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Mode Localization Induced by a Nonlinear Control Loop

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Abstract. We present the analysis of a nonlinear control system that is used to excite and maintain a specified amplitude of oscillation in the Jet Propulsion Laboratory vibratory gyroscope. This experimental application shows that nonlinear localization through active means can be implemented in a practical system when it is desirable to confine the response to a favorable mode. The closed-loop system response predicted by the model shows very close agreement with the experimental results for a significant range of controller parameters. We also experimentally demonstrate that the actively localized motion is eliminated through bifurcation, similar to what was observed in previous passive localization studies applied to extended flexible oscillators.

Key words: nonlinear localization, nonlinear control, automatic gain control, vibratory gyroscope.

1 Introduction

We consider active nonlinear localization of energy in an oscillator. Our main application is the micromachined vibratory rate sensor designed by the Jet Propulsion Laboratory (JPL) for future spacecraft missions [1,2]. This sensor's mechanical structure possesses three dominant elastic modes representing translational motion and rotation of the micro-structure. Proper sensor operation requires that one of the rotational degrees of freedom be excited to a periodic oscillation with a stable, specified amplitude. We approach this problem by designing a nonlinear feedback control loop, termed Automatic Gain Control (AGC), that selectively localizes energy into the desired mode. We demonstrate that by proper design of the AGC, a single-degree-of-freedom (SDOF) oscillator model driven by the control loop accurately describes this active nonlinear localization phenomenon in the physical device. The SDOF case is thoroughly analyzed using standard averaging and phase-plane techniques. Simulations of the averaged equations show excellent agreement with reality, especially given the complexity of the physical device and the fact that the nonlinear controller is implemented on a microprocessor and is governed by discrete-time equations.

Passive nonlinear localization was extensively studied in previous works for discrete [3,4] and continuous systems [3,5,6]. In these works it was shown that weakly coupled nonlinear oscillators can possess strong and weak localized modes corresponding to energy confinement in one or more substructures of the system. In contrast to the well known linear (Anderson) localization, nonlinear localization occurs even in the absence of structural disorder. The generating mechanisms for localization in spatially periodic systems were also studied in previous works. These can either be mode bifurcations, or smooth transitions of non-localized branches of nonlinear modes. To the authors' knowledge, few previous studies exist addressing energy localization in systems with active nonlinear control. In two such works [7,8], nonlinear motion confinement induced by active nonlinearities was investigated in a system of coupled beams. These works demonstrated that active control can be used to "alter" the linear dynamics of a structure and induce nonlinear confinement phenomena that are favorable to the design objectives.

The structure of the paper is as follows. In Section 2 we introduce and analyze, using the method of averaging, the SDOF oscillator with automatic gain control. The AGC can be physically implemented using rectifiers, low-pass filters, and comparators and this simplicity make it attractive for use in applications such as the JPL sensor. A description of the sensor is given in Section 3.1 and experimentally identified models of the sensor and a simplified SDOF model are presented in Section 3.2. This section also presents experimental results of active localization in the sensor and makes a comparison with theoretical predictions based on the SDOF model. Excellent agreement is observed, validating the SDOF simplification. Limitations of the SDOF model, however, are revealed in Section 3.3 through additional experiments where quasi-periodic response of the closed-loop system is observed after a modest increase in the sensor transfer function gain. Indeed, these experiments show what appears to be a Hopf bifurcation that destabilizes the localized motion and gives rise to nonlocalized multi-mode response. This behavior is not reproducible with the SDOF model and motivates the need for developing multi-degree-of-freedom models to provide a more thorough understanding of the sensor/AGC dynamics. Section 4 discusses the predictive qualities of the SDOF model and proposes some topics for further investigation.

2 Analysis of an Oscillator with Automatic Gain Control

This section presents the analysis of an AGC loop regulating a single degree of freedom oscillator. The operation of the AGC is intuitive and is explained with the assistance of Figure 1. In this configuration, the velocity of the oscillator is applied to the oscillator input after being multiplied by a gain so in essence the AGC manipulates the damping of the oscillator: if the desired velocity

amplitude is less than the *reference amplitude* then the multiplicative gain is chosen to be negative in order to dampen the oscillations; conversely, if the reference amplitude is larger than the current amplitude then the AGC gain is positive and the “anti-damping” causes the oscillation to grow. This idea forms the basis of a simple automatic gain control, however, implementation in the JPL sensor requires additional compensation as discussed in Section 3.1.

The equations governing the oscillator/AGC system are

$$\begin{aligned}\ddot{x} + c\dot{x} + \Omega^2 x &= K(r - z)\alpha\dot{x} \\ \dot{z} &= \lambda(|\alpha\dot{x}| - z),\end{aligned}\tag{1}$$

where x is the oscillator degree of freedom, and z is the state of the low-pass filter which estimates the amplitude of a scalar multiple of the oscillator velocity $\alpha\dot{x}$ (the filter has corner frequency λ). The AGC gain is simply the difference between the reference amplitude, r , and filter state, z , multiplied by the gain K .

Though this model is a SDOF oscillator with essentially nonlinear damping, the control method is applied to the JPL rate sensor that has three modes that dominate the frequency response. By proper design of linear filters in series with the sensor’s inputs and outputs, the AGC excites only the selected mode, thereby localizing the energy in this mode. The analysis of the SDOF oscillator model is shown to predict with great accuracy the response of the physical system in most cases.

Exact analysis of Equation (1) is difficult because of the nonlinear amplitude detection. Perturbation analysis using periodic averaging, however, leads to simplified nonlinear differential equations. The separation of time scales between the oscillator dynamics and the low-pass filter and AGC gain may be used as the perturbation parameter. The analysis proceeds in several steps. First, a change of coordinates is performed, then the equations are averaged to remove the fast time scale and, lastly, the approximate system of equations is analyzed.

A coordinate change transforms the oscillator states x and $\dot{x}$ into slowly varying amplitude and phase variables. Define

$$x(t) := a(t) \cos(\Omega t + \phi(t)),\tag{2}$$

where a and ϕ are amplitude and phase functions, respectively. Differentiating Equation (2) with

respect to t yields

$$\dot{x} = -a\Omega \sin(\Omega t + \phi) + \dot{a} \cos(\Omega t + \phi) - a\dot{\phi} \sin(\Omega t + \phi). \quad (3)$$

The sum of last two terms in this expression is set to zero and is used in determining the equations for a and ϕ :

$$\dot{a} \cos(\Omega t + \phi) - a\dot{\phi} \sin(\Omega t + \phi) \equiv 0. \quad (4)$$

Differentiate Equation (3) once more (keeping in mind Equation (4)):

$$\ddot{x} = -\dot{a}\Omega \sin(\Omega t + \phi) - a(\Omega + \dot{\phi}) \cos(\Omega t + \phi). \quad (5)$$

Substituting Equations (2), (3) and (5) into the oscillator equation yields

$$(-\Omega\dot{a} - \Omega a(c - \alpha K(r - z))) \sin(\Omega t + \phi) + (-\Omega a(\Omega + \dot{\phi}) + \Omega^2 a) \cos(\Omega t + \phi) = 0. \quad (6)$$

Differential equations for a and ϕ are obtained by pre-multiplying [(4) (6)]^T by the non-singular matrix

$$\begin{bmatrix} \cos(\Omega t + \phi) & -\frac{1}{\Omega} \sin(\Omega t + \phi) \\ \frac{1}{\Omega} \sin(\Omega t + \phi) & \frac{1}{\Omega^2} \cos(\Omega t + \phi) \end{bmatrix}.$$

Simplifying the equations after this step and including the equation for z yields the system

$$\begin{aligned} \dot{a} &= [\alpha K(r - z) - c] a \sin^2(\Omega t + \phi) \\ \dot{\phi} &= [\alpha K(r - z) - c] \sin(\Omega t + \phi) \cos(\Omega t + \phi), \quad a \neq 0 \\ \dot{z} &= \lambda (| -\alpha \Omega a \sin(\Omega t + \phi) | - z). \end{aligned} \quad (7)$$

No approximations are made for these equations. We may apply averaging [9] by changing the time scale to $\tau = \Omega t$ and noting that vector field is 2π -periodic and scaled by a small parameter if the following relation is satisfied

$$\frac{K}{\Omega} \propto \frac{\lambda}{\Omega} \propto \frac{c}{\Omega} \ll 1. \quad (8)$$

These conditions restrict the AGC gain K , low-pass filter corner frequency λ , and the damping c to be small compared to the oscillator frequency Ω . Since K and λ are parameters that the designer can specify, the analysis really hinges on the requirement that c/Ω be sufficiently small.

Fortunately, this requirement is not difficult to satisfy in our application because the damping in sensor is very small. Thus, the quantitative behavior of Equation (7) can be described by the averaged system if Equation (8) is satisfied.

The averaged equations are simply,

$$\begin{aligned}\dot{\bar{a}} &= \frac{1}{2} [\alpha K (r - \bar{z}) - c] \bar{a} \\ \dot{\bar{\phi}} &= 0 \\ \dot{\bar{z}} &= \lambda \left(\frac{2}{\pi} \alpha \Omega \bar{a} - \bar{z} \right).\end{aligned}\tag{9}$$

The averaged variables are denoted with the bar. Note that $\bar{\phi}(t)$ is constant and may be set to zero for the analysis.

The equilibria of Equations (9) are

$$\begin{aligned}\bar{a}_0 &= 0 & \bar{a}_1 &= \frac{\pi}{2\Omega} \left(r - \frac{c}{K} \right) \\ \bar{z}_0 &= 0 & \bar{z}_1 &= \frac{1}{\Omega} \left(r - \frac{c}{K} \right).\end{aligned}$$

The Jacobian linearizations at these equilibria are

$$\begin{aligned}D_x f(\bar{a}_0, \bar{z}_0) &= \begin{bmatrix} \frac{1}{2}(\alpha K r - c) & 0 \\ \frac{2}{\pi} \alpha \Omega \lambda & -\lambda \end{bmatrix} \\ D_x f(\bar{a}_1, \bar{z}_1) &= \begin{bmatrix} 0 & -\frac{\pi K}{4\Omega} \left(r - \frac{c}{K} \right) \\ \frac{2}{\pi} \alpha \Omega \lambda & -\lambda \end{bmatrix}.\end{aligned}\tag{10}$$

We must assume $\alpha K r > c$ for perturbations near equilibrium $(\bar{a}_0, \bar{z}_0)$ to grow, i.e., the linearized system has one unstable eigenvalue and the AGC loop is excited if $\bar{a}(0) \neq 0$. This condition merely states that the anti-damping added by the AGC loop must overcome the natural damping present in the system.

The $(\bar{a}_1, \bar{z}_1)$ equilibrium point is stable when $\alpha K r > c$ since the eigenvalues of $D_x f(\bar{a}_1, \bar{z}_1)$ are

$$-\frac{\lambda}{2} \left(1 \pm \sqrt{1 - \frac{2}{\lambda} (\alpha K r - c)} \right).$$

In this case, the response of the linearized equations is critically damped with rate of convergence $\lambda/2$ when the system parameters satisfy $\lambda = 2(\alpha K r - c)$. The eigenvalues are complex conjugates

when $\lambda < 2(\alpha K r - c)$ and oscillations in the amplitude are observed. The design parameters in the AGC are λ and K and these may be used to alter the response the system. Increasing K reduces the effect of the natural system damping in the steady state amplitude but produces a more oscillatory response. Note that the response depends on the reference amplitude r and a larger reference amplitude tends to cause oscillation.

A global picture of the $\bar{a}$ - $\bar{z}$ dynamics is desirable. In particular it is of interest to determine the existence of limit cycles. A limit cycle indicates periodic solutions of $\bar{a}$ and this implies the existence of quasi-periodic motion of the oscillator. This situation is undesirable since a constant oscillator amplitude is required in applications such as the JPL rate sensor. We now show that limit cycles cannot exist for the averaged equations.

Recall that the sum of indices of equilibrium points enclosed by a limit cycle must be one [10]. This implies that a limit cycle, if it exists, must enclose the equilibrium point $(\bar{a}_1, \bar{z}_1)$. The closed set $Z =: \{(\bar{a}, \bar{z}) : \bar{a} \geq 0, \bar{z} \geq 0\}$ is invariant under the action of Equation (9) because $\dot{z} \geq 0$ when $z = 0$ (the vector field points into Z along the $\bar{a}$ axis, for $\bar{a} > 0$). Thus, any limit cycle that passes through part of Z must actually lie entirely in the interior of Z and enclose $(\bar{a}_1, \bar{z}_1)$. We discount this last possibility by applying Dulac's Criterion [11]. Define

$$\begin{aligned} f(\bar{a}, \bar{z}) &:= \frac{1}{2} [\alpha K(r - \bar{z}) - c] \bar{a} \\ g(\bar{a}, \bar{z}) &:= \lambda \left(\frac{2}{\pi} \alpha \Omega \bar{a} - \bar{z} \right), \end{aligned}$$

i.e., $\dot{\bar{a}} = f$ and $\dot{\bar{z}} = g$, and let $\rho = \bar{a}^m$, where $m \in \mathbb{R}$ is chosen below. Dulac's Criterion states that if the sign of $\frac{\partial(\rho f)}{\partial \bar{a}} + \frac{\partial(\rho g)}{\partial \bar{z}}$ is constant in a simply connected region of the phase space then limit cycles do not exist in the region. Computing this quantity yields

$$\begin{aligned} \frac{\partial(\rho f)}{\partial \bar{a}} + \frac{\partial(\rho g)}{\partial \bar{z}} &= \left[\frac{m+1}{2} (\alpha K(r - \bar{z}) - c) - \lambda \right] \bar{a}^m \\ &= -\frac{m+1}{2} \alpha K \bar{z} \bar{a}^m \quad \text{when } m := \frac{2\lambda}{\alpha K r - c} - 1 \\ &< 0 \quad \text{on the interior of } Z. \end{aligned}$$

Thus, the interior of Z cannot contain a limit cycle. Further arguments are necessary to determine whether $(\bar{a}_1, \bar{z}_1)$ is globally attractive (except when $\bar{a}(0) = 0$) but the fact that limit cycles cannot occur in this system is encouraging and shows that the AGC is robust to initial conditions—under practical circumstances the AGC loop always drives the oscillator to a constant amplitude..

3 AGC Implementation in a Novel Angular Rate Sensor

3.1 The JPL vibratory gyroscope

The Jet Propulsion Laboratory micromachined gyroscope is an angular rate sensor whose operation depends on the coupling of energy between two degrees of freedom within the sensor via coriolis terms in a sensor-fixed frame. Vibratory “gyros” such as the JPL instrument do not employ a spinning mass but rather require a selected degree of freedom in an elastic structure to be driven to a constant amplitude of periodic oscillation. An introduction to the principles of vibratory gyroscopes is covered in [12]. Like most vibratory gyros, the JPL prototype resides in a high vacuum to reduce dissipation and maximize the response of the mechanical structure to the relatively weak electrostatic excitation forces.

The task of driving a very lightly damped degree of freedom to a constant amplitude may be accomplished in many ways. The modal frequencies of the JPL sensor are temperature sensitive, however, so it is desirable to use a method which successfully excites the selected mode without requiring a precise estimate of the modal frequency. This is the primary motivation for using automatic gain control in the JPL sensor since it is essentially a nonlinear velocity feedback scheme that always excites an oscillator at its modal frequency.

The drive and sense electrodes of the JPL microgyro (denoted D_1 and D_2 , and S_1 and S_2 , respectively, in Figure 2) are suspended above four matching electrodes by silicon springs visible in the figure along the x and y axes. The sensor dynamics are excited by applying potentials between the drive electrodes and their corresponding base-plate electrodes. The potentials create an electrostatic force that pulls the opposing electrodes towards each other, thus rotating and translating the cloverleaf formed by D_1 , D_2 , S_1 and S_2 .

The large central post, which is rigidly attached to the cloverleaf, adds inertia and increases sensor sensitivity. Displacements of the cloverleaf structure are measured using the S_1 and S_2 electrode pairs. The sensor dynamics are dominated by the three rigid body degrees of freedom of the cloverleaf corresponding to axial motion along the z -axis and rotation about the x - and y -axes. The response amplitude is maximized by driving the sensor at the natural frequencies of these degrees of freedom.

The degree of freedom corresponding to rotation about the y -axis is referred to as the *drive mode*. The *sense mode* corresponds to rotation about the x -axis. A measurement proportional to the sense mode velocity is provided by taking the difference of the S_1 and S_2 electrode voltages, which are related to the capacitance between the electrodes pairs. Similarly, a measurement

proportional to the drive mode velocity is provided by summing the S_1 and S_2 voltage.

3.2 Experimental results

The frequency response of a linear auto-regressive (ARX) model fit to input-output data taken from a JPL microgyro prototype is shown in Figure 3. Details of the ARX model identification procedure applied to this sensor may be found in [13,14]. The four magnitude plots represent the frequency responses of the identified model from each drive input to each sense output. It is evident from these plots that there are three lightly damped modes in the sensor in the frequency range shown. The lowest frequency mode near 2000 Hz is the axial mode in which the post-cloverleaf assembly plunges up and down in the z -axis direction. The lower of the two high frequency modes corresponds to the drive mode (rotation about the y -axis in Figure 2) and is denoted $\Omega_{drv} \approx 2431$ Hz. The remaining high frequency mode, denoted $\Omega_{sen} \approx 2492$ Hz, is the sense mode. The rounded peak near 3.5 kHz in the frequency response plots is due to the pre-amplifier used in the capacitive measurement scheme. The sense mode can be nearly eliminated from the frequency response by driving D_1 and D_2 with the same input and summing the measurements of S_1 and S_2 . This single input/single output configuration, which we denote as the *drive configuration*, primarily excites and measures the axial and drive mode, although the sense mode is present in the response to some extent.

The drive configuration is used to test the AGC, however, due to the signal rectification, the controller cannot distinguish between the relative contribution of the three modes so it is necessary to additionally filter the drive and sense signals as shown in Figure 4. These analog filters are designed to provide the controller with the measurement proportional to the drive mode velocity as well as adjust the phase of the remaining modes as described below.

The frequency response of an identified model of the sensor drive configuration with the filters at the input and output is shown in Figure 5 (gray plot). This figure demonstrates that the sense mode has almost been eliminated (compare to the individual channels in Figure 3). The most informative plot for purposes of the AGC design, however, is the Nyquist plot of the drive configuration in Figure 6 (gray plot). The large lobe encircling -1 is the drive mode whose amplitude the AGC is designed to regulate. The small lobe in the upper half-plane is the axial mode while the smallest lobes can be attributed to the sense mode and pre-amplifier dynamics. The most important role of the analog filters now becomes clear: the filters are used to adjust the phase of the drive mode to create a measurement proportional to its modal velocity while also adjusting the phases of the axial and sense modes such that the AGC will not excite these modes.

The AGC loop is implemented with a digital signal processor sampling at 25 kHz. Unfiltered quantized drive signals excite the transient dynamics of the sensor leading to high harmonic content in the sensor response. Thus, another advantage of using filters for the sensor drive electrode signals (denoted LPF1 in Figure 4) is to smooth the quantized signals from the digital-to-analog converters (D/A). The filters at the sensor output, denoted LPF2 in Figure 4, are also used for anti-aliasing, in addition to adjusting the phase of the drive configuration transfer function.

An intuitive explanation of the AGC operation is that it adjusts the magnitude, but not the phase, of the Nyquist diagram in Figure 6. For example, if the drive mode amplitude is smaller than the desired amplitude, the AGC adjusts the loop gain so that the drive mode lobe encircles -1 . This has the effect of destabilizing the drive mode by adding anti-damping in the closed-loop system. Conversely, if a smaller drive amplitude is desired, then the AGC changes the sign of the loop gain so that the drive mode lobe no longer encircles -1 . This has the effect of adding damping to the drive mode to decrease its amplitude. In steady-state operation the AGC adjusts the loop gain so that the drive mode lobe passes through -1 on the Nyquist plot. The AGC will not excite the other modes unless a large feedback gain is used with a phase change, i.e., the smaller lobes should not encircle -1 . The black graphs in Figures 5 and 6 are explained below.

The estimation of Ω_{drv} , c , and α in Equation (1) from experimental data is required to compare this model with the actual device response. In principle these parameters may be extracted from the identified model in Figure 5. This model, however, was obtained from transient response experiments where the input was a chirp signal of short duration. More accurate parameter values may be obtained by using a function synthesizer to excite the drive mode to a constant amplitude and measuring the steady-state gain, decay time when the excitation is removed, etc. Using this method yields $\Omega = 2431$ Hz. The damping is so small in the sensor that the frequency where the transfer function achieves its maximum response may be taken to be the natural frequency in the model. The damping coefficient c is determined from Figure 7. The damping coefficient is twice the magnitude of the exponent used to create the envelope, i.e., $c \approx 2 \cdot 15 = 30$. Figure 7 also illustrates that the damping in the sensor is nonlinear because the exponential envelope does not fit the entire time plot. The gain α is also determined by observing the response amplitude relative to the input amplitude, denoted H_{max} , when driving the sensor with a sinusoid at Ω . The approximate relationship that holds for small damping ratio is $H_{max} = \alpha/c$. H_{max} is determined to be 2.6 and so $\alpha = 2.6c = 78$. Thus, the simplified oscillator model used in Equation (1) is

$$\ddot{x} + 30\dot{x} + (2431 \cdot 2 \cdot \pi)^2 = K(r - z) \cdot (78\dot{x}). \quad (11)$$

The Bode and Nyquist plots of this model from the perspective of the AGC are the black graphs in Figures 5 and 6. We see that the ARX model overestimates the peak amplitude by about a factor of two. The ARX model is still indispensable, however, in showing us the phase relationships of the drive mode and the other two modes —facts which are used to argue that the AGC loop regulates the drive mode yet does not excite the axial mode or sense mode.

The experimental closed-loop responses of the sensor are the solid graph traces shown in Figures 8 and 9. These figures illustrate how the AGC loop excites the drive mode to a steady-state amplitude. Each column of sub-plots represents one experiment with a fixed set of AGC control parameters (α, K, r, λ) . These parameters are given in the title of the top sub-plots. Each sub-plot row represents a particular variable in the closed-loop system. The first row represents the measurement $S_1 + S_2$ which is the signal that the AGC receives (“ y ” in Figure 4). The units are in volts normalized with respect to the A/D range of ± 10 volts, i.e., 1 represents 10 volts, 0.5 represents 5 volts, etc. The second row represents the state of the filter, z , used in the AGC. There are no physical units attached to this quantity since it represents a variable within the computer. The third row represents the signal that drives the sensor (“ u ” in Figure 4). Symbolically this is written as $D_1 = D_2 = K(r - z)(S_1 + S_2)$. The units are volts normalized with respect to the D/A range of ± 10 volts.

The black traces in Figures 8 and 9 are the variables from time simulations of the averaged model in Equation (9) with the same control parameters as its corresponding experiment and with Ω , α and c as determined above. The black traces in the first row of sub-plots represent the amplitude, $\alpha\Omega a(t)$, of the measured velocity. The black traces in the second rows are $z(t)$. Finally, the black traces in the third row of sub-plots are amplitudes of the oscillator input: $|K(r - z(t))\alpha\Omega a(t)|$. Thus, the black traces are the correct analogs from Equation (9) of the measured experimental variables. Figure 10 is the power spectral density of the constant amplitude portion of the sensor response from the top plot in column two of Figure 8. This figure confirms that the AGC is regulating the drive mode at 2431 Hz as it was designed to do (the PSDs of the other experiments look identical to this figure).

3.3 Limitations of the AGC/SDOF oscillator model

The robustness of the AGC scheme is experimentally tested by perturbing control system parameters and this section demonstrates that a modest increase in the sensor transfer function gain from u to y in Figure 4 can produce complex behavior in the closed-loop system. In particular we perturb the gain parameter $\alpha = 78$ in Equation (11) to $\alpha = \{117, 128, 131\}$. These perturbations

represent an increase in the transfer function gain by the approximate factors $\{1.5, 1.64, 1.68\}$. These gain changes are implemented by modifying a parameter in the signal processing software so even though the gain of the *physical* device is only known approximately, we can modify the parameters in the controller with high precision. The reference signal, AGC gain, and low pass filter corner frequency are fixed at $r = 0.4$, $K = 10$, and $\lambda = 200$ Hz. Figure 11 shows a transition from the stable limit cycle oscillation of the drive mode to a stable quasi-periodic response as α is increased. The sensor response power spectrum in the left-most column in Figure 11 corresponds to $\alpha = 117$ and shows that the sense mode is exited to a very small amplitude (the bump at $\Omega_{sen} \approx 2492$ Hz). The bump near 2370 Hz is a harmonic whose frequency is related to Ω_{drv} and Ω_{sen} according to $2\Omega_{drv} - \Omega_{sen}$. As α is further increased we observe a strong modulation in the sensor input and output signals with period equal to the difference between Ω_{drv} and Ω_{sen} (approximately 61 Hz). Power spectra show the presence of harmonics at $(m + 1)\Omega_{drv} - m\Omega_{sen}$ and $(m + 1)\Omega_{sen} - m\Omega_{drv}$, $m = 1, \dots, 4$, in the sensor output.

These experimental results indicate that nonlinear active localization (that is, the desired mode of sensor operation) is possible only for certain ranges of parameters. This nonlinear localization is destroyed through what appears to be a Hopf bifurcation of the closed-loop dynamics that leads to quasi-periodic, nonlocalized response. Similar elimination of passive nonlinear localization through bifurcation has been reported in earlier work [3,4].

4 Discussion

Comparison of the experimental data and the simulations in Figures 8 and 9 reveal that the averaged two state model in Equation (9) accurately predicts the response of the physical device. We conclude that the SDOF model in Equation (1), though simple, captures the salient features of the automatic gain control loop for the range of parameter values considered. The close fit of the simulations to the experimental sensor data is further evidence that neglecting the dynamics of the axial and sense modes is justified so long as the design of the drive and sense signal filters yields a Nyquist plot similar to Figure 6. In other words, the sensor input and output filters are designed so that the mode to be regulated by the AGC encircles -1 on the Nyquist plot while the other modes avoid encirclement for all likely values of feedback gain.

The AGC control parameters are chosen to provide different responses depending on the eigenvalues of the linearization in Equations (10). The eigenvalues of $D_x f(\bar{a}_1, \bar{z}_1)$ are plotted in Figure 12 for the six different sets of gains considered in the experiments. Even though the simulations in Figures 8 and 9 are of the nonlinear model in Equation (9), the eigenvalues of the

linearizations are a good indicator of oscillations in the response envelope. For example, the sensor response in the first two columns of Figure 8 exhibit over-damped envelopes; the eigenvalues of the linearizations for these cases are the “ $\times$ ” and “ $+$ ” in Figure 12 where the $\times$ corresponds to the first column with the longer rise time (the other eigenvalues corresponding to these cases are negative, real, and lie much further in the left half-plane and hence do not dominate the response). The response with the most oscillation is the experiment in the first column of Figure 9. The eigenvalues of the linearization for this case are the $\diamond$ ’s in Figure 12 –note that the ratio of imaginary-to-real parts is the largest for this parameter combination. The reader may confirm that the remaining experiments correlate well the linearizations given in Figure 12.

The robustness of this control method is tested through modification of the sensor transfer function gain. It is always desirable to design the controller such that the closed-loop response is insensitive to reasonable perturbations in the dynamics of the controlled device. Perturbations in the gain characteristics of the sensor are not uncommon since electronic components age, and changes in the operating environment, such as ambient pressure and temperature, can modify the sensor dynamics. Experiments reveal that an increase in the sensor gain by less than a factor of two over the nominal cases produces a quasi-periodic response that involves both the sense and drive modes. The quasi-periodic response is very undesirable and renders the sensor useless for its intended task. This set of experiments reveals the limitations of the AGC/SDOF oscillator model since the nonlinear control loop clearly excites the sense mode in addition to the drive mode. Thus, nonlinear localization through active control in the sensor is effective only for a carefully designed range of the control parameters and modest deviations in the sensor dynamics from the design condition can eliminate active localization and produce quasi-periodic nonlocalized response.

A topic of current research considers a two degree-of-freedom model with AGC where the objective is to explore how the sensor and controller parameters influence the stability of the drive mode limit cycle. A bifurcation study of the multi-mode system can be used to design a controller that avoids or delays the onset of the quasi-periodic response thereby producing a more robust system.

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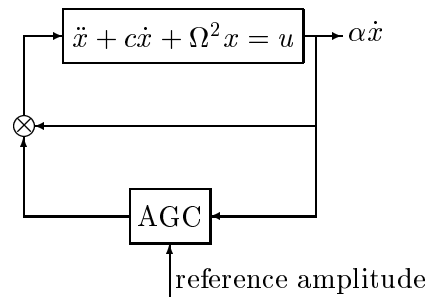


Figure 1: Automatic gain control loop.

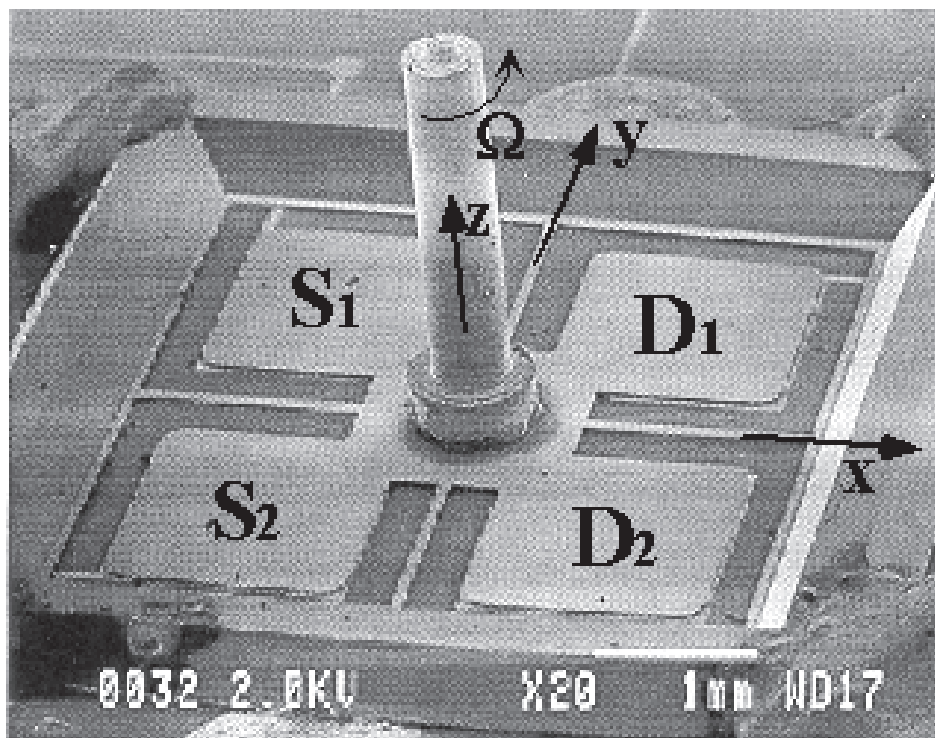


Figure 2: Photo of Jet Propulsion Laboratory microgyro with sense and actuator electrodes labeled (courtesy of T. Tang).

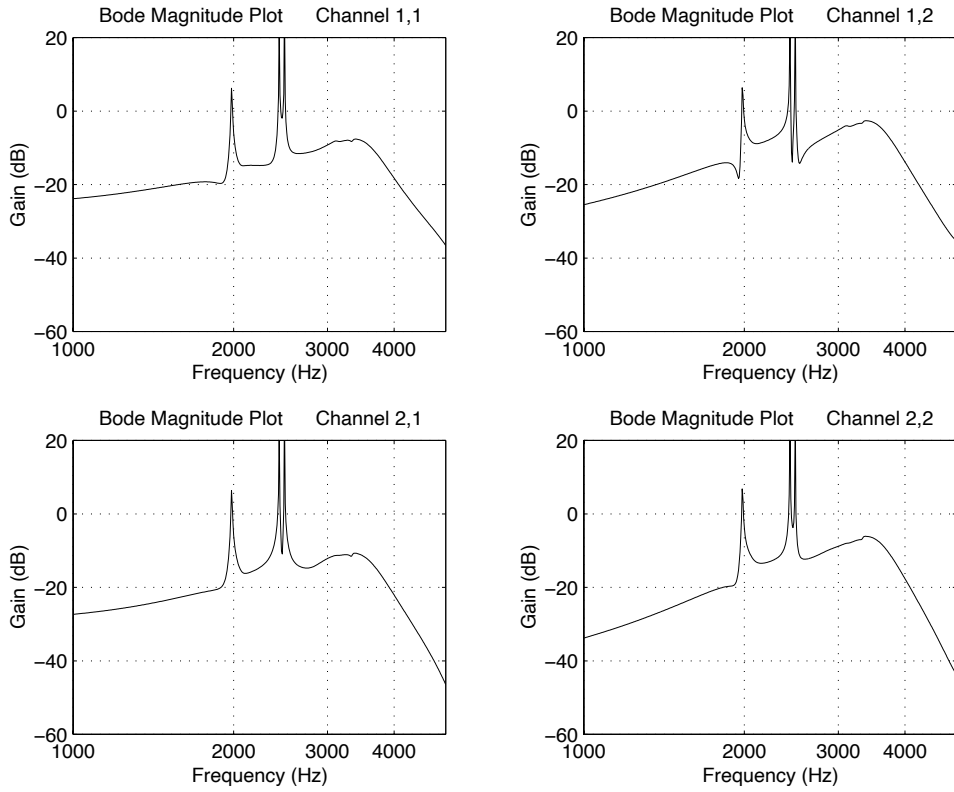


Figure 3: The four frequency responses from D_1 and D_2 to S_1 and S_2 .

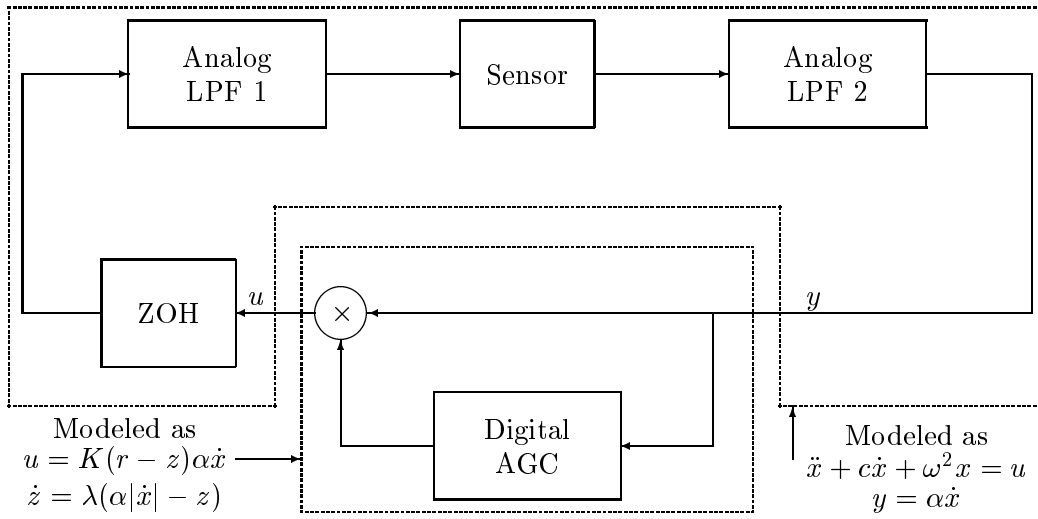


Figure 4: Feedback configuration of sensor, AGC, and analog filters. The zero-order-hold (ZOH) device, which converts the discrete-time signals from the microprocessor into continuous-time signals, is not explicitly modeled.

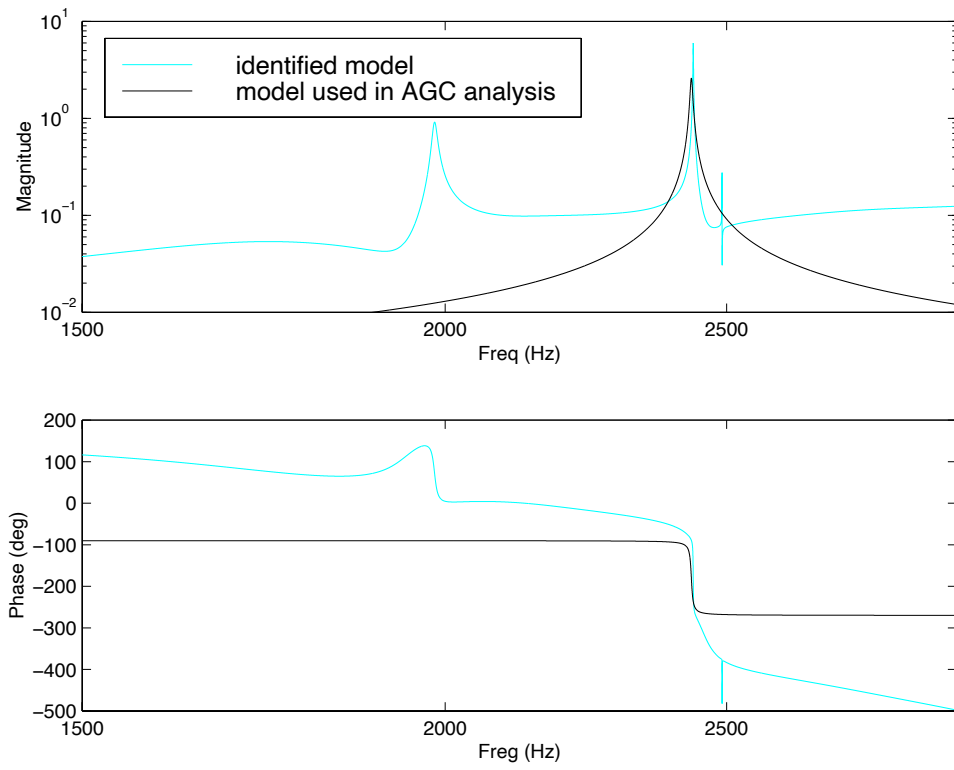


Figure 5: Bode plot of the *drive configuration* including analog filtering.

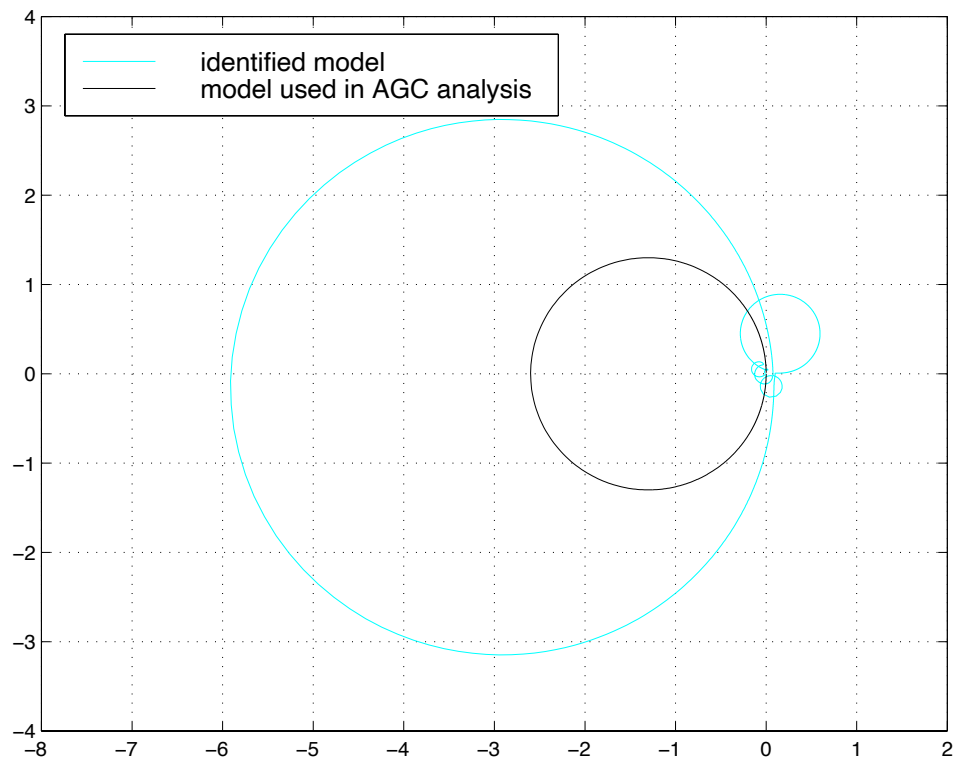


Figure 6: Nyquist plot of the *drive configuration* including analog filtering.

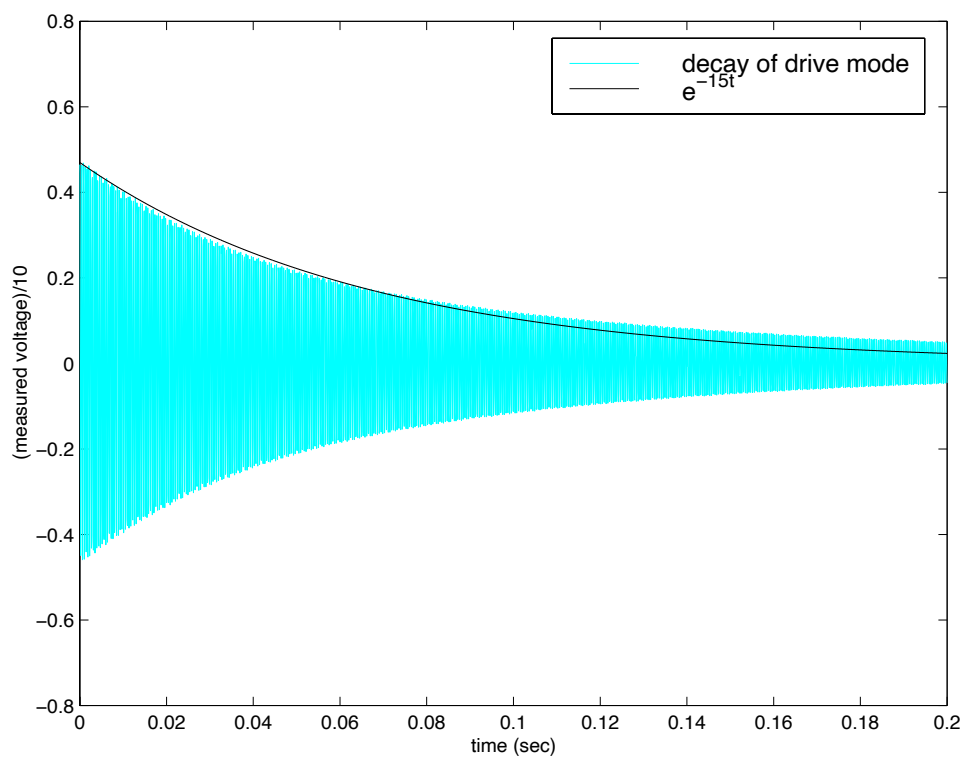


Figure 7: Determining the damping coefficient associated with the drive mode.

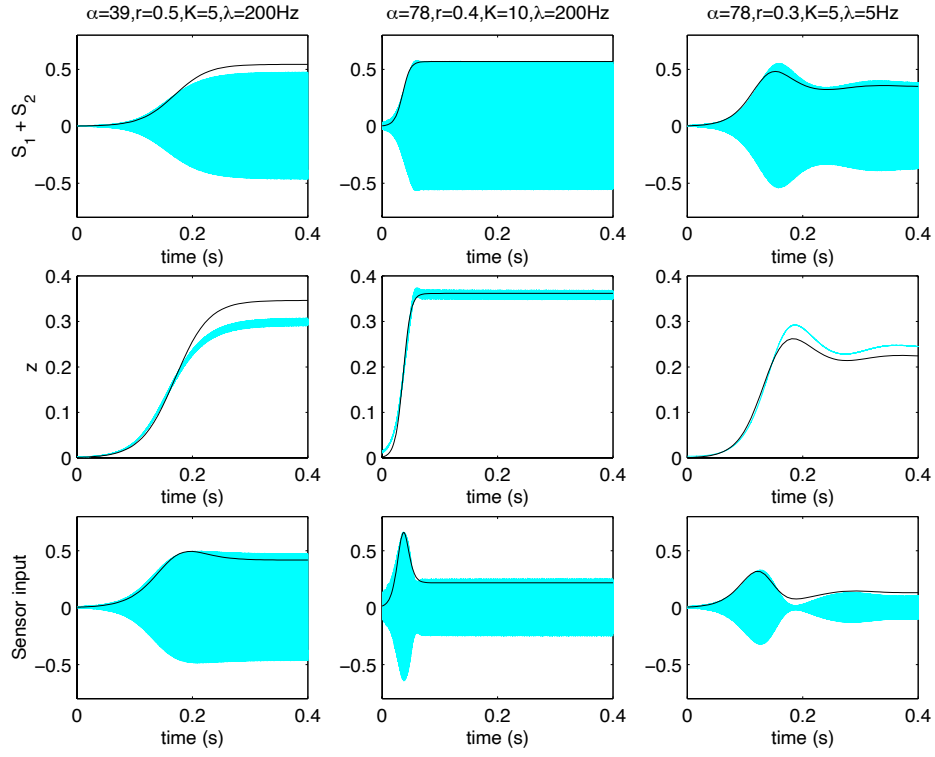


Figure 8: Closed-loop response (gray) and averaged equation simulation (black)

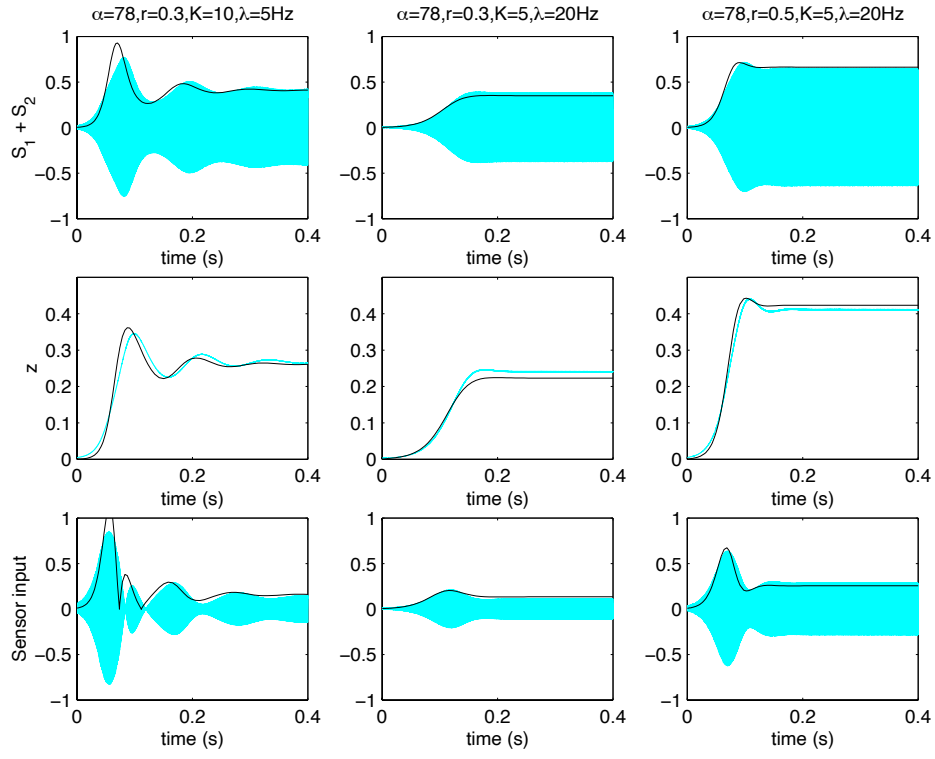


Figure 9: Closed-loop response (gray) and averaged equation simulation (black).

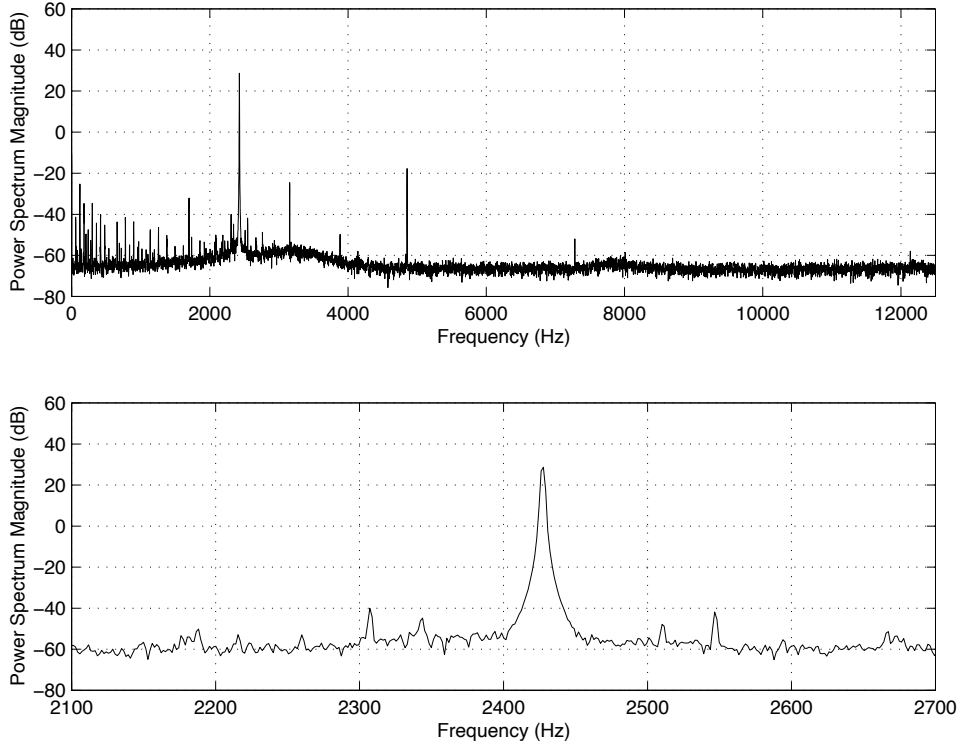


Figure 10: Power spectral density of the sensor output at equilibrium amplitude demonstrating excitation of the drive mode (top plot shows spectrum up to Nyquist frequency and the bottom plot is the detail in a neighborhood of the drive mode).

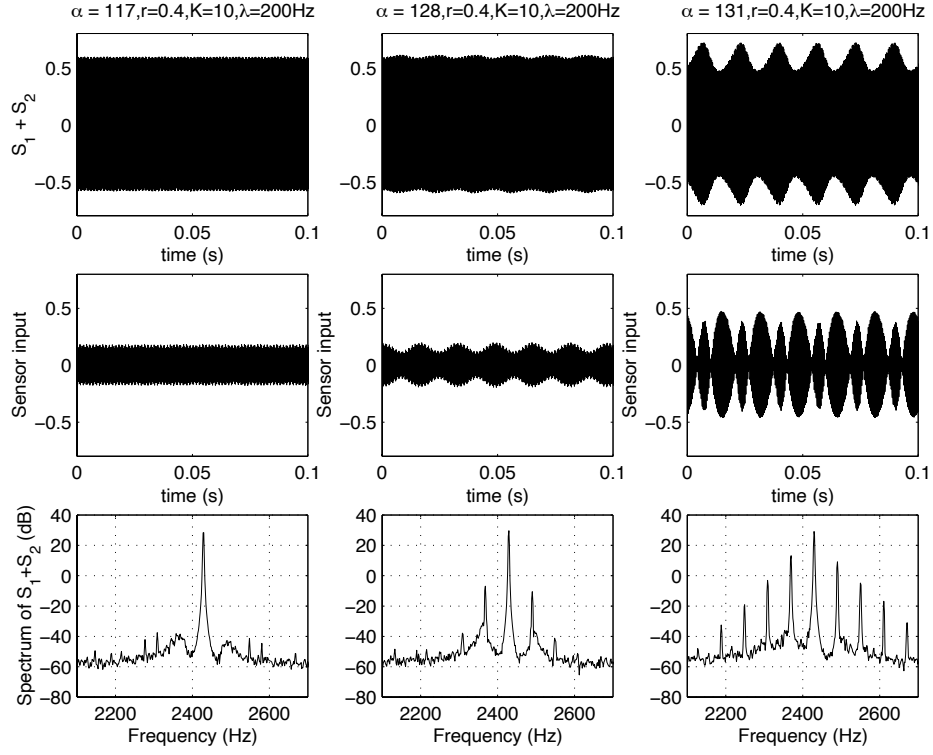


Figure 11: Quasi-periodic response of sensor for different values of sensor gain.

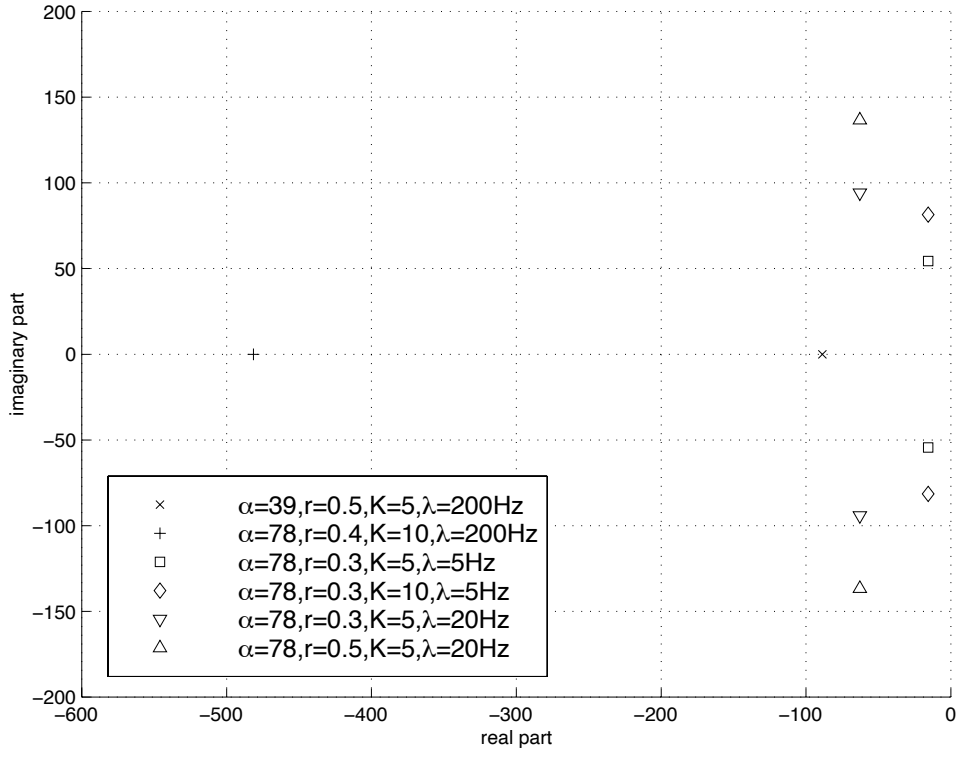


Figure 12: Locus of eigenvalues of closed-loop system linearized about equilibria.

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