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Fission of a Chromoelectric Flux Tube and Meson Radiation from a Quark-Gluon Plasma

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Abstract

The chromoelectric flux tube model is used to obtain a dynamical description of the evaporation of mesons from a quark-gluon plasma. The radiation pressure is computed to assess whether this process is an important mode for the disassembly of a compressed plasma. Of broader interest, we derive a new result for $q-\bar{q}$ pair creation in the chromoelectric field, which includes the mutual interaction of the pair.

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The lattice gauge solutions of a QCD description of matter provide fairly convincing theoretical support for the notion that hadrons will dissolve into a quark-gluon plasma at a sufficiently high energy density.[1] If such a plasma is formed in a high-energy hadron-hadron or nucleus-nucleus collision, the high temperature and internal pressure of the plasma will cause its rapid disassembly. Therefore, evidence for its prior existence must be found in its decay products, hadrons, dileptons, and photons. To assess whether signals of the plasma survive the evolution and what they are, a dynamical description of the disassembly is needed. One facet has been studied by Bjorken, who proposes a hydrodynamical model for its expansion.[2] As it expands it will cool, and the conditions for condensation back to the hadronic phase will be attained. In this note we address another facet of the disassembly, the formation and radiation of mesons at the surface of the hot plasma. The pre-freezeout radiation, which has also been emphasized in connection with the disassembly of a hadronic fireball,[3] plays two roles. It carries signals of the state of the plasma as it evolves. Second, it is coupled to the hydrodynamical expansion of the plasma, which may be inhibited by the backward radiation pressure. In the extreme, the plasma may evaporate mesons rather than expand collectively as a plasma.

The chromoelectric flux tube model provides a phenomenological description of the confinement of color. We show below how it can be used to obtain a dynamical description of the materialization of mesons and their radiation at the surface of a quark plasma.

Consider a quark (or antiquark) of momentum $\underset{\sim}{k}_0$ that is outward directed with respect to the surface within which the color field of the plasma is confined. Such quarks are assumed to be in thermal and chemical equilibrium. As the quark passes through the surface, a tube of chromoelectric flux is built up behind it, out of its kinetic energy. The energy per unit length that is stored in the tube is $\sigma = \varepsilon^2 A/2$, where ε and A are the field strength and cross section of the tube. Gauss' law relates the flux εA to the quark charge, g/2 through $\varepsilon A = g/2$, yielding

$$\sigma = g\varepsilon/4 \tag{1}$$

This string constant can also be related to the Regge slope and so is essentially a known parameter, $\sigma = 0.177\ (\mathrm{GeV})^2$. We assume that the flux tube that shields its color will connect it to the plasma by the shortest path. The motion of the quark will be governed by the equations expressing the conservation of energy and of momentum parallel to the surface. Choose the coordinate system so that the z-axis is normal to the surface, the origin is on the surface, and the motion of the quark is in the z-y plane. Then the conservation laws read

$$\left(\underset{\sim}{k}^2 + m^2\right)^{1/2} + E_s = E_o = \left(\underset{\sim}{k}_o^2 + m^2\right)^{1/2} \quad , \quad k_y + k_s = k_{yo} \tag{2}$$

where $\underset{\sim}{k}$ is the instantaneous quark momentum, k_y its component parallel to the surface, and m its mass. Its velocity in the y direction is

$$v_y = k_y/E \quad , \qquad E = \left(\underset{\sim}{k}^2 + m^2\right)^{1/2} \tag{3}$$

The "rest mass" of the string is σz where z is the coordinate of the emitted quark. So the energy and momentum of the string are

$$E_s = \sigma z\left(1 - v_y^2\right)^{-1/2} \quad , \qquad k_s = (\sigma z) v_y \left(1 - v_y^2\right)^{-1/2} \tag{4}$$

These equations, (2)-(4), govern the motion of the quark and can be solved analytically. In particular

$$\left.\begin{aligned} & v_x = 0 \quad , \quad v_y = v_{yo} = \text{const} \quad , \quad v_z = k_z/E \\ & k_z = k_{zo} - (E_{zo}/E_o)\sigma t \quad , \quad E_{zo} = \left(k_{zo}^2 + m^2\right)^{1/2} \\ & \sigma z = E_{zo} - \left(k_z^2 + m^2\right)^{1/2} \end{aligned}\right\} \tag{5}$$

and the instantaneous energy of the quark is

$$E = \left(\underset{\sim}{k}^2 + m^2\right)^{1/2} = E_o\left(1 - \sigma z/E_{zo}\right) \tag{6}$$

The motion of the quark normal to the surface is decelerated until it stops at time

$$t_c = (E_z/E_{zo})(k_{zo}/\sigma) \tag{7}$$

Thereafter it is accelerated back into the plasma.

The above equations describe the motion of a typical quark as it penetrates the surface of the plasma, provided the string or color tube does not break. The tube can break as the result of the creation of a quark-antiquark pair in the constant chromoelectric field ε inside the tube. If such a pair is created, say at a distance z' from the surface of the plasma and at a time t, a meson consisting of the original quark together with the antiquark of the created pair can evolve to a physical meson. Its momentum perpendicular to the surface will be that possessed by the leading quark at the time t, given by (5). Its energy will be E_0 less the energy carried back into the plasma by the fragment of string of length z' and the quark contained in it. Thus, the meson momentum and energy are

$$k_z^M = k_{zo} - (E_{zo}/E_0)\sigma t \quad , \quad E^M = E_0 - (\sigma z' + m)(E_0/E_{zo}) \tag{8}$$

where $\left(1 - v_y^2\right)^{1/2} = E_{zo}/E_0.$

Pair creation in a constant external field is similar to the QED process solved by Schwinger[4] and employed by Casher, Neuberger, and Nussinov[5] to describe particle production in high-energy e^+e^- annihilation. However, these authors neglect the interaction of the pair with each other. This is a serious neglect in QCD, where the chromoelectric flux created by the pair is of the same strength as the original field in which they were created. Suppose a virtual pair is created at some point in the tube, and suppose each component has oppositely directed transverse* momentum p_T. We wish first to calculate the probability that each component will tunnel from the virtual state to a real state having the same energy as the original tube. Initially, the longitudinal momentum of quark and antiquark is $p_L = iE_T = i\left(p_T^2 + m^2\right)^{1/2}$. As they move apart, their mutual interaction produces a field equal in magnitude but opposite in direction to the field in the tube, so that the field between the pair is cancelled. After they have each moved a distance r from the point of creation, the energy balance reads

$$\left(p_L^2(r) + p_T^2 + m^2\right)^{1/2} = g\varepsilon r/4 \tag{9}$$

where the right side is the energy that was contained in the destroyed field. Hence

$$p_L(r) = i\left(E_T^2 - (g\varepsilon r/4)^2\right)^{1/2} \tag{10}$$

The action of each quark, integrated from the creation point to the point where they materialize, given by $p_L(r) = 0$, is

*We use longitudinal to denote the orientation of the tube and transverse to denote an orthogonal direction.

$$S = \int_0^{4E_T/g\varepsilon} |p_L|\, dr = \frac{\pi E_T^2}{g\varepsilon} \tag{11}$$

The probability that a virtual pair can tunnel to a real state in the field of the tube, with each component having transverse momentum, p_T, is therefore

$$P(p_T) = \left|e^{-2S}\right|^2 = \exp(-4\pi E_T^2/g\varepsilon) \tag{12}$$

Our action, and hence the exponent in P, differs by a factor of 2 from the result of Schwinger[4] and of Casher et al.,[5] because we include the effect of the mutual interaction. It is worth emphasizing that it is the confinement of the flux to a tube that makes the calculation so simple. Effectively the quark charge is reduced by a factor 1/2. Knowing the tunneling probability, one can compute the vacuum persistence probability, the probability that no such tunneling event has occurred, in the manner of Casher et al.[5] This probability is evaluated as

$$|\langle 0_+|0_-\rangle|^2 = \exp\left(-\int p\, d^4x\right) \tag{13}$$

where p is given by

$$p = \sum_{\text{flavor}} \frac{g^2\varepsilon^2}{64\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{4\pi m_f^2 n}{g}\right) \tag{14}$$

and can be interpreted as the probability per unit four-volume that a quark pair will be created with any transverse momentum in the field ε. Again this differs from the earlier results and can be obtained from them by reducing the charge by 1/2. We retain in our calculations only the dominant contributions of the massless u,d quarks.

Having calculated the string dynamics and the probability per unit four-volume that a real pair will be formed and that hence the string will break, we can now calculate the probability that the string will break at the time t measured from the time that the outward-moving quark crosses the surface and in the interval dt. This is given by

$$dP(t) = pAz(t)\, dt\, [1 - P(t)] \tag{15}$$

where z(t) is the coordinate of the leading quark at time t and hence the length of the string. This integrates to

$$R(t) \equiv 1 - P(t) = \exp\left(-pA \int_0^t z(t)dt\right) \tag{16}$$

$$= \exp\left(-\frac{pA}{\sigma^2} E_o \left\{ k_{zo} - k_z + \frac{1}{2E_{zo}} \left[k_z E_z - k_{zo}E_{zo} + m^2 \ln\left(\frac{k_z+E_z}{k_{zo}+E_{zo}}\right)\right]\right\}\right)$$

$$\rightarrow \exp\left(-\frac{pA}{\sigma^2}\frac{k_o}{2k_{zo}}(k_z - k_{zo})^2\right) \quad , \quad (m \to 0) \tag{17}$$

Next we use this to calculate the probability that the string will break at the point z' at the time t. The probability is distributed uniformly along the string. Therefore,

$$d^2P(z',t) = pA\, dz'\, dt\, R(t)\, \theta(z' - z(t))\, \theta(-z')\, \theta(t - t_c)\, \theta(-t) \tag{18}$$

where θ is the step function $\theta(+) = 0$, $\theta(-) = 1$. Now we have all the ingredients required to calculate the radiated spectrum, radiation pressure, and surface brightness of the plasma. We fold d^2P with the flux of quarks of momentum $\underset{\sim}{k}_o$ and sum over all such momenta to obtain the number* of mesons radiated per unit time per unit surface area of the plasma having normal momentum k_z^M and energy E^M,

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*For non-zero chemical potential, multiply our result by cosh μ/T.

$$\frac{d^5N}{dS\ dt\ dk_z^M\ dE^M} = \frac{\gamma}{(2\pi)^3} \int d^3k_o\ e^{-E_o/T} \frac{k_{zo}}{E_o} \frac{pA}{\sigma^2} R(k_z^M)\ \theta(E_z - m - (E_{zo}/E_o)E^M)$$
$$\times\ \theta(m + (E_{zo}/E_o)E^M - E_{zo})\ \theta(-k_z^M)\ \theta(k_z^M - k_{zo}) \tag{19}$$

(We employ (8) to write $dz'dt = dk_z^M\ dE^M/\sigma^2$). The degeneracy factory γ is $\gamma = \gamma_C \times \gamma_S \times \gamma_F \times 2 = 24$ where 2 counts quarks and antiquarks. $R(k_z^M)$ means that (16) is evaluated at the time (8) that yields the momentum k_z^M.

The radiation pressure exerted on the plasma by the radiation of mesons, and the energy flow per unit surface area per unit time carried in the radiation follow immediately:

$$P^M = \frac{d^3k}{dS\ dt} = \int_0^\infty dk_z\ k_z \int_{E_\pi}^\infty dE^M \frac{d^5N}{dS\ dt\ dk_z\ dE^M} \tag{20}$$

$$\frac{d^3E}{dS\ dt} = \int_0^\infty dk_z \int_{E_\pi}^\infty dE^M\ E^M \frac{d^5N}{dS\ dt\ dk_z\ dE^M} \tag{21}$$

where $E_\pi = \sqrt{k_z^2 + m_\pi^2}$

The above formulae characterize the hadronization at the surface of a plasma when confinement is described by the chromoelectric flux tube model. We can compare the above with the case of vanishingly small confinement through

$$\frac{d^5N}{dS\ dt\ dk_z^M\ dE^M} = \frac{\gamma}{(2\pi)^3} \int d^3k_o\ e^{-E_o/T} \frac{k_{zo}}{E_o}\ \delta(k_{zo} - k_z^M)\ \delta(E_o - E^M) \tag{22}$$

Our main result is exhibited in Fig. 1, which, as a function of temperature, shows the ratio of radiation pressure and internal quark pressure. Since this ratio is less than 20% up to T = 500 MeV, we conclude that the radiation due to this process will be a minor perturbation on the collective hydrodynamical disassembly of the plasma. If the gluon pressure is included, then the internal pressure is increased by (16 + 24)/24 for the Boltzmann statistics or by (16 + 21)/21 for the Fermi and Bose statistics, which further reduces the role of the radiation pressure.

In summary, the hadronization at the surface of a quark-gluon plasma has been studied in the framework of a chromoelectric flux tube model. As a quark passes through the boundary of the region in which the color field of the plasma is confined, a tube of chromoelectric flux connects it to the plasma. We have solved the equations of motion of such a tube. It can fission as a result of pair creation. This is the mechanism for hadronization. We have obtained a new result for q-$\bar{q}$ pair creation in a uniform chromoelectric field that includes the effect of the mutual interaction of the pair. This can be folded with the tube dynamics and the thermal distribution of quarks in the plasma, to calculate the radiation pressure and spectrum of emitted mesons. We find that the radiation pressure is sufficiently small compared to the internal pressure that it can be ignored to first approximation in the disassembly of the plasma. Thus our solution for meson radiation can be folded with a solution to the hydrodynamical expansion to obtain the spectrum of radiated mesons emitted over the history of the expansion. Of special interest is the distinction between strange and non-strange mesons, which is explicit in the theory through the dependences on the quark masses and the thermal populations in the plasma.

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Figure Caption

Fig. 1. Pressure acting on the surface of the quark-gluon plasma due to the radiation of mesons as a function of plasma temperature.

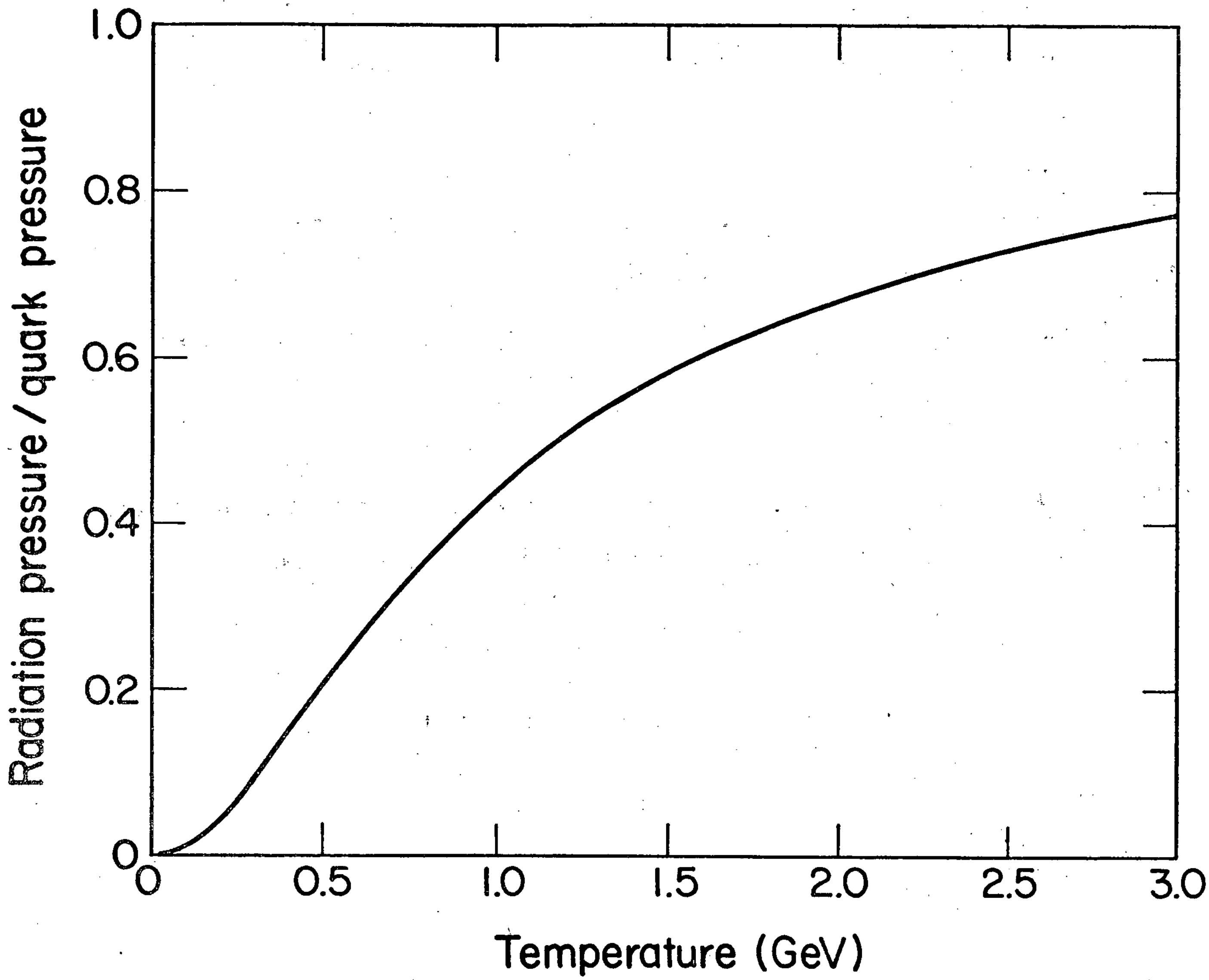
1.0
0.8
0.6
0.4
0.2
0
Radiation pressure / quark pressure
0
0.5
1.0
1.5
2.0
2.5
3.0
Temperature (GeV)

Fig. 1