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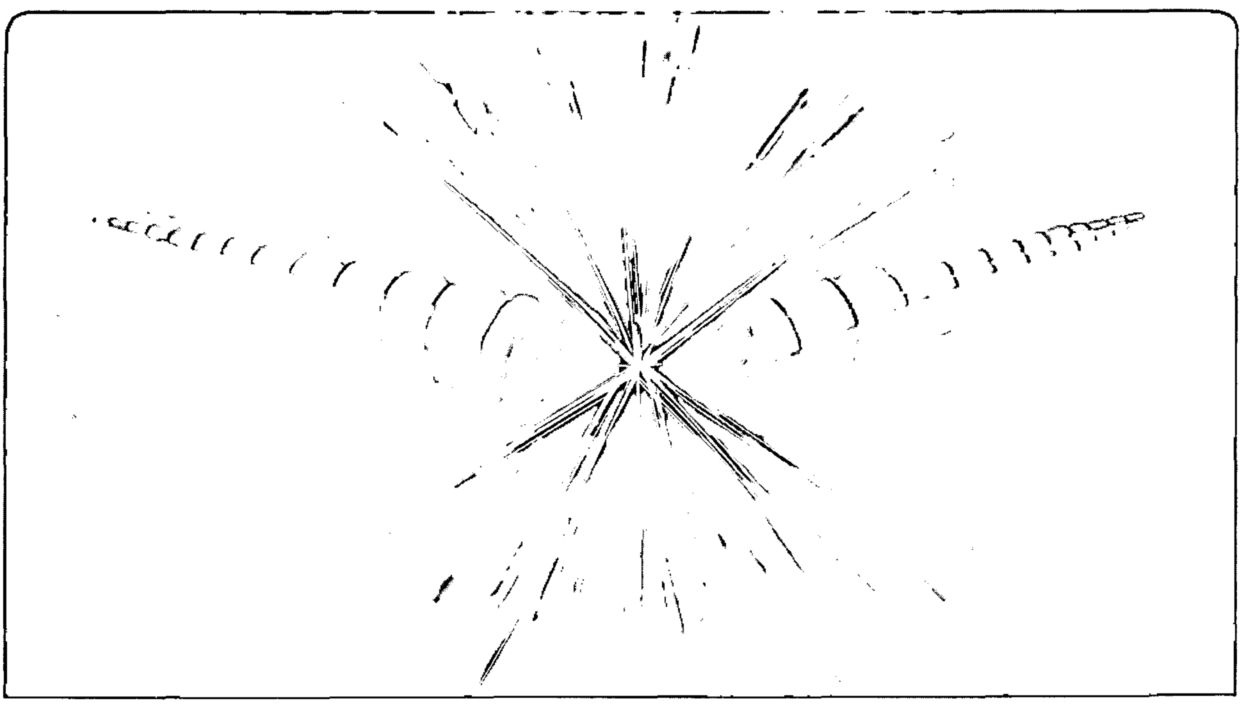

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"Investigation of Focusing of Relativistic Electron and Positron Bunches Moving in Cold Plasma"

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FINAL REPORT

ON THE WORK "INVESTIGATION OF FOCUSING OF RELATIVISTIC ELECTRON AND POSITRON BUNCHES MOVING IN COLD PLASMA"

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a) First stage - "FOCUSING IN DIFFERENT MODELS OF COLD OVERDENSE PLASMA-ELECTRON BUNCH SYSTEM".

Focusing of relativistic electron (positron) bunches is considered in three different descriptions of cold overdense plasma-rigid electron bunch system.

In all three models Coulomb component of field exists. For large values of the bunch Lorentz-factor it is negligible in comparison with the wake field component.

Total charge and current densities in general are not compensated. For narrow bunches they are nearly proportional to each other. The resulting focusing force is a complex combination of magnetic and electric forces, which relative strength depends on bunch parameters.

The obtained results in case of narrow bunches are practically independent from the considered models.

The general formulae for focusing force are obtained, which can be used for estimates in planned experiments.

Particular cases of narrow, short and long bunches discussed and focusing gradient calculated for the experiments performed at ANL, Tokyo University-KEK and UCLA.

Results of these stage were presented to LBL in Interim Report on March 1994, in talk on 6th Workshop on Advanced Accelerator Concepts, Abbey on Lake Geneva, Wisconсовnsin, June 1994, and also presented for publication in "Particle Accelerators".

b) Second stage - "INVESTIGATION OF THE FOCUSING AND WAKE FIELDS OF THE SEQUENCE OF RELATIVISTIC ELECTRON AND POSITRON BUNCHES MOVING IN COLD PLASMA".

We considered the focusing of the bunch of electrons (positrons) moving in wake field generated by sequence of N-1 cylindrical bunches with uniform distribution of the charge with the densities n_{bk} $(k=1,2,\dots N-1)$, lengths d_k, radius a and the distances in between l_k. We used the linear approximation when the condition $n_{bk}/n_0 \ll 1$ is fulfiled. (n_0 is the density of plasma electrons in equilibrium, plasma is neutral, ions are immobile).

1. THE GENERAL SOLUTION OF THE LINEAR EQUATIONS.

The rigid bunch of electrons with the density $n_b(r,z)$, $z=Z-V_0t$ is moving in cold neutral plasma with the velocity $\vec{V_0}(0,0,V_0)$. In linear approximation the components of the momenta of plasma electrons $\vec{P}(P_r,0,P_z)$ obeyed the system of equations, formulated in [1-3].

$$\frac{\partial^2 P_z}{\partial z\,\partial r}-\frac{1}{\gamma^2}\frac{\partial^2 P_r}{\partial z^2}+k_p^2\beta^2 P_r=0\,, \tag{1}$$

$$\frac{\partial}{\partial z}\left[\frac{1}{r}\frac{\partial}{\partial r}(rP_r)\right]-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial P_z}{\partial r}\right)+\beta^2\frac{\partial^2 P_z}{\partial z^2}+k_p^2\beta^2 P_z=$$
$$=-k_p^2\beta^2 m V_0 n_b(r,z)/n_0\,,$$
$$k_p=\omega_p/V_0\,,\quad \omega_p^2=4\pi n e^2/m\,,\quad \beta=V_0/c\,,\quad \gamma=(1-\beta^2)^{-1/2}.$$

Introducing in (1) the Hankel transformations:

$$L(\xi,z)=\int_0^\infty P_z(r,z)\,r\,J_0(r\xi)\,dr\,,\quad P_z(r,z)=\int_0^\infty L(\xi,z)\,\xi\,J_0(r\xi)\,d\xi\,,$$

$$T(\xi,z)=\int_0^\infty P_r(r,z)\,r\,J_1(r\xi)\,dr\,,\quad P_r(r,z)=\int_0^\infty T(\xi,z)\,\xi\,J_1(r\xi)\,d\xi\,, \tag{2}$$

where $J_0(r\xi)$ and $J_1(r\xi)$ are the Bessel function of the zero and first order, the system (1) become

$$-\xi\frac{\partial L}{\partial z}-\frac{1}{\gamma^2}\frac{\partial^2 T}{\partial z^2}+k_p^2\beta^2 T=0,$$

$$\xi\frac{\partial T}{\partial z}+\xi^2 L+\beta^2\frac{\partial^2 L}{\partial z^2}+k_p^2\beta^2 L=-k_p^2\beta^2 m V_0 A(\xi,z), \tag{3}$$

$$A(\xi,z)=\int_0^\infty \frac{n_b(r,z)}{n_0}\,r\,J_0(\xi r)\,dr.$$

Excluding the function $T(\xi,z)$ from system (3) it is possible to obtain the following eq. for $L(\xi,z)$

$$\frac{\partial^4 L}{\partial z^4}+(k_p^2-\gamma^2\varkappa^2)\frac{\partial^2 L}{\partial z^2}-k^2\gamma^2\varkappa^2 L=$$

$$=k_p^2 m V_0\left(k_p^2\beta^2\gamma^2 A-\frac{\partial^2 A}{\partial z^2}\right), \tag{4}$$

where $\varkappa^2=k_p^2\beta^2+\xi^2$.

Solving the eq. (4) by Gteen function method for finite solution with the boundary condition $L(z\to+\infty)\to 0$ from (2) we get

$$P_z = k_p^2 m V_0 \int_{-\infty}^{\infty}\int_0^{\infty} \frac{A(\xi, z')}{k_p^2+\xi^2}\,\xi J_0(r\xi)\Big[-k_p \sin k_p(z'-z)\,\Theta(z-z') +$$

$$+ \frac{\xi^2}{2\gamma\varkappa}\, e^{-\gamma\varkappa|z'-z|}\Big] d\xi\, dz', \qquad (5)$$

$$\frac{\partial P_r}{\partial z} = -k_p^2 m V_0 \beta^2 \int_0^{\infty} A(\xi, z) J_1(\xi r)\, d\xi - k_p^2 m V_0 \int_{-\infty}^{\infty}\int_0^{\infty} \frac{A(\xi, z')}{k_p^2+\xi^2}\cdot$$

$$\cdot \xi^2 J_1(\xi r)\Big[-k_p \sin k_p(z'-z)\,\Theta(z-z') + \frac{\gamma\varkappa}{2}\, e^{-\gamma\varkappa|z'-z|}\Big] d\xi\, dz',$$

where

$$\Theta(z-z') = \begin{cases} 0, & z \geq z' \\ 1, & z < z'. \end{cases}$$

The components of electromagnetic fields and force, acting on electron bunch are defined through (5) by formulae

$$E_z = \frac{V_0}{e}\frac{\partial P_z}{\partial z}, \quad E_r = \frac{V_0}{e}\frac{\partial P_r}{\partial z}, \quad B_\theta = \frac{c}{e}\left(\frac{\partial P_r}{\partial z} - \frac{\partial P_z}{\partial r}\right),$$

$$F_r = -e\,(E_r - \beta B_\theta) = -V_0 \frac{\partial P_z}{\partial r}, \qquad (6)$$

$$F_z = -e E_z = -V_0 \frac{\partial P_z}{\partial z}.$$

2. SELF-FOCUSING OF THE SINGLE BUNCH.

For completness and comparison with the results obtained for the sequence of driving bunches, let us present here the expression for focusing force in case of the single uniform electron bunch, when

$$n_b(r,z)=\begin{cases} n_b, & z\in[-d,0], \ r\leq a,\\ 0, & z\notin[-d,0], \ r>a; \end{cases}$$

$$A=\frac{n_b}{n_0}\frac{a}{\xi}J_1(a\xi),$$

and from (5-6) focusing force is

$$F_r=\omega_p^2 m a\frac{n_b}{n_0}\left[-g(z)\int_0^\infty\frac{\xi}{k_p^2+\xi^2}J_1(a\xi)J_1(r\xi)d\xi+\right. \tag{7}$$

$$\left.+\frac{1}{2\gamma^2}\int_0^\infty\xi^3\frac{J_1(a\xi)J_1(r\xi)}{(k_p^2+\xi^2)(k_p^2\beta^2+\xi^2)}\cdot h(z,\xi)d\xi\right],$$

where

$$g(z)=\begin{cases} 0, \quad z>0,\\ 1-\cos k_p z, \quad -d\leq z\leq 0,\\ -2\sin k_p\frac{d}{2}\cdot\sin k_p\left(z+\frac{d}{2}\right), \quad z<-d; \end{cases}$$

$$h(z,\xi)=\begin{cases} e^{-\gamma\varkappa z}-e^{-\gamma\varkappa(z+d)}, & z>0,\\ 2-e^{\gamma\varkappa z}-e^{-\gamma\varkappa(z+d)}, & -d\leq z\leq 0,\\ e^{\gamma\varkappa(z+d)}-e^{\gamma\varkappa z}, & z<-d. \end{cases}$$

Consider the case when $\exp(-\gamma\beta k_p d/2)\ll 1$ we have $h(z,\xi)\simeq 2$ in the middle of the bunch and $h(z,\xi)\simeq 1$ at the edges of the bunch (see fig.1). Using this approximation and performing the integration in (7) for $r\leq a$ we obtain

$$F_r=\omega_p^2 m\frac{n_b}{n_0}a\left\{-g(z)I_1(k_p r)K_1(k_p a)+\binom{1}{1/2}\left[I_1(k_p r)K_1(k_p a)-\right.\right.$$

$$\left.\left.-\beta^2 I_1(k_p\beta r)K_1(k_p\beta a)\right]\right\}, \tag{8}$$

where I_1 and K_1 are the modified Bessel functions. In (8) the factors $\binom{1}{1/2}$ are taken for the middle of the bunch and for the edges consequent ly. For the case of narrow bunch when focusing force is linear on r, we have

$$F_r = -\omega_p^2 m \frac{n_b}{n_0} \frac{r}{2} \left[1 - \cos k_p z - \binom{1}{1/2} \frac{1}{\gamma^2}\right]. \qquad (9)$$

As it is evident from (9) for the region of the bunch, where

$$|z| \leqslant d_* = \frac{\lambda_p}{2\pi} \arccos \frac{1+\beta^2}{2}, \qquad (10)$$

the force is defocusing, $F_r \geqslant 0$ (see fig. 2) due to the presence of the Coulomb field, so self-focusing is absent for the short bunches $d \lesssim d_*$. From (9) follows that Coulomb component of the force, proportional to γ^{-2}, in ultrarelativistic limit is small, and can be neglegted, as it was done, for example, in the primary work [4].

3. FOCUSING IN THE WAKE FIELD OF THE TRAIN OF BUNCHES.

For the chain of cylindrical bunches with the radius a

$$n_b(r \leqslant a, z) = \begin{cases} n_{bk}, & -\left(d_k + \sum\limits_{s=1}^{k-1} L_s\right) \leqslant z \leqslant -\sum\limits_{s=1}^{k-1} L_s, \\ 0, & -\sum\limits_{s=1}^{k} L_s \leqslant z \leqslant -\left(d_k + \sum\limits_{s=1}^{k-1} L_s\right), \end{cases} \qquad (11)$$

$$L_k = d_k + \ell_k,$$

where $n_{\beta k}$ -the electron density of k^{th} bunch, d_k -the length of k^{th} bunch, ℓ_k is the distance between k^{th} and $(k+1)^{th}$ bunches. Radial force, acting on electrons moving behind the chain of N-1 bunches with the velocity V_0 from (5-6) is

$$F_r^{N-1} = \omega_p^2 m \frac{a}{n_0}\Big[-I_1(k_p r)\cdot K_1(k_p a)\cdot G(z) + \qquad (12)$$

$$+\frac{1}{2}\int_0^\infty\Big(\frac{1}{k_p^2+\xi^2} - \frac{\beta^2}{k_p^2\beta^2+\xi^2}\Big)\xi J_1(a\xi)\cdot J_1(r\xi)\cdot H(z,\xi)d\xi\Big],$$

where

$$G(z)=\sum_{k=1}^{N-1} n_{\beta k}\, g_k(z)\ , \quad H(z,\xi)=\sum_{k=1}^{N-1} n_{\beta k}\, h_k(z,\xi),$$

$$g_k(z) = -2\sin\frac{k_p d_k}{2}\sin k_p\Big(z+\frac{d_k}{2}+\sum_{s=1}^{k-1} L_s\Big),$$

$$h_k(z,\xi) = e^{\gamma\varkappa(z+d_k+\sum_{s=1}^{k-1}L_s)} - e^{\gamma\varkappa(z+\sum_{s=1}^{k-1}L_s)}.$$

When N^{th} bunch, which must be focused, is located on such a distance that $\exp(-\gamma\beta k_p \ell_N)\ll 1$ the second term in (12) contained integrals is negligible and for the simple case, when $n_{\beta k}=n_\beta$, $d_k=d$, $\ell_k=\ell$ from (12) follows

$$F_r^{N-1} = \omega_p^2 m \frac{n_\beta}{n_0} a I_1(k_p r) K_1(k_p a)\cdot 2\sin\frac{k_p d}{2}\cdot \qquad (13)$$

$$\cdot\frac{\sin k_p(N-1)\frac{\ell+d}{2}}{\sin k_p\frac{\ell+d}{2}}\sin k_p\Big[z+\frac{d}{2}+\frac{N-2}{2}(\ell+d)\Big].$$

From (13) it is evident that focusing force reached its maximum value when

$$\frac{\ell + d}{2} = n_1 \lambda_p , \qquad d = \frac{\lambda_p}{2} + n_2 \lambda_p \tag{14}$$

where $n_1, n_2 = 1, 2, \dots$

For the case of narrow bunches, $k_p a \ll 1$, when conditions (14) are fulfilled, from (13) follows (see fig. 3):

$$\begin{aligned} F_r^{N-1} &= \omega_p^2 m \frac{n_b}{n_0} r (N-1) \cos k_p z = \\ &= 4\pi n_b e^2 r (N-1) \cos k_p z , \end{aligned} \tag{15}$$

so the focusing force is enhanced by a factor N-1. It is necessary to add to (15) self focusing force of the single bunch, given by (9), which for large enough N can be neglected.

By the same way it is possible to obtain the longitudinal force acting on the Nth bunch which follows the chain of N-1 bunches. Neglecting the Coulomb component from $F_z = - V_0 \, \partial P_z / \partial z$ and (5) we obtain

$$F_z^{N-1} = m v_0^2 \frac{n_B}{n_0} \left[1 - k_p a K_1(k_p a) I_0(k_p r)\right] \partial G(z)/\partial z. \qquad (16)$$

In case of narrow bunches which are suitable for transversal focusing (16) have the following form

$$F_z^{N-1} = 4\pi n_B e^2 (N-1) k_p a^2 \left[\ln\left(\frac{2}{k_p a \varepsilon}\right) + \left(1 - \frac{r^2}{a^2}\right)\right] \sin k_p z, \quad (17)$$

where $\ln \varepsilon \simeq 0{,}577$ —is the Euler constant.
The focusing bunch must be placed in the middle of the region with the length $\lambda_p/2$, located on the distance $3\lambda_p/4 + n\lambda_p$ $(n = 0, 1, 2, \ldots)$ from the end of the bunches chain (see fig. 3). As it is seen from (17) and fig 3. at this case the longitudinal contraction of the Nth bunch is present, which however is weaker than than transwearsal one. It is also nessery to remember that all this results are valid only in the linear approximation. This assumption puts certain restriction on the number of driving bunches in the chain

$$N n_B / n_0 \ll 1.$$

This condition is restrictive enough and in order to avoid it, we must consider the nonlinear effects.

It is interesting to mention here also that when $d_k = n_k \lambda_p$ $(n_k = 1, 2, \ldots)$ the wake fields are absent.

c) IN ADDITION TO STAGES a) AND b) DESCRIBED ABOVE THE ATTEMPTS HAVE BEEN MADE TO ELABORATE SUITABLE TECHNIQUE FOR TREATMENT THE NONLINEAR EFFECTS IN PLASMA FOCUSING AND WAKE FIELD GENERATION.

In particular, the analytical treatment of the case of underdense plasma was undertaken.

It was also noticed that in all three models of cold plasma-rigid electron bunch system, considered in [3] in linear approximation the wake field component of magnetic field is zero. The physical reason for this effect is the absence of energy flow in wake fields, (see also results of work [5]).

If magnetic field is zero, in non linear approach too, electric field is potential.

If plasma electrons are nonrelativistic (which also means that we neglect a significant part of nonlinearities, (caused by the relativism) and driving bunch is flat with the horizontal dimension much larger than vertical one, it is possible to show that potential function $\varphi(y,\ z = Z - V_0 t)$ obeys the following nonlinear equation:

$$\frac{\partial^2 \varphi}{\partial z^2} + \frac{1}{\beta^2} \varphi \frac{\partial^2 \varphi}{\partial y^2} = 0 \qquad (1)$$

(The Breizman-Tajima-Fisher-Chebotaev transformation of variables is used [6,2-3]).

Exact solution of this eq. with separating arguments is the following

$$\begin{aligned} \varphi(y, z) &= \varphi_1(y)\varphi_2(z), \\ \varphi_1(y) &= \frac{\varkappa y^2}{2} + C_1\ , \end{aligned} \qquad (2)$$

where $\varkappa$ is the separation constant and $\varphi_2(z)$ must be found from equation:

$$\varphi_2'' + \frac{\varkappa}{\beta^2}\varphi_2^2 = 0. \qquad (3)$$

First integral of eq. (3) is

$$\varphi_2' + \frac{2\varkappa}{3\beta^2}\varphi_2^3 = C_2. \qquad (4)$$

The constant C_2 is connected with the boundary conditions: $\varphi_2(-d) = \varphi_0$, $\varphi_2'(-d) = \varphi_0'$,where -d is the position of the rare end of the driving bunch,

$$C_2 \equiv \varphi_0' + \frac{2\varkappa}{3\beta^2}\varphi_0^3.$$

If $\varkappa > 0$ and $C_2 > 0$, the implicit solution of eq. (3) is

$$z + d = \frac{3^{1/12}}{C_2^{1/6}}\left(\frac{\beta^2}{2\varkappa}\right)^{1/3}\left[F(\psi_0, \alpha) - F(\psi, \alpha)\right], \qquad (5)$$

where

$$\alpha = 75^\circ, \qquad \sin\alpha = \frac{1}{2}\sqrt{2+\sqrt{3}},$$

$$\cos\psi = \frac{\sqrt{3}-1+\left(\frac{2\varkappa}{3\beta^2 C_2}\right)^{1/3}\varphi}{\sqrt{3}+1-\left(\frac{2\varkappa}{3\beta^2 C_2}\right)^{1/3}\varphi};$$

and F is an elliptic integral of the first order. Solutions (2),(5) (3) represent the nonlinear two dimensional wake field in the vicinity of the bunch symmetry plane y=0. ("Paraplanar" solution; it means that separation of variables is possible only in this region of space).

Further investigations of this particular solution as well as other solutions of eq. (1) will be the subject of the future work. The possibility of taken into account the relativism of the plasma electrons also will be discussed.

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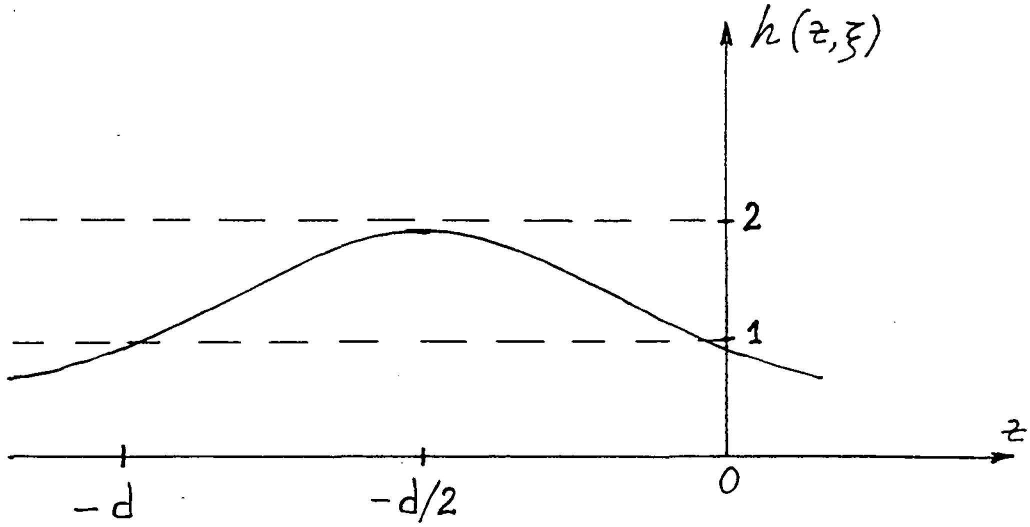

Fig. 1

Function $h(z,\xi)$.

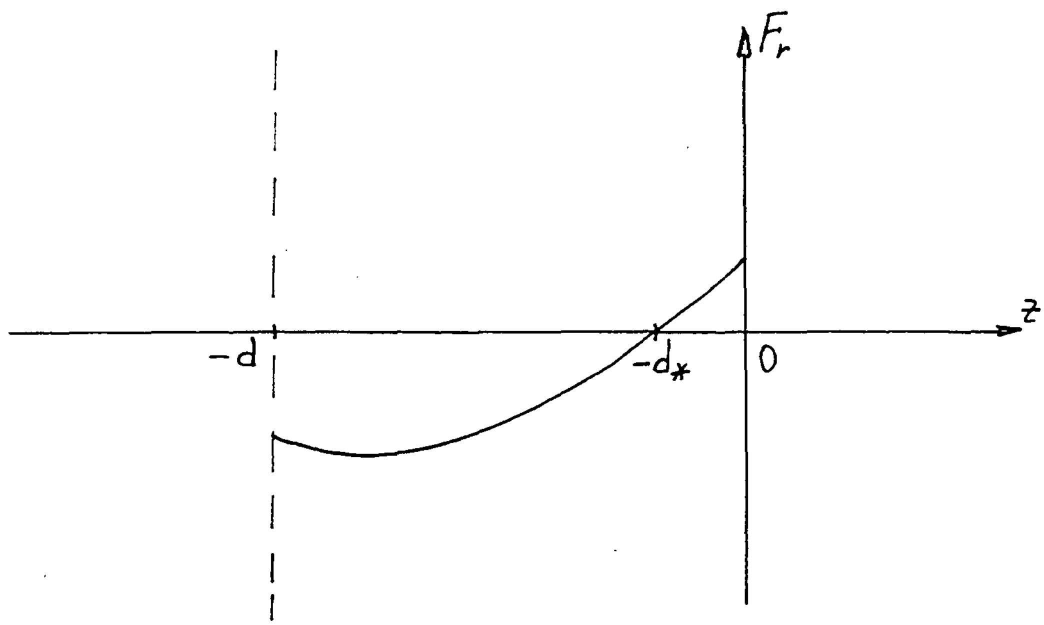

Fig. 2

Focusing force for single bunch.

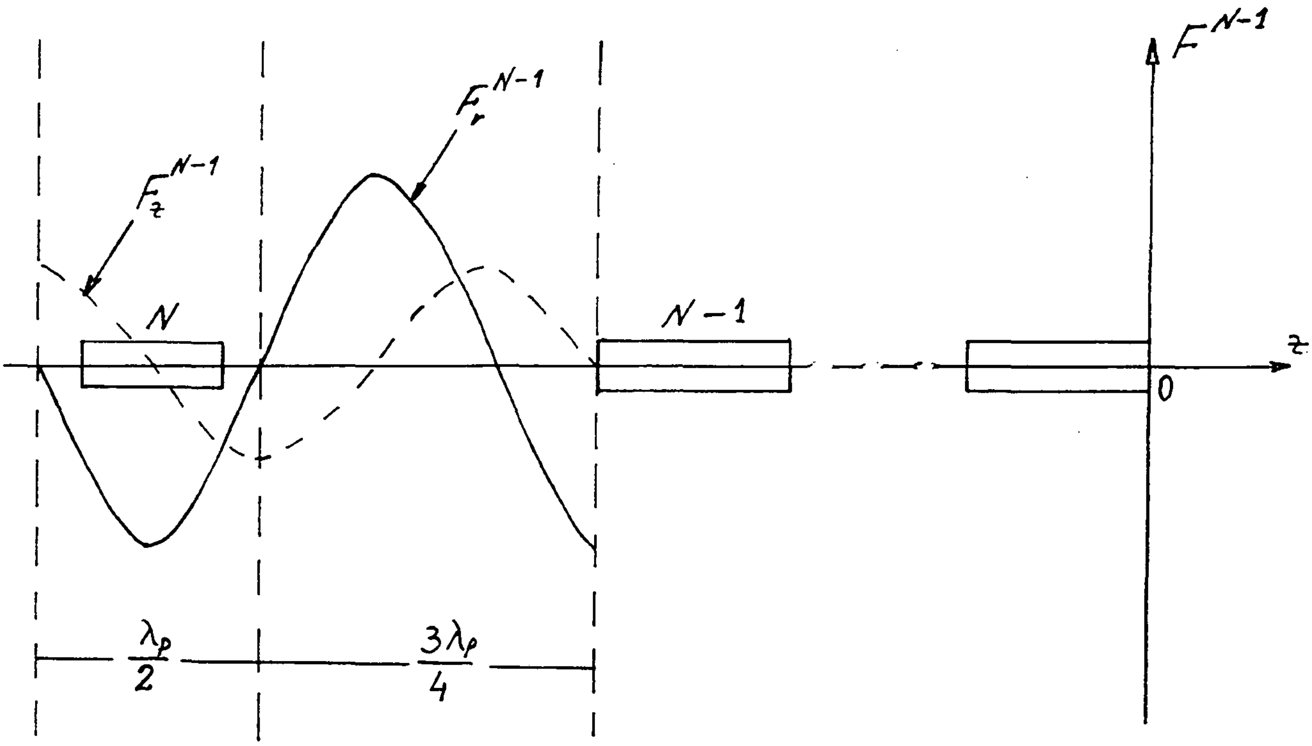

Fig. 3

Forcies acting on Nth bunch,

following the chain of N-1 bunches.

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