with half life 4×10^{-9} sec, as measured by McGowan,[48] by Graham and Walker,[49] and by Vergnes and Marty.[51] Its K-conversion coefficient has been variously measured as 1.19 (Dubey et al.[52]), 1.2 (Marty[51]), 1.1 (McGowan[48]), and 1.2 (Cohen et al.[54]). Bhattacherjee and Raman[57] reported α_K = 0.67, and Bisi et al.[58] reported 0.68, but these two last-mentioned studies apparently failed to take into account the presence of the 98-kev E1 transition, which would not have been resolved from the 102-kev in the photon spectrum. From the L-subshell conversion ratios of the 102-kev transition it is established[75] that there is less than 5% E2 admixture. Sliv's theoretical K-conversion coefficients for this case are 1.48 for M1 and 1.10 for E2. The experimental ICC seems significantly lower than the theoretical by about 20%. However, we have classified the conversion coefficient as normal in Table X, since the analysis by Wapstra and Nijgh[3] on a number of M1 transitions, most of which were not of the retarded nature considered here, showed their conversion coefficients to be systematically still somewhat lower than the Sliv theoretical values.

Table XI Cases (K-Forbidden M1)

Tm^{169}, 178 and 199 kev

In the first section of this Appendix, where the 63-kev E1 transition in Tm^{169} was discussed, we listed the references to experimental work elucidating the Tm^{169} level system. In Tm^{169} there is a level at 316.2 kev with a half life of 6.4×10^{-7} sec. The level is assigned 7/2, 7/2 + [404],[19] and it decays by K-forbidden ($\Delta K = -3$) transitions to states of the ground rotational band, 7/2 + and 5/2 +, K = 1/2 [411]. The analysis by Mihelich et al.[37] determines the M1-E2 mixing ratios of the 178- and 199-kev transitions on the basis of L-subshell conversion coefficients, and they give the value 5×10^5 as factors of retardation for the M1 components of both transitions.

Using the Mihelich et al. mixing ratios and Sliv ICC values, we obtain the theoretical α_K for the 178-kev transition of 0.49, while Mihelich et al. find experimentally 0.49 ± .10 and Hatch et al.[34] find 0.51. For the 199-kev transition, the corresponding theoretical value is 0.35 and the experimental values 0.45 ± 0.09[37] and 0.40[34]. Thus, we conclude that the ICC's are normal.

Pu^{239}, 277.9, 228.2, and 209.7 kev

There are three prominent M1 transitions seen following beta decay of Np^{239} or Am^{239} and following alpha decay of Cm^{243}. These transitions of 277.9, 228.2, and 209.7 kev arise from a level at 285.8 kev with half life 1.1×10^{-9} sec. The experimental work relevant to this level scheme has been reviewed by Perlman and Rasmussen,[76] and we refer to this review work for the original references. The level at 285.8 kev has been assigned 5/2, 5/2 + [622], and the three M1 transitions go to excited states of the ground rotational band of spins 3/2, 5/2, and 7/2. The ground state of this latter band has been assigned K = 1/2 + and the orbital [631]. Hence, the retardation of the M1's may be largely ascribed to K-forbiddenness ($\Delta K = 2$). The best determinations of conversion coefficients of these three transitions are probably those of Ewan and Knowles.[70] Limits of < 10% E2 admixture were set on the basis of L-subshell conversion coefficients, and Table XII lists their experimental α_K values and Sliv theoretical values for pure M1 and for the upper limit of 10% E2 admixture. We conclude that these α_K values are probably normal although the 210-kev conversion coefficient shows a discrepancy with the theoretical values by twice the probable error.

Table XII

K-forbidden M1 transitions in Pu^{239}

Gamma energy (kev)	α_K (Exptl.)	α_K Theoretical: Pure M1	α_K Theoretical: 90% M1–10% E2	Retardation factor
278	1.16±.12	1.18	1.07	6.0×10^4
228	1.60±.16	2.04	1.85	4.7×10^4
210	1.76±.30	2.58	2.33	9.0×10^3

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