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R.H. Muller and M.L. Sand

July 1987

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OPTIMUM ANGLE OF INCIDENCE FOR
OBSERVING THIN-FILM INTERFERENCE COLORS

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ABSTRACT

A colorimetric analysis of white-light interference colors for transparent films on absorbing substrates has shown that specific angles of incidence exist for which the purity of interference colors is predicted to be greatest for s- or p-polarized light. These optimum angles of incidence depend primarily on the optical constants of the substrate and they decrease from the first to the third order of interference. They provide the best conditions for the determination of film thicknesses from interference colors and their use is essential for making interference colors visible with highly reflective substrates. As an alternate measure of the observability of interference colors, a polychromatic fringe visibility has been introduced. Its use provides similar predictions of optimum angles. The computations are in good agreement with experimental observations.

The observation of interference colors provides a simple means to determine the distribution of film thickness over extended surfaces. For certain thickness regimes, interference colors can be extremely sensitive to small changes in thickness and several interference orders may be identified visually. This combination of high resolution (down to $100\text{\AA}$) and large range (up to $10,000\text{\AA}$) of film thickness distinguishes white-light from monochromatic thin-film interference. To make full use of interference colors for film thickness measurements, however, it is necessary that the interference colors be of high color saturation.

Kubota¹ recognized that, in order for the saturation of the interference color of thin dielectric films on metallic surfaces to be sufficiently high for observation, the reflection coefficients at both interfaces must be nearly equal. However, calculations for specific film-metal substrate pairs were not given as evidence. He did demonstrate, though, that the color saturation decreases for increasing reflection coefficients.² The effects of substrate reflectivity and angle of incidence on the observability of thin-film interference color has also been discussed by Gaudin.³ The present authors previously used the criterion of equal reflection coefficients at the two interfaces to calculate the optimum angle of incidence for the observation of monochromatic interference fringes for dielectric films on metal substrates.⁴ For monochromatic interference, this optimum angle, at which complete extinction occurs under conditions of destructive interference and Michelson fringe visibility is therefore a maximum, is independent of the interference order.

It is the purpose of this paper to show under which conditions thin-film interference colors of highest color saturation can be obtained. Colorimetric quantities have been derived from computed interference spectra for transparent films on substrates of different reflectivities as a function of film thickness and angle of incidence. Optimum angles derived from the observation of interference colors on tapered films of cryolite on chromium and aluminum are compared to predictions.

The colorimetry of interference colors has been reviewed by Kubota.² His calculations, which were conducted for the standard daylight source C, did not use the complete optical equations. In the present study, multiple-beam (true thin-film) interference is used for the computations, which consider all phase changes in oblique reflection, particularly at absorbing substrates.

For each polarization and interference order, an angle of incidence is established which results in interference colors of greatest color saturation possible for given optical properties of substrate and film materials. At other angles, colors of lower saturation, which may be difficult to interpret or which are not at all observable, are obtained. Color purity, derived from 1931 C.I.E. chromaticity diagrams, is used as a measure of color saturation.

Polychromatic fringe visibility is introduced as an alternate means for optimizing the observability of white-light interference, and the results are compared to those based on color purity. Previously derived optimum conditions for monochromatic interference⁴ based on Michelson fringe visibility are also compared as a practical

yet even more approximate means.

I. Thin-Film Interference

Interference spectra have been computed by means outlined previously⁴ in which the reflectance of the film-substrate combination has been determined by use of the multiple-beam Eq. (1).

$$R = \frac{r_1^2 + r_2^2 + 2r_1r_2 \cos (\delta_1 - \delta_2 + \delta)}{1 + r_1^2 r_2^2 + 2r_1r_2 \cos (\delta_1 - \delta_2 + \delta)} \quad (1)$$

The meaning of terms is summarized in Fig. 1: r_1 and r_2 are the amplitude reflection coefficients of the two film surfaces with the associated phase changes for reflection δ_1 and δ_2 ; $\delta_1 - \delta_2$ is the total phase change due to reflection. Values for these quantities depend on the optical constants of the media at each interface, polarization and angle of incidence and can be determined with the complex Fresnel equations. The phase delay δ due to propagation through the film is given in Eq. (2).

$$\delta = \frac{4\pi}{\lambda} n_f d \cos \phi' \quad (2)$$

This quantity depends on the wavelength λ , the refractive index n_f and thickness d of the film and the angle of refraction ϕ' which is derived from the angle of incidence by use of Snell's law.

The optical path difference ΔS between two consecutive rays in multiple reflection is contained in Eq. (2). It is due to the presence of the film and is proportional to the geometric film thickness d but depends on angle of incidence:

$$\Delta S = 2 n_f d \cos \phi' \quad (3)$$

The optical path difference will be used extensively throughout this paper as a measure of optical film thickness. Optical constants used for all the computations are given in Table I. Air has always been the incident medium ($n_0 = 1$).

II. Chromaticity of Interference Colors

All the colorimetric computations have been conducted for the standard tungsten source A. Interference spectra have been converted to 1931 C.I.E. tristimulus values and chromaticity coordinates by use of colorimetric distribution coefficients tabulated by Wright⁵ for 380-780 nm with 5 nm increments.

The hue and purity of colors are given by the coordinates in the chromaticity diagram. Hues are approximately represented by lines of constant dominant wavelength which extend from the chromaticity coordinates of the white-light source. Color purity has been used as a convenient, well-defined, although approximate measure of color saturation.

The definition of color purity⁶ is illustrated in Fig. 2: A straight line, drawn in the chromaticity diagram from the achromatic (white) point through the coordinates (x,y) of the color in question intersects the spectrum locus at the dominant wavelength (line of constant dominant wavelength). The color purity a/b represents the relative separation of a color locus from white point and pure spectral color. It should be recognized that for different dominant wavelengths, equal purities do not necessarily designate equal saturations⁷ and the least perceptible difference in purity also varies.⁸

To examine the effect of angle of incidence on chromaticity, it has been found useful to plot chromaticities for constant values of the optical path difference ΔS (Eq. 3), in order to minimize variations in hue. The reasons are best understood by considering the double-beam equation, which approximates the multiple-beam equation (Eq. 1) for low values of r_1 and r_2 ; the double-beam equation is equivalent to the numerator of Eq. 1. For the sake of explanation, the phase change ($\delta_1 - \delta_2$) due to reflection is momentarily assumed to be constant with changing angle of incidence.

For the double-beam equation, curves of constant ΔS in the chromaticity diagram are straight lines radiating from the achromatic point. These are lines of constant dominant wavelength and therefore of similar hue. The point furthest from the achromatic point and therefore of highest purity corresponds to $r_1 = r_2$. Kubota² has outlined this derivation omitting the effect of oblique incidence on the phase delay δ . Thus, curves of constant optical thickness nd rather than constant ΔS were presented.

III. Optimum Angle of Incidence Based on Color Purity

The effect of angle of incidence on the chromaticity of first-order interference colors is illustrated in Figs. 3-9. In general, the locus of the chromaticity coordinates for a fixed optical path difference ΔS (shown in nm) follows a radial curve which recedes from white point and returns to it as a function of angle of incidence (shown in deg.) The concentric curves connect chromaticity loci for constant angle of incidence. They illustrate the change in color purity with angle of

incidence. The angle associated with the greatest departure from the white point is taken here as the optimum angle.

For the sake of simplicity, s-polarization which does not show the effect of Brewster's angle, has been chosen in Figs. 3-6. These figures illustrate how the optimum angle of incidence shifts to higher (more grazing) angles with increasing reflectance of the substrate. Also, the maximum departure from the white point and the range of angles over which chromaticities close to those of greatest color purity are obtained becomes smaller with increasing reflectance.

Chromaticities for a film-substrate combination with low values of r_1 and r_2 are illustrated in Fig. 3. Such a system is reasonably well represented by the double-beam interference model and, as expected, these curves for constant ΔS are nearly straight lines. Also, the highest color purity on each constant ΔS curve occurs close to the angle at which $r_1 = r_2$. For a film of refractive index 1.35 on a substrate of refractive index 2.0, the optimum angle for observing interference fringes in s-polarized light is found to be 44° (to the nearest degree). Under these conditions the reflection coefficients for the two interfaces are $r_1 = 0.233$ and $r_2 = 0.237$.

In this specific case (low reflectance), the color purity at normal incidence (0°) is an appreciable fraction of that at the optimum angle which is not the case for more highly reflecting (metallic) substrates. The optimum angle is nearly independent of film thickness within the same interference order for all the substrates considered. For dielectric substrates having low reflectance, interference colors can be observed satisfactorily over a range of angles including normal incidence.

Curves of somewhat similar appearance have been presented by Kubota² for constant optical thickness and constant angle of incidence for dielectric films on dielectric substrates. The analysis appeared to be most appropriate for films of constant thickness. For a film with refractive index equal to the square root of the substrate refractive index, the hue varied with the direction of observation with less variation in the purity. On the other hand, for a film with refractive index nearly equal to the square of the substrate refractive index, the purity varied with the direction of observation with little effect on the hue.

For observations of films of uniform thickness, these considerations are important. It is particularly important to adjust the angle of incidence for the purpose of viewing certain hues that are most sensitive to film thickness. Kubota² referred to this as the "sensitive color." However, for observations of a series of color fringes in a tapered film, the optimum angles of incidence derived in the present study should be used for the purpose of maximizing color purity.

For more highly-reflecting substrates, the chromaticity curves of constant ΔS are open loops rather than straight lines. Increasing reflectance results in a shift in the optimum angle to higher angles, a decrease in the maximum color purity, and an increased dependence of the color purity on angle of incidence. For a substrate such as silver, it would be impossible to observe interference colors at normal incidence. In comparing the chromaticities for silver with those for less reflecting substrates, note the different scales in Figs. 3-6.

The use of wavelength-dependent optical constants has been found to

affect the results only to a minor extent and is illustrated for a substrate of medium-high reflectance in Fig. 7 by comparison with Fig. 5.

A similar comparison for a substrate of high reflectance (Figs. 8 and 6) indicates a lowering of the optimum angle over a segment of the first color order.

For p-polarized light, examination of the chromaticities yields similar results as shown by a comparison of Figs. 9 and 5. Since phase changes for the air-film interface are different by 180° for the two polarizations, (angles of incidence of interest are typically above Brewster's angle) similar chromaticities are associated with different optical path differences ΔS (or film thickness). A distinguishing feature in the use of p-polarized light is that an optimum angle always exists above Brewster's angle. For substrates of low reflectance the color purity at normal incidence can be high and it decreases with increasing angle of incidence up to Brewster's angle, where reflection from the film is lost. A second optimum angle of incidence may therefore exist below Brewster's angle, as had been found for monochromatic interference.⁴

Second and higher order interference colors are successively less saturated at the optimum angle. The optimum angle for each color order is lower than that for the first order and is less sharply defined. In comparing Figs. 10 and 5, note the difference in scales.

The effect of angle of incidence on color purity for several color orders can be illustrated by examining a single hue. A red hue represented by a line of complementary dominant wavelength 506.4 nm was selected. Plots of color purity as a function of angle of incidence are

shown in Figs. 11 and 12 for a substrate of medium-high reflectance (platinum). For both polarizations, the highest color purity (attained at the optimum angle of incidence) decreases with increasing interference order, the maxima become broader and the optimum angle decreases for the first three interference orders. For angles far removed from the optimum angle, the relative magnitudes of the color purities in the different color orders can be reversed.

The principal reason for the variation of color purity with angle of incidence is the strong dependence of the reflection coefficient r_1 for the air-film interface on angle of incidence. This effect is well demonstrated in Fig. 13, which also illustrates the difference between multiple-beam (thin-film) interference and double-beam interference (observed, e.g., in the polarizing microscope or some interferometers). It is a special feature of multiple-beam interference that even with equal reflection coefficients for the two interfaces color purity decreases with increasing values of the reflection coefficients. For double-beam interference, though, color purity is independent of the absolute value of the matched amplitudes (Fig. 14).

IV. Polychromatic Fringe Visibility

Tristimulus values of thin-film interference colors illustrated in Fig. 15 exhibit periodic variations with optical path difference ΔS (or film thickness). The decreasing amplitude of the variations reflects the decrease in color purity with increasing interference order. By use of the Y tristimulus value as a measure of fringe brightness (relative luminance), a polychromatic fringe visibility V_{poly} analogous to the

monochromatic Michelson fringe visibility may be defined as an alternate means to characterize the observability of white-light interference.

$$V_{\text{poly}} = \frac{Y_{\text{max}} - Y_{\text{min}}}{Y_{\text{max}} + Y_{\text{min}}} \quad (4)$$

In the following application of this concept, pairs of adjacent minima and maxima have been chosen in such a way that the maximum was associated with the lower optical path value. Fringe visibilities thus derived for different interference orders are shown in Fig. 16.

The positions of the maxima in the polychromatic fringe visibility curves decrease from the first to third interference order and reverse the trend for the fourth order in the same manner as the maxima in the color purity curves (Fig. 13). All maxima occur at r_1 less than r_2 . In contrast, the double-beam equation, which is valid for low reflection coefficients, produces maxima at $r_1 = r_2$.

V. Comparison of Methods for Calculating Optimum Angles

Among the criteria investigated to judge the quality of interference colors, color purity is the most valid. Its use is, however, quite cumbersome. It is, therefore, of practical interest that the optimum angles of incidence derived from the use of polychromatic fringe visibility, which is easier to determine, are almost the same as those obtained from the color purity analysis for the first three color orders. Even the optimum angles of incidence obtained on the basis of monochromatic fringe visibility do not differ greatly from the colorimetric results for the first interference order, being in general

1-2° higher (Table II).

The difference can widen, though, for the higher orders as indicated by some of the entries in Table II; therefore, optimum angle charts presented previously for monochromatic interference⁴ must be used with these differences in mind. Since the color purity is less sharply peaked as a function of the angle of incidence in the higher orders, the use of the precise optimum angle is less important.

The substrates represented in Table II are the same as those on which Figs. 4-6 and Figs. 9-10 are based. The optical constants for silicon, though, in Fig. 4 are slightly different from those used in the table. Some difference is evident in the optimum angles given in the figures and in Table II. In the figures, an angle close to the optimum angle is chosen. The outer constant angle curve is an appropriate choice for the small number of constant ΔS curves shown in the figures. The optimum angle does vary slightly within each color order and is more closely estimated in the table from many values throughout the color order.

VI. Comparison of Predicted Optimum Angles with Experiments

Tapered cryolite films on chromium and aluminum substrates were observed both photographically and visually on an optical bench with a tungsten source over a range of angles of incidence to determine the angles at which the interference colors were most saturated. The optimum angles of incidence thus determined are compared with the calculated values in Table III.

Relative color saturations were qualitatively the same as those expected from the calculations. The interference colors of cryolite on aluminum were of low saturation even at the optimum angle, whereas those for cryolite on chromium were very much higher. Below Brewster's angle, the colors were similar for the s- and p-polarizations. At Brewster's angle, interference colors were absent for the p-polarization. As the angle of observation was increased further, the color saturation reached an optimum and decreased at still higher angles.

Photographic slides were taken in 2° increments for the chromium-supported film and in 1° increments for the aluminum-supported film. The camera exposure had been adjusted to compensate (at least approximately) for the changing luminance of the brighter colors. The relative saturation of interference colors was judged by viewing the slides on a light-table and in projection.

Difficulties were encountered in separating the effect of exposure on color saturation and brightness in the photographs. Other uncertainties can be expected due to the color temperature of the light sources used for exposure and viewing, as well as the color-correction of the film. As shown in Table III by the good agreement between photographically and theoretically-determined optimum angles, these uncertainties appear to have only a minor effect.

The visual observation of interference colors on the optical bench (with tungsten source) to establish an optimum angle produced results which differed somewhat with different observers, but were close to the photographic observations (Table III). The changing brightness of the fringes (particularly the dark fringes) at different angles could have

affected the judgement of color saturation. The time delay in adjusting the angle of incidence on the optical bench was also a factor.

These observations demonstrate the validity of the optimum angle calculations. For a comparison of observed and calculated values, it should be first noted that the series of colors associated with these observations did not always belong entirely to one interference order. For example, colors of the cryolite film on chromium viewed in s-polarized light belonged to the second and third orders. Consequently, the observed optimum angle was several degrees below the value based on matched reflection coefficients. From calculations for a substrate of similar reflectance (silicon in Table II), the difference would be expected to be $5-6^{\circ}$, which is the same as that determined on the optical bench. The difference for all other observations, except for the anomalously low angles reported by one of the observers for the aluminum substrate, was about 1° . These differences would also be expected by examining calculations in Table II for substrates of reflectance comparable to that of chromium and aluminum (silicon and silver, respectively).

VII. Summary and Conclusions

The observability of thin-film interference colors for the determination of film thickness depends on the color saturation. Color saturation depends on the reflectance of the film-substrate combination, polarization and angle of incidence. The colorimetric color purity is found to be a good measure of color saturation.

Interference colors of thin films on highly reflective substrates

cannot be observed unless viewed very near angles of incidence chosen to maximize color purity. Color purities are found to decrease not only for more highly reflective substrates but also for higher interference orders near the optimum angle of incidence.

The optimum angle for observing white-light interference colors can be estimated by different means. The most valid method, which is based on maximum color purity at the optimum angle, requires extensive calculations. A second method yielding very similar results but requiring fewer calculations is based on luminance contrast within each color order. A "polychromatic fringe visibility," similar in definition to the Michelson fringe visibility in monochromatic interference, is highest at the optimum angle. In both of these methods, the optimum angle has been found to decrease from the first to the third order.

The simplest means of estimating the optimum angle is to use previously-established charts⁴ based on a Michelson fringe visibility $V = 1$. For monochromatic interference, the optimum angle is, however, the same for all interference orders. The criterion of maximum color purity yields an optimum angle which is $1-2^\circ$ lower than that determined for monochromatic interference for the first order and as much as several degrees lower for the second and third orders. These values have been confirmed by observations of a low-index film on chromium and aluminum. Since the color purity is less sensitive to angle of incidence in the higher orders, optimum angles estimated for the first order can also be used for higher orders (thicker films).

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TABLE I. Optical Constants of Films and Substrate Materials.

Material	Symbol	Value	Source
Incident medium, air	n_o	1.00	Assumed
Films: Cryolite	n_f	1.34 [*]	Measured
Aqueous solution	n_f	1.35	Assumed
High index glass	n_f	2.0 ^{**}	Assumed
Substrates: Dielectric	$\hat{n}$	2.0	Assumed
Silicon	$\hat{n}$	4.14 - 0.03i	Ref. 9 (546nm)
		4.05 - 0.028i ^{**}	Ref.11 (546nm)
Platinum	$\hat{n}$	2.07 - 4.40i	Ref.10 (590nm)
		1.43 - 3.03i + (0.000308 - 0.000605i) x (λ - 380nm)	Ref.10 own fit
Silver	$\hat{n}$	0.18 - 3.71i	Ref.10 (590nm)
		0.155 - 1.76i + (0.000125 - 0.00905i) x (λ - 380nm)	Ref.10 own fit
Chromium	$\hat{n}$	2.23- 1.23i [*]	Measured (546nm)
Aluminum	$\hat{n}$	0.8 - 5.4i [*]	Measured (546nm)

^{*}Used only in experimental confirmation of optimum angle.

^{**}Used only in Table II.

Table II. Optimum Angle of Incidence (deg), based on the following criteria: a- Michelson fringe visibility $V=1$, values of $n=r_1=r_2$ at optimum angle given in parentheses; b - Maximum color purity; c- Maximum polychromatic fringe visibility (estimated to nearest 0.5 deg).

Substrate refractive index	Criteria	Film refractive index 1.35						Film refractive index 2.0					
		s-polarization Interference order			p-polarization Interference order			s-polarization Interference order			p-polarization Interference order		
		1	2	3	1	2	3	1	2	3	1	2	3
Silicon 4.05-0.028i*	a	77.2 (0.62)			76.1 (0.36)			12.0 (0.34)			76.1 (0.29)		
	b	76.0	72.5	70.8	75.7	75.2	75.1	0**	0**	0**	75.9	75.6	75.5
	c	76.5	73	72	76	75	75	10**	0**	0**	76	75.5	75.5
Platinum 2.07-4.4i	a	86.2 (0.86)			85.4 (0.72)			76.7 (0.77)			85.6 (0.70)		
	b	84.6	81.5	80.6	84.0	82.8	82.5	74.4	67.0	64.0	84.5	83.4	83.1
	c	85	82	81	84	83	83	75	69	66	84.5	84	83
Silver 0.18-3.71i	a	89.5 (0.98)			89.4 (0.96)			88.3 (0.97)			89.4 (0.95)		
	b	88.2	86.8	86.2	88.0	87.2	87.1	85.9	82.4	81.4	88.2	87.5	87.4
	c	88.5	87.5	87	88.5	87.5	87	86.5	83.5	82.5	88.5	88	87.5

* Ref. 10, value used elsewhere in this paper 4.14 - 0.03i.

** Criteria not sensitive to angle of incidence.

Table III. Comparison of experimental and theoretical optimum angles of incidence.

Specimen 1: Cryolite on chromium

Specimen 2: Cryolite on aluminum

Specimen	Polarization	Interference order	Optimum angle (deg)			
			Visual observer 1	Visual observer 2	Photographic**	Theoretical
1	s	2,3	67.5		70	73*
	p	1,2	72.5		74	73*
2	s	1,2	83	87	87	88
	p	1	84	87	87	88

*Based on $r_1 = r_2$; angle lower based on color purity.

**Both deservers.

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Figure Captions

1. Definition of terms for interference in a transparent film (thickness d , refractive index n_f) on an absorbing substrate (complex refractive index $\hat{n}$); angle of incidence ϕ , angle of refraction ϕ' ; complex reflection coefficients for top and bottom film interfaces $\hat{r}_1$ and $\hat{r}_2$. The incident medium is assumed to be air, $n_0 = 1$.
2. Definition of color purity a/b in the C.I.E. chromaticity diagram.
3. Chromaticity coordinates of thin-film interference colors ($n_f = 1.35$) as a function of angle of incidence (0° - 90°) and optical path difference due to the film (0-400nm), Dielectric substrate, ($\hat{n} = 2.0$), s-polarized light, first interference order (optimum angle 44°).
4. Chromaticity coordinates of thin-film interference colors ($n_f = 1.35$) as a function of angle of incidence and optical path difference due to the film. Silicon substrate ($\hat{n} = 4.14 - 0.03i$), s-polarized light, first interference order (optimum angle 77°).
5. Chromaticity coordinates of thin-film interference colors ($n_f = 1.35$) as a function of angle incidence and optical path difference due to the film. Platinum substrate ($\hat{n} = 2.07 - 4.40i$), s-polarized light, first interference order (optimum angle 85°). Expanded scale.
6. Chromaticity coordinates of thin-film interference colors ($n_f = 1.35$) as a function of angle of incidence and optical path difference due to the film. Silver substrate ($\hat{n} = 0.18 - 3.71i$), s-polarized light, first interference order (optimum angle 88.6°). Expanded scale.
7. Effect of wavelength-dependence of optical constants of substrate (given in Table I) on chromaticity of thin-film interference colors ($n_f = 1.35$). Platinum substrate, s-polarized light, first interference

order (optimum angle 85^0). Compare to Fig. 5.

8. Effect of wavelength-dependence of optical constants of substrate (given in Table I) on chromaticity of thin-film interference colors ($n_f = 1.35$). Silver substrate, s-polarized light, first interference order (optimum angle 88.6^0). Compare to Fig. 6.

9. Effect of p-polarization on chromaticity of thin-film interference colors ($n_f = 1.35$). Platinum substrate ($\hat{n} = 2.07 - 4.40i$), first interference order (optimum angle 84.5^0). All angles of incidence are above Brewster's angle for reflection at the film-air interface. Compare to Fig. 5.

10. Chromaticity coordinates of second order thin-film interference colors ($n_f = 1.35$). Platinum substrate ($\hat{n} = 2.07 - 4.40i$), s-polarized light (optimum angle 82^0). Expanded scale. Compare to Fig. 5.

11. Color purity of thin-film interference colors ($n_f = 1.35$) as a function of angle of incidence for interference orders 1 to 4. Platinum substrate ($\hat{n} = 2.07 - 4.40i$); red interference color of complimentary dominant wavelength 506.4 nm for $\Delta S = 225, 735, 1270$, and 1800 nm in first 4 interference orders; s-polarized light.

12. Color purity of thin-film interference colors ($n_f = 1.35$) as a function of angle of incidence for interference orders 1 to 4. Platinum substrate ($\hat{n} = 2.07 - 4.40i$), red interference color of complimentary dominant wavelength 506.4 nm for $\Delta S = 450, 950, 1500$, and 2040 nm in first 4 interference orders. p-polarized light; ϕ_p = Brewster's angle for reflection of the film-air interface.

13. Effect of reflection coefficient r_1 for reflection from the air-film interface on color purity of thin-film interference colors of

interference order 1 to 4. Reflection coefficient for the film-substrate interface assumed constant $r_2 = 0.8$ (e.g. platinum). Phase change due to reflection $\delta_1 - \delta_2$ assumed zero. Red interference color of complimentary dominant wavelength 506.4 nm. Difference between double-beam and multiple-beam interference.

14. Effect of equal reflection coefficients r_1 and r_2 on color purity of thin-film interference colors for interference orders 1 to 4. Phase change due to reflection $\delta_1 - \delta_2$ assumed zero. Red interference color of complimentary dominant wavelength 506.4 nm. Difference between double-beam and multiple-beam interference.

15. Variation of tristimulus values with optical path difference S for multiple-beam interference. Reflection coefficients for both interfaces (from top to bottom curves) 0.95, 0.9, 0.8, 0.7, 0.6, 0.4 and 0.2. Phase change due to reflection $\delta_1 - \delta_2 = 0$.

16. Polychromatic fringe visibility for thin-film interference colors (based on luminance, Eq. 4). Effect of reflection coefficient r_1 with constant $r_2 = 0.8$ (e.g. platinum) for interference orders 1 to 4. Phase change $\delta_1 - \delta_2 = 0$.

Fig. 1

XBL 785-873 A

XBL 755-6355

Fig. 2

XBL752-5694

Fig. 3

XBL752-5695

Fig. 4

XBL752-5696

Fig. 5

XBL752-5698

Fig. 6

XBL 752-5697

Fig. 7

XBL 752-5699

Fig. 8

XBL 752-5807

Fig. 9

XBL 752-5805

Fig. 10

XBL 752-5820

Fig. 11

XBL 752-5821

Fig. 12

XBL 752-5883 A

Fig. 13

XBL 752-5881

Fig. 14

XBL 754-6065A

Fig. 15

XBL 764-6064A

Fig. 16

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