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September 1964

ENERGIES OF ISOBARIC MULTIPLETS IN A=16 AND 20\*

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September 1964

If the specifically nuclear part of nuclear interactions is charge independent, then the energies of the various members of an isobaric multiplet will differ only because of the neutron-proton mass difference and the electromagnetic interactions among the protons. These electromagnetic contributions are calculable<sup>1</sup> and could be removed; the remaining masses of the multiplets thus could be examined with regard to charge independence of nuclear forces. These calculations however are somewhat model dependent, but to first order in the Coulomb energy it has been shown,<sup>2</sup> quite generally, that the masses within an isobaric multiplet are characterized by

$$M(A, T, T_z) = a(A, T) + b(A, T)T_z + c(A, T)T_z^2 \quad (1)$$

where  $M$  is the mass of a member of the multiplet,  $A$  is the number of nucleons,  $T$  is the isobaric spin and  $T_z = (n-p)/2$ ;  $a(A, T)$ ,  $b(A, T)$  and  $c(A, T)$  are taken as constants within a given multiplet. The adequacy of this formula has never been tested because at most three members of a multiplet are known, so that no verifiable predictions can be made using Eq. (1).

Recent (p,t) experiments<sup>3</sup> however, have been able to find a  $T=2$  level in certain  $T_z=0$  nuclei. These  $T=2$  levels are the isobaric analogs of the ground states of the  $T=2$ ,  $T_z=2$  isobars. For example a  $T=2$  level is found in

$Mg^{24}$  that is the analog of the ground state of  $Ne^{24}$ . In the mass 24 system the three lowest lying  $T=1$  levels with  $T_z=0, \pm 1$  are also known. In a recent paper<sup>4</sup> D. H. Wilkinson suggested that one assume the coefficients  $b(A,T)$  and  $c(A,T)$  within the same  $A$  to be  $T$  independent. With this assumption, using the levels mentioned above he was able to show that the resulting prediction for the mass of the ground state of  $Al^{24}$  is in agreement with the observed mass,<sup>5</sup> though the experimental uncertainties are large.

We have recently completed a study<sup>6</sup> of  $(p,t)$  and  $(p,He^3)$  reactions on  $O^{18}$  and  $Ne^{22}$ . These reactions on  $O^{18}$  allowed us to locate the analog of the ground state of  $C^{16}$  in  $N^{16}$  ( $9.91 \pm 0.1$  MeV) and in  $O^{16}$  ( $22.9 \pm 0.1$  MeV). Similarly with the  $Ne^{22}$  target the analogs of the  $O^{20}$  ground state were located in  $F^{20}$  ( $6.43 \pm 0.1$  MeV) and in  $Ne^{20}$  ( $16.8 \pm 0.1$  MeV). Thus in each case we have 3 members of a  $T=2$  isobaric multiplet and in each case a set of  $T=1$  isobars is also known.<sup>7,9</sup> Therefore the coefficients  $b(A,1)$ ,  $c(A,1)$ ,  $b(A,2)$ , and  $c(A,2)$  can be determined from the data and compared for  $A=16$  and 20. Table I shows the values obtained. Two values are given for the  $T=1$ ,  $A=16$  multiplet because the spin of the ground state of  $F^{16}$  is not known<sup>9</sup> with certainty. The ground state of  $N^{16}$  is  $2-$ , but in  $O^{16}$  the lowest lying  $T=1$  state is  $0-$ . This inversion is probably due to the Thomas-Ehrman effect<sup>8</sup> which would be most pronounced for the  $s$  state proton. Thus it would seem that the ground state of  $F^{16}$  is also  $0-$  but values for the coefficients assuming it to be  $2-$  are also included. From the values given in Table I it would seem that the assumption suggested by Wilkinson is not generally valid.

It is however instructive to take a more detailed look at the factors which bring about this disagreement between the coefficients. In each case it seems to be the position of the  $T=1$ ,  $T_z=-1$  member of the multiplet. Using the above assumption (that  $b(A,1) = b(A,2)$  and  $c(A,1) = c(A,2)$ ) a prediction can

be made which does not involve the mass of the  $T=1, T_z=-1$  member.

$$E(A,2,1) = E(A,2,0) - E(A,1,0) \quad (2)$$

where  $E$  stands for the excitation energy above the respective ground states and the terms in the parentheses have the same meaning as before. The results of this prediction are shown in Table II and show much better agreement than the coefficients in Table I. Furthermore, to demonstrate that the  $T=1, T_z=-1$  levels are mainly responsible for the disagreement between the two sets of coefficients, one can use  $b(A,2)$  and  $c(A,2)$  to predict the mass differences in the  $T=1$  multiplets. The resulting values obtained for  $N^{16}-O^{16}$  and  $F^{20}-Ne^{20}$  are in excellent agreement with experiment whereas the value obtained for  $F^{16}-O^{16} = 16.07 \pm 0.27$  MeV (experimentally  $15.43 \pm 0.05$  MeV is obtained) and for  $Na^{20}-Ne^{20} = 14.33 \pm 0.27$  (reference 5 gives  $15.3 \pm 0.3$ ).<sup>\*</sup> The lower value observed for the fluorine-oxygen mass difference is most certainly due to the Thomas-Ehrman shift as the  $F^{16}$  ground state is unbound to particle decay.

In mass 20 a different situation holds: If the mass of  $Na^{20}$  is calculated by adding to the lowest  $T=1$  level in  $Ne^{20}$  the Coulomb energy difference obtained from  $Na^{21}-Ne^{21}$ , adjusted for the radius change, one obtains  $Na^{20}-Ne^{20} = 13.86$  MeV, which is in substantial disagreement with both the prediction in the paragraph above and the value currently accepted for this mass difference.<sup>5</sup> Considering the magnitude of the disagreement, the current value for the mass of  $Na^{20}$  is probably too high by at least 1 MeV. It should be noted that it has only been reported once,<sup>10</sup> and therefore certainly should be remeasured.

<sup>\*</sup>We believe the best number at present to be  $Na^{20}-Ne^{20} = 15.0 \pm 0.3$  based on a recent remeasurement of the mass of  $N^{12}$  (B.A.P.S. 8, 598 (1963)) on which the  $Na^{20}$  mass determination was based.<sup>10</sup>

It appears that one will certainly need an isobaric quartet to test Eq. (1) to a reasonable degree of exactness. Using this equation, which is presumably good within a given multiplet, we predict  $Mg^{20}-Ne^{20} = 25.6 \pm 0.7$  MeV and  $Ne^{16}-O^{16} = 30.2 \pm 0.7$  MeV.

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FOOTNOTES AND REFERENCES

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Table I. Table comparing the coefficients of Eq. (1) for A=16 and 20 in the T=1 and 2 multiplets. The two values shown for the A=16, T=1 case use the levels with the spin-parity listed before the set of coefficients.

	T=2	T=1
A=16	$b = -2.93 \pm 0.25 \text{ MeV}$ $c = 0.35 \pm 0.12 \text{ MeV}$	(2-) $b = -2.61 \pm 0.03 \text{ MeV}$ $c = 0.05 \pm 0.025 \text{ MeV}$  (0-) $b = 2.45 \pm 0.03 \text{ MeV}$ $c = 0.19 \pm 0.025 \text{ MeV}$
A=20	$b = -3.69 \pm 0.25 \text{ MeV}$ $c = 0.35 \pm 0.12 \text{ MeV}$	$b = -4.15 \pm 0.15 \text{ MeV}$ $c = 0.91 \pm 0.15 \text{ MeV}$

Table II. Table showing the results obtained using Eq. (2). The 2- states are used in the A=16 case as they are not shifted by the Thomas-Ehrman effect as severely as the 0- states.

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	$E(A,2,1)$	$E(A,2,0)-E(A,1,0)$
A=16	$9.91 \pm 0.1$ MeV	$9.93 \pm 0.1$
A=20	$6.43 \pm 0.1$	$6.53 \pm 0.1$

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