Title
GENERAL PROPERTIES OF AN ASYMMETRIC B-FACTORY LATTICE

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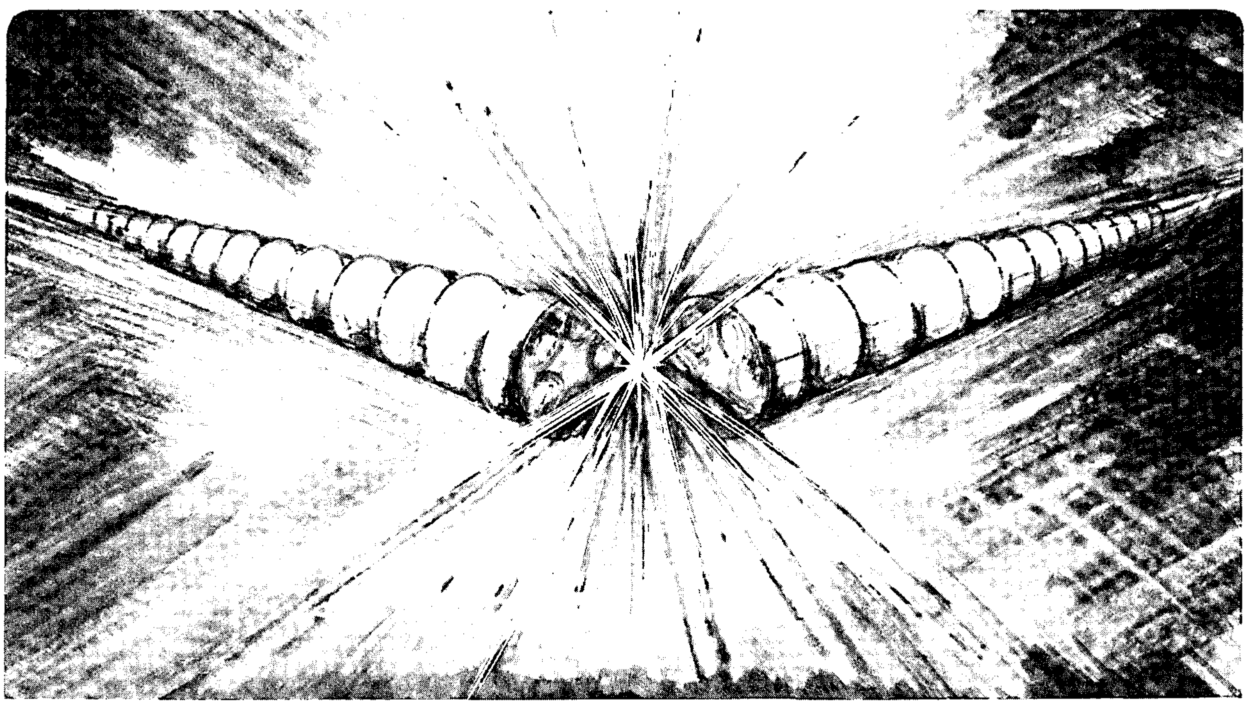

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GENERAL PROPERTIES OF AN ASYMMETRTIC B-FACTORY LATTICE*

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General properties of an asymmetric B - factory lattice

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Rules of the game

The number of parameters involved in the design of a collider is so large that it is useful to establish rules which limit the number of degrees of freedom to a minimum, define the designer' s options clearly and serve as a basis for the appraisal of different machines. The rules which are proposed here are based on a principle of simplicity and, hopefully, of efficiency. For beams of unequal energies, the following quantities are supposed to be the same: area of their cross - section at the collision point, beam - beam tune shift, damping time between two collisions and the phase modulation due to the synchrotron motion [1].

Beam size

The beam size is

$$\sigma = \sqrt{\varepsilon \beta^*} \tag{1}$$

where ε is the emittance and β^* the value of the β-function at the interaction point, subscripts x or y for the horizontal or vertical plane respectively have been omitted for brevity.

Beam-beam tune shift

The vertical beam - beam tune shift is

$$\Delta\nu_{y1} = \frac{r_e}{2\pi} \frac{\beta^*_{y1} N_2}{\gamma_1 (\sigma_x + \sigma_y)\, \sigma_y} \tag{2}$$

where r_e is the classical radius of the electron ($2.8\ 10^{-15}$ m), N the bunch population and γ the ratio of the particle total energy to its rest energy. The two beams are distinguished by the subscripts 1 and 2. In the case of a flat beam

$$\frac{\sigma_x}{\sigma_y} = \frac{\beta^*_x}{\beta^*_y} = \frac{\varepsilon_x}{\varepsilon_y} >> 1 \tag{3}$$

and the expression of the beam-beam tune shift can be reduced to

$$\Delta\nu_{y1} = \frac{r_e}{2\pi} \frac{N_2}{\varepsilon_{nx1}} \tag{4}$$

where we use the normalized emittance

$$\varepsilon_{nx} = \gamma\,\varepsilon_x \tag{5}$$

in the sense of a linear accelerator. For a round beam, the expression (4) is replaced by

$$\Delta\nu_1 = \frac{r_e}{4\pi}\frac{N_2}{\varepsilon_{n1}} \tag{6}$$

Emittance

In a circular machine, the vertical equilibrium emittance is zero in the absence of any coupling between the horizontal and vertical motions and the horizontal equilibrium emittance is [2]

$$\varepsilon_x = \frac{\lambda_c}{2\pi\, J_x}\frac{\gamma^2}{Q_x^3}\frac{R}{\rho} \tag{7}$$

where λ_c is the Compton wavelength (2.4 10^{-12} m), J_x the horizontal damping partition number which is usually of the order of 1, Q_x the horizontal betatron tune, ρ the radius of curvature in the bending magnets and R the mean radius.

As a matter of fact, the machine has not to be circular, the emittance would be the same if straight sections were inserted. R is thus to be understood as the radius of the arcs and Q_x as the horizontal betatron tune in the arcs only. The design of the straight sections depends on different considerations like the luminosity lifetime or the topology of the intersection regions.

Damping time

The damping per turn, also called *damping decrement*, is the ratio of the damping time τ to the revolution period T

$$\frac{\tau}{T} = \frac{3\rho}{2\pi\, r_e\, J\, \gamma^3} \tag{8}$$

where c is the light velocity and J a damping partition number supposed to be independent of the general lattice structure. As T is proportional to R, ρ has necessarily to scale like

$$\rho \sim \gamma^3 \tag{9}$$

and the same rule is true for R

$$R \sim \gamma^3 \tag{10}$$

The expression (9) implies that the dipole field B is given by

$$B \sim \gamma^2 \tag{11}$$

Synchrotron motion

The synchrotron motion modulates the path length [3] by a quantity Δl which is related to the momentum compaction factor α and to the relative momentum spread $\Delta p/p$ through the relation

$$\frac{\Delta l}{2\pi R} = \alpha \frac{\Delta p}{p} \tag{12}$$

Because this path length changes at each turn, the collision between the test particle and the opposite bunch does not occur always at the same place and the phase $\Delta\mu$ of the betatron oscillation is shifted by $\Delta l/\beta^*$. The expression of the emittance (7) is deduced from the general expression by replacing the β-function by its mean value

$$\bar{\beta} = \frac{R}{Q} \tag{13}$$

and the α-function by

$$\alpha = \frac{1}{Q_x^2} \tag{14}$$

The parameter which controls the synchrotron motion and is maintained independent of the particle energy is the ratio of the maximum tune modulation to the relative momentum spread

$$p\frac{\Delta\mu}{\Delta p} = \frac{2\pi R}{\beta^* Q_x^2} \tag{15}$$

We have assumed the β-function constant and equal to β^*, the exact expression of $\Delta\mu$ is Arctan ($\Delta l/\beta^*$). However, the scaling laws which are derived here are only guidelines and are amenable to slight variations, it is then sufficient to reduce transcendental expressions to their first order expansion. Let us note that the source of the phase modulation is in the arcs, the remark made about the shape of the ring in the case of the equilibrium emittance remains true in the case of the synchrotron motion.

By multiplying (7) and (14) together and taking the conditions of equal beam sizes, equal synchrotron parameters and the laws (9) and (10) into consideration, it turns out that

$$Q_x \sim \gamma \tag{16}$$

and

$$\bar{\beta} \sim \gamma^2 \tag{17}$$

By substituting this dependence into (7), the emittance varies like

$$\varepsilon \sim \gamma^{-1} \tag{18}$$

and, from the conditions of equal beam sizes at the interaction point, β^* varies necessarily like

$$\beta^* \sim \gamma \tag{19}$$

In other terms, the normalized emittance is constant and the bunch population N has also to be constant in order for the beam-beam tune shift to be independent of γ.

Head on versus crossing angle collisions

The scaling laws which have been derived are quite compatible with head on collisions [4] whose configuration is ideal for having the particle detector as close to the beam as possible but suffers from the drawback of unequal β^* values because the final focusing quadrupoles cannot be at the same distance from the interaction point in the two rings. The natural alternative consists of accepting a finite crossing angle at the cost of the complications inherent to the *crab crossing* scheme and to a bigger vacuum pipe. One would then prefer [5] to have

$$\beta^* \sim \varepsilon \sim \text{constant} \tag{20}$$

and

$$N \sim \gamma^{-1} \tag{21}$$

from beam size and beam-beam tune shift requirements. The conditions (9) and (10) which are related to the beam decrement can stay unchanged. The horizontal tune in the circular part of the machine has to vary like

$$Q_x \sim \gamma^{2/3} \tag{22}$$

according to (7) to satisfy (20) but the synchrotron motion parameter (15) can no longer be kept constant

$$p\frac{\Delta\mu}{\Delta p} \sim \gamma^{5/3} \tag{23}$$

The variation is actually a little smoother if one chooses $\Delta\mu$ as the relevant parameter because $\Delta p/p$ is proportional to $\gamma/\sqrt{\rho}$

$$\Delta\mu \sim \gamma^{7/6} \tag{24}$$

Conclusion

Scaling laws consistent with general rules of optimization have been established for colliders of unequal energy beams. They are valid for any ring deduced from the circular shape by insertion of straight sections. The constraint on the synchrotron motion seems to be met more easily for the head on configuration than for a finite crossing angle. In any case, the equal damping decrement requirement leads to the use of *high field dipoles in the low energy ring.*

If the rules on equal beam size and equal beam-beam tune shift are generally accepted, those on equal damping decrements and equal amplitude of the betatron phase modulation by the synchrotron motion are still controversial matters: below a certain threshold which is still undefined, they may be unimportant. Finally, additional flexibility could be provided by wigglers and radio frequency adjustments.

References

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