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John Ellis[a], N.E. Mavromatos[b] and D.V. Nanopoulos[c]

Abstract

We discuss the possibility that CPT violation may appear as a consequence of microscopic decoherence due to quantum-gravity effects, that we describe using a density-matrix formalism motivated by our studies of non-critical string theory. The maximum possible order of magnitude of such decohering CPT-violating effects is not far from the sensitivity of present experiments on the neutral kaon system, and we review a simple parametrization for them. We also review a recent data analysis carried out together with the CPLEAR collaboration, which bounds any such decohering CPT-violating parameters to be $\lesssim 10^{-19}$ GeV.

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1 CPT Invariance

This is a fundamental theorem of Quantum Field Theory, which follows from the basic requirements of locality, Lorentz invariance and unitarity [1]. Among the consequences of the CPT theorem are equal masses for particles and their antiparticles, equal lifetimes, equal and opposite electric charges, and equal magnetic moments. None of these properties need hold in non-relativistic quantum mechanics, which need not obey all the constraints of local quantum field theory. However, what interests us is the possibility that CPT invariance may be violated as a consequence of microscopic decoherence in the context of quantum gravity in general, and specifically in the context of string theory.

A number of experimental upper limits on violations of the CPT theorem can be found in the Particle Data Book [2]. They include the following measurements on electrons and positrons:

$$|\frac{\Delta m_e}{m_e}| < 4 \times 10^{-8}, |\frac{\Sigma Q_e}{Q_e}| < 4 \times 10^{-8}, \frac{\Delta(g-2)_e}{g_e} = (-0.5 \pm 2.1) \times 10^{-12}, \tag{1}$$

the following test with muons:

$$\frac{\Delta(g-2)_\mu}{g_\mu} = (-2.6 \pm 1.6) \times 10^{-8} \tag{2}$$

and the following test with protons:

$$\frac{\Delta m_p}{m_p} = (2 \pm 4) \times 10^{-8}. \tag{3}$$

These are all very impressive limits, but they all pale in precision compared with the bound from the neutral kaon system:

$$\frac{\Delta m_K}{m_K} < 9 \times 10^{-19} \tag{4}$$

obtained by the CPLEAR collaboration [3].

In view of the impeccable credentials of the CPT theorem, and the tremendous accuracy with which it has been verified, why would any theorist challenge it, and why should any experimentalist want to try any harder than in (1,2,3,4) above? One possible motivation is provided by the apparently unrelated theoretical problem discussed in the next section.

2 Do Topological Space-Time Fluctuations Destroy Quantum Coherence?

It is known [4] that a macroscopic four-dimensional black hole has non-trivial entropy S related to the area A of its event horizon, and hence to its mass M in the case of a black hole with no additional quantum numbers such as electric charge Q or spin J [1]:

$$S = \frac{1}{4}A = M^2 \tag{5}$$

[1] Here and subsequently, we use natural units in which the Planck mass $M_P = 1$

A macroscopic black hole also has an effective temperature T:

$$T = \frac{1}{8\pi M} \tag{6}$$

as manifested, for example, by its Hawking radiation. The facts that the entropy (5) and temperature of the macroscopic black hole are non-zero tell us immediately that it *must* be described by a mixed quantum-mechanical state. At first sight, this is surprising, because we could certainly imagine having made our black hole by colliding particles in a pure initial state, and conventional quantum mechanics and quantum field theory forbid pure states from evolving into mixed states.

The basic intuition behind the appearance of a mixed state is the following: imagine that a pure state with two components $|A, B>$ is prepared near the event horizon of a macroscopic black hole, and that one of the components, $|B>$ say, falls inside the horizon. Conventional semiclassical quantum gravity, as embodied in Hawking's original calculation of the radiation that bears his name, would suggest that all information I about the component $|B>$ is lost to an exterior observer, including for example information about its quantum-mechanical phase, and that all one can therefore observe is a mixed external component with a density matrix

$$\rho_A = \Sigma_I |A>_I \, _I<A| \tag{7}$$

suggesting the "forbidden" evolution of a pure state into a mixed state.

This proposal is already controversial with many quantum field theorists, but what makes them really see red [5] is the further suggestion of Hawking [6] that such evolution from pure into mixed states might also occur at the microscopic level. Here the hypothesis is that information about particle wave functions, etc., might be lost across microscopic event horizons associated with topologically non-trivial fluctuations in the space-time background taking place on the Planck distance scale $L_P \simeq 10^{-33}$ cm within the Planck time scale $t_P \simeq 10^{-43}$ seconds, called generically space-time foam. The intuition behind any such proposal is an analogy with the conventional quantum mechanics of open systems coupled to undetected degrees of freedom, represented in this case by physics at the Planck scale L_P, t_P. Here, however, the suggestion is that this might be an intrinsic and fundamental limitation of laboratory physics, rather than an artefact of technological limitations or laziness in not measuring the state of some communicating reservoir.

Any such fundamental loss of information, and hence transition from pure to mixed states at the microscopic level, would entail a modification of conventional quantum field theory and quantum mechanics, which is repugnant to many theorists enamoured of the great beauty of these theories.

Concretely, Hawking has proposed [6] that one should abandon the conventional S-matrix description of asymptotic particle scattering, and replace it by a density matrix description, in which scattering occurs via a superscattering operator $\not{S}$:

$$\rho_{out}{}^A_B = \not{S}^{AD}_{BC} \, \rho_{in}{}^C_D \tag{8}$$

where, unlike in conventional field theory, $\not{S}$ does not factorize into a product of matrix elements of S and its hermitian conjugate:

$$\not{S}^{AD}_{BC} \neq S^A_C \, S^{+D}_{\ \ B} \tag{9}$$

One way of thinking about this proposal may be that, whereas $\not{S}$ factorizes in any fixed space-time background, this would no longer be the case if one sums over many such backgrounds, as may be appropriate to take into account fluctuations in the space-time foam.

If the proposal (8,9) is correct, a corollary would be that the Liouville equation that describes the time evolution of a quantum system must also be modified. When integrated over all time, the usual Liouville equation $\partial_t \rho = i[\rho, H]$ becomes the normal S-matrix description of asymptotic scattering. In order to avoid this so as to accommodate a lack of factorization (9), we need an extra term [7]:

$$\partial_t \rho = i[\rho, H] + \not{\delta} H \rho \tag{10}$$

Just such a modification of the quantum Liouville equation is characteristic of open quantum-mechanical systems. Here, the 'openness' would be due to the coupling of the observable system to unseen (unseeable?) modes of the theory within the microscopic event horizons believed to infest space time. Any modification (10) would necessarily cause pure states to evolve into mixed states, with the collapse of off-diagonal entries in the density matrix. One can also show that symmetries no longer correspond to conservation laws, in general [7].

Clearly, any modification of the type (10) should respect probability conservation, which means that $\mathrm{Tr}\rho$ should be time-independent, and it should also conserve energy, at least to a very good approximation. Shortly after the proposal (10) was made, the concern was expressed [8] that energy non-conservation might be the Achilles heel of this idea. However, we have shown that energy is conserved, in the sense that

$$\partial_t < E > = 0 \ : < E > \equiv \mathrm{Tr}(E\rho) \tag{11}$$

in our non-critical string approach [9], as a consequence of the renormalizability of the two-dimensional field theory on the string world sheet. Moreover, Unruh and Wald have recently given general arguments why energy conservation need not be an essential difficulty [10].

In order to gain some intuition how solutions to a modified quantum Liouville equation of the type (10) behave, it is instructive to consider [7] the simplest possible example of a two-state system with two energy levels $E \pm \Delta E/2$:

$$H = E + \Delta E \sigma_z \tag{12}$$

whose normal quantum-mechanical evolution is unitary:

$$\rho(t) = \frac{1}{2} \begin{pmatrix} 1 & e^{-i\Delta E t} \\ e^{i\Delta E t} & 1 \end{pmatrix} \tag{13}$$

as seen in Fig. 1(a).

In the open quantum-mechanical formalism (10), it is convenient to parametrize ρ in the four-dimensional Pauli σ-matrix basis σ_α for 2×2 hermitian matrices $(\mathbf{1}, \sigma_x, \sigma_y, \sigma_z)$: $\rho = \rho_\alpha \sigma_\alpha$. In this formalism, one can regard $\not{\delta} H$ as a symmetric 4×4 matrix $\not{h}_{\alpha\beta}$. Probability conservation then requires

$$\not{h}_{0\beta} = 0 = \not{h}_{\alpha 0} \tag{14}$$

and energy conservation requires

$$\not{h}_{3\beta} = 0 = \not{h}_{\alpha 3} \tag{15}$$

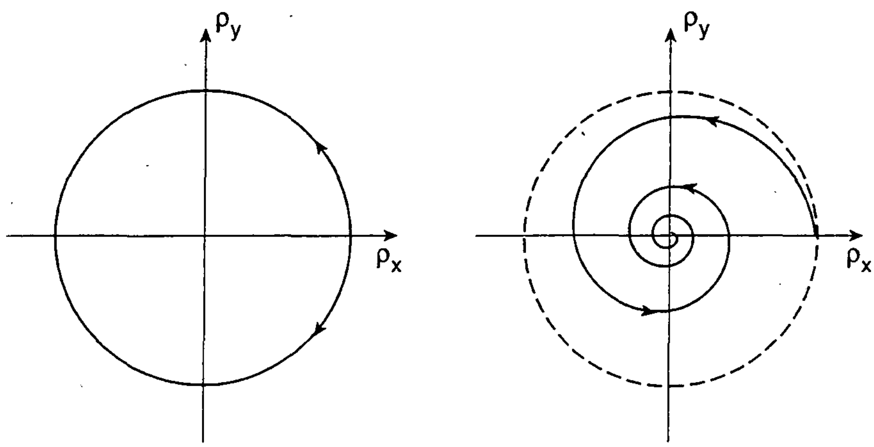

Figure 1: The unitary evolution of a simple two-dimensional quantum-mechanical system is contrasted with the spiralling behaviour induced by the parameters α, β, γ, representing evolution towards a completely mixed diagonal density matrix [7].

so that its general form is

$$\not{h}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\alpha & -\beta & 0 \\ 0 & -\beta & -\gamma & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

where the three free parameters must obey the positivity conditions

$$\alpha, \gamma > 0, \qquad \alpha\gamma > \beta^2 \tag{17}$$

It is easy to verify that the corresponding evolution of the density matrix is non-unitary: with the addition of the terms (16), ρ evolves as

$$\rho(t) = \frac{1}{2}\begin{pmatrix} 1 & e^{-(\alpha+\gamma)t/2}e^{-i\Delta E t} \\ e^{-(\alpha+\gamma)t/2}e^{i\Delta E t} & 1 \end{pmatrix} \tag{18}$$

which spirals into the origin as in Fig. 1(b). approaching the completely mixed form

$$\rho(\infty) = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{19}$$

at large times, with a decay time scale $\tau = 2/(\alpha+\gamma)$. This toy example turns out to be directly applicable to the 2×2 neutral kaon system which we discuss later.

3 Decoherence and CPT Violation

The non-unitary evolution in the simple two-state example above manifests an arrow of time: the system spirals in, not out. Everyday experience tells us that an arrow of time is present macroscopically: our bit (at least) of the Universe is expanding, and we are all of us getting older. On the other hand, no such arrow of time is visible in our accepted fundamental laws of physics: Quantum Field Theory is invariant under CPT, and time t is just a coordinate in General Relativity - the motion of the Earth around its solar orbit could be reversed with no apparent problem. On the other hand, an arrow of time appears in thermodynamics via the second law, which states that entropy increases monotonically. Is it possible that this has a microscopic origin?

It has been pointed out by Wald and Page [11] that a microscopic arrow of time must appear if pure states evolve into mixed states as suggested above, in the sense that the strong form of the CPT theorem must be violated. Suppose there is some CPT symmetry transformation Θ which maps initial-state density matrices into final-state density matrices:

$$\rho'_{out} = \Theta \rho_{in} \tag{20}$$

and correspondingly

$$\rho'_{in} = \Theta \rho_{out} \tag{21}$$

where

$$\rho_{out} = \$ \rho_{in}, \; \rho'_{out} = \$ \rho'_{in} \tag{22}$$

It is easy to deduce from these equations that $\$$ must have an inverse:

$$\$^{-1} = \Theta^{-1} \$ \Theta^{-1} \tag{23}$$

which cannot be true if pure states evolve into mixed states, entropy increases monotonically and the density matrix collapses as in the simple two-state example given above.

Although there are many people in the Quantum Gravity community who suspect that some modification of Quantum Mechanics may be necessary so as to incorporate decoherence associated with black holes, there is disagreement whether this is necessarily accompanied by CPT violation. This division of opinion is exemplified by the viewpoints of Hawking and Penrose in [12]: Hawking is very reluctant to give up CPT, whereas Penrose accepts it as a likelihood. The formalism we have developed definitely points in the latter direction, as we see explicitly in connection with the neutral kaon system in section 5.

4 CPT Violation and Decoherence in String Theory

We have already reminded ouselves that CPT invariance is a fundamental theorem of string theory, following from its locality, Lorentz invariance and unitarity. It is clear why one should re-examine the theorem's validity in string theory. Strings are described by a local field theory on the two-dimensional world sheet, but are not local in four-dimensional space time, being extended objects with sizes of the order of the Planck length. Moreover, although Lorentz invariance is a property of classical string vacua, corresponding to conformal field theories on the world sheet known as critical string theories, this is no longer guaranteed when one ventures 'off shell' into non-critical string theories [9]. Various authors have studied conditions under which CPT violation could indeed occur in string theory. It has been shown that they are not met by closed strings in conventional fixed flat space-time backgrounds [13]. One possibility is that CPT might be violated spontaneously in certain string backgrounds [14], but this would not be correlated with a possible breakdown in quantum coherence in the way described in the previous section.

It seems to us that such a breakdown is possible when one treats quantum fluctuations in space time using non-critical string theory [9]. We argue that this leads generically to decoherence and CPT violation through parameters analogous to the α, β, γ in the simple two-state system described above. These parameters are theoretically distinct and experimentally distinguishable from the $K\bar{K}$ mass and lifetime difference parameters $\delta m, \delta\Gamma$ and the decay amplitude difference $\delta A_{2\pi}$ discussed by other authors [15].

Before we develop our point of view, we recall that it is very controversial, and is disregarded by most string theorists [16]. They usually think that the study of black holes in string theory will reveal how quantum-mechanical purity can be maintained. This belief is often based on indications that the black-hole entropy (5) can be understood as the number of distinct string states, as we suggested in 1992 on the basis of our studies of two-dimensional string black holes [17]. However, the possibility that pure states might evolve into mixed states does not shock people who analyze the black-hole information problem from a quantum-gravity point of view [12]. The reason we advocate this point of view is that the types of measurements needed to identify the black-hole state and all its quantum numbers, such as generalized Aharonov-Bohm phase measurements using highly-excited string states [18, 19], are not carried out in practice and are very probably impossible in principle. We believe that the full enormity of this problem will become apparent to other string theorists when they get as far as treating the back reaction of low-energy particles on black-hole backgrounds, and discussing quantum fluctuations in the background space time, as we have been attempting for some time using the two-dimensional black hole as a representative example.

Our string approach starts from the remark that laboratory experiments are all carried out using light particles such as K, n, π, ν, which consist of lowest-level string degrees of freedom. Laboratory experiments do not use the full infinite set of higher-level degrees of freedom that complete the string spectrum. This would not be a problem if the higher-level states decoupled from the lowest-level states, as they do in a fixed, flat space-time background. However, it is known that the higher-level states are indeed coupled to the lowest states in a generic curved background such as the two-dimensional black hole [20], as a result of the same infinite set of symmetries that provide black-hole quantum numbers and label the distinct black-hole states (W_∞ in the two-dimensional case [9]).

The effective low-energy theory is obtained by integrating over the unseen higher-level states:

$$\tilde{\rho}(light, t) = \int d(higher)\rho(light, higher) \tag{24}$$

where the *higher* states play a role analogous to those of the unseen states $|B>_I$ inside the black-hole horizon in (7). The integration over *higher* in (24) ensures that the reduced density matrix $\tilde{\rho}$ is mixed in general, even if the full $\rho(light, higher)$ is pure. We have argued that $\tilde{\rho}$ obeys a modified quantum Liouville equation (10) of the form [9]

$$\partial_t \tilde{\rho} = i[\tilde{\rho}, H] + \not{\delta} H \rho \qquad : \qquad \not{\delta} H = -i\Sigma_{i,j}\beta^i G_{ij}[\ , q^j] \tag{25}$$

where H is the usual light-particle Hamiltonian, the indices (i, j) label all possible microscopically-distinct string background states with coordinate parameters q^i, and G_{ij} is a metric in the space of such possible backgrounds [21]. We argue that these are not conformally invariant once one integrates out the *higher* degrees of freedom, and the β^i are the corresponding renormalization functions. These are non-trivial to the extent that back reaction of the light particles on the background metric cannot be neglected. Equations of the form (25) are quite generic in the context of non-critical string theories [9, 22].

The maximum effect that we can imagine is of order

$$\not{\delta} H \simeq H^2/M_P \tag{26}$$

which would be around $10^{-19}...10^{-20}$ GeV for the neutral kaon system. A contribution to the evolution rate equation (10) of this order of magnitude would arise if there were some Planck-scale interaction contributing an amplitude $A \simeq 1/M_P^2$ and hence a rate $R \simeq 1/M_P^4$, to be multiplied by a density $n \simeq L_P^{-3} \simeq M_P^3$, yielding the overall factor of $\simeq 1/M_P$ shown in (26) [23, 24]. A similar estimate was found in a pilot study of a scalar field in a four-dimensional black-hole background [25].

The origin of the time-dependence on the left-hand side in (24), which is crucial to the derivation of (25), merits further discussion at a more technical level. In order to describe microscopic quantum fluctuations in the space-time background, we need to go 'off shell', so as to be able to interpolate between different conformal (critical) string backgrounds, which is necessary for the description of transitions between them. Non-conformal backgrounds necessarily lead to divergences which must be regularized by introducing a cutoff or renormalization scale. *We identify* this with the Liouville field ϕ, a scalar field that sets the scale for the world-sheet metric [9]:

$$\gamma_{\alpha\beta} = e^{\phi}\,\tilde{\gamma}_{\alpha\beta} \tag{27}$$

where $\tilde{\gamma}$ is a reference metric. In a conformal background, the dynamics of the Liouville field ϕ is trivial, and it decouples from the rest of the theory. This is no longer true in a non-conformal background, and one can show that ϕ has *negative* metric. *We therefore identify* ϕ with the target time variable t. Motion in the space of non-conformal field theories is governed by the renormalization-group flow in the t variable, which is controlled classically by the Zamolodchikov function and the β^i [21], but is subject to quantum fluctuations induced by higher-genus configurations of the world sheet [26]. The non-conformal divergences induce t dependences that cannot be subsumed in the usual Hamiltonian evolution of the effective theory of the light string degrees of freedom, and the extra terms take the form shown in (25). We have exhibited explicit terms of this type associated with transitions between two-dimensional black holes of different masses, and with the creation and destruction of two-dimensional black holes [9].

One note of clarification is perhaps useful: we are not arguing that Quantum Field Theory should be abandoned. In our approach, it applies with all its normal rules to physics on the world sheet. Our point is that problems may arise in the elevation of quantum physics to target space, if the latter has singularities and/or a classical event horizon. The 'ugliness' of the $\tilde{\rho}(light, t)$ density matrix formalism we propose in (24) is not intrinsic: as we remarked earlier, the full $\rho(light, higher)$ may well be pure.

Even if you do not follow the arguments leading to the string version (25) of the modified Liouville equation (10), the latter still provides an interesting phenomenological framework in which one can parametrize possible decoherence and CPT-violating effects with a view to the experimental tests in the neutral kaon system, which are reviewed in the next section.

5 Testing Quantum Mechanics and CPT in the Neutral Kaon System

This audience does not need convincing that the neutral kaon system has an enviable track record as a probe of fundamental physics, ranging from P violation (the τ-θ puzzle) and CP violation to the motivation for charm coming from the absence of strangeness-changing transitions. It is also known to provide very elegant tests of quantum mechanics, and provides the most stringent available test of CPT at the microscopic level, as we saw in section 1. How can the formalism of decoherence and related CPT violation developed above be applied to the neutral kaon system, and how sensitively can we test them?

The quantum-mechanical Hamiltonian for neutral kaons can be written as

$$H = \begin{pmatrix} (M + \frac{1}{2}\delta M) - \frac{1}{2}i(\Gamma + \frac{1}{2}\delta\Gamma) & M_{12}^* - \frac{1}{2}i\Gamma_{12} \\ M_{12} - \frac{1}{2}i\Gamma_{12} & (M - \frac{1}{2}\delta M) - \frac{1}{2}i(\Gamma - \frac{1}{2}\delta\Gamma) \end{pmatrix} \tag{28}$$

in the $K, \bar{K}$ basis, where M and Γ are the common mass and width of the K and $\bar{K}$, and M_{12}, Γ_{12} are the complex off-diagonal mixing parameters. In writing (28), we have allowed for quantum-mechanical parameters δM and $\delta\Gamma$ that violate CPT [27] but preserve coherence. In the density matrix formalism, the time evolution is given by

$$\partial_t \rho \; = \; -i(H\rho - \rho H^+) \tag{29}$$

which preserves the purity of the intial state: it is easy to see that the asymptotic form of the density matrix in the $K_{1,2} = (K \pm \bar{K})/\sqrt{(2)}$ basis is

$$\rho \simeq e^{-\Gamma_L t} \begin{pmatrix} 1 & \epsilon^* + \delta^* \\ \epsilon + \delta & |\epsilon + \delta|^2 \end{pmatrix} \quad : \quad \epsilon = \frac{\frac{1}{2}iIm\Gamma_{12} - ImM_{12}}{\frac{1}{2}\Delta\Gamma - i\Delta M}, \qquad \delta \simeq -\frac{1}{2}\frac{\frac{1}{2}\delta\Gamma - i\delta M}{\frac{1}{2}\Delta\Gamma - i\Delta M} \tag{30}$$

where $\Delta M = M_L$ - M_S is positive and $\Delta\Gamma = \Gamma_L - \Gamma_S$ is negative.

In our approach, the quantum-mechanical evolution equation (29) is modified to become

$$\partial_t \rho = -i(H\rho - \rho H^+) + \not{\delta} H\rho \tag{31}$$

where we parametrize $\not{\delta} H$ in a similar way to the simple two-state system discussed earlier, namely as

$$\not{H}_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -2\alpha & -2\beta \\ 0 & 0 & -2\beta & -2\gamma \end{pmatrix} \tag{32}$$

where the indices α, β label Pauli matrices $\sigma_{\alpha,\beta}$ in the $K_{1,2}$ basis, and we have assumed that $\not{\delta} H$ has $\Delta S = 0$. It is easy to verify that the asymptotic form (30) of the density matrix is now replaced by[2]

$$\rho_L \propto \begin{pmatrix} 1 & \frac{-\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma + i\Delta M} \\ \frac{\frac{1}{2}i(Im\Gamma_{12}+2\beta)-ImM_{12}}{\frac{1}{2}\Delta\Gamma - i\Delta M} & |\epsilon|^2 + \frac{\gamma}{\Delta\Gamma} - \frac{4\beta ImM_{12}(\Delta M/\Delta\Gamma)+\beta^2}{\frac{1}{4}\Delta\Gamma^2 + \Delta M^2} \end{pmatrix} \tag{33}$$

This is clearly a mixed state: the parameters α, β, γ can be regarded as causing "regeneration in vacuo".

[2] We now set $\delta M = \delta\Gamma = 0$ for simplicity: if desired, they may be retained in a combined discussion of coherent and decohering CPT violation.

It is easy to see that these parameters also violate CPT [28]. In the neutral kaon system, the CPT transformation acts as follows on K and $\bar{K}$ wave functions:

$$CPT|K> = e^{i\phi}|\bar{K}> \qquad CPT|\bar{K}> = e^{-i\phi}|K> \tag{34}$$

The CPT transformation for density matrices may be represented by

$$CPT \equiv \begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix} \tag{35}$$

in the $K, \bar{K}$ basis, which is proportional to a linear combination of $\sigma_{1,2}$. We see in this representation that the CPT operator does not commute with $\delta m, \delta\Gamma$, whose contributions to the Hamiltonian are proportional to σ_3 in the $K, \bar{K}$ basis. Transforming to the $K_{1,2}$ basis used in (32) above, the CPT operator becomes proportional to a combination of $\sigma_{2,3}$. Since the new terms α, β, γ in (32) couple to the indices 2, 3, they clearly violate CPT, though in a different way from the quantum-mechanical parameters $\delta m, \delta\Gamma$.

Experimental observables may be represented in the density matrix formalism by expectation values of operators:

$$< O > = \mathrm{Tr}(O\rho) \tag{36}$$

Common observables are represented by the operators

$$\begin{aligned} O_{2\pi} &= \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ O_{3\pi} &= (0.22)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ O_{\pi^- l^+ \nu} &= \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ O_{\pi^+ l^- \bar{\nu}} &= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \end{aligned} \tag{37}$$

(strictly speaking, there should be a corresponding prefactor of 0.998 in the formula for the $O_{2\pi}$ observable.). Then, for example, the semileptonic decay asymmetry

$$\delta \equiv \frac{\Gamma(\pi^- l^+ \nu) - \Gamma(\pi^+ l^- \bar{\nu})}{\Gamma(\pi^- l^+ \nu) + \Gamma(\pi^+ l^- \bar{\nu})} \tag{38}$$

becomes

$$\begin{aligned} \delta_L &= 2Re[\epsilon(1 - \frac{i\beta}{ImM_{12}})] \\ \delta_S &= 2Re[\epsilon(1 + \frac{i\beta}{ImM_{12}})] \end{aligned} \tag{39}$$

in the short- and long-lived kaon limits, respectively. A difference between $\delta_{S,L}$ is often mentioned [27] as a possible signature of the CPT-violating parameter δm, and here we see how it could also appear in our different formalism for CPT violation.

Table 1: Qualitative comparison of predictions for various observables in CPT-violating theories beyond (QMV) and within (QM) quantum mechanics. Predictions either differ ($\neq$) or agree ($=$) with the results obtained in conventional quantum-mechanical CP violation. Note that these frameworks can be qualitatively distinguished via their predictions for the asymmetries A_{T}, A_{CPT}, $A_{\Delta m}$, and the interference coefficient ζ discussed in [29].

Process	QMV	QM
$A_{2\pi}$	$\neq$	$\neq$
$A_{3\pi}$	$\neq$	$\neq$
A_{T}	$\neq$	$=$
A_{CPT}	$=$	$\neq$
$A_{\Delta m}$	$\neq$	$=$
ζ	$\neq$	$=$

The asymmetries which have been used so far in experimental probes of this formalism are the 2π decay asymmetry

$$A_{2\pi} = \frac{\mathrm{Tr}(O_{2\pi}\bar{\rho}(t)) - \mathrm{Tr}(O_{2\pi}\rho(t))}{\mathrm{Tr}(O_{2\pi}\bar{\rho}(t)) + \mathrm{Tr}(O_{2\pi}\rho(t))} \tag{40}$$

where $\rho, \bar{\rho}$ denote the density matrices of states that are tagged initially as pure $K, \bar{K}$ respectively, and the double semileptonic decay asymmetry (in an obvious short-hand notation for the rates of different semileptonic decays)

$$A_{\Delta m} = \frac{R(K^0 \to \pi^+) + R(\bar{K}^0 \to \pi^-) - R(\bar{K}^0 \to \pi^+) - R(K^0 \to \pi^-)}{R(K^0 \to \pi^+) + R(\bar{K}^0 \to \pi^-) + R(\bar{K}^0 \to \pi^+) + R(K^0 \to \pi^-)} \tag{41}$$

in which various systematic effects cancel. As can be seen in Fig. 2, $A_{2\pi}$ is sensitive to the presence of α, β and γ, whereas $A_{\Delta m}$ is particularly sensitive to α. The Table shows how the form of decohering CPT violation that we propose here may be distinguished in principle from "conventional" quantum-mechanical CPT violation via the parameters $\delta m, \delta\Gamma$, by measuring the full gamut of observables discussed in [29, 30].

6 Analysis of CPLEAR Data

Together with the CPLEAR collaboration itself, we have published a joint analysis of CPLEAR data [32], constraining the CPT-violating parameters α, β, γ. Fig. 3 compares the data for $A_{2\pi}$ and $A_{\Delta m}$ compared with a conventional quantum-mechanical fit (which is, of course, perfectly good) and a fit in which our parameters are taken to have values a factor of 10 larger than the experimental limits we quote. Imposing the positivity constraints (17), we find the upper limits (see also the talk here by Pavlopoulos [15])

$$\alpha < 4.0 \times 10^{-17}\mathrm{GeV}, \qquad \beta < 2.3 \times 10^{-19}\mathrm{GeV}, \qquad \gamma < 3.7 \times 10^{-21}\mathrm{GeV} \tag{42}$$

We cannot help being impressed that these bounds are in the ballpark of m_K^2/M_P, which is the maximum magnitude (26) that we could expect any such effect to have. Perhaps, with a bit more effort by CPLEAR [31] or at DAϕNE [30], ...?

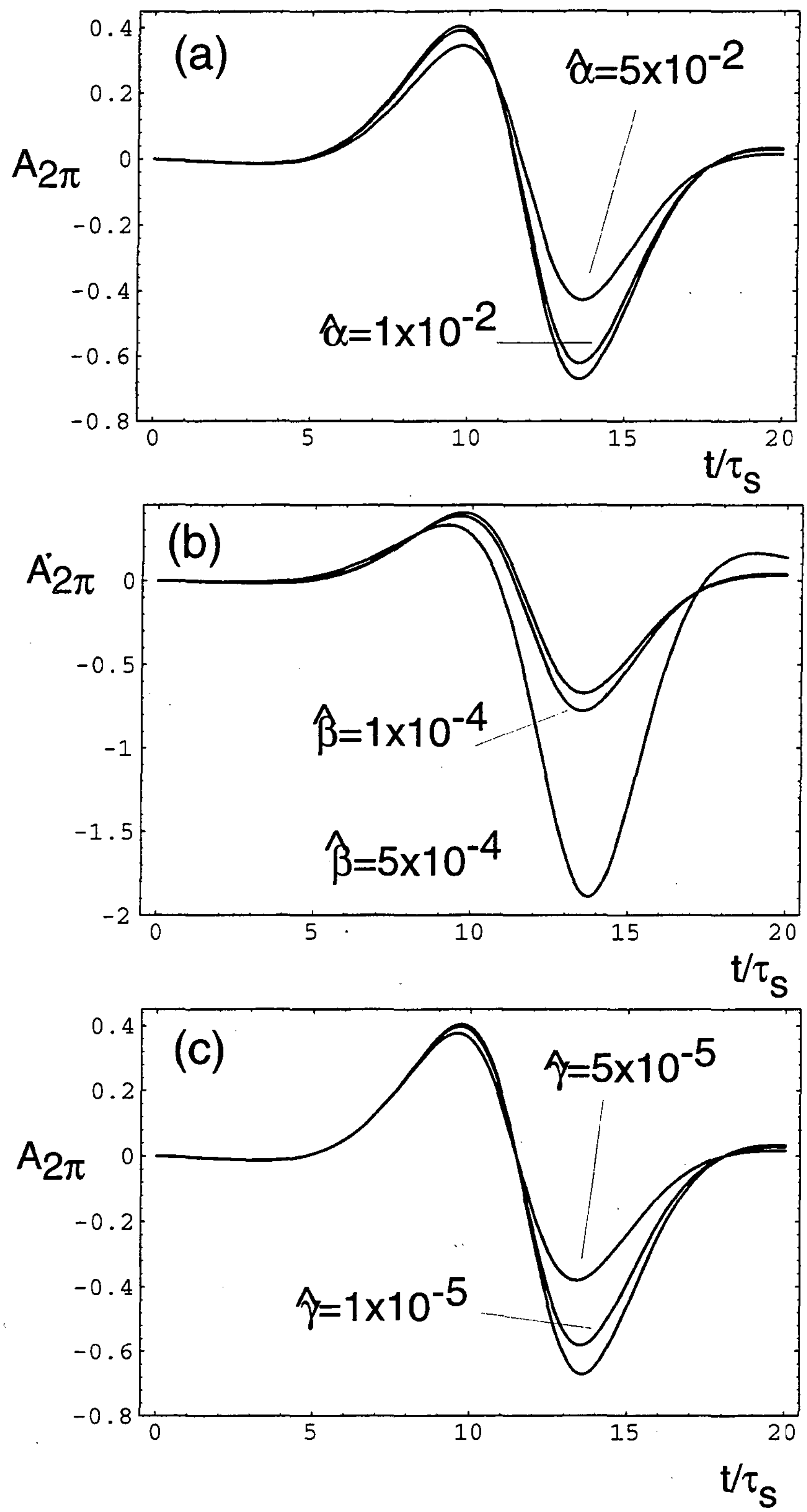

Figure 2: The time-dependence of the 2π decay asymmetry of neutral kaons, $A_{2\pi}$, computed with non-zero values of (a) α, (b) β and (c) γ. We use the notation $\hat{\alpha} = \alpha/|\Delta\Gamma|$, etc., where $\Delta\Gamma = \Gamma_S - \Gamma_L$ [29].

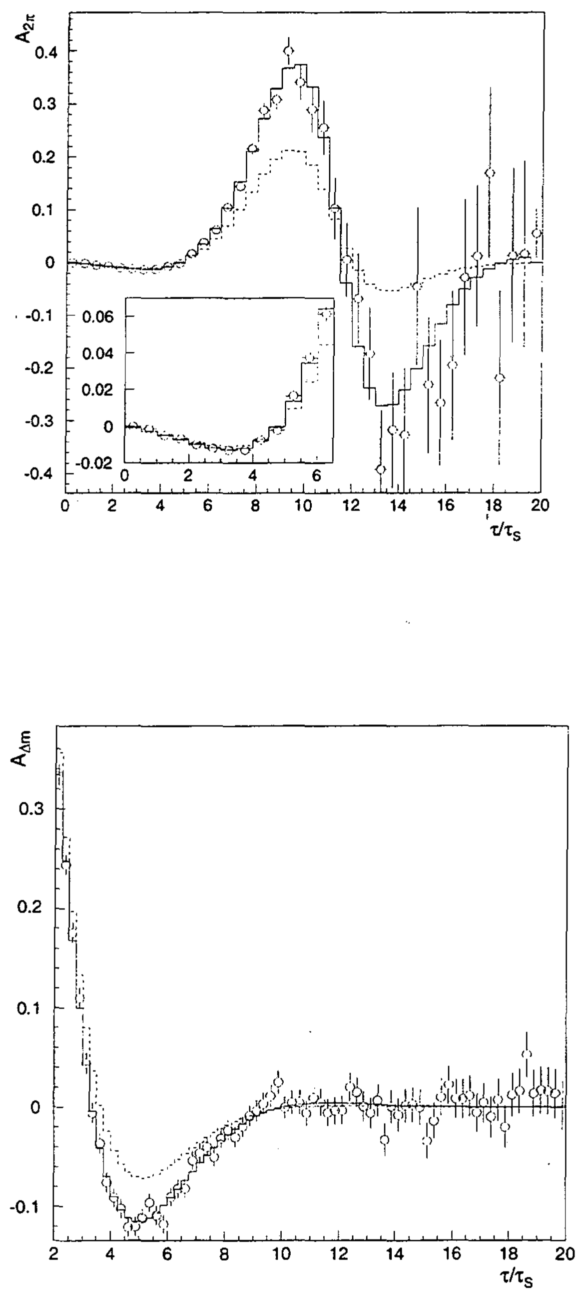

Figure 3: The measured time-dependences of the neutral kaon decay asymmetries (a) $A_{2\pi}$ and (b) $A_{\Delta m}$, compared with a CPT-invariant fit (solid lines) and a fit in which our bounds on the CPT-violating parameters are each relaxed by a factor of 10 (dashed lines) [32].

7 Outlook

We believe there are indications from quantum gravity that the effective quantum-mechanical description of low-energy observable laboratory systems may need to be modified, but we remind you that this suggestion is controversial. We have found evidence in an analysis of non-critical string theory that supports this suggestion, but we remind you that this is even more controversial. We welcome the fact that the opposing views on this issue are clearly delineated.

Our particular approach is based on the density-matrix formalism developed above, which we have applied specifically to the neutral kaon system. Our approach and formalism can in principle be distinguished from others by measuring a number of different $K, \bar{K}$ decay asymmetries [29]. Although we have presented an estimate (26) of the largest possible magnitude that any such decohering and CPT-violating effects might have, we are not in a position to calculate its magnitude. Even if this order of magnitude is attained for some other microscopic system, it is conceivable that it might be suppressed in the neutral kaon system for some 'accidental' reason associated with the way the K is made out of light string modes. And, of course, we cannot exclude the possibility that the decohering and CPT-violating effects we discuss may be suppressed by additional powers of m_K/M_P (or even an exponential!) below our maximal estimate (26). Therefore we can offer our experimental colleagues no guarantee of success. Nevertheless, we think that the importance of the issues discussed here motivate a new series of microscopic experiments to test quantum mechanics and CPT. We are glad that other speakers at this meeting agree with us, even if their approaches to the issues differ from ours.

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