Radial Basis Leaky Competing Accumulator Model: A Biologically Plausible Framework for Decision-Making in a Continuous Option Space

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Abstract

In many real-life situations, we make decisions between a defined set of options, which can be either discrete (as when deciding between going on driving and stopping the car) or continuous (as when stirring the wheel, the possible range of angles goes from -30 to 30 degrees). However, most computational models for decision-making focus on decisions between a discrete set of options. While there are a few sequential sampling models that can explain behavioral patterns (i.e., choices and response times) of decisions in a continuous option space (i.e., the CDM and the SCDM), these models have a few limitations. For example, these models assume no leakage in the evidence accumulation process and no spatial inhibition (i.e., inhibition among different areas of the option space depending on their distance to each other). In this paper, we propose a novel sequential sampling model based on an existing computational model (i.e., the leaky competing accumulator model) for decisions in a continuous option space. Our proposed model includes leakage and spatial inhibition and is thus more biologically plausible.

Keywords: Continuous Decision Task; Sequential Sampling; Cognitive Modeling; Leaky Competing Accumulator; Radial Basis function;

Introduction

Decision-making is the process of selecting one among different, available options. Until recently, most decision-making research focused on decisions in a discrete option space (e.g., choosing between different gambles, discriminating between perceptual stimuli, choosing between altruistic vs. egoistic options). This includes many well known paradigms, such as the Go/No-Go task (Gomez, Ratcliff, & Perea, 2007; Ratcliff, Huang-Pollock, & McKoon, 2018), n-alternative forced-choice tasks (Bogacz, Brown, Moehlis, Holmes, & Cohen, 2006; van Ravenzwaaij, Brown, Marley, & Heathcote, 2020), the accept/reject task (Zhao, Walasek, & Bhatia, 2020; Mallahi-Karai & Diederich, 2019, 2021). Although these kinds of decisions have helped better understanding the cognitive and neural bases of decision-making (Bogacz, 2007; Forstmann, Ratcliff, & Wagenmakers, 2016), they exclude many situations in our daily life in which we are instead confronted with a range of options on a continuous scale (e.g., when setting the price of an item we are planning to sell (Kvam & Busemeyer, 2020)). Moreover, there are many laboratory tasks with continuous scale report stimuli in different areas such as visual working memory (Lilburn, Smith, & Sewell, 2019) and perceptual decision-making (Ratcliff & McKoon, 2020). For more applications of decisions in continuous space, interested readers can see (Yoo, Hayden, & Pearson, 2021).

Recently, a few sequential sampling models (Smith, 2016; Ratcliff, 2018) have been developed to handle both choices and response time in decisions between a continuous set of options. The circular diffusion model (CDM) (Smith, 2016) assumes that the process of evidence accumulation progresses following a Brownian motion within a circle or semi-circle and it terminates whenever the accumulator reaches any point on the perimeter (and, thus, a decision is made). This model has been used for modeling visual memory tasks with a continuous reporting scale (e.g., remembering the color of a previously presented stimulus and selecting it on a color wheel). While this model is good at capturing different behavioral phenomena, such as the color bias (Smith, Saber, Corbett, & Lilburn, 2020), it has a few limitations. For example, the CDM cannot capture the heavy-tailed response error distribution. Moreover, the original CDM can only be used to model choices in a one-dimensional (1D) option space and not in two- or three-dimensional (2D or 3D) option spaces. But there are some extended versions of CDM which is called hyper-spherical diffusion model (Smith & Corbett, 2019) and it can be applied for higher dimensional spaces like 2D or 3D. The other problem with CDM is that it can not produce multi-modal choice distributions. In order to address this issue, Ratcliff introduced the spatially continuous diffusion model (SCDM) (Ratcliff, 2018). This model is the generalized form of the diffusion decision model (Ratcliff & McKoon, 2020) and is based on a Gaussian process in a 1D or Gaussian field process in 2D spaces. The SCDM considers the accumulation process in both the spatial and the temporal domain continuous. Therefore, by running a Gaussian (field) process at each time step and accumulating the obtained distributions, the first location which reaches the decision threshold is selected. So the process is similar to a multi-dimensional Brownian motion and the sampling process is done continuously through time. The SCDM has some advantages compared to the CDM. For example, it can be utilized for both 1D and 2D option spaces. However, the SCDM assumes total inhibition between the different locations (i.e., evidence one of each location is against the evidence for all other locations) and it does not depend on distance from the other locations (Ratcliff, 2018; Ratcliff & McKoon, 2020). Moreover, both models do

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not include leakage parameter and they have low biological interpretation. In this paper, we are going to present a computational model for decisions in a continuous option space that attempts to overcome some of the limitations of CDM and SCDM discussed above, in particular:

1. the fact that they do not have a mechanism for spatial inhibition,
2. the fact that they do not include leakage and are thus less biologically plausible,
3. the fact that they both have some problems with modeling random decisions, which typically cause heavy-tailed response error distribution.

More specifically, we propose a sequential sampling model based on the leaky competing accumulator (LCA) model (Usher & McClelland, 2001), in which each accumulator corresponds to a segment of the continuous option space.

In the following sections, we are first going to present our proposed model, the ”Radial Basis LCA” (RB-LCA). Then, we make a detailed comparison between our proposed model and the two previous models (CDM and SCDM), and, finally, we make some concluding remarks and discuss ideas for future work.

Radial Basis Leaky Competing Accumulator Model

In this section, we are going to present a sequential sampling model based on the LCA model for decisions in a continuous option space. In the original LCA model, the evidence accumulation process is described by the following stochastic differential equation (Usher & McClelland, 2001):

$$dX_i = \Big(I_i - \kappa X_i(t) - \beta \sum_{j \neq i} X_j(t)\Big)dt + \sigma dW_i, \qquad (1)$$

where $x_i(t)$ is the i-th accumulator and shows the accumulated evidence to the benefit of the i-th option, I_i is the drift rate of the i-th accumulator, κ is the leakage parameter, β is the inhibition parameter, σ is the noise coefficient, and σdW_i represents Gaussian white noise with mean 0 and variance $\sigma^2 dt$.

In order to extend the original LCA model for decisions in a continuous option space, we first discretize the option space to N segments and assign an accumulator to each part. Then, based on the distance of each segment to each other, we assign excitatory/inhibitory values of each corresponding accumulator to the others. In order to better understand the underlying mechanisms of such spatial excitation/inhibition, let's first recall the definition of the radial basis function: **Definition:** A function $\Phi : \mathbb{R}^d \to \mathbb{R}$ is called radial basis, if there exists a univariate function $\rho : [0,\infty) \to \mathbb{R}$, such that

$$\Phi(x) = \rho(r),$$

where $r = \|x\|$ and $\|.\|$ is a p-norm on $\mathbb{R}^d$ (Wendland, 2005; Kazem, Rad, & Parand, 2012).

In other words, a radial basis function is a function in which all points that have the same distance from a center point have equal function values. The formulation of some well-known radial basis functions is presented in Table 1, where ε is a shape parameter in such a way that $\varepsilon \to 0$ corresponds to the basis functions becoming flat (as discussed extensively in, for example, (Fornberg & Wright, 2004)):

Table 1: Formulation of some radial basis functions.

Radial basis function	Formulation
Gaussian (GA)	$e^{-(\varepsilon r)2}$
Multiquadric (MQ)	$\sqrt{1+(\varepsilon r)^2}$
Inverse quadric (IQ)	$\frac{1}{1+(\varepsilon r)^2}$
Inverse multiquadric (IMQ)	$\frac{1}{\sqrt{1+(\varepsilon r)^2}}$

We can add an excitatory-inhibitory weight to the LCA model based on the main property of radial basis functions (i.e., $\rho(x) = \rho(y) \Leftrightarrow \|x\| = \|y\|$; which means that the same distance from a center yields same function value) by assuming that each accumulator excites the accumulators that are close and inhibits the ones that are far (Seeholzer, Deger, & Gerstner, 2019). Now, consider a semi-circle as the presented option space. Then, the RB-LCA model assumes that the decision space $[0,\pi]$ is discretized into N segments: Each segment has a length equal $\Delta\theta = \frac{\pi}{N}$, and the location of i-th segment is obtained directly by $\theta_i = (i-1) * \Delta\theta$ for $i = 1,2,\cdots,N$. Then, the i-th accumulator $x_i(t)$ accumulates evidence to the benefit of the i-th segment and has a spatial dimension that is determined by its index. The accumulation process in the RB-LCA model is thus defined as:

$$dX_i = \Big(I_i - \kappa X_i(t) - \sum_{j \neq i} \beta^i_j X_j(t)\Big)dt + \sigma dW_i, \qquad (2)$$

where β^i_j is equal to $\rho(|i-j|) - 0.5$, and ρ is a radial basis function. As an example, let's consider $\rho(r) = \frac{1}{\sqrt{1+(\varepsilon r)^2}}$, (inverse multiquadric formulation), then $\beta^i_j = \frac{1}{\sqrt{1+(\varepsilon|i-j|)^2}} - 0.5$. If $\beta^i_j > 0$, then the j-th accumulator $x_j(t)$ has an excitatory impact on the i-the accumulator $x_i(t)$. Similarly, when $\beta^i_j < 0$ the j-th accumulator $x_j(t)$ inhibits the i-the accumulator $x_i(t)$. The RB-LCA model has an extra parameter ε, in comparison with the original LCA model, which determines the radius of excitation. In the inverse multiquadric formulation of β^i_j, the maximum value for the fraction part is equal to 1, and it occurs when $(\varepsilon r)^2 = 0$. Therefore, when the distance between two accumulators reduces, $r = |i-j|$ tends to zero and the fraction part tends to its maximum value. Consequently, β^i_j increases. A schematic view of how an accumulator can excite and inhibit the other accumulators based on the distance between two accumulators is illustrated in Figure 1.

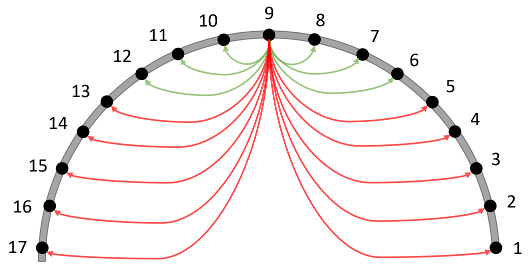

Figure 1: A schematic view for how location of an accumulator can affect the excitatory/inhibitory impact of that accumulator to the other accumulators.

On the other hand, when ε tends to zero, $\varepsilon \to 0$, β^i_j is less sensitive to the distance value $|i-j|$ and the excitatory radius becomes wider. The effect of ε value on the response error distribution is illustrated in Figure 2. When ε has higher values, RB-LCA yields more precise decisions.

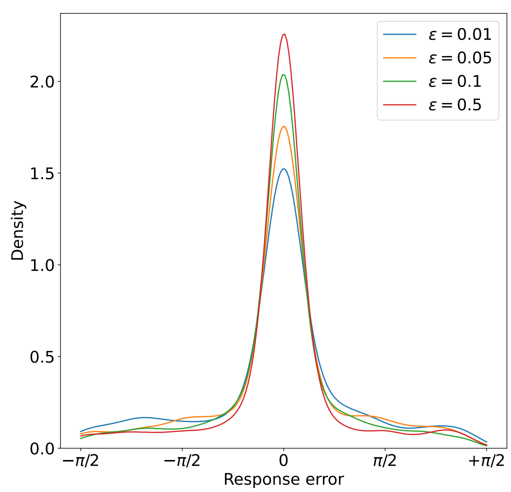

Figure 2: Plot of response error distributions for different values of ε parameter.

As mentioned above, one of the limitations of the CDM is its inability to explain the heavy-tailed response error distribution which is typically observed in visual working memory tasks (Zhang & Luck, 2008). Basically, CDM utilizes drift rate variability to generate heavy-tailed response error (Smith, 2019). The other way which is more popular in the literature is considering the response error distribution as the mixture of a uniform and von Mises distribution (Zhang & Luck, 2008; Kvam & Turner, 2021). But empirically, these methods are not successful to capture the guest very well. The RB-LCA model is able to generate heavy-tailed response error distributions thanks to the across-trial drift rate variability η. Figure 3 exhibits the effect of the across-trial drift rate variability η parameter on the response error distribution.

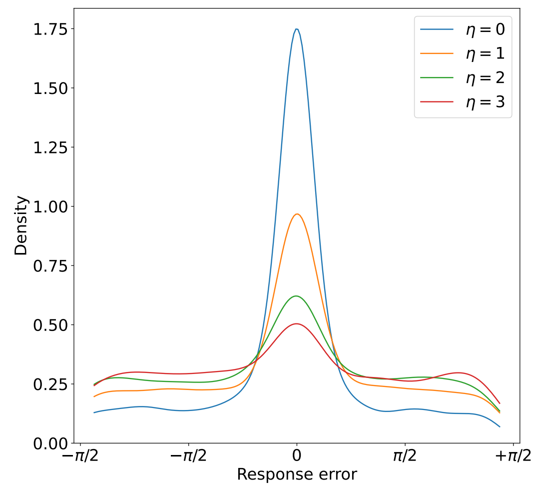

Figure 3: Plot of response error distributions for different values of η parameter.

As it is clear by increasing the η value, RB-LCA predicts more random decisions, and also the response error distribution becomes more heavy-tailed.

One dimensional decision space

The RB-LCA model can be applied to both 1D and 2D continuous option spaces. In this part, we explain how this model can be used in 1D spaces. Mainly, there are three types of 1D continuous option spaces: 1) interval $[a,b]$, 2) semi-circle, and 3) circle. Since each interval $[a,b]$ can be shifted to another interval, any value in the interval $v \in [a,b]$ corresponds to $v' = \frac{\pi}{b-a}(v-a) \in [0,\pi]$. Therefore, there is no difference between the interval $[a,b]$ and the semi-circle $[0,\pi]$ in the RB-LCA model. And what about the circular option space? The formulation for the circular option space is a bit different because there are two possible distances between two points on a circle (i.e., clockwise and anticlockwise) and the actual distance is the smaller one. Thus, after discretizing the circle into N parts, each part is assigned to two indexes (i.e., one positive and one negative). Then, the actual distance is the minimum of the absolute difference between the positive indexes of two parts and the absolute difference between the negative indexes of them. For simplicity, the positive index is assigned anticlockwise and the negative index is assigned clockwise. Figure 4 illustrates how this model works for a circle.

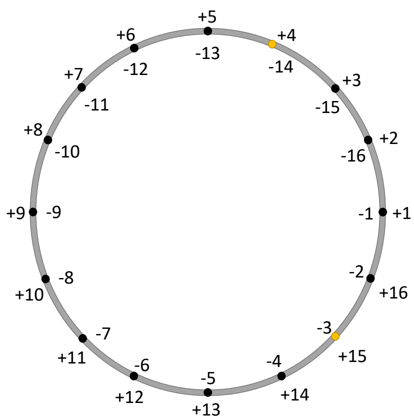

Figure 4: A schematic view of how to index each segment in a circular decision space. For example the distance for two yellow points on the circle is equal to 5.

Thus the distance of two points on a circle can be obtained by:

$$min\Big\{|i-j|,|N-|i-j||\Big\}. \quad (3)$$

It should be mentioned here that, empirical studies have illustrated that there are some differences between interval and circular option spaces. One of the main effects which are reported for interval option spaces is the *bow* effect. This effect states that the options that are located at the ends of the interval are identified more precisely (Lacouture & Marley, 1995). RB-LCA is also able to capture this effect because the endpoints are infected less by the inhibitory effects of the middle points. Hence, by considering proper drift rates for the endpoints they can be identified more precisely. In other words, the selection of the middle points is more competitive than the endpoints and it causes a higher level of accuracy in the selection of endpoints. While this inhibitory impact is symmetric in a circular option space and there is no difference between the points of the circular perimeter.

Two dimensional decision space

As in the 1D case, the 2D option space is discretized first, and one accumulator is assigned to each part. Then, each accumulator has an excitatory impact on the close accumulators and has an inhibitory impact on the other accumulators. While in 1D case we used absolute difference between two points, in 2D case we can use p-norms (i.e., if $z=(z_1,\cdots,z_N)$, then the p-norm is defined by $\|z\|_p = \sqrt[p]{\sum_{i=1}^{N}|z_i|^p}$ where $p \geq 1$). When p is equal to 1 (L_1 norm), it is called the Manhattan norm and equal distance points are located on the perimeter of a square. But when p is equal to 2 (L_2 norm), it is called Euclidean norm and equal distance points are located on the perimeter of a circle. Thus, by choosing different norms, accumulators may have different effects on each other. The mechanism of allocating accumulators to a 2D continuous option space is shown in Figure 5.

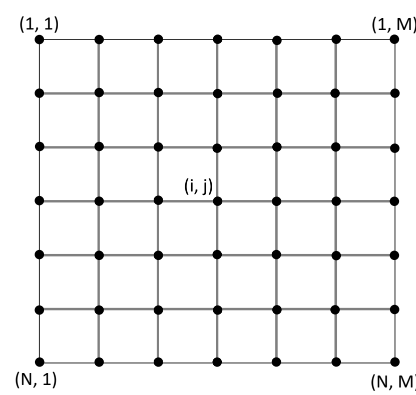

Figure 5: A schematic view of a discretized 2D decision space by $N \times M$ parts.

Similarly to 1D situation, in 2D cases each part can be indexed by two indexes (i, j). Thus, the corresponding stochastic differential equation of accumulation process at point (i, j) is as follows:

$$dX_{i,j} = \Big(I_{i,j} - \kappa X_{i,j}(t) - \sum_{l\neq i}\sum_{s\neq j}\beta_{l,s}^{i,j}X_{l,s}(t)\Big)dt + \sigma dW_{i,j}, \quad (4)$$

and $\beta_{l,s}^{i,j}$ is defined by $\rho(\|(i,j)-(l,s)\|)-0.5$, where ρ can be one of the Gaussian, inverse quadric, or inverse multiquadric functions. It is worth mentioning that the definition of the norm function can affect the performance of the RB-LCA model. Difference of L_1 norm and L_2 norm is presented in Figure 6.

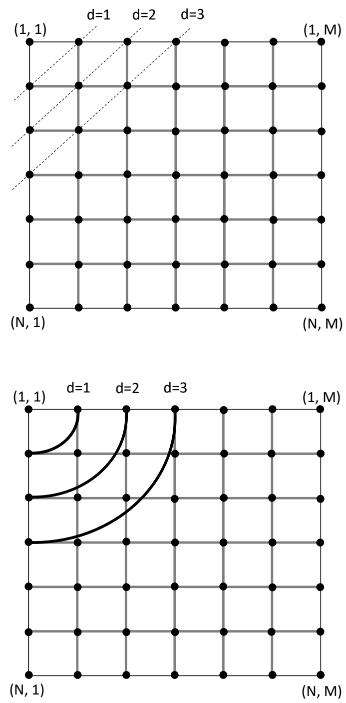

Figure 6: Plot of difference between L_1 and L_2 norms. For example the top panel shows different L_1 distances from point $(1,1)$, and the bottom panel shows L_2 distance from $(1,1)$.

It is worth mentioning that the RB-LCA model is not limited only to 1D or 2D option spaces and it can be generalized to higher dimensions easily due to the nature of radial basis functions.

Diffusion Properties of RB-LCA

The problem of the likelihood approximation of LCA models is one of the challenging problems of cognitive modeling literature. Various methods have been developed for this approximation problem, such as the sampling method (Miletić, Turner, Forstmann, & van Maanen, 2017), the neural network method (Radev, Mertens, Voss, & Köthe, 2020), and the Lie-algebraic group method (Lo & Ip, 2021). The Lie-algebraic method is a very powerful method because it can obtain a close form solution for partial differential equations and is based on solving the corresponding Fokker-Planck equation of joint transition distribution of the accumulators. The corresponding partial differential equation of joint transition probability distribution of the accumulators is one of the main properties of this model. Similarly to (Lo & Ip, 2021), by defining $x = \ln X$, the corresponding Fokker-Planck equation is obtained as follows:

$$\frac{\partial p(\{x_i\},t)}{\partial t} = \sum_{i=1}^{N} \frac{\partial}{\partial x_i}\left\{ \left[\frac{\sigma^2}{2}\frac{\partial}{\partial x_i} - \left(I_i - \kappa x_i + \sum_{j\neq i}^{N} \beta_{i,j} x_j\right)\right] p(\{x_i\},t)\right\}. \tag{5}$$

This equation is very important for studying the diffusion properties of the RB-LCA model because it gives us the joint transition probability distribution of all accumulators at each time. So we can have the moment-to-moment dynamic of the accumulators by solving this equation.

Discussion

In this paper, we have tried to overcome the limitations of the two main computational models for decisions in continuous space (i.e., the CDM and the SCDM) by introducing a new computational model based on the LCA model. The behavior of the RB-LCA model depends on a few crucial features, that we explain here below.

The first important feature of our proposed model is given by the parameter ε, which regulates the spatial excitatory/inhibitory role of the accumulators. When linked to neural data, this parameter could relate to the behavior of competing neuronal populations (Smith & Ratcliff, 2004). For example, when it has a high value, a long range of accumulators excite each other and more precise decisions are made. In contrast, when it has a low value, the RB-LCA model approximates to the original LCA model in which all accumulators inhibit each other. Therefore, the RB-LCA inhibition mechanism can vary depending on ε.

The type of radial basis function and norm function in our model is also crucial for the behavior of RB-LCA. As mentioned before, the type of norm (L_1 or L_2 norm) is not important in 1D option spaces but different norms have different behavior in 2D option spaces. The norm selection should be done based on the properties of the stimuli representing the participants. Moreover, the type of radial basis function can change the shape of the error distribution. The inverse multiquadric function has a heavy-tailed behavior and can generate the observed heavy-tailed distribution in the visual memory tasks. Furthermore, the Gaussian functions have normal-like behavior and are good when the response error as a normal distribution.

RB-LCA model has some shared properties with the CDM and the SCDM, but it is different in some properties. The inhibition mechanism is one of the key mechanisms of the RB-LCA model which is shared with CDM and SCDM models. But the inhibition mechanism in SCDM is a bit different from the two other models. In SCDM, it is assumed that evidence for one location is evidence against all the other locations (Ratcliff & McKoon, 2020), while in both RB-LCA and CDM, evidence for one location is against only some other locations but not all, depending on their distance. The other difference between the models is the number of drift rates. CDM model has only one drift rate, while RB-LCA and SCDM both consider a drift rate for each location. The other difference between models is in the noise-adding mechanism. Since in CDM, there is only one accumulator, the whole process consists of only one normal Gaussian noise. In both RB-LCA and SCDM, each location has its own normal Gaussian noise and these noise distributions are considered identical independent distribution (i.i.d) in the RB-LCA but in SCDM, the noise distribution are considered correlated. A summary of the model mechanisms is presented in Table 2

Table 2: Summary of different mechanisms which are included in three computational models of decisions in continuous space.

Model	1D space	2D space	inhibition	leakage
CDM	✓	–	✓	–
SCDM	✓	✓	✓	–
RB-LBA	✓	✓	✓	✓

Moreover, the bimodal distribution of responses cannot be handled by the lone CDM of (Smith, 2016), as it predicts unimodal distributions of responses on the circle, but bimodality is accounted for in the SCDM model of (Ratcliff & McKoon, 2020) by virtue of competing accumulation processes (see Experiment 3 and 4 of SCDM article), a similar approach can be gainfully applied here in our RBF LCA model. Our idea could be that we can either use the Bessel radial basis functions (because these radial functions are multi-modal, see (Fornberg, Larsson, & Wright, 2006) or (Roger, Moreau, & Marsan, 2014)), or we can use a combination of several radial basis functions depending on the nature of the behavior task in the β formula in RBF-LCA.

Another important thing to consider is the dimensionality issue. In particular, the dimension of the Fokker-Planck equation of RB-LCA can be very huge. For example, in a semi-circle situation, if we discretize the arc into 36 parts, the corresponding partial differential equation has 36 spatial dimensions. Thus, solving this problem becomes increasingly challenging. Recently, however, new powerful methods for solving high dimensional partial differential equations have been proposed. One of the most powerful ones is the deep splitting method (Beck, Becker, Cheridito, Jentzen, & Neufeld, 2021). There are also some other deep learning based methods for solving the Eq (5) and approximating the likelihood function. But solving Eq (5) is not the only way for fitting this model on experimental data. There are also some other deep learning methods such as deep inference network (Radev, Mertens, Voss, & Köthe, 2020) and Bayesfolow network (Radev, Mertens, Voss, Ardizzone, & Köthe, 2020) which can learn the behavior of the model in simulated parameter space and then fit the model on behavioral data based on the learned simulations data. Generally, likelihood approximation is the main problem of sequential sampling models for decisions in continuous space. SCDM does not have a close-form solution for its likelihood function. On the other hand, while CDM has an analytical form for its likelihood function but it is so time-consuming to compute. Thus, maybe using a deep learning method for fitting these models is a good solution that can be tested.

The final point of discussion is about the discretizing algorithm. In this paper, we considered that the points are distributed in whole the decision space uniformly. But it is more realistic to add some attention mechanisms to the model which implies that in the focusing locations there should be more accumulators. In this way, the model is able to overcome the dimensionality issue. Because there is no need to allocate so many accumulators to process whole the decision space.

It is worth mentioning that there are also some other computational models for some kinds of continuous outcome decisions. One of these models is the geometric framework which is developed recently (Kvam, 2019). This framework can be applied to any number of alternatives (discreet or continuous) in an optimal way. But it has two main limitations. Firstly, similar to SCDM, there is no exact formulation for its likelihood function. On the other hand, since the aim of this framework is to accumulate the information in an optimal way, the procedure of extending this framework to multi-dimensional continuous spaces is not straightforward.

Conclusion

To conclude, we here proposed a new sequential sampling model for decisions in a continuous option space. Compared to previously proposed models for decisions in a continuous option space, our model has the advantage of modeling guess decisions by adding drift rate variability, it can be applied to both 1D and 2D decision space, and it is more biologically plausible (Bogacz, Usher, Zhang, & McClelland, 2007). Note that the proposed model's fit to experimental data is not illustrated in this paper yet and it is left to future studies.

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