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UNIVERSITY OF CALIFORNIA, SAN DIEGO

What It Means to Be an Elementary Mathematics Teacher: Changing Practices and Understandings over the Course of Preservice Professional Development

A Dissertation submitted in partial satisfaction of the requirement for the degree

Doctor of Education

in

Teaching and Learning

by

Susan Michelle Scharton

Committee in charge:

Claire Ramsey, Chair
Gerald Balzano
Paula Levin
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2007

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Chair

University of California, San Diego

2007

DEDICATION

to those who prepare preservice teachers
to make mathematics meaningful
for our elementary students

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1999 "Conversations with a Colleague," *Creative Classroom Magazine*, 14(1), 64-68

ABSTRACT OF THE DISSERTATION

What It Means to Be an Elementary Mathematics Teacher: Changing Practices and Understandings over the Course of Preservice Professional Development

by

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This study incorporated a cross-sectional design to approximate a longitudinal study to examine the ways in which professional development influences the understanding of preservice elementary mathematics teachers from the beginning to the end of preservice preparation. While some quantitative methods were used, this study focused primarily on qualitative methodology to examine the beliefs, knowledge, and practices of preservice mathematics teachers. The methods used included questioning, interviewing, video recording, and observation. Both large and small groups of individuals from three courses representing points during the course of preservice professional development participated in this study; small groups were composed of three focus participants chosen from the large participant groups. Large scale instruments included a questionnaire, quickwrite, and reflection on teaching. Small scale instruments included interviews, video recording teaching and learning events, and video-elicited

interviews about the teaching and learning event. Analysis of data from questionnaires, quickwrites, reflections, interviews, and video tapes focused on similarities and differences among the beliefs about, knowledge of, and practices toward mathematics teaching and learning in individuals of varying quantity and type of pedagogical content knowledge and teaching experience.

Four findings emerged from data analysis: 1) with more professional development, preservice teachers' beliefs become increasingly aligned with a reform perspective; 2) with more professional development, preservice teachers' views of mathematics teaching and learning become more complex; 3) there is a change in the support preservice teachers provide students from using a few teacher strategies that lead students toward a correct answer to a wider range of strategies that elicits students' processes; and 4) teaching practices are differentiated by the degrees to which preservice teachers focus on process and product while engaged in teaching and learning.

CHAPTER 1: INTRODUCTION

Mathematics instruction in the United States is under constant public scrutiny. Results from national and international assessments continue to remind the public that student achievement is not what it could be. Student achievement results from the complex interaction of many factors; mathematical understandings and pedagogical practices of the classroom teacher are two that have been shown to be predictive of student performance.

Professional development in teaching mathematics can provide opportunities for elementary teachers to examine and add to their understandings about mathematics content and pedagogy as well as consider the effectiveness of current and potential instructional practices. Understandings of and practices in teaching mathematics change as beliefs about, knowledge of, and practices in learning and teaching mathematics continue to accrue and shape subsequent understandings and practices. An examination of preservice preparation outcomes could assist those responsible for inservice professional development get a sense of how these understandings and practices begin and are shaped in the early stages in becoming a mathematics teacher. Those responsible for preservice preparation benefit from knowing what prior understandings and experiences prospective teachers bring to credentialing programs as well.

People develop particular beliefs about what mathematics is and how it ought to be taught during early experiences as mathematics students. These beliefs follow prospective teachers into the preservice professional development classroom where they encounter experiences intended to shape their knowledge of mathematics content

and pedagogical practices. If beliefs are the lenses through which these experiences are filtered, the persistent and stable nature of beliefs about mathematics teaching and learning pose significant challenges to teacher preparation. If instructional practices emerge from the complex interaction between beliefs toward and knowledge about teaching and learning, those who train teachers must consider the ways in which beliefs and knowledge interact and how this interaction manifests in teaching practices.

Depth of mathematics content knowledge impacts instructional practices. While Lampert (1988) points to evidence that the number of college mathematics courses does not necessarily correlate with teaching quality, work by Brown and Borko (1992) and Hart (2002) provide evidence to indicate that the depth of content knowledge allows preservice teachers to more effectively respond to students and can result in ways of teaching that may emphasize conceptual understanding rather than just procedural knowledge. Because elementary teachers typically teach in self-contained classrooms, preservice preparation in mathematics competes with the time and attention afforded other required disciplines. According to Smagorinsky et al. (2003), these limitations can contribute to conceptual understandings that are both superficial and fragile.

Smagorinsky et al (2003) outlines the various characteristics that typify the practices of preservice elementary mathematics teachers. Traditional beliefs about mathematics content and pedagogical practices are developed during their own experiences as students. These beliefs tend to follow preservice teachers into the field placement classroom and can be resistant to change, especially if student teachers are placed in classrooms with cooperating teachers who have similar beliefs and employ instructional approaches consistent with these beliefs. Preservice professional

development serves to expand pedagogical content knowledge and can influence belief change in the direction of reform. However, field experiences may make it difficult to translate their knowledge and beliefs into practice. Inconsistency among the approaches espoused in the university classroom and those in place in the field setting may impact understanding, trigger reversion to previously held beliefs, and result in lack of transfer between these two settings.

A Personal Journey

My interest in the beliefs, knowledge, and practices of elementary mathematics teachers is a reflection of my own evolution as a teacher and learner. My early practices as an elementary mathematics teacher involved helping my students ‘cover’ the content in their mathematics textbook. The discomfort with and apathy that I felt from implementing this type of instruction pushed me to learn more. Early professional development experiences forced me to question my beliefs about mathematics teaching and learning, the nature of my conceptual understanding, and my pedagogical practices. I made changes to my own beliefs, knowledge and practices in service of student learning. These changes eventually led me towards providing preservice and in-service professional development experiences that would challenge teachers to question their own classroom practices as well.

As I challenge educators to question their teaching practices, I continually question my own. My ongoing questioning, my teaching experience, and my evolution as a teacher both qualify and motivate me to pursue my own research. A study of the complex interaction among beliefs about mathematics teaching and learning, the influences of professional development on knowledge of content and pedagogy, and

teaching practices will serve to benefit my own teaching practices as well as the practices of preservice and inservice teachers.

The Nature of this Study

The practices of preservice elementary mathematics teachers result from the interaction among numerous factors: the knowledge and beliefs from early experiences as students of mathematics, content and pedagogical knowledge from professional development, and practice in field settings. Preservice elementary programs intend to influence these contributing factors so that new teachers acquire the best skills and knowledge to enter their careers adequately prepared to face the challenges of teaching subject matter content in a number of areas, including mathematics.

In order for preservice professional development to best match the need of those entering, it is critical to understand the beliefs, knowledge, and practices that prospective teachers bring with them. A number of studies reveal findings about the early development of beliefs about mathematics teaching and learning, the stability of these established beliefs, the complex types of knowledge required to teach mathematics, and the ways in which knowledge and beliefs impact the instructional practices of preservice teachers (Artzt & Curcio, 2003; Ball, 1990a; Crespo, 2003; Foss & Kleinsasser, 1996; Gill, 2004; Hart, 2000, 2002; Linek et al., 2003; Lubinski & Otto, 2004; Seaman et al., 2005; Stuart & Thurlow, 2000; Vacc & Bright, 1999; Wilkins & Brand, 2004).

While previous studies examine this complex interaction among beliefs, knowledge and practices, these studies did not address knowledge of how past experiences influence the beliefs, knowledge and instructional practices of preservice

teachers; the distinctions between the content and pedagogical knowledge held by individuals at different stages in their process toward becoming mathematics teachers; and the nature of preservice teachers' instructional practices as they develop during the course of professional development. This study will first account for the literature focusing on beliefs, knowledge, and practices. It will then detail an approach to examine some of the experiences, understandings, and practices of those individuals that are on the path toward becoming elementary mathematics teachers in an attempt to address some aspects of the literature that remain to be examined.

Paradigm and Assumptions

This study reflects certain beliefs about preservice mathematics professional development and educational research that are most closely aligned with a constructivist paradigm. According to Merten (2005), this approach assumes that reality is socially constructed. In order to make sense of what is occurring in an educational setting, it is critical to understand that setting from the perspective of those who take part in it and to realize that these meanings constantly evolve, rather than remain in a static state. Multiple meanings of the same event make objectivity a challenge, so multiple data sources contribute to the confirmability, rather than the objectivity of a study.

This study used primarily qualitative methods that included interviews, reflections, and observations. Participants gave their own historical accounts as both a learner and teacher of mathematics and interpreted and reflected on the practices of others. Analysis of multiple data sources examined patterns in, classifications for, and descriptions of the ways in which beliefs about, knowledge of, and practices toward

teaching and learning mathematics interacted and evolved, and was expected to reveal how these understandings and practices influenced the development of teacher thinking that resulted in enhanced knowledge of content, pedagogy, and students.

CHAPTER 2: BELIEFS, KNOWLEDGE AND PRACTICES OF PRESERVICE ELEMENTARY MATHEMATICS TEACHERS: A LITERATURE REVIEW

The space race that began in 1957 instigated a number of studies of mathematics and science education. These studies largely examined the role of curriculum in U.S. mathematics and science classrooms, prompting efforts to create, implement, and assess new curriculum that would make American students competitive with their international peers (Lagemann, 2000). According to Shulman (2000), research during the 1950s and 60s focused on problems of learning, rather than problems of teaching. Researchers studied the effect of curriculum and instruction by measuring student outcomes. Examining the ways in which instruction influences both the implementation of curriculum and resulting student outcomes was largely absent.

Efforts to enhance mathematics education by focusing on curriculum development without consideration of professional development were both limited and ineffective: teachers lacked training in how to implement the curriculum they were given. The research on mathematics teaching and learning that followed during the 1970s and 80s moved from a focus on curriculum to a focus on instruction. Work in a variety of disciplines examined the ways in which teacher knowledge, understanding, and beliefs interact with mathematics instruction. Cognitive psychology and educational psychology contributed in a number of ways, helping the educational community understand how the qualities of experienced teachers differ from those of novices, as well as explain the development of teaching expertise. A number of constructs that frame the domains of teacher knowledge emerged from these intersecting fields, as did framework

and typologies that help us understand the development of teacher knowledge, the development of beliefs about teaching and learning, and the impact of beliefs on instructional practices. In the last twenty years, researchers from these disciplines have been joined by classroom practitioners, teacher researchers, and those involved with preservice education to study how beliefs about and knowledge of mathematics teaching and learning affect classroom instruction.

This chapter examines the complex interaction among beliefs and attitudes about, knowledge of, and practices in teaching elementary mathematics by examining the patterns revealed in the research on 1) the nature and stability of preservice elementary candidates' beliefs about and knowledge of mathematics teaching and learning, 2) the content and structure of preservice mathematics professional development, and 3) the teaching practices of preservice elementary mathematics teachers.

Current State of Research in the Field

Research by Peterson et al (1989b) has examined how teachers' knowledge and beliefs affect their instructional practices. For this reason, it is important for this study to consider the research that examines how preservice elementary credential candidates progress in their development as teachers of mathematics, in terms of their beliefs about, knowledge of, and practices in teaching elementary mathematics. Research in this area has produced a number of findings. These findings have important implications for the structure and content of preservice mathematics professional development as well as the teaching behaviors that result from the interaction among existing beliefs, prior knowledge, and professional development experiences.

Preservice Teachers' Beliefs about and Knowledge of Mathematics

Elementary preservice teachers enter teacher education programs with definitions about what it means to know, learn, and teach mathematics from their early experiences as students. These beliefs are constructed from a variety of influences and are quite complex. Research on mathematics inservice and preservice has shown that beliefs are difficult to shape and change (Charalambous et al., 2002; Pajares, 1992; Richardson & Placier, 2001; Stuart & Thurlow, 2000), although research in elementary mathematics preservice professional development confirms that shaping is, indeed possible (Crespo, 2003; Hart, 2002; Lubinski & Otto, 2004; Muis, 2004; Seaman et al., 2005; Stuart & Thurlow, 2000; Vacc & Bright, 1999; Wilkins & Brand, 2004). In studies that focus on belief change in the process of changing preservice teachers' content and pedagogical knowledge, beliefs have been shown to change (Gill, 2004; Stuart & Thurlow, 2000).

Most elementary teacher candidates regard mathematics as a formal, difficult and rule bound content area, made up of a static, unchanging collection of facts, procedures, and formulas (Muis, 2004; Stuart & Thurlow, 2000). Studies by Ball (1990a), Foss and Kleinsasser (1996) and Seaman et al. (2005) found that, rather than consider the range of topics that exist within the discipline, elementary preservice teachers narrowly equate mathematics with arithmetic, an interplay of numbers, skills, and operations that produce a unique result: the answer. Ball (1990b) found that preservice teachers associate memorization and use of procedural rules with understanding and believe that learning skills and procedures should happen quickly; they tend to compartmentalize mathematical knowledge, rather than see the relationship between concepts and topics.

Instead of viewing logic, reasoning, and problem solving as embedded aspects of all mathematics lessons, Cooney (1985) found that they consider them discrete and separate topics to teach.

Elementary teacher candidates connect their conceptions of what it means to learn and teach mathematics to their conceptions of what elementary mathematics is. Studies by Foss and Kleinsasser (1996) and Seaman et al. (2005) found that they tend to rely on knowledge from their past when defining both the role of the teacher and the role of the student in the mathematics classroom. Rather than view mathematical understanding as a creation of the learner, preservice and beginning teachers tend to look upon mathematics as a body of facts and skills that teachers dispense to students (Cooney, 1985). There is a tendency for prospective and new teachers to rely on traditional models of instruction: instructional methods are used to assist students in acquiring understanding of mathematics content, concepts, and skills by way of the teacher explaining and students practicing a particular skill or procedure (Ball, 1990a; Cohen & Ball, 1990; Hart, 2002; Smagorinsky *et al.*, 2004; Spillane, 2001). Muis' work (2004) points to two complementary beliefs that result in this common conception of mathematics instruction: because mathematical knowledge consists of a stable set of facts, skills and procedures, it is inappropriate to expect students to construct mathematical knowledge and students are incapable of doing so. It is not surprising, then, that other research studies (Foss & Kleinsasser, 1996; Seaman et al., 2005) confirm that preservice teachers believe that mathematical understanding is equated with memorization and teaching with telling.

Researchers have studied how preservice teachers relate these ideas to those that define the role of the teacher and the role of the student. Many preservice teachers regard

mathematical ability as an innate capacity of the student (Muis, 2004) and believe that students come to school with little or no mathematical knowledge (Stuart & Thurlow, 2000); mathematical ability is part of the student, rather than something that can be learned through experience. These beliefs can have a significant impact on the ways preservice teachers conceptualize their role as elementary mathematics teachers. If they believe that innate ability is required for learning to take place, they regard mathematical understanding as more closely tied to student ability rather than to the teachers' content and pedagogical decisions (Foss & Kleinsasser, 1996; Seaman et al., 2005). Although preservice professional development interacts with these beliefs about mathematics teaching and learning while engaged in the process of shaping the knowledge of its potential teachers, studies by Vacc and Bright (1999) confirm that it may be difficult to put recently modified beliefs into practice.

Nature and Effectiveness of Preservice Professional Development

According to Hiebert (1999), American mathematics instruction in both elementary and secondary classrooms has changed very little in the last fifty years, so it is not surprising that the type of mathematics instruction currently evident in most elementary classrooms resembles the type of instruction that most elementary credential candidates—and their methods instructors--experienced during their own elementary education. Interestingly, reform approaches to elementary mathematics instruction dominate the mathematics methods courses of most university teacher education programs and these approaches suggest alternatives to the type of instruction currently in place in most elementary classrooms.

The influence of beliefs. As compared to previous experience as elementary students as well as other classroom experiences prior to student teaching, the content encountered in preservice professional development can be dissonant with the experiences of those who enter it (Foss & Kleinsasser, 1996; Pajares, 1992; Seaman et al., 2005). If beginning elementary mathematics teachers enter preservice education with firmly established beliefs about mathematics teaching and learning based on a lifetime of experiences as students of mathematics, preservice professional development is faced with a series of challenging tasks: 1) to cultivate an awareness of the assumptions preservice teachers have formed, 2) challenge the accuracy and effectiveness of these assumptions, 3) use these assumptions as working hypotheses, and 4) replace these assumptions with some more productive alternatives.

Content and pedagogy in preservice professional development. Research on preservice and inservice mathematics professional development has examined how attention to content, pedagogy, and the relationship between them are important characteristics of effective professional development. Content knowledge is individualized and complex. In order for professional development to effectively impact content knowledge and classroom practice, it must attend to 1) where and when teacher knowledge originates, 2) the nature and form of teachers' current content knowledge, and 3) the process involved in retrieving existing knowledge, combining it with new knowledge and creating a new knowledge base.

Studies by Lave and Wenger (1991), Wertsch (1998), Shulman (1987), and Shulman and Shulman (2004) have examined how knowledge is situated in specific social contexts and question the ways in which learning transfers when contexts change.

Prospective teachers enter preservice preparation programs with notions about what it means to teach and these notions are situated in the settings in which they originated. Knowledge about teaching and learning is based on and in past teaching and learning experiences. Representations extracted from these experiences affect the ways in which preservice teachers filter professional development experiences. Attention to these factors illuminates the variability in content knowledge from individual to individual, suggesting implications on pedagogical practice.

Research by Ball (2000) and her colleagues (Ball, 2000; Ball & Bass, 2000; Graham & Fennell, 2001; Hill & Ball, 2004; Lampert, 1988) have confirmed that developing content knowledge in conjunction with pedagogical knowledge is an important outcome of methods courses. Preservice mathematics instructors must think about what content knowledge teachers must have, how it is most effectively held by prospective teachers, and what is needed to put that knowledge into practice. The work of these individuals have examined the role that content knowledge plays in helping to bridge the gap between content and pedagogy in learning to teach. Complex content knowledge is required for a teacher to make sense of what students know and can do in order to make spontaneous—yet sound—instructional decisions that further student understanding. These decisions rest on the teacher’s conceptual understanding of the mathematics being taught as well as an ability to break down complex concepts into simpler critical components.

Hill and Ball (2004) examined the relationship between the organization of mathematics inservice and what people learned. They found four key components of effective mathematics professional development: 1) a focus on mathematics content, 2)

the provision of opportunities for teachers to address problems that arise when teaching, 3) consideration of experiences teachers bring, and 4) experiences that require making sense of mathematics. Experiences in which teachers build on existing knowledge, construct new mathematical understandings, and use this sense making for epistemological purposes, result in changes to basic epistemological perspectives, knowledge of what it means to learn, and conceptualizations of classroom practice (Franke et al., 1998). As a result of these experiences, teachers alter their existing knowledge of—as well beliefs about and attitudes toward—what it means to learn and teach mathematics.

Brown and Borko (1992) reviewed studies that examine the relationship between content and pedagogy in experienced versus novice teachers. Their findings argue that the routines invoked more or less automatically by experienced teachers impose nontrivial cognitive demands on novices. The representations, examples, and demonstrations used to teach mathematics are routine to the experienced teacher, but must be created by the novice. An important goal of preservice professional development is to provide experiences that will help prospective teachers establish new schema for teaching mathematics and modify and elaborate existing schema.

According to studies by Graham and Fennell (2001), creating explicit connections between previous experiences and new ones provided in methods courses is a difficult but necessary process for preservice elementary mathematics teachers. Shulman (2000) proposes that teacher preparation programs have limited time, fiscal resources and perspectives. These constraints can result in methods courses that emphasize breadth over depth and student teachers who leave with partial or superficial understanding of

mathematics concepts (Smagorinsky et al., 2003). Teachers' knowledge of content can be fragile and illusory: understanding of mathematics concepts can be incomplete or inaccurate and they appear to know something they do not (Shulman, 2000). Conceptual understanding is, therefore, a difficult goal.

Much research has examined how teaching mathematics goes beyond knowledge of subject matter content (Ball, 1990a, 2000; Ball & Bass, 2000; Brown & Borko, 1992; Foss & Kleinsasser, 1996; Graham & Fennell, 2001; Hill & Ball, 2004; Linek et al., 2003; Munby et al., 2001; National Council of Teachers of Mathematics, 2000; Peterson et al., 1989a; Peterson et al., 1989b; Putnam & Borko, 2000; Schoenfeld, 2002; Shulman, 1986, 1987; Shulman & Shulman, 2004); there is a difference between knowing and helping another to know (Ball, 2000; Shulman, 2000). Teaching requires knowledge of pedagogy as well as a deep conceptual understanding of content. Ball (1990a) and others (Ball & Bass, 2000; Graham & Fennell, 2001; Munby et al., 2001; Shulman, 1987; Staub & Stern, 2002) have found that effective mathematics teachers make use of multiple types of knowledge to plan, implement, and evaluate instruction and they are able to understand, hear, represent, and connect student ideas. Teacher preparation programs must balance attention to content and pedagogy during an all too abbreviated teacher preparation program. New teachers must determine what to teach as and how to teach it (Shulman, 1987).

How Preservice Teachers Teach Mathematics

During the field placement component of preservice professional development, student teachers are introduced to the many and varied cognitive demands of teaching as

they integrate knowledge from a number of sources: 1) their elementary, undergraduate, and preservice education, 2) content across disciplines, and 3) connections between theory and practice (Smagorinsky et al., 2004; Smagorinsky, et al. , 2003). Methods courses may integrate the content and pedagogical knowledge required to teach mathematics, but student teaching requires preservice teachers to apply that knowledge in the context of a field setting. Much research has explained the challenge that confronts preservice teachers: how to transfer knowledge from the university into the elementary classroom (Ball & Bass, 2000; Graham & Fennell, 2001; National Council of Teachers of Mathematics, 2000).

Initial focus. Research that has examined the practices of preservice teachers has found that they initially focus on classroom management (Brown & Borko, 1992; Foss & Kleinsasser, 1996; Mewborn, 1999; Stuart & Thurlow, 2000). Refinement of conceptual content knowledge develops after addressing these initial demands; attending to student learning follows next, if at all, during student teaching. The behavior of preservice and beginning elementary mathematics teachers indicates how they think about teaching and learning in general and how they think about teaching and learning mathematics. If teaching is equated with telling, and student learning is equated with memorization and practice, archaic teaching methods are often employed (Foss & Kleinsasser, 1996). Cooney (1985) argues that, if preservice teachers assume mathematics to be difficult, they often opt to lessen the drudgery by using interesting, engaging and enjoyable tasks, games, and activities to motivate students rather than use these same tasks as a context for learning something new.

How knowledge gets operationalized. While it is a commonly held belief that teachers learn how to teach through the act of teaching, it is not clear how this development happens. Shulman (1987, 2000) examines the ways in which pedagogical reasoning and subsequent teaching action involve a cycle of comprehension, transformation, instruction, evaluation, and reflection to “make the internal external, work on it, and put it back in” (Shulman, 1987, p. 133). Steps in this process presume that teachers understand the content they teach, can organize the knowledge to make it more accessible to students, can assess instructional effectiveness and adjust subsequent lessons accordingly. A teacher’s ability to incorporate student feedback and improvise instruction is necessary to keep this cycle moving. This ability tends to be more advanced in experienced teachers than in novice ones. Brown and Borko (1992) found that experienced teachers are more able to respond flexibly to content, students, and purpose than novice teachers.

When preservice elementary mathematics teachers begin student teaching, their pedagogical content knowledge tends to be limited; knowledge of mathematics content can be superficial or riddled with misconceptions and is paired with instructional strategies that are “under construction” (Vacc & Bright, 1999). Student teachers tend initially to regard teaching as a custodial task and Crespo (2003) found that they do a number of things to ensure student success: alter problems to make them unnecessarily familiar, pose ones that are inappropriately difficult, adjust problems to make them easier to solve than necessary, and alter practices so that student confusion and errors are minimized or avoided altogether. Many elementary preservice teachers focus on teaching arithmetic. While they may integrate other topics and application into situations from

everyday life, their lessons frequently center on innovative ways to practice and memorize arithmetic skills and procedures (Foss & Kleinsasser, 1996). Student teachers tend to replicate lessons and experiences from their methods courses and their instructional style often mirrors their own experience or that of their cooperating teachers. Crespo (2003) found that, while these tendencies may lessen over time, they usually do not disappear altogether.

Teaching practice is guided by theory. Russell and Munby (Russell & Munby, 1991) address the gap between knowing-in-action and reflection-in-action by building on Schön's (Schön, 1987) notion of "reframing" (Schön, 1987, p. 29). Puzzling experiences that occur during practice prompts the teacher to not only question what has occurred but to reconsider the theories on which the original practice is based. This action prompts teachers to question, hear, or see these formative theories differently than before (Dewey, 1910). Questioning previously held theories allows teachers to make important connections among beliefs, knowledge, and practice. While beliefs stay intact, the teacher is able to view a problem in a new and/or different way. This new perspective is an important step in solving teaching dilemmas and prompts an adjustment to the theory as a result of the reframing experience. While prospective and preservice teachers may be skeptical of the content and pedagogical knowledge espoused in the university setting, their teaching can continue to develop and improve through practice.

New teachers have difficulty in productively reframing problems and addressing inconsistencies in practice. This difficulty can be attributed to a new teacher's proximity to the teaching task. With additional experience, novice teachers are able to evolve from distancing themselves from immediate teaching actions, concern themselves with the

effect of teaching actions on students, and to finally to focus on issues of mathematics content (Mewborn, 1999). Because reflection-in-action is difficult to detect and challenging to document, research studies have examined ways for teachers to analyze data sources to develop and enhance reflective practices. Use of video-recorded accounts of both expert and novice teaching seem to be promising methods for teacher to view teaching practices and begin to focus on student learning (Carboni & Friel, 2005; Lambdin et al., 1997; Sherin & Van Es, 2005; Wang & Hartley, 2003).

Interpreting teaching practices. One critical aspect of teaching expertise is the ability to detect and repair teaching dilemmas. Unexpected outcomes to the implementation of carefully planned lessons provide opportunities for practitioners to reconsider assumptions, adjust practices, and experiment with new theories about instructional effectiveness (Schön, 1987). The demands of teaching practice are overwhelming for new practitioners. While implementing a lesson, teachers must do such different tasks: 1) break down a concept into manageable chunks, 2) contextualize the concept in a task that makes it accessible to students, 3) introduce information to students in chunks of appropriate depth and breadth, 4) monitor student understanding of concepts and the directions for the focus task, 5) keep students engaged in the lesson, and 6) attend to classroom management (Mewborn, 1999; Munby et al., 2001; Shulman, 1986; Stuart & Thurlow, 2000). While varied types of teaching dilemmas confront preservice teachers, those related to student learning present particular challenges to the pedagogical content knowledge of a developing professional. Effective instructional practices require the teacher to make lesson adjustments based on student feedback. Because preservice teachers must attend to this aspect of teaching along with many

others, it is important to provide opportunities for developing practitioners to witness learning dilemmas outside of their field service teaching demands. Researchers have found ways to use video recordings in order to provides such opportunities (Sherin & Van Es, 2005; Wang & Hartley, 2003).

Sherin and Van Es (Sherin & Van Es, 2005; Van Es & Sherin, 2002) have documented the development of teachers' ability to notice what goes on in the mathematics classroom. Whether watching video-recorded accounts of their own teaching or those of others, teachers' early interpretations attend to what the teacher, rather than the student is doing. In order for elementary mathematics teachers to affect student learning, they must account for how students respond to instruction. Linek et al. (2003) have found that attending to dilemmas of teaching related to student understanding requires teachers to shift their focus from "how teachers teach" to "how students learn." Helping preservice teachers to learn about this trajectory of noticing while providing them with opportunities to notice and interpret teaching practice should help them build and refine an ability to analyze teaching practice and find evidence of student learning.

Relationships among Beliefs, Professional Development and Teaching Practices

Beliefs and professional development. Student teachers can change their self-efficacy (Wilkins & Brand, 2004) as well as their beliefs about mathematics teaching and learning through methods courses that have a reform focus (Muis, 2004; Wilkins & Brand, 2004). However, Seaman et al. (2005) found that contradictory beliefs tend to remain after methods courses. While teacher beliefs eventually tended toward the more

constructivist leanings of their methods courses, contradictory beliefs made overall philosophies inconsistent. In order to make beliefs more consistent, preservice elementary mathematics methods courses must adopt a goal of challenging the contradictory beliefs of their preservice teachers (Hart, 2002; Seaman et al., 2005; Wilkins & Brand, 2004).

Beliefs and teaching practice. Prospective teachers gain content knowledge as well as a more positive perception of their own mathematical ability during methods courses but this knowledge is not necessarily carried into the teaching context (Brown & Borko, 1992; Vacc & Bright, 1999; Wilkins & Brand, 2004). Traditional definitions of what constitutes mathematics can remain stable (Foss & Kleinsasser, 1996; Seaman et al., 2005) and belief change from the beginning of preservice professional development to the end of student teaching may be minimal (Seaman et al., 2005). Gill et al. (2004) studied student teachers' general epistemological beliefs and epistemological beliefs specific to mathematics. Student teachers who regard mathematical knowledge as "specific and certain" (Gill, 2004, p. 165) may be less likely to think deeply about mathematics, have difficulty understanding how students' mathematical thinking develops, and are more resistant to challenge their own beliefs. Those who think that knowledge is "changing and tentative" (Gill, 2004, p. 165) are more willing to change beliefs. These findings suggest that the stability of beliefs about mathematics teaching and learning may be reflective of general epistemological beliefs: a change in general epistemological beliefs may be necessary for changes in beliefs that are specific to mathematics concepts.

Consistency between the mathematics teaching and learning espoused in the university methods classroom and that of the field placement classroom can influence change in the beliefs of preservice elementary mathematics teachers. Stuart and Thurlow (2000) found that student teachers who are placed in partnership with cooperating teachers who have adopted a reform approach tend to change beliefs in the direction of reform. Munby et al. (2001) suggest that field placements foster an “authority of experience” (Munby et al., 2001, p. 896) that is critical for belief change to happen. Through interaction with students and realizing the effect of their teaching practices on student learning, student teachers may alter previous beliefs to be more consistent with the teachings of their methods courses (Stuart & Thurlow, 2000). Hart (2002) argues that change in beliefs can occur, it takes time and Crespo has found that, among individual student teachers, belief change happens at different rates, and follows different trajectories (Crespo, 2003).

The beliefs and assumptions that follow prospective teachers into the preservice professional development classroom are part of what composes beginning teachers’ espoused theories about what teaching actions to take, given certain classroom conditions. Many studies have indicated how actual teaching practice —theories-in-use— are often incompatible with espoused theories even though individuals may not be aware of this contradiction (Argyris & Schön, 1974; Carboni & Friel, 2005; Cooney, 1985; Foss & Kleinsasser, 1996; Richardson & Placier, 2001). While preservice teachers may be able to explain their espoused theories, observations of teaching can reveal theories-in-use. Teacher reflections on teaching can allow these theories to be questioned, explained and adjusted.

Relationship between professional development and teaching practice. Learning is an aspect of all activity and all learning is situated in the context where the learning occurred; likewise, learning to teach (Lave & Wenger, 1991; Shulman & Shulman, 2004) and learning to teach mathematics (Munby et al., 2001; Smagorinsky et al., 2003) are situated activities. This aspect of learning complicates the process of becoming a teacher. Putnam and Borko (Putnam & Borko, 2000) suggest a definition of transfer that is specific to the professional development of preservice elementary mathematics teachers: professional knowledge is developed in context, stored together with characteristic features of classrooms and activities, organized around tasks that teachers accomplish in class settings, and accessed for use in new settings. The relationship among these various contexts is critical to understanding the ways in which knowledge is transferred from one setting to another.

Theory about and practices in teaching are interdependent. Conceptual understanding develops over time and in context. Due to temporal and structural constraints, conceptual understanding of mathematics teaching and learning is a particularly challenging goal for preservice professional development programs. The content encountered in preservice professional development is virtually meaningless without opportunities to apply content in practice, yet attempts to apply theory from the university classroom to field placement settings results in conceptual understandings that are manifested in unique and incomplete ways.

Activity theory (Wertsch, 1998) can be used to understand the unique and incomplete nature of teaching practice. Teaching practice is in constant flux: in any individual teacher, present practice is an outgrowth of previous experience. In order to

understand present teaching practice, it is necessary to abandon the perspective that teaching is composed of a set of cemented, permanent skills and understand that any observable act of teaching has a particular origin and distinctive trajectory. It is a complex human action that is difficult to understand without understanding each of the elements: the agent (the teacher), tools (the means or instruments used by the teacher), scene (where the teaching took place), purpose (why the teaching happened), and the act itself (Wertsch, 1998). Understanding how these elements are unique in isolation as well as in combination with each other makes the multifaceted nature of teaching all the more apparent.

Research related to methodology. Teacher interviews may provide a way to better understand how individual teachers make distinctions among the various elements that constitute a teaching act, as well as explain the dynamics among these elements. Many aspects of learning complicate the process of becoming a teacher: prospective teachers enter preservice preparation programs with notions about what it means to teach and these notions are situated in the settings in which they originated (Brown & Borko, 1992; Franke et al., 1998; Harel, 2001, 2005). Knowledge about teaching and learning is based on and in past teaching and learning experiences. Representations extracted from these experiences affect the ways in which preservice teachers filter professional development experiences and this filtering process ultimately impacts instructional practices. Early notions of what teaching is and what it looks like are combined with new ideas gained from preservice preparation development experiences. These later experiences are situated as well.

Knowing the stories in which early representations are developed contextualizes past, present and future meanings in important ways. The context within which these meanings were constructed is as important as understanding the actual meanings.

Interviews that focus on mathematics autobiographies can uncover these stories, reveal the pedagogical content underpinnings that develop from them, and allow preservice professional development instructors to consider the ways in which context has and will continue to effect the ways in which knowledge is acquired. Teacher descriptions of contexts where beliefs and knowledge about mathematics teaching and learning both originate and develop will provide information about these aspects of teacher learning.

Reflections on their own teaching and that of others may allow prospective and preservice teachers the opportunity to explain how understandings formed in early contexts are manifested in teaching practice, shape interpretations of practice, and are related to the practices of others. Because new teachers have particular difficulty in productively framing problems and addressing inconsistencies in their teaching practices, opportunities to reflect on teaching actions are promising ways for teachers to ameliorate both. Requiring prospective teachers to reflect on their teaching actions is common among preservice professional development programs. It is used to help novice teachers to consider their teaching effectiveness and promote the ability to reflect-in-action, a critical attribute of the expert teacher. However, reflection-in-action is not only challenging to develop, but difficult to detect and document. Use of video-recorded accounts of teaching provides a medium for prospective teachers to reflect on their practices and call out points where reflection-in-action occurs.

Surveys, questionnaires and interviews have been used to better understand the beliefs, knowledge and practices of preservice and practicing teachers (Hart, 2002; Lubinski & Otto, 2004; Stipek et al., 2001; Vacc & Bright, 1999) and video-based pedagogy has been incorporated into preservice professional development (Carboni & Friel, 2005; Lambdin et al., 1997; Sherin & Van Es, 2005; Wang & Hartley, 2003). However, no studies have triangulated data from first person accounts of mathematics autobiographies, video taped teaching and learning events, and video-elicited interviews about these situations.

The transition from the university to the elementary classroom is affected by the consistency in approaches used in each (Hart, 2002; Vacc & Bright, 1999). The goals of university mathematics methods can be at odds with those of the field setting (Smagorinsky et al., 2003). For example, a methods focus on deep understanding of a few topics tends to be replaced by field-setting focus on covering a breadth of mathematics topics. Experiences and approaches modeled in mathematics methods can be discordant with the scope and sequence and instructional methods taken from the textbook curriculum currently in place (Lampert, 1988). Beliefs and values central to the core of mathematics methods may be in conflict with those where student teachers work (Hart, 2002). Student teachers may abandon what they have learned in the university classroom and appropriate the values and instructional style used by their cooperating teacher (Smagorinsky et al., 2003). Conflicts between the university and field setting may result in limited implementation of university methods content and pedagogy (Hart, 2002).

Consistency of approach used in these settings makes for a different kind of transition for the student teacher (Hart, 2002; Vacc & Bright, 1999). The knowledge taken from methods and fieldwork is situated in the settings within which it occurs (Munby et al., 2001; Putnam & Borko, 2000; Smagorinsky et al., 2003). Preservice teachers are more likely to transfer knowledge gained from preservice experiences when practices in that context are closely aligned to those of real practitioners in the field. It is critical to maintain approach consistency to maximize the potential for transfer and prevent the pull of more traditional school cultures (Putnam & Borko, 2000). Research by Hart (2002) and Vacc and Bright (1999) found that, when university methods instructors and cooperating teachers hold similar beliefs, attitudes and dispositions toward mathematics, student teachers are more inclined to successfully implement methods content into student teaching practice. What they experienced as learners can now be used in their role as teachers. Artzt and Curcio (2003) found that reflection on teaching and learning facilitates this process as does mentoring by the cooperating teacher and other experienced teachers at the school site (Putnam & Borko, 2000).

Munby et al. (2001) point to the dilemma in acquiring the necessary knowledge needed for teaching given the many challenges that exist in the divide where the knowledge is acquired and the place where it is intended to be put into practice:

To the uninitiated, teaching unfolds as a set of skills, but to the initiated, teaching depends on, is grounded, and constitutes knowledge. The character of this knowledge poses the irony for teacher education: The knowledge is, in part, practical, and that part can only be learned in practice, the very setting over which teacher educators have little direct control. (p. 896)

While it is critical that preservice professional development assists prospective teachers in acquiring both content knowledge and pedagogical content knowledge, both develop through teaching practice (Brown & Borko, 1992).

Relationship between content knowledge and teaching practice. Many researchers have examined the effects of content knowledge on student teaching practice (Ball, 2000; Ball & Bass, 2000; Graham & Fennell, 2001; Shulman, 1986). Knowledge gleaned from experience in undergraduate mathematics courses is necessary but insufficient preparation for teaching elementary mathematics (Ball, 1990b). Preservice teachers tend to draw on content knowledge learned as elementary students rather than as undergraduates (Ball, 1990a). While it is important for teachers to understand mathematics beyond the level they teach (Ball, 2000), Lampert (1988) concludes that it has been difficult to determine the relationship between college mathematics courses and future teaching quality. The content and form of college mathematics courses vary greatly. Grades from these courses as well as Graduate Record Examination scores may not reflect the extent of students' mathematical content knowledge.

Research by Crespo (2003) indicates that the number of undergraduate mathematics courses may not reliably predict a student teacher's ability to pose sound problems and Lampert (1988) found a weak correlation between content knowledge and teaching effectiveness. However, Hart (2002) have found that strong preparation in one's content area prior to student teaching is especially important for student teachers who adopt a reform approach. Research studies have identified a number of positive teaching outcomes correlate with high levels of mathematics content knowledge: 1) greater confidence when teaching, 2) less time spent on learning content for lessons and more

time on planning instructional strategies, 3) greater flexibility in planning, 4) more responsiveness to student input while teaching, 5) an ability to make connections between mathematical topics, 6) and greater use of explanations that are conceptual, rather than procedural (Brown & Borko, 1992; Hart, 2002). Two separate research studies (Artzt & Curcio, 2003; Crespo, 2003) show a positive correlation between grade point averages from undergraduate mathematics courses and knowledge of pedagogy and knowledge of student understanding; lower grade point averages in undergraduate mathematics courses have been associated with difficulty in representing mathematics concepts, appropriate choice of lessons, and lesson sequencing. The debate between the level of content knowledge and effective pedagogical practices deserves further examination in order to more appropriately prepare preservice teachers with the content knowledge needed to teach effectively.

Research Questions

The practices of an expert elementary mathematics teacher emerge from a complex interaction between beliefs, knowledge, and teaching experiences. An investigation of the research literature on these aspects of preservice elementary mathematics teachers has revealed a number of findings that help preservice instructors to best prepare future elementary mathematics teachers, yet significant gaps in understanding continue to exist. Exploring ways that preservice professional development influences changes in beliefs, knowledge, and practices over the course of preservice professional development and investigating the complex dynamic among these aspects of preservice elementary mathematics teacher is the focus of this study. For this reason, the larger question I addressed in this study was:

- How does professional development affect preservice elementary mathematics teachers' beliefs, knowledge, and practice?

Research findings indicate that beliefs about mathematics teaching and learning are established early and tend to be relatively stable. Beliefs can change if professional development focuses on belief change, so it is beneficial to understand how professional development influences the beliefs of preservice teachers over the course of professional development. It is helpful to investigate the experiential contexts in which these beliefs originated and what subsequent preservice experiences influence these beliefs as well. For these reasons, I will investigate the larger question by answering a related question that documents changing beliefs and knowledge:

- Beliefs and knowledge differ depending on the amount of professional development. How?

Beliefs influence the ways preservice teachers incorporate new knowledge about mathematics teaching and learning and teaching practices are the manifestation of both. Examination of preservice teachers' formative teaching will provide important evidence to better understand changing practices, as well as the beliefs about and knowledge of teaching and learning that is embedded within those practices. For these reasons, I will this study will consider:

- Teaching practices differ depending on the amount of professional development. How?

A primary goal of preservice professional development programs is to help preservice teachers develop the content and pedagogical knowledge that will most

effectively result in sound instructional practices and positive student outcomes. Because teacher beliefs and knowledge strongly affect instructional practices, it is critical to understand how beliefs, knowledge, and teaching practices of those who enter credential programs develop and change throughout the course of preservice professional development in elementary mathematics. This research study has enhanced this understanding by pursuing the answers to these questions. While an examination of teaching practice holds important information about the beliefs and knowledge of preservice teachers, investigations on how they interpret practice holds additional information about beliefs about, knowledge of, and practice in teaching and learning mathematics.

I conducted a mixed methods study to address each of these questions through the implementation of various instruments with both large and small groups of participants who represent various stages in the course of preservice profession development. In the following chapter, I explain how these participants were chosen, what the instruments were designed to measure, how the project was implemented, and how the data were analyzed.

CHAPTER 3: DESIGN, IMPLEMENTATION, AND ANALYSIS

My experience teaching elementary students, leading mathematics professional development, and my work as a lecturer and supervisor of student teachers influenced my choice of theoretical frameworks, the research questions I proposed to address, and my research design. I am one of four lecturers in a small elementary program at a large state university who teach a year-long methods sequence which all students in the elementary credential program are required to take. As well, I am one of two people who supervise the student teaching of those involved in an optional program designed to further knowledge of mathematics content and pedagogy. The evaluative role I play as both a lecturer and supervisor required me to be particularly sensitive to the ways in which these professional roles interact with my role as a researcher.

Overview of Research Design

I implemented a cross-sectional design to approximate a longitudinal study to describe the ways in which professional development influences the thinking and practices of preservice elementary mathematics teachers from the beginning of their foundational courses required for admission into a teacher education program until the end of their student teaching. Various instruments were used to understand the beliefs, knowledge, and practices of preservice teachers at the beginning and end of professional preparation. While this study used some quantitative methods, qualitative methods predominated, including questioning, interviewing, video recording, and observation. I distinguish between large scale instruments used with all participants that represent

various stages along a professional development continuum and small scale instruments used with a subset of focus participants representing the same stages. Data from large scale instruments were intended to provide information about the following aspects of preservice teachers' beliefs about, knowledge of, and practices in elementary mathematics: 1) range and number of mathematics content courses, 2) beliefs about mathematics teaching and learning, 3) interpretations of teaching practice, and 4) personal experiences related to both teaching and learning mathematics. Data from small scale instruments were used to detail beliefs about, experiences with, knowledge of and interpretations of teaching and learning for individuals at the beginning and end stages of preservice mathematics professional development.

Large scale instruments included a questionnaire, quickwrite, and reflection on teaching. Participants from three courses representing extremes along the preservice professional development continuum completed these instruments: an undergraduate mentoring course that serves as a prerequisite course for those students considering entering the elementary credential program (Education 101), a graduate level course required of all students enrolled in the elementary credential program (Education 201), and a graduate level mathematics content course required of all students that opted to pursue additional training in elementary mathematics (Education 301). Small scale instruments included interviews, a video-recorded teaching and learning event, and video-elicited interviews about the teaching and learning event. I used these instruments with focus participants, three of whom were chosen from each of the aforementioned groups.

Focus participants, chosen from the larger group of consenting students, each completed three instruments. In semi-structured interviews, I asked participants to give accounts of teaching and learning mathematics, as well as beliefs about “good” and “bad” mathematics teaching practices. These interviews were intended to expose information about the origination of beliefs about mathematics teaching and learning, knowledge of mathematics, and understanding of pedagogical practices. I video recorded participants in a one-one-teaching and learning event with a second grade student; in addition to video recording the lesson, I observed and took field notes to document the learning segment and focus on the participant’s instructional practices. After the video recording, I conducted a video-elicited reflective interview with each participant to facilitate the participant’s explanations of what occurred during their own lesson. Data from questionnaires, quickwrites, reflections, interviews and video tapes were analyzed to examine the intersections among the beliefs about, knowledge of, and practices toward mathematics teaching and learning in individuals of varying quantity and type of pedagogical content knowledge and teaching experience.

Participants

In order to examine how professional development affects the thinking and practices of preservice elementary mathematics teachers, I considered what type of university students would reflect individuals progressing through stages along a professional development continuum. Three categories of participants were used in this study to represent stages in the process of becoming elementary mathematics teachers: 1) undergraduate students who are considering becoming teachers (Education 101

participants); 2) preservice teachers finishing the standard methods sequence while completing the second of two ten-week student teaching field placements (Education 201 participants); and 3) preservice teachers in an optional mathematics emphasis program who had completed a mathematics content and pedagogy course in addition to all the professional development of their 201 peers (Education 301 participants). Table 1 summarizes the content of courses from which these participants were chosen.

Education 101 participants. Students who are considering applying to this university's graduate level elementary credential program take a number of foundations courses before the culmination of their undergraduate education and beginning of their graduate level coursework. Students complete six courses before they begin their graduate level preservice preparation. Five of these courses are required and are open to all students at the university. University students who enroll in Education 101 do so for a variety of reasons; while many students enroll in Education 101 before considering a career in teaching, some consider this professional path as a result of their experiences in this course. Education 101 exposes students to a survey of topics pertinent to teaching elementary students, including one to two lectures devoted to issues related to teaching and learning elementary mathematics. Because this course is typically one of the first that students take to learn about issues related to teaching and learning elementary mathematics, it is an appropriate group from which to sample participants that represent the beginning stages of preservice professional development in elementary mathematics.

Education 201 participants. All students in the elementary credential program take a required three-quarter methods sequence that addresses content and pedagogy in various subject matter areas; Education 201 (Instructional Practices) meets Monday,

Wednesdays and Fridays for 20 weeks during the academic year. The subject matter area that is the content and pedagogical focus for Wednesdays changes each quarter; however, Mondays are devoted to language arts content and pedagogy and Fridays are devoted to

Table 1 *Courses from which participants were chosen for this study.*

Course	Characteristics of participants
EDUCATION 101	Undergraduates take this class as a possible prerequisite to enter the elementary credential program. Students mentor intermediate elementary students who have college potential. Students also take this class out of interest or to fulfill college graduation requirements.
EDUCATION 201	Graduate students admitted to the elementary credential program are required to take this three-quarter methods sequence. Students attend 20 weeks of teaching methods in various content areas. During each of 20 weeks, three three-hour class sessions per week focus on content and pedagogy; one session per week is devoted to elementary mathematics.
EDUCATION 301	Graduate students who have applied and been accepted into a mathematics emphasis (ME) program are required to take this mathematics content course in addition to the standard methods sequence (Education 201).

mathematics content and pedagogy during each quarter throughout the year. Because Education 201 is a required course for all students pursuing an elementary teaching credential, sampling from this group of individuals during the final quarter of the methods sequence was the only sampling option available from which to draw in order to examine the beliefs, knowledge, and practices of those individuals who are in the final stages of their preservice preparation.

Education 301 participants. Some elementary credential candidates have opted to complete additional coursework and specialized field placement experiences in order to receive a certificate of Mathematics Emphasis. For ME students, the final phase of preservice professional development culminates with a summer content course in elementary mathematics (Education 301). Students enrolled in this course represent those individuals farthest along in the course of their professional development. This continuum is represented in Figure 1.

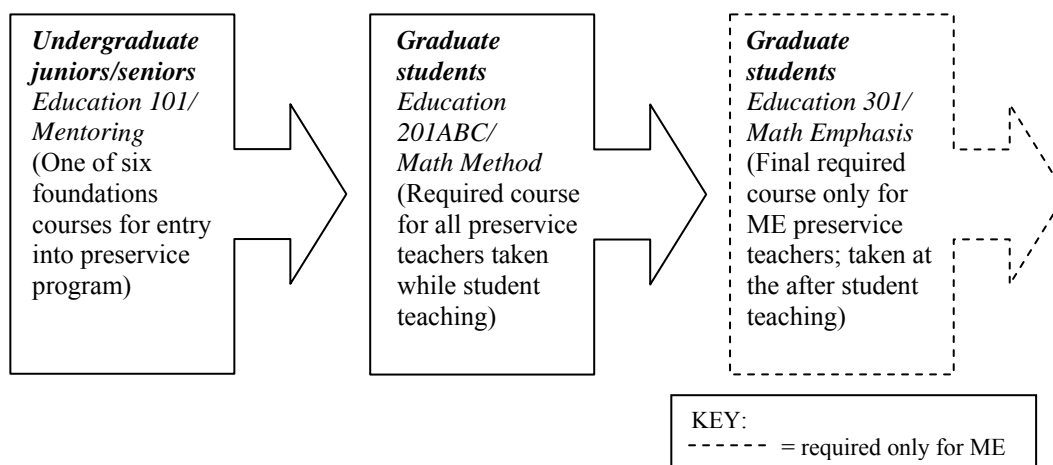


Figure 1 The course of preservice professional development in elementary mathematics

Amount and Type of Experience Working with Elementary Students

Knowledge of elementary school-aged students varies greatly from individual to individual within each group. The experiences of Education 101 students ranges from those who have spent little or no time with elementary-aged students to those who have worked as camp counselors or volunteered to work in elementary classrooms for several years.

Preservice teachers enrolled in Education 201 and Education 301 have experience of varying type and length. During this study, both the larger group of Education 201 participants and the smaller group of focus participants were finishing their second of two ten-week student teaching field placements. The larger group of Education 301 participants were finishing their second of two ten-week field placements as well; however, I did not administer the small scale instruments to focus participants from the Education 301 group until the end of summer, when their ME preparation had finished¹. Prior to entering the teaching program, Education 201 and Education 301 students were required to complete at least three courses that include a field placement component that requires a minimum of 40 hours in the field.

Depth of Content and Pedagogical Knowledge Encountered in These Courses

The amount of mathematical content and pedagogical knowledge addressed in these courses varies as well. Two lectures and follow-up section meetings for the mentoring course (Education 101) focus on elementary mathematics instruction. Credential students complete a standard mathematics methods course sequence (Education 201ABC) that meets for 20 three-hour weekly classes throughout the academic year. Student teachers admitted to the ME program take two courses in addition to the standard methods sequence. A course in advanced mathematics teaching

¹ Participants in all three groups completed three large scale instruments (questionnaire, quickwrite, and reflection on teaching). The smaller group of three focus participants chosen from each of the larger groups completed three additional small scale instruments (interview, teaching and learning event, and video elicitation on the teaching and learning event. Education 101 and Education 201 focus participants completed these instruments in the spring. To determine the effect of the additional content course on the preparation of the Education 301 focus participants, these three instruments were not administered until the end of the summer, after they had completed the content course.

practices is a seminar that includes approximately 15 three-hour class meetings that examine topics from the methods component in further depth while also helping students bridge their field service experiences with these topics; it is taken concurrently with the methods sequence. An additional course (Education 301) addresses elementary mathematics content and pedagogy in greater depth than the standard methods sequence over ten three-hour class meetings; it meets after students have completed the standard methods sequence (Education 201), the seminar in advanced mathematics teaching practices, and their second of two ten-week student teaching experiences.

Who Takes These Courses

The students who are enrolled in the courses chosen to represent stages in the course of preservice professional development vary somewhat in terms of gender, ethnic background and criteria they must meet in order to be enrolled. This information is summarized in Table 2. The criteria for inclusion in each group become increasingly more selective from the group listed at the top of the table to that listed at the bottom.

Table 2 *Who takes these courses*

Course	Who can enroll	Gender	Ethnic make up	Criteria for inclusion
ED 101	Sophomore-Senior undergraduates	75% female, 25% male	55% Caucasian, 25% Asian/Pacific Islander 15% Hispanic	None
ED 201	Graduate students accepted into credential program	95% female, 5% male	57% Caucasian, 22% Asian/Pacific Islander 10% Hispanic	Previous experience, undergraduate GPA, letters of introduction
ED 301	Graduate students accepted into both and credential and ME program	99% female	45% Caucasian 43% Asian/Pacific Islander	College mathematics courses

University Student Participant Selection

Two groups of university student participants were used for this study: larger groups of students enrolled in courses that represent phases in the course of preservice professional development in teaching elementary mathematics (the beginning, the end, and the end with additional and specialized training) and smaller groups of three focus participants that were chosen from each of these three larger groups. While a study of data collected from the large groups would give me a breadth of information about what characterizes the beliefs, knowledge, and practices of individuals from larger pools, studying individuals from each of these groups through additional instruments allowed me to explore patterns in these characteristics in greater depth.

Purposive and convenience sampling strategies were used to select the larger groups of participants. My research design used groups of individuals representing the beginning and end of preservice professional development in teaching elementary mathematics. I collected data from students enrolled in one of the foundations courses that covered course material related to the elementary mathematics content and pedagogy; there was only one course offered from which to solicit participants during the spring quarter (Education 101). The only students at the end of their preservice professional development were graduate students enrolled in the standard methods sequence (Education 201ABC). To examine the influence of additional coursework intended to advance knowledge of mathematics content and pedagogy, I collected data from Mathematics Emphasis (Education 301) participants at two different junctures: large scale data was gathered from all ME students at the same time that I gathered this data from their non-ME graduate student peers (spring, 2006) and small scale data from

focus participants at the conclusion of their final ME content/pedagogy course. Possible issues related to instrument reliability and validity arise, considering that large scale data were collected from these focus participants three months prior to small scale data instrument implementation; likewise, data gathered from small scale instrument implementation may have been influenced by maturity during that three month time span. These issues will be addressed in sections devoted to instrument design and implementation.

I selected focus participants through purposive sampling strategies. Specific criteria included input from instructors and teaching assistants, as well as proximity to and familiarity with the elementary school site used for the video-recorded teaching and learning event. Having three participants from each group (Education 101, Education 201 and Education 301) allowed me to search for trends among groups of participants.

Undergraduate Students from Education 101: Participant Selection

I used purposive sampling strategies to select the larger group of Education 101 participants. This course content addresses pedagogical practices in teaching reading, writing, mathematics and science, as well as educational issues such as diversity, equity, classroom management, and the effect of the mentor. Lectures and sections are designed to engage students in experiences related to the course content and course requirements include active participation in these contexts. At the time when data were collected for this study, this course was the only foundations course (one of the prerequisite options students could choose from before applying to this university's elementary credential program) in which potential candidates could enroll that addressed content and pedagogy

related to teaching elementary students. As such, this was the only group I could use to represent those individuals at a beginning stage of preservice professional development in mathematics.

While I do teach the course from which these participants were chosen (Education 101), I was not the instructor during the quarter in which this project was implemented so I did not personally know any of the enrolled students. While the instructor taught lectures on a weekly basis, he had far less contact with or specific information about individual students than the teaching assistants who conducted the weekly section meetings immediately following the lectures. For these reasons, I sought input for focus participant selection from the teaching assistants. Because the active participation required in Education 101 is a relatively unusual practice in the university classroom, I asked the teaching assistants to consider which students expressed interest toward and enthusiasm about course content and field experiences. Teaching assistants gave me the names of six students who fit these criteria and who might be willing and able to serve as focus participants.

I began my focus participant selection by contacting two students who were recommended by teaching assistants and who also had field placements at the school site attended by the elementary students who were a part of this study. While these Education 101 students were placed in intermediate (fourth through sixth grade) classrooms rather than in the second grade classroom used for this study, these Education 101 undergraduate student mentors were familiar with the school site and would likely be more comfortable participating in this project than Education 101 students placed at another school. In addition to these potential participants, I contacted two other

individuals recommended by teaching assistants but placed in field placements at other participating Education 101 schools. I emailed all potential participants to briefly remind them about my study and arranged informal meetings during a time and at a place chosen by the potential participants. I began each meeting by thanking potential participants for their time, inquiring about their experiences, and briefly explained the study. At the end of each meeting, I inquired about each person's willingness and desire to participate. Three out of the four individuals showed immediate interest and were willing to participate; these individuals were chosen as Education 101 focus participants. All participants are identified by pseudonyms names and course numbers. Characteristics of these individuals are summarized in Table 3.

Sampling Implications: Education 101 Participants

Prior to introducing my project to these students and obtaining their consent to participate, I had given a guest lecture during which I was introduced as a faculty member from this university's teaching program. Because some of the Education 101 participants were potential applicants to the credential program at which I work, it is possible that these participants may have felt some pressure to comply with my request for participation to ensure their successful admission.

Graduate Students from Education 201: Participant Selection

My relationship with the other participants was different than the relationships I had with the Education 101 participants. All candidates accepted into this university's elementary credential program are required to complete a three-quarter sequence in

Table 3 *Education 101 focus participants*

Name/ Identifier	Age	Gender	Ethnicity	Major	Year	Characteristics
DA/Dalene Education 101 #1	23	female	Asian	Human Development	Jr	Will apply to elementary credential program next year
HS/Hilary Education 101 #2	21	female	Caucasian	Political	Jr	Plans to become a school psychologist at elementary level
TN/Tiffany Education 101 #3	21	female	Asian	Human Development	Jr	Will apply to elementary credential program next year

teaching methods; during this sequence, students examine subject matter content and pedagogical practices that are appropriate for the elementary classroom. As a lecturer for this teaching program, I teach all Education 201 students during their year-long language arts methods sequence. In sum, all 201 students know me as a language arts methods instructor.

Purposive sampling strategies were used to enlist the larger group of Education 201 students as participants in this study: these individuals were the only group of elementary preservice teachers at this university that were finishing their preservice professional development in mathematics. Logistical concerns necessitated use of convenience strategies to choose the smaller group of Education 201 students as focus participants for this study. Data from Education 201 students was collected during the middle part of the spring quarter of the academic year. In addition to those assignments associated with completion of M.Ed. degree requirements, student teachers are required to assume responsibility for planning and implementing all lessons for the last two to

three weeks of their student teaching placement as part of their credential program requirements. Lesson planning and M.Ed. assignment completion fill much of student teachers' time after their teaching day has ended; in addition to these responsibilities, most student teachers are researching and applying for teaching positions during the spring quarter. While I needed to be sensitive to the demands that consume much of the graduate student participants' time, it was also important to consider the elementary-aged students who would be involved in the teaching and learning event. In order to make these second grade students as comfortable as possible during a simulated learning experience, it was important to schedule these video-recorded sessions during or immediately following school hours. Because of these various factors, I solicited the participation of Education 201 student teachers currently assigned to field placements in schools located within a thirty minute drive from the campus at which these second grade students were enrolled. I contacted three student teachers who had demonstrated interest during the explanation and implementation of the large scale instruments. The first three student teachers I contacted agreed to be Education 201 focus participants and are identified in Table 4.

Sampling Implications: Education 201 Participants

My most recent relationship with the larger group of 201 participants was primarily as one of three methods instructors. In this role, I contribute evaluative information about their preparation to that of two other instructors for a final course grade. All elementary credential candidates had finished their final quarter of methods courses during the time in which this study was conducted. While all elementary

Table 4 *Education 201 focus participants*

Name/ Identifier	Age	Gender	Ethnicity
KE/Karen Education 201 #1	24	female	Caucasian
SJ/Sarah Education 201 #2	24	female	Vietnamese
BA/Belinda Education 201 #3	23	female	Caucasian

credential candidates still attended courses at the university to complete other requirements necessary for professional certification, my evaluative role as a methods instructor had terminated at the point during which their involvement was solicited. While they may have felt some pressure to participate due to the evaluative role I played earlier in the academic year, I assume this pressure was minimal at the time I asked them to participate.

Similar to the relationship that characterized my connection to the larger group of Education 201 participants, my most recent relationship with this smaller group was as a methods instructor. At the time of data collection, all preservice students were completing their second of two student teaching field placements. While my evaluative role with them had ended at the point at which this study was conducted, these individuals may still have felt some pressure to be involved in this project due to this previous relationship. Because I did not supervise any of these three individuals during their field service student teaching, I was not responsible for evaluating this aspect of

their professional preparation so it is unlikely that my role as a student teaching supervisor had any impact on their decision to be involved.

Graduate Students from Education 301: Participant Selection

Students enrolled in Education 301 are preservice teachers who had apply to and are selected for admission into the Mathematics Emphasis program that requires additional coursework and specialized field placements in addition to the satisfactory completion of the Education 201 methods sequence. Because these students were the only group who represented those at the end of the preservice preparation with additional experience in teaching and learning elementary mathematics, purposive sampling was used to select from this larger group of Education 301 participants. Other issues complicated my choice of Education 301 focus participants as well. These participants needed to complete a course in mathematics content and pedagogy in early August. Supervising these individuals during their student teaching made me aware of some details of their personal and professional lives that would impact their involvement in this research project: some were getting married, some were moving from the area soon after their final course ended in order to begin their careers as elementary teachers, some were uncertain about their future, and some had procured positions in the county and were sure they were remaining in the area once their university responsibilities had ended. Five out of eleven Education 301 students were remaining in the local area; I contacted individuals to inquire about their willingness and ability to be involved and chose the first three individuals who agreed. These individuals are represented in Table 5.

Sampling Implications: Education 301 Participants

In addition to my teaching responsibilities, I am a supervisor of student teachers; because of my experience providing mathematics professional development outside of this university setting, I am responsible for supervising Education 301 students. All student teachers have two different field placements. For both placements, Education 301 students completed their student teaching with a cooperating teacher that had specialized expertise in teaching elementary mathematics. Due to my supervision case load and the number of students in the ME program, I supervised the Education 301 students during one of their two student teaching placements. At this point in these individuals' preservice preparation, my evaluative role as a supervisor was no longer a factor. All preservice courses that involved graded assignments and student teaching evaluation had ended; because most students exiting our program request supervisors to

Table 5 *Education 301 focus participants*

Name/ Identifier	Age	Gender	Ethnicity
GM/Gabrielle Education 301 #1	25	female	Hispanic
RH/Reese Education 301 #2	30	female	Caucasian
BA/Brianna Education 301 #3	25	female	Caucasian

either write letters of recommendation or to serve as a reference during the job application process, participants may have felt some pressure to serve as participant to increase the likelihood of being hired as a teacher.

Elementary School Students

In addition to the other participants, nine second grade elementary students were involved in this study for the implementation of one instrument: a video-recorded teaching and learning event during which each focus participant was paired with one second grade student, gave the elementary student some word problems to solve, then supported the student in solving these problems. Due to the varying amount and type of experiences that participants have had with elementary students and teaching elementary mathematics, it was important to choose a) a grade level where mathematic content would be interesting with which to work yet comprehensible to the focus participants, and b) students from a classroom in which solving word problems was a familiar experience.

I chose a second grade class of students who had opportunities to use their own strategies to solve word problems. I asked the teachers of these students to choose individuals that would be comfortable working in a teaching and learning event with someone they had not previously met. To ensure that the elementary students would feel comfortable participating in this project and being video-recorded, I visited the class on three occasions to work with students and observe their classroom routine to ensure that the elementary students would be familiar with me on the day of the teaching and learning session. During one of my visits, I introduced myself to the students to briefly explain the study to them. I informed them about my former work as a second grade classroom teacher in this district as well as my work helping college students become elementary teachers. I explained my interest in mathematics and my desire to learn more about how student teachers learn how to be teachers of elementary mathematics. I

composed a letter to inform the parents of these students about my research project. In addition, the classroom teachers had informal conversations with these parents to convey their support of my project. The letter and consent form were sent home with each student in this second grade class. Nine second graders, along with their parents, gave written consent to participate in this study.

Data Collection Procedures

For each research question, multiple data sources were used to ensure that outcomes were confirmed across triangulated sources. Large scale instruments were administered to entire classes of consenting students who were enrolled in preservice courses completed by individuals at various points during professional development in becoming elementary mathematics teachers. These instruments consisted of a questionnaire about the type and number of courses taken in high school and college, attitudes about the teaching and learning experienced in each of these courses, and attitudes about and beliefs toward mathematics teaching and learning; a quickwrite that required students to provide a written response to prompts that focused on the ease/difficulty required to teach and learn mathematics; and a brief video clip of mathematics teaching, followed by form that elicited a written reflection on teaching to capture interpretations of what was noteworthy, potential teaching decisions that could be made, and evidence of student understanding.

In addition to the large scale instruments, three small scale instruments were used with each of three focus participants selected from each of the larger three groups: an audio-recorded interview about experiences both teaching and learning mathematics that

included explanations of how these experiences affect feelings and thoughts about mathematics teaching and learning; a video and audio-recorded teaching and learning event with one elementary student; and a video and audio-recorded video elicitation session about the teaching and learning event during which the participant viewed clips from the teaching and learning session and stopped the recording at points to talk about what she was thinking at the time. Data were gathered from all instruments to better understand the beliefs, knowledge, and practices of preservice elementary mathematics teachers as they progress through the course of professional development in teaching elementary mathematics. The groups of instruments (large and small scale), what was included in them, and the participants who completed them are summarized in Table 6.

Table 6 *Instruments and participants*

Instrument Group	What Was Included	Participants Who Completed
Large Scale	Questionnaire Quickwrite Reflection on Teaching	All consenting Education 101, 201 and 301 participants
Small Scale	Interview Teaching and Learning Event Video Elicitation on the Teaching and Learning Event	Three focus participants chosen from each of the large groups: Participant 101 #1, #2, #3 Participant 201 #1, #2, #3; Participant 301 #1, #2, #3

Large Scale Instruments

Midway through the spring quarter of the 2006 academic year, I presented an outline of my study to the students enrolled in Education 101 and Education 250 (because Education 201/methods had ended at the time data collection began, it was necessary to

work with the same participants enrolled in a different class that continued to meet through the project implementation). I contacted the instructors for these courses, explained my project to them, and requested time during a few class sessions during which I could 1) introduce my research project and obtain consent from students to be involved as both participants and focus participants; 2) explain the general content of and give direction for completing the questionnaire; and 3) introduce a video recorded clip of an elementary mathematic teacher, play the clip and then have participants complete a structured written reflection on teaching to account for their interpretations of what they viewed.

Instructors for both courses provided me five to ten minutes of class time to explain my project and request consent from students participate in my project. I briefly explained my project, each of the three large scale and small scale instruments I intended to use, and the amount of time it would take participants in each group to complete each instrument. I explained the process for giving me consent to participate, then handed out forms to all class members. Twenty-two participants from Education 101, 31 participants from Education 201/250 and 10 participants from Education 301 signed consent forms agreeing to participate in this study. Participants kept participant information sheets.

Questionnaires

I constructed a two-part questionnaire (see Appendix A) to gather various types of information about these larger groups of participants. The first part focused on the levels and types of math classes taken in high school and college, general perceptions of the content and teaching practices experienced in each class, and descriptions of memorable

math experiences and qualities of effective mathematics teaching. The second part required participants to indicate the degree of agreement toward 28 statements about mathematics teaching and learning, as measured on a Likert scale. The validity of these statements were tested in previous studies (Hart, 2002; Schoenfeld, 1989; Zollman & Mason, 1992). The questionnaire was field-tested with approximately 45 graduate students enrolled in the single-subject credential program prior to implementation. Upon viewing these responses and considering feedback from those completing the initial questionnaire, the questions and format were revised prior to implementation with participants.

Questionnaires were administered two slightly different ways to the different groups. Whereas Education 201/250 students (in which Education 301 students were subsumed) completed questionnaires during the same class session in which the project was explained and signed consent was obtained, questionnaires were completed by Education 101 students approximately two weeks after the introduction of my project. In each instance, I explained the format of and content contained in parts of the questionnaire. Questionnaires were handed out to Education 201/250 students during class; Education 101 students received these questionnaires during their section meeting. Participants were given approximately 15 minutes to complete the questionnaire. To ensure instrument validity, I reminded participants in both groups that this task was intended to be completed individually by each participant without discussing questionnaire items or responses with classmates. After collecting questionnaires, I informed students that they would receive additional information about other instruments at other class sessions. Thirty-one questionnaires were completed by Education 101

participants, 33 by Education 201 participants, and nine by the Education 301 participants.

Quickwrite

In order to examine ideas about the ease/difficulty of teaching and learning mathematics, I wrote two prompts to which participants responded in writing. These prompts were intended to capture first impressions about knowledge, beliefs, and attitudes associated with how difficult it is to both teach and learn mathematics content through relating an experience that contextualized these impressions. Prompts were field tested in an email exchange with approximately ten of my colleagues, peers and friends; I examined written responses and adjusted subsequent prompts to more accurately capture beliefs and experiences about teaching and learning mathematics (see Appendix B).

Quickwrites were implemented in different ways depending on participant group. Students in Education 101 complete required weekly written journals within which they reflect on and integrate their field experiences as mentors of elementary students, content addressed in lectures and sections, and assigned readings. For one of the two weeks during which lecture and section content focused on teaching and learning elementary mathematics, the course instructor posted these prompts as the alternative to the journal prompts, explaining that students should spend approximately ten minutes writing responses to the prompts. The following week, students turned in their journals for grading. I made copies of those journal pages containing responses to the quickwrite before teaching assistants collected them for grading.

A different process was used to obtain quickwrite responses from the Education 201 group. At a previous Education 250 class session, I explained my research project and obtained signed consent; I then handed out the questionnaire, briefly explained the intent and format, and collected them upon completion. Before leaving this class session, I explained the process for completing the quickwrite. I informed participants that they would receive the quickwrite in an email and I requested them to spend no more than a total of ten minutes writing their answers to both questions before emailing responses back to me. While I received approximately one third of these participants' quickwrite responses within a week, I provided weekly reminders through email and in teaching seminars to ensure receipt of additional data. I stopped providing reminders four weeks after the initial solicitation. Seventeen quickwrites were completed by Education 101 participants, 20 by Education 201 participants, and eight by Education 301 participants.

Reflection on Teaching

The final large scale instrument involved having participants view a predetermined video clip of exemplary teaching practice before completing a structured reflection on teaching that required participants to 1) call out noteworthy aspects of teaching practices, 2) describe what occurred, 3) interpret the actions of both teacher and students, and 4) explain appropriate "next steps" and a rationale for them (see Appendix C). A number of criteria were used to select an appropriate video clip, including duration of the clip, the mathematics content addressed by the teacher in the clip, and age/grade level of the students featured. Participants viewed this clip during seminar and lecture sessions. Since course instructors would be adjusting their course schedules to

accommodate the implementation of this instrument, the duration of the clip, followed by time to record written reflections about what participants viewed needed to be taken into account. Because some of these participants would later be chosen as focus participants and would be involved in a video-recorded teaching and learning event working on word problems with a second grade student, it was important to choose a clip in which a teacher worked with students from a grade level other than second grade, teaching content from a topic other than number and operations. I selected a clip of a teacher introducing fraction concepts to fifth grade students using of pattern blocks.

Both Education 201/250 and Education 101 participants were introduced to this instrument during a class session that followed the introduction of my project and the completion of the questionnaire. Because pattern blocks were used in the actual clip, I put some each table at which students were sitting. I explained that they would view a clip of a fifth grade class working on fractions and described what had occurred prior to the clip I had selected for them to view. I played the clip, then passed out the reflection on teaching sheet after the clip had played. To minimize the influence of a question and/or answer on a question/answer that proceeded, I asked participants to read and answer the questions in order that the questions appeared. It took participants approximately fifteen minutes to complete these reflection forms. This instrument was implemented with Education 101 participants in a similar fashion; however, the experience was contextualized within a lecture that addressed teaching fractions. I followed the sequence for the implementation of this instrument in a similar manner as I had done with the previous group. Nineteen reflections were obtained from Education 101 participants, 28 from the 201 participants and eight from the 301 participants.

Small Scale Instruments

Three instruments were used with focus participants. They were designed to provide additional, more comprehensive understanding of how the thinking and practices of preservice elementary mathematics teachers change through professional development. They included an interview, a teaching and learning event, and a video elicited reflective interview about the teaching and learning event.

Interviews

I designed a protocol to guide individual, semi-structured interviews intended to elicit experiences about both teaching and learning mathematics that influence current thinking about and feelings towards this content area. This interview protocol was field-tested with some of my teaching colleagues and revised in order to more accurately elicit beliefs about, knowledge of, and experiences teaching and learning mathematics. I began the audio-recorded interview sessions by asking the participant to think about some experiences related to teaching and learning mathematics; as the interview progressed, I elicited positive and negative experiences (see Appendix D). At the end of each interview, I provided each participant with a set of the word problems they would use in the teaching and learning event with a second grade student. I explained that we would discuss one or two of these problems before that session began, when I would ask them to discuss noteworthy characteristics of the problem and make predictions about what the student might do when given this problem to solve. I kept field notes to track information not captured by audio recordings, such as interruptions in recording, participant gestures, and my unvoiced responses to or questions about participant input.

Video and Audio Recordings of a Teaching and Learning Event

Prior to the video-recorded sessions, the participating teachers suggested I distribute an informal letter to explain my project to the parents, rather than hold a more formal meeting to accomplish the same end. I wrote and submitted a letter for their review; they provided feedback and revisions to this letter were made. During this time, I visited the classroom on three occasions so that the students would be familiar and comfortable with me. On my first visit, I reminded students that had I visited their classroom before, when the Education 301 participants and I observed their teachers model a mathematics lesson. I explained my interest in learning and teaching mathematics to these second grade students and briefly explained how my research project would help me learn how student teachers learn how to be good math teachers. I explained that I hoped to work with some of them by having them work on some word problems with a student teacher. I visited these students on two other occasions. During these times, I circulated the room, engaging in informal conversations with students while they worked. The cooperating teachers volunteered to distribute my revised letters of explanation in person to the parent(s) of those children they believed would be willing and able to work with a relative stranger (Education 101, 201, and 301 focus participants). The parents of nine children consented to have their child participate.

Focus participants were contacted by phone or email to schedule a one-hour block of time during which they could be involved in the teaching and learning event with a second grade student. These video recordings were scheduled to take place after the interview so that the teaching and learning event would not influence the content of the

interview. All video recording sessions for Education 101 and Education 201 focus participants were scheduled during the latter half of the spring 06 quarter; Education 301 focus participants were scheduled to be video-recorded subsequent to their completion of the final mathematics content course during the summer 06 quarter.

On scheduled video recording days, I set up audio and video recording equipment in an unused classroom at the school site attended by the second grade students. I met with the focus participant prior to the session to briefly discuss one or two of the word problems and possible methods that the student might use to solve them. These sessions were audio recorded. I reminded the participant to read the word problem to the student, make sure the student understood the problem, and support the student in solving the problem without telling him/her the answer. I asked each participant to address the problems in order. The problems were roughly ordered by difficulty to ensure that the second grade student would feel initially feel confident toward the problems and this event. I marked problems that I wanted to participants to do first and other problems were explained as optional, as long as time permitted and the second grade student wanted to continue. I took notes about this conversation with each focus participant.

After this conversation, the participant and I walked to the classroom. The teacher chose one of the consenting second grade students with whom the participant would work. The second grade student, preservice teacher participant, and I had an informal conversation prior to the video recording so that the participant and second grade student would feel as comfortable as possible before the recording began. I reminded the student about the video recording process, then filmed the student so that s/he could view her/himself prior to the actual recording. I informed the student that I

would be audio and video recording her/him solving word problems with the student teacher similar to the way they solved problems in their own classroom. I provided some information about the equipment used for this event, pointing out the camera, the microphone, the digital audio recorder and the placement of my chair during the recording. I read the student information sheet and the student assent form to the student and received signed assent from each student. Base ten materials (place value manipulatives constructed to represent ones, tens and hundreds) were on the table and available for the student to use, along with blank paper and markers.

Each preservice teacher worked with a second grade student for between 20 and 35 minutes. At any point during the video recording, if the second grade student wanted to stop working, the teaching and learning session was terminated. This termination occurred with one student about 20 minutes into the event. During the teaching and learning session, the student was asked to represent solution strategies on paper. At the end of each teaching and learning event, I asked the student if I could keep the student work and each student consented. I kept field notes to track information not captured by the video recordings, such as interruptions in recording, participant gestures, and my unvoiced responses to or questions about participant input.

Video and Audio Recorded Video Elicitation on the Teaching and Learning Event

Prior to the video elicitation interview session with each participant, I reviewed the video recording from the teaching and learning event to select segments for the participant to watch and discuss. I selected segments within which the participant seemed particularly engaged with the second grade student, either guiding the student to

successfully solve a problem that proved to pose a particular challenge or questioning the student about a specific process or procedure used to solve the problem. To keep the video elicitation session within the promised length of one hour, I selected two to three segments that ranged from two to seven minutes. I wrote a protocol for the video elicitation session. In addition to questions that focused on these preselected clips, the protocol also included a request for information about how the participant prepared for the teaching and learning event ahead of time, an examination of student work not addressed during the selected clips, and an elicitation of new information about mathematics teaching and learning gained as a result of participation in both the teaching and learning event and the video elicitation interview (see Appendix E for an example of one video elicitation protocol).

Within two weeks after the video-recorded teaching and learning event, I met with each Education 101 and each Education 201 participant to conduct a video elicitation (VE) interview session. The VE interviews with Education 301 participants ranged one to four weeks following the teaching and learning event due to scheduling conflicts brought about by participants beginning new jobs as classroom teachers. Video elicitation interviews were held in a variety of places to accommodate participant preferences. These locations included my office or a classroom at the university, the participant's own classroom, and my home. At the onset of this interview, I engaged the participant in conversation to help them feel comfortable with me in order to make this experience as natural a conversation as possible. I explained the placement of the camera, microphone, laptop computer, and audio recorder. I arranged the camera to

capture the laptop screen image as the video recording of the teaching and learning event played.

I began the formal part of the interview by asking about the way in which the participant prepared for the teaching and learning event. I then explained the process for reviewing the clips I had selected. The participant and I would view a segment without commenting and I would then rewind the clip back to the beginning; during the second viewing, I asked the participant to stop the recording at any point during which she would like to tell me what she was thinking. Each VE interview progressed in this manner. At times I would ask additional questions to clarify my own confusion about a participant's explanation. I video and audio recorded each of these video elicitation sessions. I kept field notes to track information not captured by audio or video recordings, such as interruptions in recording, participant gestures, and my unvoiced responses to or questions about participant input.

Data Analysis Procedures

Data gathered from both large and small scale instruments included were reduced in number of ways. Both qualitative and quantitative methods were used to search for patterns that characterized points in the course of preservice professional development in teaching elementary mathematics. These procedures are described according to the scale of the participant group.

Data Reduction: Large Scale Data

Three instruments were used to understand beliefs and attitudes about, knowledge of, and practices toward learning and teaching elementary mathematics of the larger

group of Education 101, Education 201, and Education 301 students. They included a questionnaire, a quickwrite, and a reflection on teaching. Procedures used to reduce the data from these instruments are described in the following sections.

Data Reduction: Questionnaire

Participants were asked to respond to 28 statements designed to measure beliefs about teaching and learning mathematics. Fifteen statements required participants to identify the degree to which they agreed with a statement and 13 statements required participants to identify the degree to which a statement was true. A four-point Likert scale was applied to each set of statements. Responses for each statement were compiled for each participant within each of the three participant groups. I found the mean response for each statement for each group. I adjusted scores to accurately reflect a unidirectional trend: a score of '1' being most reform-oriented and a score of '4' to be the least reform-oriented. I figured the difference between the mean and one (the most reform-oriented stance) by statement for each group, as well as the standard deviation for each statement by group. Statements were compared from group to group to examine differences in central tendency between participant groups that represent points in the course of professional development.

Data Reduction: Quickwrite

The larger groups of participants were asked to complete a ten minute quickwrite responding to two prompts that asked them to consider how easy/difficult it is to learn

mathematics and how easy/difficult it is to teach mathematics, along with an account to illustrate each response.

I created a 1-5 scale and applied a rating to score statements of ease/difficulty. If the participant wrote that mathematics was easy to learn/teach, that portion of the quickwrite was given a score of 1; if the participant wrote that mathematics was both easy and difficult to learn/teach, that portion of the quickwrite was given a score of 3; and if a participant wrote that mathematics is difficult to learn/teach, that portion of the instrument was given a score of 5 (If a participant commented in a quickwrite that mathematics was fairly easy to learn/teach it was given a '2' and fairly difficult to learn/teach, that part of the quickwrite received a score of '4').

Participants were listed in a spreadsheet and two scores were entered for each participant: one score for the responses regarding difficulty/ease of learning mathematics and one for difficulty/ease of teaching mathematics. Writing from the nine focus participants was entered into HyperRESEARCH and these quickwrite responses were coded for the following:

- ease/difficulty learning elementary mathematics,
- an experience that illustrates ease/ease learning mathematics,
- ease/difficulty teaching elementary, an experience that illustrates ease/difficulty teaching mathematics,
- beliefs and attitudes about teaching or learning mathematics,
- knowledge of students, knowledge of pedagogy,
- statements about understanding, and

- vocabulary specific to teaching in general and/or teaching mathematics in particular.

In addition to two scores related to expressed ideas about the difficulty/ease learning/teaching mathematics, quickwrites from the larger groups of participants were coded for language used when detailing the experience that was illustrative of their ease/difficulty in learning/teaching mathematics.

Data Reduction: Reflection on Teaching

Six prompts/questions were included in the reflection on teaching that was completed by the large groups of participants. The response to one question was particularly valuable for analysis: Question 1: Everyone views instructional situations differently. What three aspects of this video did you find noteworthy?

I reduced data from responses to this question/prompt in two ways. I used the Trajectory of Development on Learning to Notice (Sherin and Van Es, 2005) as a guide (see Table 7) and constructed a rubric to categorize the responses that participants gave to this question (see Table 8).

I applied a score of one to a response if participants described and/or evaluated what they noticed, a score of two if the response included description and/or evaluation and general evidence from the video, and a score of three if the response contained description and/or evaluation and specific evidence from the video. Each participant received three scores, one for each response; I calculated the mean score for each participant and for each group. In addition, I listed the vocabulary that participants used when writing about what was most noteworthy. I examined the vocabulary that was used

to describe noteworthy aspects found in the clip and to determine patterns in the type of words chosen to characterize learning and teaching.

Table 7 *Trajectory of development on learning to notice*

	Level 1	Level 2	Level 3	Level 4
Dominant pattern in writing	Describe and evaluate	Mixture of describe and evaluate and complete analytic chunks or Incomplete analytic Chunks	Complete analytic chunks and Evaluate	Complete analytic chunks Connections among call-outs and evidence Identify pedagogical decisions

Table 8 *Rubric used for reflection on teaching, question 1*

Score	Characteristics of writing
1	Describe and evaluate
2	Describe, evaluate, provide general evidence
3	Describe, evaluate, provide specific evidence

Data Reduction: Small Scale Data

Initial data analysis focused on searching for patterns in the data generated by three small scale instruments with each of nine focus participants: audio recorded interview accounts of teaching and learning mathematics, as well as beliefs mathematics teaching practices (interview); video and audio recordings of a teaching and learning event between a participant and a second grade student; and video and audio recordings from a video elicited interview about the teaching and learning event (VE).

Data Reduction: A First Pass through the Small Scale Data

Each of the three small scale instruments was analyzed multiple times to ensure instrument validity and reliability. After listening to each audio recorded interview two to three times, I created complete transcripts of each interview. I watched and listened to video recordings from the teaching and learning events and video elicitations on the teaching and learning events three to four times; these data sources were transcribed, indexed and annotated. Thus, I transcribed the audio and video-recorded data and made multiple passes through each data source to define, create, revise, and apply codes.

At the beginning of small scale instrument data analysis, I made a thorough examination of the first instrument implemented by group, beginning with the interviews from the group representing the beginning phase of preservice professional development in mathematics teaching and learning (interviews from focus Participants 101 #1, #2, #3) before looking at the data from the same instrument implemented with the group representing the end of preservice preparation (interviews from Participants 201 #1, #2, #3); finally I analyzed the same instrument data from the group representing those who pursued additional mathematics professional development beyond that from the standard methods sequence (interviews from Participants 301 #1, #2, #3). This process was completed with each successively implemented instrument (interview, teaching and learning event, and video elicitation) for one group at a time and in order (Education 101, 201, 301), to get a sense of the trajectory of development as represented by data from each instrument. These nine data sources were transcribed and annotated to get a sense of the characteristics of each individual representing a point in the course of professional development and to guide my search for patterns when examining the remaining

interviews, teaching and learning events, and video elicitation sessions for other individuals.

Data reduction: interviews. Interviews with nine focus participants were transcribed using voice recognition software (Dragon NaturallySpeaking 7). Snapshot descriptions of each participant and each participant group were written upon completion of each transcription to capture my impressions about beliefs, knowledge and practices in teaching and learning mathematics. Transcripts for each interview were entered into HyperRESEARCH; codes for beliefs and attitudes, content knowledge, pedagogical knowledge, and understanding were developed, tested, defined, revised, and applied.

Data reduction: Teaching and learning events and video elicitation of the teaching and learning events. Video recordings of the teaching and learning events from focus Participants 101 #1, 201 #1, and 301 #1 were uploaded into InqScribe software. Talk between each of these participants and the second grade student with whom each worked was indexed, annotated and transcribed; annotations included observation notes about the participant's body movements, any interruptions to the teaching /learning event, and my spontaneous impressions of and questions about the participant's teaching decisions. Video recordings of the elicitation on the teaching and learning event from Participants 101 #1, 201 #1, and 301 #1 were entered into InqScribe software as well. I annotated transcripts with impressions of the participant's responses. Text from these six data sources were entered into HyperRESEARCH. The following ten codes were developed, tested, defined, revised, and applied to the transcribed text: 1) beliefs and attitudes; 2) content knowledge; 3) pedagogical knowledge; 4) teacher comments on student knowledge; 5) teacher comments on student notation; 6) teacher comments on

student processes; 7) teacher comments on teacher's role; 8) teacher questioning student; 8) teacher redirecting student; and 10) teacher suggestions to students.

I viewed the video recordings of the six remaining teaching and learning events and corresponding video elicitation on the teaching and learning events (from Participants 101 #2 and #3; Participants 201 #2 and 3; and Participants 301 #2 and #3). I wrote indexed field notes while viewing these recordings of the teaching and learning events and video recordings of the video elicitation sessions from the remaining participants to capture impressions of themes that emerged from viewing. The purpose for this sequence was to develop a more detailed depiction of each participant as a preservice elementary mathematics teacher.

By examining each data source in the order that the data was collected, I could get a sense of how the data gathered from one instrument is informed by and contextualized within the experiences provided by the implementation of the next instrument in the sequence. In the next stage, looking at the entire corpus of small scale instrument data from one participant would give me a more detailed picture of each participant as a preservice elementary mathematics teacher, while serving to better define that point in the course of professional development more fully before moving onto the data from the next individual. In sum, this next stage in small scale data analysis was intended to both define each participant as a professional and further define each point along the continuum before moving onto the next.

Creating and revising coding categories: interviews. A priori codes were developed upon review of the literature that focuses on the knowledge, beliefs and practices of preservice elementary mathematics teachers (see Table 9). I created code

definitions through applying them to the interviews with focus participants about their beliefs, experiences in, and practices with teaching and learning mathematics; constant comparison was used to revise the accuracy of definitions. As I coded each interview, the definitions for each code became more specific. Key words were identified and expanded and examples for definition features were identified and added. During this second round of analysis, data was examined in a different sequence.

Table 9 *Interview coding categories and definitions*

Coding	Definition
beliefs and attitudes	expressed feelings about experiences teaching or learning mathematics, whether positive or negative, as well as expressed rationale for teaching practices
content	references to mathematics topics, such as algebra, or subtopics, such as multiplication tables
pedagogical knowledge	discussion of which instructional practices were used or abandoned in order to teach content, including reference to pacing, materials, tools, and preparation
knowledge of students/ understanding	comments about whether or not student understanding was assumed or evident

Rather than examine each type of data (e.g., interviews from Participant 101 #1, #2, #3, Participant 201 #1, #2, #3, Participant 301 #1, #2, #3 before moving on to examine teaching and learning events and video elicitation in this order), I examined the entire corpus of data from a participant (e.g., interview data from participant 101 #1, video and audio recording data from participant 101 #1's teaching and learning episode, and participant 101 #1's video and audio recording of the video elicitation session about

the teaching and learning event). Cycling through each participant's data occurred two more times with data from the second and third participant from each group: I examined data gathered from a participant at the beginning point on the preservice professional development continuum and ending with an examination of the same data sources from participant representing the end of preservice professional development with an emphasis in mathematics teaching and learning. These cycles are represented in Figure 2.

Using HyperRESEARCH, I created reports of the interviews from each of the three participants that represented groups on the professional development continuum. These reports were organized by the following codes: beliefs and attitudes, content, pedagogy and understanding. I read the coded interview reports for one group at a time and listed patterns I found across individuals within each group. This process was repeated for a participant group for each code. Once patterns were listed, I read through each report, highlighting text that corresponded to one or more of the listed patterns (See Figure 3).

All highlighted text from interview reports by code was read for patterns; four to seven patterns were found in each report and patterns differed across groups. Text in each report was highlighted according to these codes. Highlighted text for each pattern within each code within each group was entered into a spreadsheet. I read across each row/pattern in order to synthesize/summarize the row of coded text that corresponded to the text highlighted for that pattern. This synthesis/summary was reported in the final

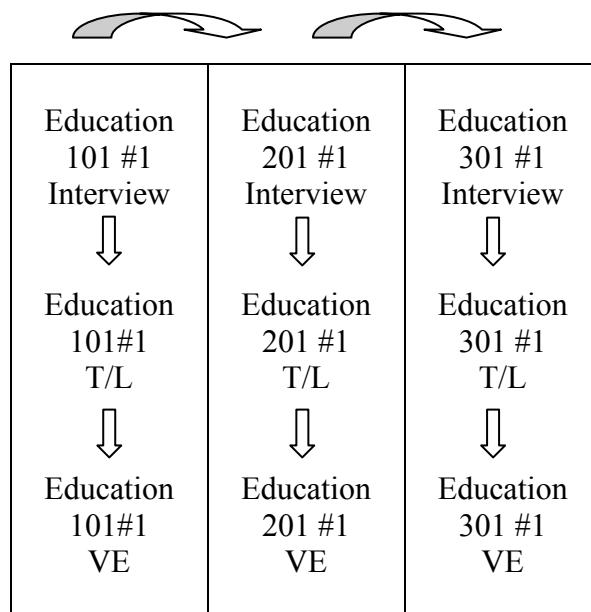


Figure 2 Sequence for initial data analysis, first round

column for each worksheet. I printed out each worksheet and summarized each worksheet by code and by group; I report these summaries in the section that details outcomes of interview data reduction.

Teaching and learning events. During initial examinations of video data from the teaching and learning Events, I continually revised transcripts to thoroughly and accurately document talk between teacher and student as well as participant behavior (e.g., counting out math manipulatives, gestures included in explanations, change of voice volume). I annotated transcripts when I viewed instances in which I questioned the purpose and/or intent behind enacted instructional practice and to capture noteworthy aspects of interactions between the teacher and student (such as instances where I was

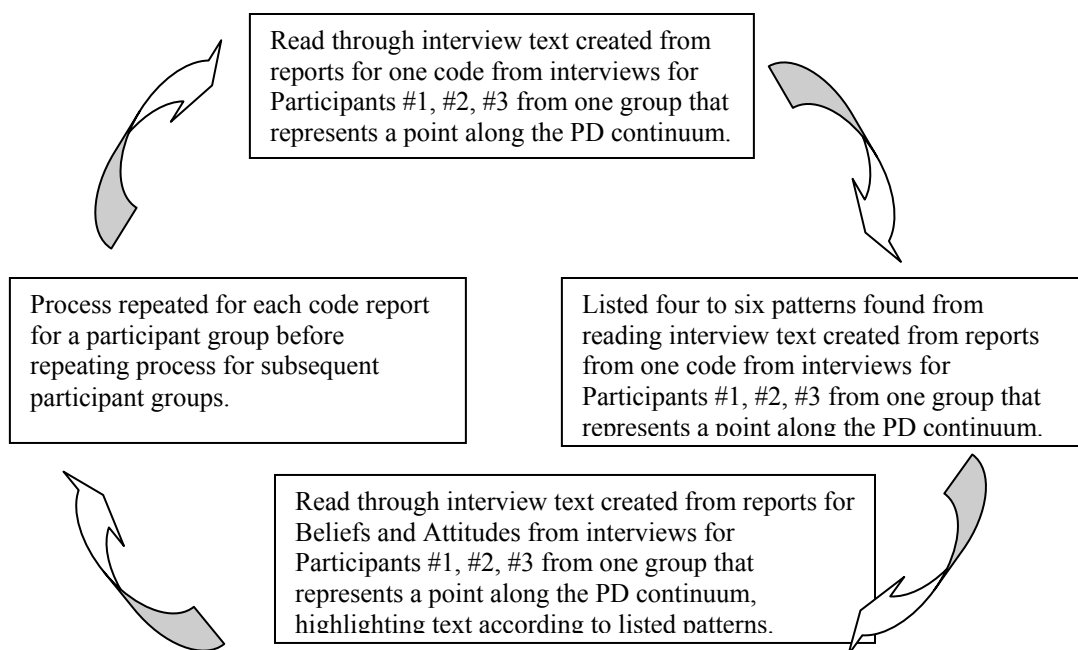


Figure 3 Sequence for data analysis, second round

surprised by instructional decisions made by the teacher, places where teacher insights indicated less/more advanced insights than I had previously witnessed, examples where the interaction took an unexpected turn). Upon completion of subsequent viewings of a teaching and learning video recording, I added to my notes on each participant. I looked for patterns in the instructional practices of the preservice teachers.

In the first step of data reduction for this instrument, I culled the transcripts from the teaching and learning events for a chronology of teacher talk throughout a video-recorded session. I inserted this talk into a spreadsheet and read through the data multiple times to look for emergent patterns. I constructed formative hypotheses about range and variety of question types. However, separating teacher talk from the rest of the transcript made me realize that 1) much information was missing from the context in which the talk

emerged, including the effect of the talk on student behavior, 2) the relationship between the teacher talk, the subsequent student response, and the teacher's interpretation of the talk and response was unclear.

The next step in data reduction was meant to keep the transcript intact and put it into a format in order to examine relationships between teacher talk and resulting student response. Because the video elicitation sessions focused on two to three clips taken from the teaching and learning events, these transcript portions were entered into a separate spreadsheet for each participant. I viewed the teaching and learning event video recordings a third time, this time viewing only those portions that were used for the video elicitation session. Each time a participant teacher made a suggestion, asked a question, or prompted the student, I made note of the resulting student behavior in an adjacent spreadsheet cell.

Video elicitation on teaching and learning events. In addition to examining the relationship between the teaching suggestion/question/prompt and the resulting student response, I was interested to see what additional information the participant provided in their interpretation of the teaching and learning event. I watched the video elicitation on the teaching and learning event. Whenever the participant made a comment about something occurring in the teaching and learning event, this portion of the video elicitation transcript was entered into the spreadsheet in a cell adjacent to that in which the original interaction took place. When this process was completed, I had a spreadsheet that contained transcript portions on the teaching and learning events, student behavior that resulted in each round of teacher talk, and teacher explanations of self-selected

portions of two to three clips extracted from the original teacher and learning event for each focus participant.

Using the Sherin and Van Es Noticing Trajectory of Developing in Learning to Notice (Sherin and Van Es, 2005) as a guide (see Table 8), I created, tested, revised and then applied a score of one to four to represent the type of noticing behavior exhibited by focus participants each time the teaching and learning video was stopped to state what they were thinking (see Table 10). I calculated a mean score for each participant.

Table 10 *Rubric used for noticing behavior during video elicitation*

Score	Characteristics of writing
1	description and/or evaluation
2	description and/or evaluation, and general evidence
3	description and/or evaluation and evidence of student behavior
4	specific evidence of student behavior, an interpretation of the event and/or provided information about the teacher's role

CHAPTER 4: RESULTS AND DISCUSSION

My larger research question was, “How does professional development affect preservice elementary mathematics teachers’ beliefs, knowledge, and practice?” This question was divided into two supporting questions: 1) How do beliefs and knowledge differ depending on the amount of professional development?” and 2) “How do teaching practices differ depending on the amount of professional development?” In this chapter, I discuss the results of my study with relation to supporting questions. Answers to the supporting question combine to answer the larger research question which I do not address here. In each discussion, I first present evidence gathered from the larger groups at each level, then by focus participants. The data illustrate the outcomes differences in how preservice elementary mathematics teachers think and teach, depending upon the amount of preservice professional development they have had.

How do beliefs and knowledge differ depending on amount of professional development?

Both beliefs about and knowledge of mathematics teaching and learning are established early on, during preservice teachers’ experiences as students. Because knowledge of mathematics content and pedagogy is addressed in preservice professional development and is filtered through existing beliefs, it is important to understand if and how beliefs about mathematics change throughout professional development. It is also important to understand how knowledge of mathematics teaching and learning changes.

Preservice teachers experience a number of changes in their beliefs and knowledge depending on the where they are in the course of their preservice professional development. Early definitions of mathematics change from arithmetic to a broader definition that includes various mathematics topics and processes. Notions that liken learning to speedy demonstrations of accurate calculation become more comprehensive, including use of multiple strategies as well as multiple models, the ability to communicate mathematically, and the facility to demonstrate understandings in a variety of contexts. Simple descriptions of mathematics as ‘easy’ or ‘hard’ to learn and teach become increasingly more complex, as preservice teachers realize the many and varied complications associated with learning and teaching. These issues include the complexities of mathematics content, complications of teaching in the school context, and the challenges of fostering students’ mathematical understandings. As preservice teachers progress through professional development, they become increasingly aware of the conflicts between their memories of learning mathematics and their goals for their own teaching.

Finding 1: With more professional development, preservice teachers’ beliefs become increasingly aligned with a reform perspective.

Participants’ responses to questionnaire statements designed to assess beliefs about mathematics learning and teaching were compared to responses reflecting a reform-oriented stance. The data provided important information about differences in

beliefs about content and pedagogy across participants who represent beginning preservice teachers (Education 101) and those finishing up preservice professional development in mathematics with additional training in content and pedagogy (Education 301). The data confirms that Education 101 participants are less aligned with a reform stance than Education 201 participants, and the beliefs of Education 301 participants are more closely aligned with a reform stance than the other two groups

Responses to Statements about Mathematics Beliefs

All participants completed a questionnaire that elicited experiences and attitudes about prior teaching and learning experiences. In addition, all participants responded to a set of statements about mathematics teaching and learning. I analyzed the second part of the questionnaire, focusing on the degree to which participants' responses tended toward a reform-oriented stance about learning and teaching mathematics. As a group, mean responses from Education 101 participants were least aligned with a reform-oriented stance, the 301 participants gave answers that were most aligned with a reform-oriented stance, and answers from the 201 participants were in between.

Two different sets of statements were included in the questionnaire. I calculated a mean score for each statement by participant, for each statement by participant group. I also calculated an overall mean score for each participant group. (see Tables 11 and 12). The beliefs of Education 101 participants are the least oriented toward a reform approach in 16 out of the 28 statements. For two of the eight statements (#13, page 3 and #4, page 4; see Appendix A) the mean response of these participants was less aligned with a

reform-oriented stance than the other groups, but responses for the other two groups (Education 201 and Education 301) did not follow a downward trend. For three

Table 11 *Mean responses for questionnaire item by group, page 3*
(*reform-oriented stance is = 1*)

101 Participants (N=31)	201 Participants (N=33)	301 Participants (N=9)
2.0	1.8	1.7

Table 12 *Mean responses for questionnaire items by group, page 4*
(*reform-oriented stance is = 1*)

101 Participants (N=31)	201 Participants (N=33)	301 Participants (N=9)
2.0	1.7	1.5

statements (#8 and #9, page 3 and #2, page 4; see Appendix A), the mean score for the Education 101 participants was the same as one of the other two participant groups) and for the remaining three (#4, #12 and #14 page 3; see Appendix A), the responses of the Education 101 groups was the most aligned of the other two groups. This outcome was unexpected (see Figures 4 and 5).

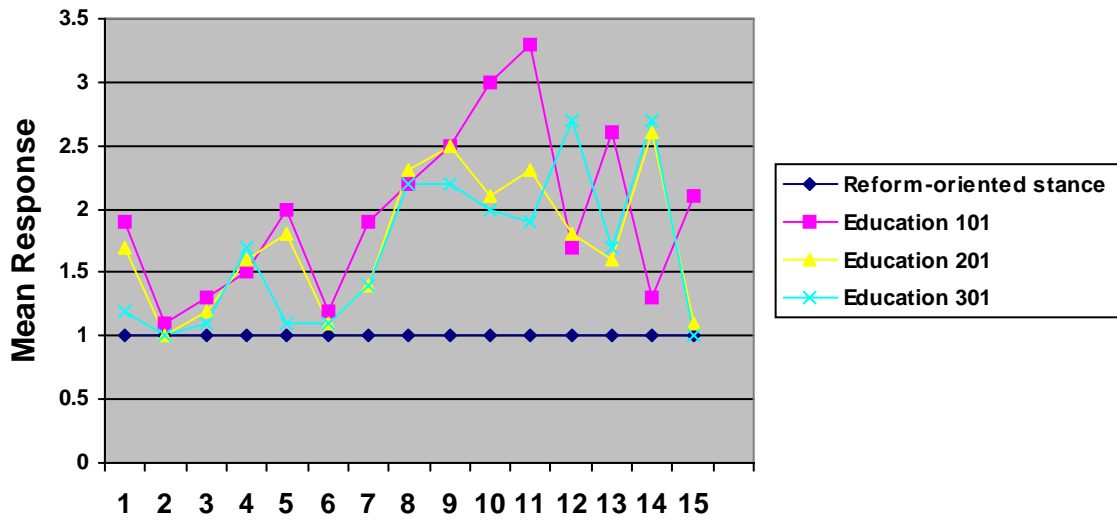


Figure 4. Questionnaire responses, page 3

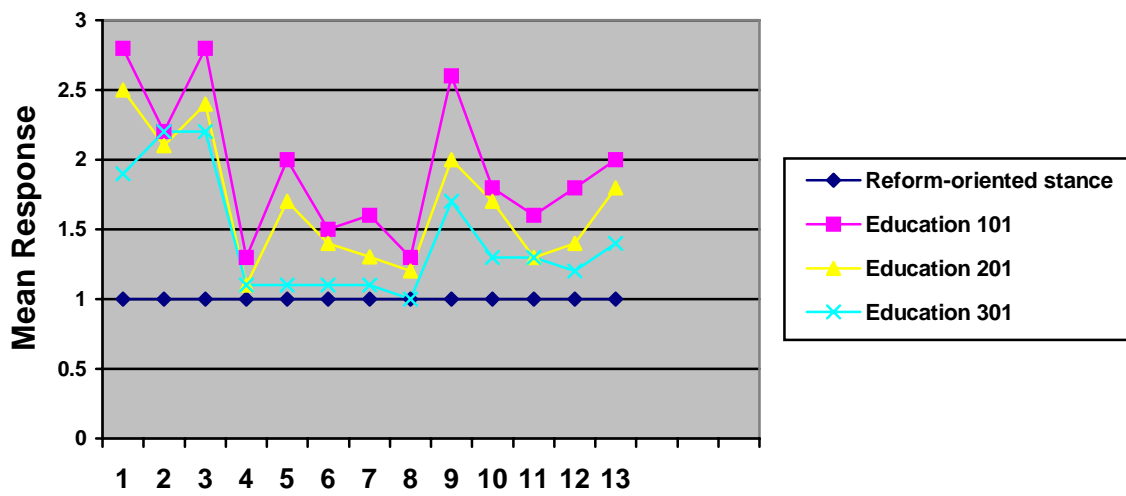


Figure 5 Questionnaire responses, page 4

When I examined how statements were worded, I understood the outcome. Statements that go against the trend are featured in Figure 6. Discussions with participants would yield important information about why responses to these statements were less

aligned with a reform-oriented stance as participants progressed through professional development, but certain explanations seem reasonable to consider.

4. A major goal of math instruction is to help children develop the belief that they have the power to control their own success in mathematics.
12. Mathematics should be taught as a collection of concepts, skills, and algorithms.
14. . Appropriate calculators should be available to all students at all times.

Figure 6. Questionnaire statements which resulted in responses that reversed trends

My knowledge of the content and pedagogical focus emphasized in this university's preservice professional development program qualifies me to explain this break from the trend. The mathematics methods courses used for this study focus on helping preservice teachers understand how some approaches to mathematics instruction emphasize speed and accuracy over reasoning and sense-making. Preservice teachers are encouraged to question instructional approaches that equate mathematics with arithmetic and focus on teaching students traditional algorithms for solving computation problems. They are asked to consider approaches in which elementary students have opportunities to first make sense of the problem and then solve it in ways they understand. Because of these emphases, preservice teachers may come to associate "algorithm" with "traditional approach" and disagree with statements that include "algorithm." Their experiences have emphasized students making use of efficient and meaningful invented computation strategies. They may deem calculators to be unnecessary tools, especially when they are

made available “to all students at all times.” Lastly, methods instructors at this university communicate that “all children can learn mathematics,” yet this phrase does not necessarily mean “all children have the power to control their own success in mathematics.” Preservice teachers are encouraged to adopt a core philosophy that all children can learn, at the same time examining and counteracting political structures that complicate and work against this philosophy. Incorporating education’s political factors may influence the way they understand and agree with statements such as these, thinking that political influences can undermine and counteract students’ sense of mathematical power.

How do teaching practices differ depending on the amount of professional development?

As they progress through professional preparation, the practices of preservice teachers change in particular ways. Initially, they focus on issues unrelated to subject matter content. Classroom management consumes much of their attention. They tend to replicate the practices of teachers they have had as well as those from field placements. As they acquire strategies to successfully deal with student behavior, they can shift their attention to crafting pedagogical aspects of their practice. Early on, they are likely to imitate lessons from methods courses and field experiences. They will teach the lessons as they observed them and have difficulty tailoring or adjusting them to fit the needs of their students.

Successful teaching incorporates several demands: student behaviors, materials management, and pacing, as well as clear directions, purposeful sequencing, and differentiated instruction are some of the many aspects teachers need to consider when implementing lessons. Over time and with experience, preservice teachers develop an awareness of teaching's demands and then transition to addressing them in practice. In this study, they demonstrate this development in three ways. Their views of mathematics teaching and learning become more complex. They acquire and use more effective strategies to support student learning. They focus their instructional attention on the thinking processes students use as well as the answers they give.

Finding 2: With more professional development, preservice teachers' views of mathematics teaching and learning become more complex.

Teaching in general and teaching mathematics in particular is a complicated undertaking. Attention to classroom management dominates the practices of preservice teachers. Through professional development in the university and in the field, they are more able to transfer attention from managing students to teaching them. In the process, they begin to realize that teaching and learning is complex in a number of ways.

Over the course of preservice professional development, preservice teachers develop an understanding of the complexity of teaching. This study documents four changes that indicate understanding of this complexity: 1) teachers change from explaining learning and teaching mathematics in relatively simplified ways to explanations that include a number of features that interact within the classroom context,

2) teachers are able to notice different aspects of teaching and learning, moving from general interpretations of teaching to interpretations of learning that are supported by increasingly more detailed evidence, 3) reflection shifts from descriptions of practice to reflections on practice (that include instructional alternatives) to reflections-in-action (where alternatives are implemented in the midst of a planned teaching and learning event), and 4) vocabulary use changes from language that is associated with layperson use and is more general to language that includes both technical language associated with the teaching profession and indicates specialized knowledge.

From Simplified Explanations of Learning and To Explanations of Features of Teaching in Classroom Contexts

Data from both quickwrites and small scale instruments interviews show that, over time and through experience, preservice teachers demonstrate an increasing awareness of and appreciation for what is involved in learning and teaching mathematics. Certain aspects of learning and teaching are shared across the three groups while others are unique to a particular group.

Education 101 participants. Preservice teachers at the beginning of their professional development named a number of features that complicate mathematics learning and teaching. Some of these features are associated with students, some with teachers, and some are features of mathematics content.

Education 101 participants completed quickwrites that required them to contextualize their stance on the difficulty learning and teaching mathematics through experiences as students and teachers. First, they reported characteristics of their classes

and what teachers did. Many accounts explained how elementary mathematics consisted of learning shortcuts, going through steps, memorizing formulas and procedures, and doing problems the ‘right’ way. Class time was spent completing many practice problems and exercises but mathematics made little sense to many Education 101 participants. In interviews, they talked about the rigorous, seemingly unreasonable pace of instruction and how difficult it was to ‘keep up’ and learn challenging mathematics content. For example, during brief class periods the teacher presented a lesson and then let students practice what was taught. If students made mistakes or did not understand, there was little class time to go back and address these issues within the class period. A fast pace usually meant an inordinate amount of content addressed within a short amount of time. Tiffany and Hilary describe similar experiences with fast-paced instruction and much content.

...when I went to trigonometry it was a negative experience because I felt like the work was just thrown out us and at a continuous pace too... So the teacher went really rapidly and we had so much homework... (Education 101 Tiffany interview, May 25, 2006)

The class was like 50 minutes. It was enough time for everyone to get settled and get your homework out, pass it to your neighbor to grade and then he tried to do a lesson but there was no time to go over anything. So you’re kinda thrown into the next lesson not knowing what was wrong with the lesson before, so it all builds up and so I just got lost...(Education 101 Hilary interview, May 22, 2006)

Next, Education 101 participants discussed the challenges of mathematics as a content area. Because it is full of abstractions, when the content becomes more challenging, mathematics is more difficult to learn. In interviews, participants mentioned specific junctures where difficulty understanding made them struggle, drop out, or get

transferred to a less advanced mathematics class; these, in turn affected their ability to progress to higher level mathematics courses. Dalene, a focus participant, enjoyed learning mathematics through much of her school career. She explains how the content became increasingly difficult, and this change affected her usually positive attitude toward mathematics.

...in middle school, math was my favorite subject... around seventh grade... because my teacher was very...um...she was very passionate about it so we were very interested because she was very interested. So from then until high school it was my favorite subject until, I think, my senior year when I took math analysis, and it started getting really, like, conceptual and stuff...it was like...I don't know...It was getting harder and like to understand...(Education 101 Dalene interview, May 22, 2006)

Many Education 101 participants reported that mathematics learning is a function of the student. They noted that foundational knowledge is critical for learning, yet teachers receive students with a variety of mathematical backgrounds. Students' mathematical ability, as well as their interest and motivation, affects learning. While the teacher's attitude toward and interest in the content are important aspects of the instructional approach, a student who comes to the elementary classroom with an interest in as well as an ability to grasp concepts and learn procedures is at an advantage.

According to a few Education 101 participants, teachers need to show students patience and give them time to learn. Teaching involves motivating students, getting them interested, and keeping their attention. Interview accounts focused on these attributes of teachers' affect. When teachers were motivated toward mathematics and interested in student understanding, it affected students' motivation and interest as well. Participants gave conflicting notions about student learning when they described their

own teaching experiences with students who appeared unmotivated and uninterested. While participants described how their own teachers' attitudes toward the content and students influenced their learning, they did not discuss the connection between their own attitudes toward learning mathematics and the effect on the students they were teaching, tutoring or mentoring. As such, there was little transfer between participants' ideas about previous teachers and their motivation toward and interest in mathematics, and the effect of the participant's motivation and interest on that of their tutees and mentees. Tiffany describes an experience in high school, where her teacher's behavior seems to blame for her own and her peers' difficulty in remaining motivated. There is a lack of connection between the teacher and students that not only contributes to disinterest but affects student understanding of the content as well:

She didn't really make an effort to invest in her students which are also, I think contributed to people not been so interested and again, you know, really structured lesson plans...I got maybe a B on one of my exams and I was so upset and she didn't have any comforting words or she wasn't motivating at all...she was like oh well just try and do better next time... there wasn't like no recognition of effort or not really nurturing to the students or cultivating their needs or anything...(Education 101 Tiffany interview, May 25, 2006)

Interestingly, Tiffany does not consider her own responsibility for motivating students when she describes frustrating teaching situations. Tiffany recalled a challenging teaching experience with a middle school student in which she had a hard time determining what made the experience so challenging. Rather than make a connection between her practice and the student response, she found the source of the problem to be the student's lack of interest and motivation.

She didn't have the discipline, struggled, no interest, lack of motivation, didn't enjoy it much. I was discouraged, frustrated that I couldn't 'get the message across' or really help. I felt helpless...(Education 101 Tiffany interview, May 25, 2006)

Education 101 participants understand that learning mathematics is dependent upon on how well the teacher's approach matches the ways students learn. They named varied explanations, as well as different methods and examples, as components effective teaching approaches. Few quickwrites contained examples of teachers who used strategies to help students understand mathematics, and in interviews, Education 101 focus participants described experiences when teachers made content accessible to students. Hilary describes the pedagogical practices her teacher used in order for students to understand.

I had a math teacher in middle school and she was just amazing...she used her like overhead projector and she umm...really made sure that everyone understood. Like she would put it in abstract form, she would use visual aids, and umm...she had set-aside time for us to work in class. And she would make sure that she went around to every student to see to see where they were at...so I really liked her...I learned a lot from her...(Education 101 Hilary interview, May 22, 2006)

Like Hilary, other 101 focus participants made references to use of materials, the overhead, flash cards, visual aides, toys, and manipulatives.

In interviews, Education 101 focus participants talked about classroom features that affect student learning but were not mentioned in quickwrites. Each participant explained that time is needed to ensure that students understand, to allow them to ask

questions to clarify confusion, and to address students' mistakes. Pacing issues have an impact student understanding, since there is little to no time for confusion or misunderstanding to be addressed during class time. Hilary summed up her ideas about what teachers need to do to ensure student understanding:

I would say that think there needs to be a lot more time spent making sure that kids understand because math, I mean, if you don't get it, you don't get it and you can't move foreword from there...(Education 101 Hilary interview, May 22, 2006)

Tiffany relayed how students are affected by a teacher's juggling of time and content. She laments, "There was just so much work that there was no way to learn all you're doing" (Education 101 Tiffany interview, May 25, 2006). She goes on to explain her teacher's way of dealing with large amounts of content to 'cover:'

...just doing his worksheets...all this independent work and not so much interacting with people or discussing how that should work but just always taught flooding with words, flooding with worksheets, flooding with things that you don't even know how to apply in the first place...(Education 101 Tiffany interview, May 25, 2006).

She paints a picture of a classroom filled with a flurry of paper, students quietly working, and the teacher doing all of the talking. Later in her interview, she provides an alternative experience where the teacher expects students to contribute to discussion, and a more collaborative atmosphere is promoted:

... at the end he would ask the person who had done the problem to talk or explain what their methodology had been and then the class could raise their hands or give opportunities to explain or to discuss. So it was really a discussion and it really made comprehension so much easier so when test time came ... (Education 101 Tiffany interview, May 25, 2006).

According to these participants, elementary teachers need to understand procedural knowledge, logic, reasoning, and critical thinking in order to teach mathematics. They must be able to know mathematics in order to teach it and the more difficult the content, the more challenging it is to teach. Teachers must have an understanding of how to teach students at different levels and must give students opportunities to practice and memorize. A few mentioned that memorization of concepts and procedures should not be equated with understanding.

In sum, Education 101 participants referenced the pedagogical practices their own teachers used to “deliver content” (Education 101 Tiffany interview, May 25, 2006) or “make content accessible” (Education 101 Dalene interview, May 22, 2006) throughout their experiences as mathematics students, “teaching student tips and tricks,” “pointing out important information,” “explaining and teaching procedures,” “breaking down information,” (Education 101 Hilary interview, May 18, 2006), and rewording word problems/stories.

They characterized their own teaching practices as 1) “Tell him to do it, show examples, explain step by step, ask questions, point out info” (Education 101 Dalene interview, May 22, 2006) 2), “break down concepts, take something that they could do and using that” (Education 101 Hilary interview, May 22, 2006), and 3) “use an example, emphasize important aspects of the problem/example, teach them, and use real life examples” (Education 101 Tiffany interview, May 25, 2006). While they realized the importance of using contexts to teach concepts, taking challenging concepts and simplifying them, using students’ prior knowledge to connect with new concepts, in many ways their approaches to teaching are quite traditional. They teach by

demonstration, point out what is important and ‘teach’ students ‘step by step.’ At their own stage of professional development, they are unable to meet their own stated expectations of good teaching.

Education 201 participants. The Education 201 and 301 participants quickwrites demonstrates an awareness of more numerous and more specific features that make learning and teaching mathematics a challenge that is more pronounced than that of their Education 101 peers’. Preservice teachers at the end of preservice professional development outline various things that can have an impact on learning mathematics (e.g., lack of student understanding, lack of teacher content knowledge, difficult content). They explained how it can be difficult to balance many aspects required of teachers.

While Education 101 participants addressed various factors that influence learning mathematics including the content itself, many Education 201 participants commented that, as students progress through the grades, teaching methods characteristically support students less and less. For this reason, rather than difficult content, mathematics can become more difficult to learn and student confidence can wane. Many explained that knowing mathematics is necessary in order to teach it but they believe that teaching mathematics gets more difficult through the grade levels as well. Thus, the content is somewhat to blame: through the grade, content is more difficult, and because it is more difficult, it is more difficult to teach that content.

Another aspect that makes learning and teaching mathematic complex are the numerous features of an instructional approach. Several participants noted that mathematics was hard to learn if procedures were the focus of instruction but students were not privy to how, why and when to use them. Some called on their own experiences

where learning was mostly about ‘trudging along’ and explained how mathematics was easy if demonstrations of memorization were the only requirements. Most reflected that true learning is difficult if instruction focused on memorization.

Procedures were the focus of early mathematics experiences for all Education 201 focus participants. Sarah recalls, “I mean a lot of my math education has just been rules, this is how you do it, go” (Education 201 Sarah interview, April 17, 2006), and Karen remembers, “growing up learning math...it was very procedural and frustrating to try to memorize procedures and get it... I didn’t fully understand what I was doing all the time” (Education 201 Karen interview, May 30, 2006). Instruction centered on demonstrating the steps in mathematics processes and algorithms. When these participants were students, they rarely inquired why or how a process or procedure worked. Sarah relayed, “You didn’t ask why... because I think when I asked why, teachers told me, that’s just how you do it” (Education 201 Sarah interview, April 17, 2006). Students reacted to this type of instructional focus with frustration.

...I remember being really frustrated. I think I may have cried once or twice, like how am I supposed to memorize all these numbers, I don’t know, I don’t think I have the greatest of memories try to remember things all that well...(Education 201 Karen interview, May 30, 2006).

Yet a focus on memorization could also be satisfying. Participants felt good about being able to quickly perform a procedure and get a correct answer even if they didn’t always understand what they were doing and why the procedures worked. Sarah explains this confusion.

I loved multiplication time tests, which I think is very crazy, but I mean...I don't know. I just remember that in third grade, we would have multiple multiplication tests and I just...I don't know...I got great satisfaction out of doing it quickly, and getting a lot right so I guess that kind of balances the rules, but I didn't know why I got those numbers. (Education 201 Sarah interview, April 17, 2006)

Education 201 participants explained how experiences in methods courses made them consider a number of features when constructing their own instructional approaches.

They realized that the features of their own formative approaches are quite different than ones used by their own teachers.

Chief among features gained from methods courses were experiences that helped Education 201 participants learn more mathematics. They explained that it was critical for them, as teachers, to learn how and why procedures work and why conceptual understanding is important. Experiences from the university methods classroom were, for many, the first times when mental mathematics, multiple strategies to solving a problem, and varied perspectives and approaches were valued, a change from early experiences that emphasized doing it “one way, the only way” (Education 201 Belinda interview, June 11, 2006).

Experiences in methods courses could be transformative. Education 201 participants gained a deeper understanding of mathematics concepts from preservice experiences in the university methods classroom that incorporated multiple strategies and valued many perspectives. These experiences resulted in sudden realizations of their own conceptual understanding, such as “Oh that's why you do that!” (Education 201 Sarah interview, April 17, 2006). Participants wonder whether the approaches from preservice professional development would have resulted in a deeper understanding of mathematics

concepts if they had been employed by their own teachers. Participants explained that earlier attitudes of not caring about mathematics and feeling unsuccessful as mathematicians were altered into feeling like capable mathematics students who had important contributions to make in classroom discussions.

... And I started becoming like ‘Wow... I’m good at math...’ If I was just taught it differently I could have gotten it... all the different types of ways that they (methods instructors) show us how to master something... it wasn’t just this is the one way and the only way... a majority of my classmates could learn it that one way, but I was the one that had my hand up in the air saying wait, what about this way? It was just nice because I felt like I could contribute... (Education 201 Belinda interview, June 11, 2006)

Other Education 201 focus participants talked about additional preservice professional development experiences that helped them realize the importance of developing conceptual understanding in mathematics. In her interview, Karen talks about such an experience when she and her classmates solved computation problems using a variety of strategies, including invented ones.

...it was more the conceptual understanding of it, rather than starting with the ones place and entering the tens place and moving to the next problem. And the other thing, going along with that one, was just how they valued all these multiple perspectives. And doing it that was really eye-opening... (Education 201 Karen interview, May 30, 2006)

While learning mathematics one way and trying to teach in a different way can be difficult (e.g., to not focus on procedural knowledge), these participants expressed how growth happens through learning more pedagogical content knowledge. Several mentioned that conceptual and procedural knowledge can be second nature to teachers and difficult but necessary to break down in order to teach students. To do so requires

teachers to rethink the instruction they experienced and retrain themselves to use new pedagogical approaches.

In interviews, preservice teachers explained how employing these approaches in field experiences may not be a simple task. A focus on memorizing computation procedures dominated their experiences as learners and understanding how and why procedures work is a focus of Education 201 participants' current mathematics learning. In order to teach students, they realize they must know why the procedure works. Participants often resort to practices they learned as students and realize their current students may come from a focus on procedures as well:

I've been programmed to do it...throughout all my schooling...at least I'm aware of it now, and so it goes across my mind, especially when I'm teaching. I feel myself slipping into my old comfortable ways, and then I might know, and the kids that you're teaching, a lot of them know those ways too, so it's hard to get them to actually understand what it is that's going on...(Education 201 Karen interview, May 30, 2006)

To counter familiar past practices, participants find it useful to be metacognitive about their understanding of the concepts they teach. They realize the need to understand concepts so that they can explain them, because students become disengaged and confused when they do not understand. Sarah addresses this transition from familiar practices to a new set of teaching behaviors:

I mean a lot of my math education has just been rules, this is how you do it, go. So, I have had a little bit of disconnect with teaching, and I've been learning a lot about math processes that way, because no I'm in a position where I don't want to teach math that way, but it's forcing me to understand the why of things...(Education 201 Sarah interview, April 17, 2006)

Karen does as well: “It’s (the procedure or algorithm) fast and its easy, but with being a teacher it’s a different story because you don’t even know the ‘why’ behind a lot of things when you’re teaching it...(Education 201 Karen interview, May 30, 2006).”

Participants at this stage of professional development know there is a difference between the skills required to “do math” and those required to “teach someone else to do it” (Education 201 Karen interview, May 30, 2006). They stated that some topics of mathematics are easier to teach than others, and some are “more procedural than others” (Education 201 Karen interview, May 30, 2006).

Education 201 participants realize the need to learn the mathematics behind the procedures students will encounter rather than have their students blindly follow them. Yet, these participants felt conflicted during field experiences when more experienced teachers talk about teaching practices that are in conflict with what preservice teachers have learned. Sarah expresses such a conflict.

And I’m asking other teachers for advice on it and they are just like ‘You know, you just need to teach the kids rules and some things are just rules,’ which is fine...Like I definitely understood that there’s just rules but what if like a kid asks why you just want to know the why so you can teach that child, too. (Education 201 Sarah interview, April 17, 2006).

The development of conceptual knowledge was an important aspect of learning and teaching explained by Education 201 participants in quickwrites. They know that students learn concepts over time; pushing students to learn content before they are ready is detrimental. Helping students acquire multiple strategies for solving problems is an important pedagogical practice. Learning standard procedures should follow understanding of other strategies. These teachers note a fine line between memorization

of a procedure and learning how and why it works. They understood that teaching new concepts requires students to have a foundation of previously established conceptual understandings.

Choice of resources to employ as a teacher is another aspect that complicates learning and teaching mathematics. These participants see teachers as able to ‘control the content.’ Their professional development experiences in the university and in the field exposed them to a variety of teaching resources that can make learning and teaching enjoyable and engaging, yet some stated that it can be difficult for them locate good resources. They understand that mathematical concepts can be learned through concrete models and use of materials. Some stated that certain instructional components (use of hands-on activities, attention to and instruction in academic language, use of varied instructional groupings) make it relatively easy to teach. Others stated that teaching from a textbook is much simpler but not as rewarding and beneficial to student learning teaching in hands-on, engaging, interactive ways. They find it challenging to make math interesting for students and help them see there are multiple ways to solve problems. Some feel that using more than a worksheet is difficult.

Education 201 focus participants explained in interviews how teaching incorporates many skills and components and this complexity makes it difficult for teachers to decide on a lesson focus. Preparing to teach is a complicated task and these teachers have much to say about what is involved in the planning process. Sarah drew on her own experience as a learner when she explained how she prepared lessons to include questions she anticipated students would ask: “So when I do lesson plans with my friend, she says, ‘Kids are not going to ask you that !’ and I say, ‘Yes they are! I think as a

learner, I didn't really question why before, when I was learning it..." (Education 201 Sarah interview, April 17, 2006). Karen, like her other 201 peers, grappled with how to give students the 'right' amount of information while teaching:

It was hard, you know...it's hard to balance between making sure that they actually understand what's behind what they're doing, and also make it easier on them to not think too much because it's like to confusing and frustrating and overwhelming. But there are so many different components to it that I didn't really think it beforehand...you know, there's a point where you feel like you're just losing them no matter how or what you say(Education 201 Karen interview, May 30, 2006).

Empathy for students was another important aspect of learning and teaching mathematics. Education 201 participants discussed how learning new content and pedagogy in methods courses gave them an appreciation for the difficulty students go through, and this experience, in turn, makes them better teachers. Students need time and practice and experiences in order to understand mathematics concepts. Students' confidence can affect their mathematical performance.

In quickwrites, Education 201 participants recognized that classrooms contain a range of students with varying experiences and abilities. They discussed how difficult it is to teach to this range. They named ways to assess students so that they can address this challenge. Examination of student work and listening to student explanations gave teachers information about what students understand and this information could be used to help teachers make future instructional decisions. During interviews, focus participants discussed field work experiences that informed their awareness of teaching a range of student abilities and experiences and helped them learn how to differentiate instruction. Belinda explains these aspects of teaching practice:

I just remember feeling really frustrated and feeling like, oh my gosh, how am I ever going to teach whole group math lessons when there's always going to be those kids at different levels and different understandings so...(Education 201 Belinda interview, June 11, 2006).

Teacher questioning can both promote and assess student learning; Education 201 participants understand how students may not be used to having teachers ask open-ended questions. Many interpret a teacher's use of questioning as an indicator that they are wrong. Focus participants explained that use of teacher questioning, allowing students to figure out their own strategies to solve problems, and assessing students while they work are important aspects of teaching they try to incorporate while instructing students about challenging mathematics content. Tapping prior knowledge, showing students different ways of approaching content, allowing students to solve problems in the ways they understand, effectively pacing a lesson, meeting with each and every student, and involving and engaging students are some of the many complex aspects of teaching that can be addressed during a mathematics lesson. Karen summed up the complexity of her teaching practices.

...asking my kids about their experiences first and involving them in any way I can, not just giving out information, but asking them what they think they might do next and things like that...And if they don't have experience with it, giving them experience with that, like how would we do this in the world or use hands-on stuff and partner work...a lot of that...Like our program always says those things are good, but I really saw how much it all comes down to it. I think a lot of it is just getting them involved and interested in it is a big thing...and giving those experiences and making it understandable and keeping their attention...(Education 201 Karen interview, May 30, 2006).

In sum, Education 201 participants developed a broad sense of what it means to learn and teach mathematics. They used their prior experiences as students as well as

experiences in the university methods classroom and field placement experiences to develop an appreciation for what is involved. They understand there are issues related to content, pedagogical practices, and knowledge of students that make elementary mathematics a difficult subject to both learn and teach. Yet, they also realize that the act of teaching is quite complex and involves a great deal of decision making: determining what components of a concept to focus on, explaining the 'why' behind concepts, addressing misconceptions or confusion, and ensuring students understand. Karen speaks to these many demands:

there's a lot to think about... a lot of different ways to do it... there's a lot of different things kids need... the pacing, to try and not make it so overwhelming, trying to get to every kid... I mean teaching in general is pretty much can be summed up as hard and frustrating and all those things, but for math especially...(Education 201 Karen interview, May 30, 2006)

Education 301 participants. Like their Education 201 peers, these preservice teachers were able to tease out the components that make teaching a complicated and challenging task. There were many fewer Education 301 participants (eight) than Education 201 participants (30), so the amount of text in the next section may make it appear that these participants had 'less to say.' Still, they were different from Education 201 participants in two ways: 1) they discussed certain features that were not mentioned by their 201 peers and 2) they understood various parts of teaching that they must attend to through the phases of planning, implementing and evaluating their instruction. They realized that assessing their own teaching happens through these various phases. While teaching, they found it necessary to monitor student learning and adjust plans

accordingly. They understood they must consider cooperating teacher feedback adjust subsequent plans.

Like their Education 201 peers, the Education 301 focus participants reflected on early experiences as both as students and as beginning teachers through lenses formed during preservice professional development. They expressed their reflections during interviews. When examining mathematics experiences as young students, Education 301 participants associated mathematics experiences with use of the textbook, performing arithmetic computation, “doing the problems and getting them right” (Education 301 Reese interview on teaching, August 7, 2006) and learning tools and tricks from the teacher, similar to the experiences of Education 201 participants. Proficiency with mathematics was equated with knowing facts and performing a procedure and knowledge and confidence were associated with getting a correct answer. Some felt they could monitor the depth of their own understanding when they were students. One participant sought help outside of the classroom when teacher support was missing, while another was able to learn content on her own. As with Education 101 participants, these participants discussed how their relationships with teachers influenced their beliefs about mathematics. It was apparent when a teacher liked a student and when s/he didn’t, and teacher affect toward the subject matter and students had an effect on beliefs and student learning.

My eighth grade algebra teacher was good...just in that he was always really encouraging...and he sort of came with the attitude that everybody can do algebra, everybody can solve these problems. I did end up learning how to do it, so I felt like I was encouraged in that class, and that I was seen. (Education 301 Reese interview on teaching, August 7, 2006)

Content was a feature of teaching and learning mentioned by this group of participants. They agreed that the higher the level of mathematics, the more difficult it is to learn. They explained that the content can become increasingly more abstract and less 'real' to students. Understanding mathematics can make it easier to teach it because content knowledge allows teachers to simplify and break down concepts. These participants mentioned that they are still learning math as adults.

Interestingly, this group discussed the students' role related to learning and teaching, as Education 101 teachers did. Education 301 participants spoke of the importance of having high expectations and concern for student learning. However, quickwrites contained comments that some kids will 'get it' while others will not, that some students are 'naturally more adept at math,' and that student motivation, interest and confidence are factors in student learning. Even though this group had significantly more professional development than the Education 101 group, they had similar ideas that mathematics ability is innate to the individual to some degree and not necessarily developed as a result of good teaching.

Awareness of the number of factors that contribute to an effective teaching approach demonstrates this group's understanding about the complexity of learning and teaching mathematics. In interviews, focus participants discussed their success learning mathematics and attributed success to their ability to memorize procedures, processes, and algorithms. New perspectives formed during professional development altered participants' previous beliefs about and attitudes toward teaching and learning mathematics. Education 301 focus participants had professional development experiences that helped them learn both mathematics content and effective pedagogical processes.

While past experiences learning mathematics content affected current practices teaching, preservice professional development and successful field placement experience enhanced confidence about teaching mathematics content. While understanding was confirmed by getting correct answers in the past, these preservice teachers now see how conceptual understanding was often missing. Reese was able to view previous learning experiences differently after her experiences in the preservice methods classroom.

...my teacher at the time...he gave me the tools to do these word problems and I think it was really like...if you see the word 'of' it means multiply. And a lot of the problems we had were very scripted. I mean they were all the same kind I don't think they gave me a deep conceptual understanding of algebra but it just helped me to do the problems and get them right. And I just remember feeling like, 'well that's OK, that's great, I am getting them all right and I'm getting a good grade' but you know, I can't see that I really felt like I deepened my understanding of math looking back (Education 301 Reese interview on teaching, August 7, 2006).

While some experiences felt good to them as learners, they realized that employing some of the same teaching practices now might result in their students' lack of confidence.

Education 301 participants explained how a number of approaches are required in order for teachers to help students learn and the use of only one can be harmful to student learning. In interview accounts, Education 301 focus participants made a distinction between 'teaching by telling' and methods that employed exploration and scaffolded instruction. They realized that student learning is greatly affected by the choices made during teaching practice. They examined ways to make instruction more effective and the content more accessible to their students, including:

- making use of their cooperating teacher's feedback;
- helping students connect new content to familiar prior knowledge;

- using familiar and multiple models to teach concepts;
- employing activities, experiences and projects that provide connection other experiences;
- allowing students to use multiple paths, strategies, and methods to solve problems;
- choosing and guiding experiences so that conceptual understanding is developed;
- abandoning plans that were not thought through;
- differentiating instruction based on student need;
- and realizing the limitations in teaching resources.

Use of only standard algorithms can have a dangerous impact on students, especially if teachers equate ‘learning’ with memorizing these procedures. These participants realize adequate preparation, use of quality resources, selection of appropriate activities, logical lesson sequencing, and deliberate scaffolding of experiences are required for teachers to meet students’ needs. They stated that students need time and numerous experiences to interact with concepts and each other. They spoke of using other students as instructional resource and how important it is for the teacher and the students to learn from each other. They named confidence in teaching and enthusiasm for mathematics as important factors in ease and effectiveness of teaching. Field experiences made these Education 301 participants realize that their teaching will improve over time and with experience and that experiences in both methods and Math Emphasis courses have prepared them to successfully teach, at the same time making content interesting and engaging to learn.

Education 101 focus participants, like their Education 201 peers, discussed the importance of attending to student needs in order to advance their student mathematical achievement but also realized that teaching is complicated by a range of student understanding in any given classroom. They understood that differentiating instruction is necessary in theory yet difficult to practice. Education 301 participants realized that teaching is complex and were able to consider choice of tasks, engaging students, monitoring understanding, and changing teaching plans based on student understanding and feedback on their feet. They could respond to student input to guide instructional decisions within and across lessons. They looked for indicators of student understanding and know that much can be learned by both students and teacher in those ‘tight spots’ when lessons unfold in ways other than they were planned. Reese explained such an experience in an interview:

Not having thought that through, I sort of got into a tight spot, which is great, actually I think because you know, it presents a learning experience for the students. But at the time I hadn’t done that, so it was a little tough for me. But again, I think that really prepares me in the future to, you know be a better teacher...
(Education 301 Reese interview on teaching, August 7, 2006)

These individuals realize the complexity of assessing students during and after instruction. They emphasize the importance of assessing their own teaching effectiveness as well. When a lesson is not thoroughly mapped out, both the teacher and students can benefit because teachers can use the unforeseen and unexpected to adjust a lesson in process and more appropriately address student need. Realizing such misconceptions within a lesson and adjusting the lesson accordingly can actually be more efficient:

future teaching time is not wasted because misconceptions have not been allowed to develop.

They reported that reform teaching practices can be scarier to implement than more traditional teaching practices. However, discovering student misconceptions early in a lesson, adjusting lessons based on student feedback and watching students construct their own understanding was both satisfying and rewarding to these preservice teachers. Gabrielle and Reese explain this cyclical process of teaching, assessing, reflecting, and adjusting.

...during that lesson, I felt as I walked around, I could see that students were just not understanding it (decimals)... that was Friday... and so Monday, we did (used) money. And it just really helped students connect to what they were learning based upon their own experiences and also, what we had gone over that day. It brought everything together, and so it just helped me realize, yeah, with these models, you know, they helped...different ways to apply to what they know...(Education 301 Gabrielle interview, July 24, 2006)

... It was clear from one of their homework activities that most of the class did not understand ...I posed a problem...I saw that there was still a lot of misconceptions. And so I pulled out the manipulatives and I talked about what 100^{th} was and I just really started from square one and related it to money. And then the students...they went back and corrected themselves. They said, 'Oh I would change my answer now and this is why.' And the thing about this whole lesson was I hadn't really planned at. It was supposed to be an intro to what we were doing that day and then I saw that, 'Oh my gosh'...nobody even knew what this quantity was. So I just sort of went into that and I just felt really good afterwards.' (Education 301 Reese interview, August 7, 2006)

In order to plan sound and effective lessons, implement them by way of appropriate instructional practices, and assess and adjust lessons in the process of teaching them, it is critical for teachers to understand student thinking. Quickwrites from

Education 301 participants demonstrated that they are aware of students' needs and were able to change practices based on student feedback while teaching. They used evidence of student learning that included examination of student work, observations of them working, and listening to their explanations. They were aware of the types, purposes and wording of questions that can be used to both advance and assess student thinking, considered points during lessons when they could appropriately insert these questions, and adjusted their questions based on their knowledge of students.

In sum, as preservice teachers progressed through professional development they were aware of increasingly more features that affect learning and teaching elementary mathematics. While Education 101 preservice teachers realized that elementary mathematics teachers must have knowledge of mathematics content, Education 201 and Education 301 participants understood that content knowledge is necessary but must be combined with knowledge of pedagogy and knowledge of students. The Education 101 group tended to teach in ways they were taught: through demonstrating procedures. Those in the Education 201 and 301 groups looked back on the early experiences as a learner with informed eyes and realization that they rarely understood; as a result of professional development experiences, they incorporated alternative practices that included allowing students to use a variety of strategies to solve problems, engaging students in lessons, and using a variety of questions. In addition, 301 participants had a greater awareness of student learning while teaching. They considered various types of questions most effective to both advancing and assessing student understanding and they were able to use student feedback to adjust lessons that were in progress.

From General Interpretations of Teaching to Interpretation of Learning Supported by Detailed Evidence

When observing teaching practice, beginning preservice teachers focused on different aspects of practice and explained what they noticed in different ways than those who completed more preservice professional development. Education 101 participants focused on aspects of teaching and describe what occurred. Over time and through experience, preservice teachers attended to student learning and provided specific evidence to substantiate their observations. Data from reflections after teaching and data from interviews and video elicitations on the teaching and learning events show these changes in preservice teachers' ability to notice.

Education 101 participants. Education 101 participants written reflections after view a video recorded segment of teaching practice were given scores from one to three, based the amount of evidence they used to elaborate descriptions of three things they found noteworthy in the video. A score was given for each response, a mean response score was figured for each participant, and a mean score was given to each participant group.

The Education 101 group had a mean score of 1.5. Their written responses were primarily a description of events bolstered by general evidence about what the teacher or students are doing, such as:

- “She praises her students.” (Education 101 CA reflection, May 3, 2006)
- “The teacher wasn’t lecturing.” (Education 101 YJ reflection, May 3, 2006)
- “She asked them to visualize.” (Education 101 FA reflection, May 3, 2006)
- “Using manipulatives” (Education 101 HN reflection, May 3, 2006)

Participants' responses largely focused on the objects and/or materials the teacher used and the level of student engagement. Typical vocabulary used in their accounts related to teaching and learning mathematics and included manipulatives, objects, participation, interest, and involvement.

In interviews with Education 101 focus participants, there was a similar tendency. Descriptions of experiences were general. Participants did not have the vocabulary to talk about their experiences nor the lenses from which to analyze the practices used by their teachers. In this example, Tiffany recounted an experience in high school. On March 14 (3/14), her teacher celebrated 'Pi Day' with the students by dressing up in a sweatshirt with a pi symbol printed on it. Tiffany talked about how much she learned because it was a "fun experience." When I asked her how she thought about that experience, looking back, she responds:

Today's thinking is so much more positive. I am so much more appreciative too of the way that he taught especially now that I'm on the verge of actually being a teacher. I definitely feel more than I did back then. Back then I thought oh that's interesting and he's really an effective teacher like I did feel very strongly about that and I did get really excited for the class, like I never once went to class dreading it. I actually went with a very positive attitude and I participated more because he facilitated that with the use of manipulatives and just with his teaching style he was a very caring and again not moving at a fast pace either, and just making his lectures very clear and audible two...it was very helpful...(Education 101 Tiffany interview, May 25, 2006)

Tiffany recalled a positive experience, yet her description lacks details about what made this experience 'interesting and effective' beyond the teacher wearing particular clothing, teaching at a reasonable pace, and making 'lectures clear.' She recalled that he used manipulatives but did not refer to what they were, how they were

used, and how they helped her learn about the mathematical relationship represented by this symbol represents.

Video elicitations of the teaching and learning events provided confirming evidence of beginning preservice teachers' general descriptions of teaching practice. This example is a typical exchange that occurred when Education 101 focus participants viewed clips of their teaching and learning event and stopped the recording to insert a comment about what they were thinking:

DA: So I suggested for him to draw because it helps me when I have a problem that I have difficulty with and he just said right now that it's kind of hard, so like, okay, what helps me is that I draw what the word problem is saying, so...just hoping that would help him.

SS: Do you think it did?

AD: I think it did...

SS: Yeah...

(Education 101 Dalene video elicitation, June 1, 2006)

Dalene gave a rationale for her teaching move, basing her suggestion on what works for her as a learner, rather on what she observed the student say or do. She did not provide specific evidence about how the student's behavior indicates he does not understand, nor does she give evidence about why she thinks her suggestion worked. This was a predominant type of comment in this group's video elicitations.

Education 201 participants. Analysis of reflection data from Education 201 participants showed that they are able to give evidence to support their description of what the teacher and/or student is doing in the video clip of exemplary teaching. There was a range of responses about what they found noteworthy, from descriptions of what they saw to providing both general and specific evidence to support the descriptions. The

group mean score was 2.1. Education 201 participants used a more keenly developed set of lenses to view instruction than the Education 101 participants. This ability was demonstrated by participants' focus on aspects of learning and teaching in general and learning and teaching mathematics in particular. The following examples typify their responses:

- “Great student-guided lesson (she let students make connections from $\frac{3}{3}$ to 1 whole).” (Education 201 AE reflection, May 8, 2006)
- “Extending student knowledge through visualization.” (Education 201 BA reflection, May 3, 2006)
- “(The teacher was) roaming around the room to monitor understanding (Education 201 CJ reflection, May 3, 2006)
- “Transitioning from manipulatives to writing fractions and doing procedures.” (Education 201 FL reflection, May 6, 2006)
- “Transition from concrete model to numeric representations.” (Education 201 HT reflection, May 6, 2006)

While they discussed manipulatives used and student engagement, the attention paid aspects of teaching (rather than aspects of learning) was most common. The predominant terminology used in their accounts included student guided lesson, extending knowledge, transition from use of manipulatives to writing fractions to doing procedures/beyond manipulatives to the abstract/concrete model to numeric representations, gauge understanding/monitor understanding, use of visuals, modeling, concept.

Interviews with the Education 201 focus participants showed that they were more descriptive in the ways they talked about their experiences than Education 101 participants. They used specific language to talk about their experiences and they were able to retrospectively notice aspects of their experiences learning and teaching with evidence of greater detail, naming practices, explaining what worked and what didn't. In the following account, Karen described an experience working on multidigit subtraction with regrouping/borrowing with second graders. She described various details about students' confusion, not only with the process but with the limitations of the manipulatives they were using:

...showing them with the base 10 blocks, what to do and that was tricky because they realized we can't take away... Even though this number is bigger, even though this number is smaller, and we're taking it away from the bigger number, we can't take that away because the blocks are like stuck together, so we have to regroup, and we have to trade in this tens stick for 10 little ones, and then show that with the problem, like with the numbers on the board, a lot of them. I felt like it went over their heads because I was only getting responses from the same group of kids. And you know, there's a point where you feel like you're just losing them the matter what you say...(Education 201 Karen interview, May 30, 2006)

While Karen's frustration is obvious, her ability to notice a number of important details is encouraging. She examined specifics of her practice but focused primarily on issues of student learning. She understood that students were confused and she understood the source of confusion. She realized only some students understand, based on formative assessment based on responses from the same students. While she did not indicate future teaching moves, she provided specific evidence of what is not working and why.

In video elicitations on their teaching and learning events, focus participants illustrated their developing ability to notice details related to teaching as well as learning. They talked about specific instructional approaches used in their teaching and learning events, the complexity involved in helping students develop conceptual understanding, and considered ways to effectively structure lessons. Education 201 focus participants noticed the questions they used and their effect on students, and they talked about previous experiences that influenced their instructional decision making. These preservice teachers talked about the importance of assessment, discussing specific representations in student work that made thinking explicit and provided details of what students said to indicate understanding.

Education 201 focus participants explained how they acquired specific knowledge of students while the teaching and learning event unfolded. They discussed how they used this knowledge to support students. In the following excerpt from her video elicitation on the teaching and learning event, Sarah stopped the recording to talk about her thinking. She had just viewed a portion in which the student was given a word problem requiring him to figure out how many total stickers he had if he had four pages of stickers with ten stickers on a page. Early in the teaching and learning clip, he seemed confused about the wording, interpreting it as if there were ten stickers distributed across four pages. Sarah questioned him to find out where he got confused. When he eventually understood she asked him for his answer. He stated “40” but she wanted to be sure that he truly understood the problem. She talked about things he said and did and she also explains what she was thinking at that time:

SJ : Well...I feel like he has it, because he says '40,' but I feel like what he articulated about being confused is, if there's two or three on a page and then getting forty... I feel like...even though he's saying just now... there's forty on one page he must not really mean that there's forty...So I am giving him the option of saying there's forty on one page or forty altogether. (Education 201 Sarah video elicitation, June 16, 2006)

She noticed and talked about many small details that could have influenced the student's understanding of the problem and how he solved it. She provided various interpretations of what was going on and provided a rationale for the questions she posed. In order to talk about her approach, she had to notice several student behaviors and consider the student thinking behind the behavior. This noticing was necessary in order for her to pose the questions she did. These aspects of her teaching are highly complex. Her interpretations of what she noticed demonstrated that she had some understandings not only of the complexity involved in helping this student successfully solve a problem but how to effectively structure her teaching to guide him once he got derailed.

In sum, Education 201 participants noticed the questions they used and their effect on the students with whom they were working. They talked about previous experiences that influenced their instructional decision making in that particular context. They talked about the importance of assessment. They discussed student work and student explanations where thinking was explicit. They noticed and discussed details of student talk that indicated understanding. Education 201 focus participants explained how they acquired specific knowledge of the students as the teaching and learning event unfolded and discussed how they used this knowledge to support students.

Education 301 Participants. In their reflections on teaching, Education 301 participants provided more specific evidence to support their descriptions of a teaching

situation and evaluations of a teacher's practice than their Education 201 peers. They received a mean group score of 2.7 on a one to three point scale. Similar to their Education 201 peers, these preservice teachers discussed manipulatives and visualization. They also explained aspects of teaching that were noteworthy, using terminology of the profession. Some examples that indicate these distinctions are:

- “She assessed student learning (‘Show me...’) before moving onto the next fraction.” (Education 301 ST reflection, May 6, 2006)
- “All students are involved because they must ‘show’ their fraction using the manipulatives.” (Education 301 EE reflection, May 6, 2006)
- “The teacher repeated what the student said. For example, ‘_____ said that these are equal.’” (Education 301 RH reflection, May 6, 2006)
- “Starting with smaller/easier numbers and increasing the difficulty as students indicated understanding.” (Education 301 DR reflection, May 6, 2006)

Terminology centered on aspects of teaching: “students come up to show ideas;” using students’ ideas to direct instruction;” “showing (representing) student thinking;” “explicit, think (wait) time;” aspects of math/teaching math (“starting with smaller numbers and increasing as students indicate understanding”); employing different student/teacher strategies/techniques; “builds the level” (increases in conceptual difficulty throughout the lesson); teaching concepts from “concrete to abstract” levels.

In their interviews, Education 301 focus participants gave descriptions of experiences similar to those given by the Education 201 group. When describing mathematics learning and teaching experiences, they talked about past experiences with

greater specificity, mentioning aspects of teaching, but focusing on student learning. In this excerpt from an interview with Reese, she discussed her realization that students did not understand the attributes of geometric shapes:

... I realized sometimes the definitions or the concepts as they're presented may be really confusing to the students and you know, if they just see that definition 'same shape, not necessarily the same size,' you know... well, they're thinking that the equilateral triangle is similar to an isosceles triangle, because they're both triangles...and if I hadn't taken them off that path, I might not have ever realized that and that it would take a lot more work on my part to get them to realize that ... if you know what I mean...(Education 301 Reese interview, August 6, 2006)

Reese noticed what specifically confused her students when she gave them the textbook definition for a geometric property. She could account for what was confusing to her students and how they over-generalized a definition because the definition was limited. She noticed students' confusion but she also realized the benefits of the confusion emerging in the lesson. She had specific evidence of students' misconceptions that she could immediately address. She appropriately acknowledged that, if this opportunity had not presented itself, the misconceptions might have remained hidden and could have resulted in even bigger misconceptions.

In video elicitations while viewing clips of their own teaching, Education 301 focus participants focused on evidence of student learning to a greater degree than their Education 201 peers. They noted instances in their professional development that specifically addressed how to gather this information from students from looking at their work, observing them solving problems, and listening to their explanations. They were conscious not only of the questions they asked, but when to ask them. These individuals already have established 'routines' for what they normally do based on previous field

experiences and talked about how they used these experiences in this teaching context. These Education 301 participants have specialized knowledge about distinctions between problem types and strategies used to solve problems. One mentioned how instructional practices might be influenced by the teacher's approach to solving the problem, as well as the teacher's confidence toward teaching mathematics. These participants, like their Education 201 peers, have experience with students. They used these experiences when they talked about awareness of and sensitivity for where and when students might struggle.

Gabrielle explained how she interpreted a student's method for solving a multidigit addition word problem. She explained what she noticed from this teaching and learning event by connecting it to her previous experiences (experiences in her methods course, her visit to this classroom, and a professional development experience when the teachers of this student gave a guest lecture to the Education 301). The understandings she developed during previous experiences is related to the strategy the student used in the teaching and learning event. She could notice his strategy because it was similar to one she had previously seen. In this transcript, Gabrielle stopped the recording to talk about the student's behavior and how she responded to it, based on her understanding of the 'pull-down' method. This method is one where the student draws a line to connect the hundreds digit in one number and the hundreds digit in the other, then recording the partial sum. The student continues to use the 'pull down' method with the other digits (the dotted line indicates a missing transcript piece).

GM: I think, if I hadn't participated in the Math Emphasis (program), I might not have known what he was doing here. But I knew exactly what kind of strategy he was using. So, in a way, it was helpful, because I can

assume that he's connecting 100s...but it could also be hurtful if I am assuming that he knows too much...I might ask less questions because I think I know what he's doing already, versus if I'd never seen that...But, this is a strategy I had seen modeled.

SS: Do you remember where you saw it modeled?

GM: Yeah...when we went to the class at this school...I believed I saw it there...and also, maybe when they (his teachers) even came in and they talked to us at that seminar. We had watched a video, so...Yeah, it looked familiar, right off that bat. I don't know...maybe in our methods courses we had seen that, but that's what it reminded me more of...working in the math emphasis program with them, the two teachers...

SS: Okay, okay....and so the thing that you saw him do there was...

GM: He was grouping the same place value...so drawing lines from the one hundred to the next hundred...

SS: to combine those quantities...

GM: to combine them...

SS: great...

GM: Right there I was just making sure he understood what he was doing by asking him, 'What do you mean, the same kind of numbers?' He might have just said, 'Oh the first two, you connect, or the two and the one, you connect.' But by asking him about the same kinds of numbers, he understood well, the same place value. So the first number in each number is hundreds and he was able to explain 100s and 10s and 1s. And then, I knew that he understood what those meant when he combined seven and eight and said it was 15. He knew that there was a ten there and a five, so...let me see...(Education 301 Gabrielle video elicitation, August 28, 2006)

Gabrielle's ability to notice is impressive, yet typical of her Education 301 peers.

She noticed and synthesized many important details from her experience and connected her understanding of these details to another context where they reappeared. These connections helped her ask particular questions at particular points to not only assess what the student understood about the problem and the process she used to solve it, but answers to the questions helped her gauge the accuracy of her interpretations. She uses disconfirming and confirming evidence to adjust her teaching plan in the rest of the teaching and learning event. This type

of teaching behavior, reflection-in-action, is an important component of teaching expertise.

In sum, as preservice teachers progressed through professional development, their ability to notice changed from noticing primarily notice features of teaching to also notice aspects of learning. Beginning professionals largely noted what was happening through general descriptions; with additional professional development, preservice teachers were able to provide more specific evidence to accompany descriptions. With more professional preparation, preservice teachers drew on professional development experiences to explain their observations of instructional decisions and assessment of students. With more professional development experience, preservice teachers become more able to deal with student confusion by naming ways to assess it, indicating evidence of its existence, and supplying reasonable interventions to minimize it.

From Descriptions of Practice to Reflections on Practice to Reflection-in-Action

When explaining what is occurring in their own practice, or in the practices of exemplary teachers, Education 101 participants described what occurred. Over the course of professional development, preservice teachers are able to reflect on their teaching practices and begin to reflect-in-action, while teaching. Engaging in reflection after teaching allows teachers to reexamine what occurred and consider alternative instructional choices. Over time and through experience, this type of reflection shifts to consideration and implementation of alternatives during the lesson. Over their professional development, preservice teachers incorporated knowledge gained from

professional development in the university classroom and in the field and put it into practice. These examples helped to explain how preservice teachers were able to evaluate how well their real teaching compared to their ideal conceptions. Data from the teaching and learning events, compared with the video elicitations on these events provide information about the ways that preservice teachers at various stages in their professional development reflected on the complexity of their own teaching practices.

Education 101 participants. Participants at this stage of professional development rarely reflected on their practice in video elicitations on the teaching and learning events. No reflecting-in-action was apparent in the teaching and learning events and focus participants did not talk about how they reflected-in-action during video elicitation sessions. Their teaching largely consisted of using leading questions with the students in order to elicit the answer, or suggesting approaches they thought would prove useful. During video elicitation sessions, Education 101 focus participants described what students were doing or what they were doing, with some justification for their ‘teaching’ behaviors.

In the following teaching and learning segment, Tiffany gave the student a word problem to figure out how many Legos a child would have if she had 80 and she lost 27. The student took base ten materials (plastic rods that represent ten and smaller units that represent one) and used them to figure out her first answer, 60. Tiffany questioned the child about parts of the model but he could not answer. Tiffany restated the problem. The following transcript portion details what happened next. It is important to know that the participant used an exaggerated tone of voice when asking a question, to the point of sounding incredulous.

Student: They all lost because 27 and oh...um.. 27 is like...if you lost 27 of them, then they would be all gone.

TN: Hmm...you think so? (the student nodded) If you had 80 and you lost 27...you would have no more Legos? Is that right? (the student nodded). You really think so?

S: Uh huh (he nodded)

TN: So is 80 more than 27? (the student nodded) So, how do you think you would have none, if you lost 27?

(Education 101 Tiffany teaching and learning, June 5, 2006)

The student appeared to be confused about either the problem, a possible strategy to solve it, or both. This was the second time he got an incorrect answer, but Tiffany seemed unsure about how to guide him, other than repeating what he said and asking him if he is sure. She articulated her assumption about what the student knew when she asked the student to compare the magnitude of the two numbers. When she asked him whether one number was more than the other, she implied that the student understood the connection between the magnitude of the numbers and the operation(s) used to determine the difference. The student eventually went back to the base ten materials but was unsuccessful again. He first stated “73” and then “82.” Each time, she repeated the answer to him using a questioning tone that conveyed ‘incorrect,’ since the student returned to solving the problem each time a question was posed. The student used the materials again, successfully. Tiffany confirmed his answer by stating, “Wow! That was really amazing!” Her role was that of ‘confirmer.’ When the student was wrong, she questioned him and when he was correct, she used a different tone of voice.

Tiffany’s video elicitation session revealed very different understandings from my own:

TN: I was just using what he already knows, when I asked him, ‘Is 80 more than 27.’ I wanted to give him a conceptualization of what...of how 80 ranks next to 27...but I was using things that I felt like he would already know...to help him apply it better...

SS: when you asked him, ‘Is 80...’

TN: ‘Is 80 more than 27?’ So it could help him to maybe solve the problem better and to understand that it was a subtracting...not adding extra things but subtracting...which is why that I thought...but I knew that this was knowledge he must have had...(Education 101 Tiffany video elicitation, June 8, 2006)

The teaching characteristics in this clip were present in most video elicitations with Education 101 focus participants. Reflections were largely descriptions of what the students did, or what they, as ‘teachers’ did (“I asked him...”). They often justified teaching moves (“I wanted to give him a conceptualization of what...of how 80 ranks next to 27”) and these justifications were not necessarily based on evidence (e.g., “I was using what he already knows”). There was little explanation of what behaviors indicated evidence of student understanding. If Education 101 participants noticed when they ‘led’ students to a process or answer, they did not mention it.

The video elicitations on the teaching and learning event revealed surprising understandings about important conditions necessary for classroom learning to happen. These conditions may have been addressed during class lectures and follow up section meetings. Attending to students’ developmental levels (“I knew that this was knowledge he must have had...”), realizing there is more than one way to solve computation problems, and connecting new content to prior knowledge (“I was just using what he already knows...’) were implied, even if further knowledge of conditions was limited.

It is one thing to understand conditions for learning. Making them a part of your instructional practice is altogether different. While all Education 101 focus participants

had previous tutoring experience, only one mentioned this experience in the video elicitation session when discussing the importance of teachers asking different types of questions. Education 101 focus participants used a limited range of questions when students needed support to solve a word problem. Most of the time participants supported students through use of leading questions. However, if participants knew their questions were leading ones, they never mentioned this knowledge in the video elicitation session.

Education 201 participants. Education 101 focus participants reflected on their teaching and learning events by describing what happened. In contrast, Education 201 participants largely reflected on their own teaching by discussing the effect of their instructional moves on student behavior and offering more effective teaching alternatives than those they actually used. They likely had more teaching experience as well as more experience with children and could draw on both to assess students. But if students did something unexpected, these preservice teachers had difficulty adjusting instruction.

Education 201 focus participants entered teaching and learning events with a plan for how an instructional sequence would likely unfold. In the following transcript, Sarah listens to the student explain his method for solving a word problem where he was asked to find the total number of students on the playground if there were 247 students initially and 138 more students came. While the student gave a clear explanation of how he used partial sums to solve the problem, the explanation was incomplete. Sarah questioned him during his explanation, first asking him how he knew to the operation to use, then inquiring about how he knew the values of the digits.

Student: I did 247 plus 138. I didn't know the answer quite yet, so I did, instead of adding these two together, at the same time, I did 100, no, 200 plus 100, 300. And I did...40 plus 30 equals 70 and I knew that seven plus seven equals 14, so seven plus eight equals 15. And I added the 300 and the 70 together to make 370, 370 plus 15 equals 185.

SJ: Equals 185?

S: 385.

SJ: 385. Wow...you did a lot of math in your head! That's very impressive. Okay, so first off it says...(She reread the problem. How did you know you needed to add these numbers together?

S: Because you said how many kids are on the playground, I heard a question like this before and I knew that how many kids are on the playground altogether (uh huh), so I started doing this.

SJ: Okay, you knew that this was asking how many girls and boys together on the playground, so you knew to add it? Okay. This is cool, how you did this strategy here. So how did you know that this two and this one in this part of the problem would make 300? How did you know to break that apart like that?

S: Because of 200 is alone it's only 200. And if 100's alone, it's only 100.

SJ: Okay, how did you know this was 200 and 100 instead of just 2 and 1?

S: Because if it was only two and one, these two numbers shouldn't even be there (referring to the rest of the digits).

SJ: Okay...and then...how did you know that this was 40 and 30?

S: Because I know that the tens come before the ones.

SJ: Okay, so this is...which is the ones place and which is the tens place?
(Education 201 Sarah teaching and learning event, June 7, 2006)

While Sarah exhibited some of the behaviors Tiffany used—repeating his answer as a question when he was wrong and confirming his answer when he was correct—she questioned him beyond the correct answer. She asked him questions that were more divergent, as well as authentic. They were questions that could be answered in a number of ways and the way the student answers them gave Sarah indications of how sound his strategy was and what place value knowledge he has.

Their previous experiences with students made Education 201 focus participants sensitive to students' nervousness, learning styles, and approaches toward learning. They mentioned their sensitivity to these individual aspects of students in the video elicitation

to express how they tried to accommodate information that emerged in the teaching and learning event. They grappled with ways to support students. They verbalized that wait time and scaffolding are critical aspects of a teaching and learning event but the balance between the two was challenging for these teachers to establish and maintain.

Education 201 participants talked about the importance of assessing students during instruction. They realized that students' representations made student thinking explicit, while student verbalization of processes and strategies not only could deepen the student's understanding but was assessment information as well. These preservice teachers noticed the questions they used and the effect of these questions on students. These teachers are able to reflect on action after teaching, providing evidence of student behavior to rationalize the instructional decisions they did or could have made.

In the following video elicitation, Sarah demonstrates many reflective behaviors of Education 201 focus participants.

SJ: Right here, I am very impressed at how he's breaking the numbers apart and doing this. But I was very interested in how he knew that 200 plus 100 is 300, because I assumed he probably just, which I asked him later about, was two plus one equals three, and this is the 100s place, so that would be 300...and then, you know...but I thought it was really interesting that he didn't just know that seven plus eight...that was 15, but he said that seven plus seven was 14, so seven plus eight was 15. Because it seems, when he was starting to tell me this, I thought he was just going to say 'Seven plus eight is 15.' So I thought it was really interesting that he broke that apart. He's also articulate about each step, you know?

SS: Why do you think he did it that way? Differently than the other two?

SJ: Um...Well, I am not sure, exactly, but maybe it's part of...maybe these numbers are...but you know, he's so good at adding numbers that this is true, but, two plus one is three, but then, if they're in the hundreds place, 200 plus 100 is 300, because you are adding those zeros...you aren't really, but you know what I mean. And then four plus three is seven and you're just adding a zero, making it 70. And seven and eight are bigger numbers than four plus three and two plus one...you would learn a bit later again, I guess...

 SJ: After he showed me all his work, I wanted to make sure he knew...obviously he knew to add the numbers together but I wanted to get a little more information about how he knew to add those numbers together. Basically I am assessing if he understands the parts of the problem and how to solve it...If he had gotten this all wrong, that's a question I would have asked him. But just because he got it right, doesn't mean that isn't an important question to ask...He explained this to me and I could have said, 'Oh, that's awesome!'...but I really wanted to make sure he understood the parts of the problem. I was pretty confident he did but I wanted to make sure that he could articulate that...(Education 301 Sarah video elicitation, June 16, 2006)

She alluded to prior knowledge of what students at this age know and can do because she was both surprised at and impressed with the student's behavior ('I am very impressed at how he...' and 'I assumed that...'). Yet her theory about what he will do is tentative. She was willing to consider why he did or did not do what she expected, drawing on specific evidence to explain her new theory. ('And seven and eight are bigger numbers than four plus three and two plus one...'). She explained multiple purposes for her questions, their use to assess what the student understands about both the problem and his strategy ('But just because he got it right, doesn't mean that isn't an important question to ask...') and to help him improve in his ability to explain his method ('...but I wanted to make sure that he could articulate that...') She even explained an alternative purpose for the same question if the outcome had been different ('If he had gotten this all wrong, that's a question I would have asked him.').

In sum, Education 201 participants used previous experiences to instruction. They understood the importance of assessing students during teaching and they grappled with how to best incorporate this information while teaching. Education 201 participants notice the effect of their instructional decisions on students. They reflect on teaching by

posing alternate teaching practices; they provide reasonable justifications for these alternatives.

Education 301 participants. Education 301 focus participants behaved differently from their Education 201 peers. Some participants at this point in their preservice professional development posed alternatives for what they might have done differently after the fact. They drew on prior experiences to make comparisons across them and this teaching event. They used these experiences to rationalize instructional decisions they made.

In the following segment from her teaching and learning event, Reese has watched the student successfully solve a word problem where students found the total number of students on a playground with 247 students, followed by 138 more. The student solved the problem using a pull-down method (drawing connecting lines from the hundreds digit in one number to the hundreds digit in the other and writing the sum, then continuing the process with the tens and ones digits before adding up the partial sums). Part of my directions to Reese prior to the teaching and learning event was to demonstrate what ‘another student did,’ a method that was similar to the method the student actually used, but represented differently. Here is what occurred:

RH: Let me grab a piece of paper. Okay...let me see if I can get this right...so...200...

Student: Oh, I've seen that way before...because that's one of the other strategies that they (her teachers) had on the things...on the wall, because they used to put ...like, in second grade they would put the strategies on the wall, so that people (the students in her class)...so if people didn't get their first one (strategy), they'd get the second one...so that's one of the strategies they did. They'd take them out and...they did that...that's my favorite kind.

RH: Wow...so let's see if you can remember how it goes from here. So you have 200 plus 100...Now what?

S: Then you'd do 400 plus 300...

RH: 400 plus...

S: I mean four...40 plus ten...I mean 30...

RH: Okay...and then...what else?

S: seven plus eight...

RH: Okay...and...what should we do now?

S: You should like maybe add 'em...up and then add those um...those answers together, like I did over here...

RH: Oh okay...so it's like that...so if I add these two up, then...

S: That would be 300...

RH: And...

(Education 301 Reese teaching and learning, August 7, 2006)

Reese did a number of things differently in this teaching and learning event that were similar to the behavior of her Education 301 peers and different than Education 201 participants. She inserted conversational markers to not only let the student know she is listening but to elicit information from her. These markers included single word prompts such as 'Okay...', 'And...', and '...then...' are some of them. She used sentence beginnings ('Oh okay...so it's like that...so if I add these two up, then...') which served to not only keep the student engaged but also provided Reese with assessment information about the student's understanding of both the process being used but also the way of representing it. Rather than generalize the meaning of the student's suggestion ('You should like maybe add 'em...up and then add those um...those answers together, like I did over here...') to create all the partial sums and then add the partial sums together to find the total, Reese required her to explain the steps in the process, step by step. She deviated from the directions that I told her to use. Rather than demonstrating the method to the student, she recorded a representation for the explanation on which she collaborated with the student, allowing the student to guide Reese's recording. While the student gave Reese incorrect information (400 plus 300, rather than 40 plus 30), Reese

may seem to lead the student to reexamine what she said by restating the first addend the student named (“400 plus...”). However, she used a kind of prompting that she had used before, so it not an unusual. Yet it served a different and effective purpose in this segment: the student was able to realize her mistake and correct herself.

The following transcript portion comes from Rees’s video elicitation on the same event. In it, she makes evident what she has noticed in the teaching and learning event:

RH: I figured at this point I don't need to tell her and show her; she can tell me.

 RH: I knew she knew but I just asked her again (400 plus 300 vs. 40 plus 30) but I wanted her to think about it again.

SS: You knew she knew because...

RH: Because she'd just done it w/the pull-down method. I know that happens to me to...I just rattle off a number and say a different number than what I mean...sort of ...she's talking faster than her mind is going or the opposite, I am not sure which. I knew she knew but I just asked her again.

(Education 301 Reese video elicitation, August 31, 2006)

Reese reflected-in-action in the teaching and learning event. Rather than do what I asked her (demonstrate the process through representing and talking it through then asking the student questions about whether it makes sense and inquiring how), she has changed her plan based on what the student demonstrated previously in the teaching and learning event and within this particular segment. She asked the student to collaborate with her, telling Reese what to do and how to record it. She noticed that the student already had some information about the digits and their value (400 vs. 40) and she used this information to insert a sentence beginning (“400 plus...”) to get the student to see her own mistake and correct it. She had faith in the student (“I knew she knew...”) and her ability to solve the problem and she drew on this ability to engage the student. But

eliciting information also provided Reese with additional assessment evidence in the growing evidence she has assembled through the event.

Similar to the reactions of their Education 201 peers, Education 301 teachers talked about being surprised by what students knew and could do. This surprise indicated that these teachers had preconceived notions about the students' level of experience that were dissonant with the student's actual behavior. This dissonance could not be considered and new/adjusted theories constructed if these preservice teachers had not noticed details of the students' behaviors to contradict or confirm their theories. Like their Education 201 peers, Education 301 participants had prior experiences with elementary students of varied ages, more experiences than the Education 101 participants had. When they adjusted instruction due to where and when students might struggle, applied appropriate amounts of wait time before calling on students to respond, and gauged student engagement by reading body language, they were effectively making use of their prior knowledge.

In sum, Education 301 teachers had content knowledge about teaching and learning mathematics, including knowledge of what students know and can do at certain grade levels. They were able to compare this knowledge to the current setting and students. They understood the differences between word problems and various strategies students use to solve them. They were able to help students realize the importance of using and seeing the connections between various mathematics strategies and considered such features as the wording of a word problem as possible sources of student confusion.

Teachers at this level of professional development focused on evidence of student learning to a greater degree than preservice teachers at earlier stages. They noted aspects

of their professional development that specifically addressed ways to gather this information from students by looking at their work, observing them solving problems, and listening to their explanations. Education 301 participants had awareness not only of the types, purpose and wording of questions to ask students, but the best points during a lesson at which to ask a particular type of question to extend or assess student thinking.

In sum, through more professional development, preservice teachers were able to use their experiences to inform interpretations of teaching events and use this information to adjust instructional practices. When viewing teaching practice, Education 101 participants primarily described what they saw. Education 201 preservice teachers were able to use prior knowledge about students to reflect on teaching practices, diagnose student behavior, and consider alternative practices that could have minimized student confusion. In addition to what their Education 201 peers could do, Education 301 preservice teachers focused their interpretations of teaching situations primarily on evidence of student learning. They were use this information to pose alternatives when reflecting on practice, but began consider and implement alternatives practices during the course of teaching.

From General Language to Technical Language that Expresses Specialized Knowledge

Another change in complexity that preservice teachers demonstrated over the course of preservice professional development was related to the type of vocabulary they employed to describe attributes of learning and attributes of teaching. There were two types of differences in the vocabulary that was used by participants: 1) vocabulary shifted from layperson vocabulary to more technical vocabulary of the profession and 2)

vocabulary use changed from general to more specific, suggesting more sophisticated vocabulary use and possibly better understanding of what these words represented. That is, participants not only acquired professional language to talk about what they noticed, but their language indicated a better understanding of certain aspects of learning and teaching elementary mathematics. Data from quickwrites, reflections on teaching, interviews, and video elicitations was used to examine vocabulary use across participant groups.

Education 101 participants. I analyzed vocabulary related to teaching and learning in general and teaching and learning mathematics in particular from Education 101 participants' reflections after viewing a video of teaching practice. Vocabulary most often used by these participants is summarized in Table 13. Education 101 participants used words in their reflections on teaching that were not only simplified and less technical but also indicated beginning understanding about the purposes and functions of teachers' actions. Most participants referenced "objects" and some used the word "manipulatives" (Other participant groups called these "manipulatives"). They mentioned "participant" and "involvement" (while other groups used the words "student engagement," "active involvement") and spoke of the teacher "demonstrating" (while other groups used the term "modeling"). While more common synonyms were used by Education 101 participants, they mentioned some of the same or similar terms that Education 201 and 301 participants include in their reflections but without a context or purpose. Some mentioned "letting kids talk about ideas" as noteworthy in the video. While Education 201 and 301 participants mentioned a similar idea, they used more specific, technical vocabulary which indicated the phenomenon's purpose in the

elementary mathematics classroom (e.g., “sharing thinking,” “using what students ask or say,” and “the teacher used the students’ questions to direct instruction”).

The quickwrite was another context in which preservice teachers progressing through a course of professional development demonstrated vocabulary related to learning and teaching elementary mathematics. Participants did quickwrites about how difficult it is to learn and teach mathematics and provided experiences to contextualize each. Quickwrites, like reflections, were analyzed for terminology used to talk about learning and teaching mathematics. In this context, like the reflection on teaching, the language used by preservice teachers changed from more general layperson terminology in the early stages of preservice professional development to language that is more specialized to professional teaching. Table 14 captures typical statement made by participants from each of the participant groups.

Some of the vocabulary used by Education 101 participants referenced ideas about how to use prior experiences to help students acquire new mathematical knowledge. A typical statement from an Education 101 quickwrite noted that she “could not understand new concepts because I did not understand old concepts,” while those in groups representing more advanced stages in preservice professional development mentioned similar aspects in more technical terms (e.g., “students should not be expected to understand more difficult problems and concepts until they have solved simpler problems and earlier concepts” and “it is necessary to know students’ prior knowledge, experiences, and math strategies”). Education 101 participants also addressed the importance of helping students understand the ways in which concepts are related, such as “math is so interconnected with each additional topic you learn.”

Education 101 focus participants talked about some important aspects of learning and teaching mathematics that are critical for preservice teachers to recognize and understand. The language that they used when they stopped the recordings to talk about what occurred in their teaching and learning events was somewhat simple but indicated beginning understandings that will grow over the course of preservice professional development to approximate the language and understandings of elementary mathematics teachers. They usually stopped recordings to describe what the students or they, as teachers, were doing. Sometimes they talked about what they thought the student was thinking at the time or provided a rationale for one of the teaching decisions they made. They talked about the materials in use (“manipulatives”) and made general references to strategies (“I could tell his methodology” and “she didn’t have a solid approach”) as well as some specific observations of strategies the student used (“counting backwards,” “discovered it on her own,” and “some way of marking in her head...leaving a mental note”). They talked about the intent of their teaching actions (“relate it to him...give him a real life experience,” “give him a conceptualization,” and “reminding him to use the visuals”). They talked about the importance of making connections (using “things he would already know to help him apply it better”) and “bringing him back to tens and the previous problem for his future reference”) and aspects of the event that seemed problematic for the student (“math can be so abstract”).

In interview accounts, Education 101 focus participants also used simplified language to talk about experiences learning and teaching mathematics. Their experiences primarily concentrated on positive experiences where the pacing was appropriate, there was regular contact with the teacher, and the teachers were caring, positive, and

passionate as well as knowledgeable about mathematics. Negative experiences were largely about difficult content, fast-paced courses, little interaction, a lot of independent work, teachers who lacked concern for students or knowledge of content, lots of practice/worksheets and students who were not motivated. When analyzing interview content for vocabulary specific to mathematics learning and teaching, I located the following: interactive, engaging, manipulatives, real-life experiences, application, discussion, concept, scaffolding, and conceptual. In sum, the language these participants used and the concepts they referenced were not necessarily that of the professional teacher, but the content demonstrates understanding of some formative concepts later developed in preservice professional development.

Education 201 participants. Like the Education 101 participants, the larger group of Education 201 participants viewed a video-recorded clip of exemplary teaching practice and completed a reflection on teaching to express what they found noteworthy. Education 201 participants used a more keenly developed set of lenses to view instruction, as evidenced by their focus on aspects of learning in general, learning mathematics in particular. While they discussed manipulatives used and engagement of students to some degree, the attention paid to aspects of teaching predominates. Data from the reflections indicated that these participants found numerous and diverse aspects of learning and teaching. They used terminology of the profession to explain what they saw. The most commonly used terminology is found in Table 13.

Education 201 participants noted general aspects of instruction that were not mentioned by Education 101 participants. They mentioned “pacing,” “wait time,” “scaffolds,” “extending knowledge,” and “gauging” and “monitoring student

understanding.” Education 201 participants also mentioned some important pedagogical practices specific to learning and teaching mathematics that were not mentioned in word or concept by Education 101 participants. These practices included “transition from use of manipulatives to writing fractions to doing procedures,” “helping students to move beyond manipulatives to the abstract,” “moving from stages of the concrete model to numeric representations,” and “teachers use of visuals, modeling, and teaching mathematics concepts.” It is clear from this language that these participants were able to not only notice certain aspects of practice, but they had complex language to attach to these complex mathematics teaching practices.

The Education 201 group’s quickwrites included tapping students’ prior knowledge when helping them acquire new conceptual knowledge (“students should not be expected to understand more difficult problems and concepts until they have solved simpler problems and earlier concepts” and “you have to listen to students and find out what they already know and need to have as background information before you proceed”) and helping students understand the relationships between mathematics concepts. A pedagogical concept that was not mentioned in Education 101 quickwrites yet is central to effective mathematics teaching is the importance of helping students grasp difficult concepts by careful and deliberate scaffolded instruction. “Students must be pushed to start solving and understanding problems that use related concepts” was a typical comment from an Education 201 participant. While “pushed” can imply “premature,” the language expresses a complex and essential practice for elementary mathematics teachers.

Video elicitations with the Education 201 focus participants provided the richest information about ways that their ideas are far more complex and sophisticated than that of Education 101 focus participants. While the Education 201 group addressed some ideas and used complex language (previously mentioned in their reflections, quickwrites, and interviews), they also mentioned concepts and language unique to video elicitations. They drew on knowledge from preservice professional development experiences in the university classroom and in the field, connecting their current understanding with previously established understandings. They explained that their knowledge of what students know and can do is continuously developing.

During video elicitations on their teaching and learning events, Education 201 participants were concerned with ensuring that students were supported throughout problem-solving and with assessing student understanding at specific points. They realized it was important that students understood the strategies and representations they were using, could articulate what parts of the problem were being represented (tallies could be ‘muffins,’ lines drawn between digits could mean addition of the quantities represented by those digits), and could tell how they solved the problem. Vocabulary in this context included “place value,” “number sense,” “algorithm,” “CGI”/“Cognitively Guided Instruction,” “problem types,” “flexible” (using lots of different strategies), “compensation” (adjusting of numbers as a computation strategy), “pull-down method” (a way of representing partial sums), “‘friendly’ numbers” (numbers that are easier than others to operate on, such as ten, 20, 25, 50, 100) and composing/decomposing numbers (‘breaking into’ 100s, tens and ones).

Interview data from the Education 201 focus participants provided additional data about how their ideas about and vocabulary used to describe learning and teaching mathematics was markedly different and far more complex than that found when examining interview data Education 101 focus participants. Education 201 focus participants discussed the importance of helping students acquire conceptual understanding and how vital it is for them to help students related to prior knowledge and if they have no prior knowledge, consider what experiences will give them access to some prior knowledge. The vocabulary they used in interviews to refer to language that was general to teaching included “tracking,” “homogeneous,” “heterogeneous,” and “inflexible groupings;” “introduction,” “procedure,” and closure;” “differentiating instruction;” “conceptual understanding” “prior knowledge.”

These teachers also used language to name specific approaches to learning and teaching mathematics attributes of reform instruction, and labels for methods students use and teachers teach. These include “CGI”/“Cognitively Guided Instruction,” “multiple perspectives,” “variety of strategies”/“multiple strategies,” “procedural thinking,” “memorization,” “hands-on,” “manipulatives,” “choosing numbers,” “lattice method,” “partial products method,” “one on one correspondence” for counting, “rules,” “algorithms,” “start unknown,” and “result unknown.” With the exception of “establishing prior knowledge” and “scaffolding instruction,” language mentioned above was not previously addressed in either the reflections or quickwrites from the Education 201 participants or the large and small scale data sources used with Education 101 participants. The diverse and varied vocabulary used by Education 201 focus participants is evidence of the how their use of language associated with learning and teaching

elementary mathematics is highly specialized and indicative of more complex understandings than those of beginning preservice professionals. Education 101 participants use primarily layperson vocabulary with limited use of vocabulary associated with teaching professionals and Education 201 participants engage in language use associated with teachers beginning the teaching profession.

Education 301 participants. In their written reflections on a video teaching, Education 301 participants used language similar to Education 101 and 201 participants (“manipulatives” and “visualization”). They mentioned general features of learning and teaching but with slightly different language than the 201 group (“explicit think time” as opposed to “wait time” and “using students’ questions to direct instruction” as opposed to “student guided lessons/using what kids say/ask”). They referred to different features as well (“structure of the lesson,” “management,” and “different answers and techniques were acknowledged”). Education 301 participants explained noteworthy features of teaching mathematics in particular that were similar to those raised by Education 201 participants but were expressed with more specific language that indicated a deeper understanding. While Education 201 participants wrote about “moving from stages of the concrete model to numeric representations,” Education 301 participants realized the teacher in the video indeed “moved from the concrete to abstract,” but she also “started with smaller numbers and increasing as students indicate understanding,” “builds the level,” “started with smaller numbers and increasing as students indicate understanding.” Surprisingly, the Education 201 participants made little mention of multiple approaches that the teachers and students used throughout the lesson, but the Education 301 group mentioned “different student/teacher strategies/techniques” and “different answers and

techniques were acknowledged”. Although Education 201 participants mentioned “student engagement” in general, Education 301 gave specific examples, including “Students come up to show ideas” and “showing (representing) student thinking”. Terminology commonly employed by Education 301 participants summarized in Table 13.

Education 201 and Education 301 participant groups both discussed the importance of building new knowledge off prior knowledge; Education 201 participants mentioned “students should not be expected to understand more difficult problems and concepts until they have solved simpler problems and earlier concepts” and “you have to listen to students and find out what they already know and need to have as background information before you proceed” while Education 301 participants wrote about it being “necessary to know students’ prior knowledge, experiences, and math strategies” and “building off of prior knowledge and experiences.” These statements do not seem to be markedly different in vocabulary and content. While Education 201 participants did not write about concrete and symbolic representations of mathematical concepts, they did find this noteworthy in the reflections on teaching. However, Education 301 participants found it important to mention in both the reflections on teaching as well as the quickwrites (“symbols and numbers should be connected to something concrete”). This type of reference is significant in that it has emerged from a quickwrite that required participants to write about the ease of learning/teaching mathematics through an experience learning or teaching, a different task than having the context chosen for them (i.e., the video recorded reflection on teaching incorporated a video I chose).

Strategies were not mentioned by Education 201 participants in their reflections on teaching, but they were mentioned by Education 301 in both their reflections in the quickwrites. While reflection data from these participants focused on different ideas, techniques and strategies shown by the teacher and the students, quickwrite data focused more on the students role (“ask students to model strategies” and “make sure it isn’t always the same students who serve as models for different types of strategies”). Education 301 quickwrites also mentioned the importance of providing students with appropriate and meaningful contexts for mathematics (“relating mathematical applications to real life situations”).

The Education 301 focus participants’ video elicitations held rich information about the vocabulary use of preservice at this level of preservice professional development. Their knowledge about elementary mathematics learning and teaching is more complex and the conceptual ideas represented in both the actual language they use and the way they use it is more sophisticated than that of the Education 201 participants. These preservice teachers were aware of problem types, the levels of difficulty associated with them and the range of strategies students were likely to use. They indicated this awareness through such comments as, “I knew exactly what kind of strategy he was using... He was grouping the same place value...so drawing lines from the one hundred to the next hundred...,” “She'd just done it with the pull-down method,” and “She is talking about the 15 being a ten and ones and breaking that apart.” During their video elicitations, they considered the progression of strategies in terms of difficulty, which strategies students use are less efficient than others, determining how ‘flexibly’ students are using a variety of strategies, and the relationship between strategies. They also

considered types of questions they could use like the Education 201 teachers (“leading” and “guiding”), but they also talked about categories of questions.

The actual vocabulary used by the Education 301 focus participants during video elicitation sessions about their teaching and learning events was more specific to elementary mathematics teaching and learning than that of the Education 201 participants. They incorporated similar language to refer aspects of the teaching and learning event that connected to the alternative type of computation instruction emphasized in methods courses, including “problem types,” “strategies,” “algorithm,” and “flexible” (use of various strategies); unique to this group was their focus on “efficiency” of strategies students used and “connections between strategies.” While they referred to similar methods used by the students to combine numbers as the Education 201 group (“pull-down,” “regrouping,” “estimation”), these individuals also talked about “working backwards.” They talked about how students approached problems in the ways they used “friendly numbers” and understood “place value,” like the Education 201 participants, but they also talked about how students showed they had “number sense. When talking about their deliberate use of questions, they used vocabulary similar to that used by their 201 peers, such as “leading” and “guiding,” but they also used knew other types of questions as well, such as “open-ended” and “clarifying.” They talked of “representations” of students’ strategies that “pictorially” depicted the processes they used. They mentioned how important it is that teachers look for and make meaning of “evidence” of student learning so that they can more effectively plan, implement and assess their own practice.

In interviews, the most impressive distinctions between these individuals and their Education 201 peers relates to their descriptions of experiences during which they made adjustments to planned lessons prior to and the midst of teaching them. Such practices require an awareness of student feedback and a repertoire of options from which to choose, neither of which the Education 101 or Education 201 participants spoke of using. Remarkably, these individuals talked about the valuable learning they experienced when lessons did not turn out in the ways they anticipated and these surprises helped them reflect on their practices and adjust their preconceived notions. They used vocabulary that was not remarkably different than Education 201 focus participants: “formulas and rules,” “making learning accessible,” “variety models,” concepts, conceptual understanding, lesson components (“launch,” “explore,” “summarize”) and “types of lessons” (reteach), “manipulatives,” “memorize,” rule, “models” among them. However, the context in which the language was embedded provided a significant and sophisticated use of this language to express aspects of learning and teaching elementary that set them apart from the Education 201 group.

In sum, a variety of contexts was used to examine the ways in which preservice teachers with varying amount of professional development talk about mathematics learning and teaching. While participants used professional language during the reflections on teaching, quickwrites, and interviews, the video elicitations on the teaching and learning event provided the richest context for vocabulary use. In essence, the closer the context resembled a mathematics teaching context that involved the participant, the more opportunities the participant had to use the language of the profession.

As preservice teachers progress through professional development, their language shifts from general, layperson language with some beginning use of professional vocabulary to more specific, technical language that is characteristic of practicing mathematics teachers, to more specialized, technical language to describe details of mathematics learning and teaching. Over time, they change from talking like a prospective teacher to talking like a beginning teacher to talking like a beginning teacher with specialized knowledge about mathematics learning and teaching.

Table 13 Commonly used vocabulary used in the Reflection on Teaching by participant group

Education 101 Participants	Education 201 Participants	Education 301 Participants
<ul style="list-style-type: none"> · manipulatives · objects · participation · interest · involvement · concept · mentally picture/visualize · demonstrate · asking children questions instead of telling · discussion/letting kids talk about ideas 	<ul style="list-style-type: none"> · student guided lessons/using what kids say/ask · extending knowledge · student engagement · pacing and wait time · active involvement · scaffolds · sharing thinking · nonverbal responses · transition from use of manipulatives to writing fractions to doing procedures · helping students move beyond manipulatives/concrete/physical to the abstract/numerical · making connections · gauging and monitoring (student) understanding · use of visuals, modeling, teaching concepts 	<ul style="list-style-type: none"> · manipulatives · visualize · management · structure of the lesson · different answers and techniques were acknowledged · students come up (to the overhead/in front of the class) to show ideas · using students' questions to direct instruction · showing (representing) student thinking · explicit, think (wait) time · starting with smaller numbers and increasing (number magnitude) as students indicate understanding; · moving from the concrete to abstract; builds the level/increases the difficulty · different student/teacher strategies/techniques

Table 14 *Typical statements illustrating vocabulary used in the Quickwrite by participant group*

Education 101 Participants	Education 201 Participants	Education 301 Participants
<ul style="list-style-type: none"> · interest in the material · keeping up 	<ul style="list-style-type: none"> · I need to break down what my brain does in a few seconds so students can understand 	<ul style="list-style-type: none"> · relating mathematical applications to real life situations
<ul style="list-style-type: none"> · math is so interconnected with each additional topic you learn 	<ul style="list-style-type: none"> · I have lots of strategies I never knew were strategies 	<ul style="list-style-type: none"> · ask students to model strategies · building off of prior knowledge and experiences
<ul style="list-style-type: none"> · teachers who explain things in many ways 	<ul style="list-style-type: none"> · must be pushed to start solving and understanding problems that use related concepts 	<ul style="list-style-type: none"> · symbols and numbers should be connected to something concrete
<ul style="list-style-type: none"> · could not understand new concepts because I did not understand old concepts 	<ul style="list-style-type: none"> · students should not be expected to understand more difficult problems and concepts until they have solved simpler problems and earlier concepts 	<ul style="list-style-type: none"> · necessary to know students' prior knowledge, experiences, and math strategies
	<ul style="list-style-type: none"> · you have to listen to students and find out what they already know and need to have as background information before you proceed 	<ul style="list-style-type: none"> · make sure it isn't always the same students who serve as models for different types of strategies

Teaching practices differ depending on amount of professional development. How?

The practices of those who are beginning and those who are completing preservice professional development in learning and teaching elementary mathematics differ in two ways: 1) there is a change in the type of support used and interventions employed and 2) there is change in the focus on process over product with more professional development. I observed teaching practices of three participants from each level of preservice preparation (Education 101, Education 201 and Education 301) by asking them to work with a second grader who was solving word problems. I provided the word problems and gave each of the nine participants instructions about the teaching and learning event. I video-recorded each event, and conducted a video elicitation interview with each participant.

Finding 3: There is a change in the support preservice teachers provide students, from using a few teacher strategies that lead students toward a correct answer to a wider range of strategies that elicit students' processes.

There is a change in support and the interventions employed as preservice teachers progress through professional development. Education 101 participants support struggling students through demonstrating or suggesting a path to the solution and use convergent, closed, leading questions to get them there. Education 201 and Education 301 participants guide students through their struggles by asking questions and using guiding statements learned from previous teaching experiences and these tend to be less

leading and more divergent in nature. While Education 101 participants, who are undergraduates, have a limited repertoire of interventions from which to draw when working with students, graduate level preservice teachers have a wider repertoire of strategies to use from past field experiences and more capable of adjusting their practices while they are teaching than Education 101 participants.

Education 101 participants. Preservice teachers representing stages in the course of professional development discussed what they have done to assist students in mathematics. When teaching dilemmas surface, Education 101 teachers found that the ‘problem’ lies in the wording of the word problem, the inappropriateness of the task, or the motivation and/or interest of the students. They tried to determine what was confusing to students and then used explanations and demonstrations to tell and/or show students where they got confused. Their repertoire of intervention strategies grew as they received more professional development. With more methods and field experiences, Education 201 and 301 teachers not only reference strategies that have worked in the past but the strategies they mention are more numerous and qualitatively different than those of Education 101 teachers.

In interviews, Education 101 focus participants were asked to talk about a time when they were teaching somebody mathematics. In the following transcript, Dalene explains what she does to help a middle school student who is having trouble solving an algebra problem.

SS: OK...how do you explain it? What do you think about when you have to explain something?

DA: I try not to hold the pencil ...I have him write it... make him do the work. If he was...I think he was solving equations like two variable equations... I try to verbally tell him step-by-step how to...I ask him questions...Like what would you do to this side...You have to do the same to the other side

SS: OK...does that work pretty well?

DA: yeah...He knows what to do...he's a lot... he's a smart kid but he's really unmotivated.

(Education 101 Dalene interview, May 22, 2006)

Dalene realizes that it is important for the student to do the work himself and her role is to explain the steps involved, yet she implies that the student's own motivation toward and interest in mathematics is to blame as well. Later in the interview, she talked about another 'teaching situation' with fourth graders, in which she explains how the wording of problems might be what is confounding to them. Her role, once again, was to carefully explain: "...when they needed help on certain problems ...most of the time I try to help them, like I try to point out important info in the word problems" (Education 101 Dalene interview, May 22, 2006).

As with Dalene, Tiffany, another Education 101 participant regarded as teaching as largely explanation and 'getting the message across.' While she expressed some remorse that her methods were not effective, she, too, conveyed that the student's lack of interest is more at fault than her own teaching practices:

I did feel discouraged and a little frustrated that I couldn't get the message across or really help her either because...I kind of felt helpless because she was just kind of like no...no...no...I'd rather watch TV...she was too distracted and too uninspired...(Education 101 Tiffany interview, May 25, 2006).

During their video recorded accounts of teaching and learning events, Education 101 participants 1) repeated parts of or the entire word problem, 2) indicated an incorrect answer by repeating the answer in a questioning tone, followed by “are you sure?” or “right?” 3) used leading questions to redirect a student toward a correct answer or strategy, or 4) suggested a solution strategy to solve the problem. In the following teaching and learning event, Dalene has given the second grade student the following problem to solve: 19 children are taking a mini bus to the zoo. There are seven seats on the mini bus. How many children can sit two to a seat and how many have to sit three to a seat? The child sits silently for a several seconds and then Dalene suggests the path for the student. When a general suggestion does not work, she makes specific suggestions:

DA: You know what helps me? I could draw it...I could draw maybe...how many seats there are, since there are seven, right? Are you having a little trouble? (The child nodded). Um...you could draw it...you could draw seven seats...could you draw seven seats? (He drew while she watched silently. After the child drew seven seats, he sat quietly). We know there are 19 children, right? And each seat could either... You could either have two children or three children. It's your choice (The child again sat quietly for several seconds). What do you think you would do with 19 children?
 Student: Add them to...to...I would put two in a seat or three in a seat.
 DA: And see how many...yeah you could try that...So if you wanna...add those to the seats and see how many seats you get?
 (Education 101 Dalene teaching and learning event, May 26, 2006)

When the student is not sure what to do, Dalene told him, first suggesting he draw, then suggesting what to draw (seats) as well as how many (seven). She then repeated the part of the problem he needed to focus on next to complete the problem. Her insertions of the word, “right?” are typical of beginning teachers. While this teacher is asking a question, in essence she is giving information and checking to see if the student is either listening

to the information or agreeing with her logic. When the student did not know what to do next, he repeated the part of the problem she reread (“I would put two in a seat or three in a seat”). The teacher confirmed that the step he stated was the next one he should take.

A different student working with another Education 101 participant had difficulty with the same problem. After the student drew seven seats as blank lines, he distributed the 19 students by ‘dealing them out’ one at a time, drawing a tally mark on each ‘seat.’ He finished drawing the problem solution, representing the six seats, but had represented three children on each and one seat with one child on it (rather than seats with both two and three children). Tiffany led the student toward the answer by telling him what he should do, first asking him to explain what he had drawn:

Student: Six children can ride on seats, three of them on each seat, and then there can only be one on that seat, because it's 19.

Tiffany: Oh...actually the problem was asking how children can sit two to a seat and how many have to sit three to a seat. So you can make seats for only two children to sit on them.

S: Well, three children are sitting on that seat and three children are sitting on that seat and three children sitting on that seat and three children sitting on that seat and three children and three children and one children because, if you count them it will be 3...'cause $3 + 3$ is 6 and that six and that six plus one is 19.

TN: That's right. Do you think that one person, though, could have sat with another person? (S nodded). Or do you think someone else could have come and moved and sat with that one person?

S: Uh huh...

TN: And what would that have done?

(Education 101 Tiffany teaching and learning event, June 5, 2006)

Tiffany confirmed the student’s arithmetic (three plus three is six and six, and six and six and six and one equals 19). At that point, her questioning tone implied something was not quite right. Her two questions—each beginning with, “Do you think that one person”—was a directive. Rather than an authentic question to which the teacher does

not know the answer, it was her way of pointing out faulty logic and provided her a way to more ‘gently’ tell the child what he should do: move a tally mark from one group of three (making it now a group of two) to the ‘seat’ with ‘one child’ (to make another group of two). She asked a leading question—“ Or do you think someone else could have come and moved and sat with that one person?”—to which the child could only reply with ‘yes’ or ‘no.’ His ‘yes’—“uh huh...”—was expressed in a tone that is unconvincing and lacks confidence. The changes he made from Tiffany’s directives result in the correct answer represented on the paper: five seats with three children and two seats with two students, but it is not clear that the child understood why he did what the teacher told him to. In sum, Education 101 teachers relied on teacher-directed instructional methods such as explanation, demonstration, leading questions and making suggestions. They used pedagogical practices that make sense to them and not necessarily the students with whom they are working.

Education 201 participants. The Education 201 and 301 participants made use of the pedagogical content knowledge they gained from their experiences in method courses and in the field. These experiences fostered more strategies to employ when supporting students, and they referenced their experiences when talking about their practice. They used fewer convergent, one-answer questions and more divergent, open-ended ones. They were more intent on ensuring that students understood what they were doing through use of different types of questions, prompts and conversation markers and focusing on students’ use of multiple solution strategies to solve problems. In addition, Education 301 participants incorporated student feedback to guide their instruction and doing so required them to deviate from the tentative plans they have in mind when they

embarked on these teaching and learning events and to use alternatives based on what students said and did.

Interviews with both Education 201 and 301 preservice teachers show the variety of resources from which these teachers can draw when supporting student learning. The following two interview segments tell of experiences in which teachers at the end of their student teaching are able to call on a variety of different strategies when they reflected on a lesson after it was over as well as while it was in progress. Karen, an Education 201 participant, tells about an event from her student teaching that occurred on the day I interviewed her. In it, she names the options she used when the second grade students in her student teaching field placement struggled with multidigit subtraction problems:

It's hard for like me...I spent a lot longer on it today than I wanted to. I thought like, 'They're not getting it...' There comes a point when you think you have to come back too, because you can't like badger it into their heads, you know? They need some time to like make some sense. I think I tried to do too much, to have a discussion, I used visuals, let's get on the floor, let's do this all. I think that maybe it was too much in one session, but I think there's a lot to think about, a lot of different ways to do it, there's a lot of different things kids need: the pacing, trying not to make it so overwhelming, and trying to get to every kid. I mean teaching in general is pretty much summed up and hard and frustrating and all those things, but for math especially...(Education 201 Karen interview, May 30, 2006).

Karen was knowledgeable about multiple support strategies she could use to make mathematics concepts accessible to her students. These included appropriately pacing her lesson, incorporating student talk, and using visual models. Yet she also revealed her sense of the many complexities of teaching, such as assessing individual students, presenting difficult concepts in smaller chunks, using

instructional time carefully and purposefully. In addition, she had the reflective sense to see that she likely used too many strategies that day.

In the following interview segment, Reese, an Education 301 participant, explained what she did to implement a lesson on decimals that was based on assumed student understandings. She detected student misconceptions, then employed a sophisticated set of strategies to deal with them.

...and so I pulled out the manipulatives and I talked about what 100th was and I just really started from square one and related it to money. And then the students, they went back and corrected themselves. They said, 'Oh I would change my answer now, and this is why.' And the thing about this whole lesson was I hadn't really planned it. It was supposed to be an intro to what we were doing that day and then I saw that, oh my gosh, nobody even knew what this quantity was. So I just sort of went into that and I just felt really good afterwards. (Education 301 Reese interview, August 7, 2006)

Both Karen and Reese assumed responsibility for the students' lack of understanding, a difference from the Education 101 teachers who equated students' lack of understanding or weak performance with their lack of interest and motivation.

Whereas beginning preservice teachers found careful explanation and demonstration to be viable pedagogical strategies, Karen realized that she could not just explain to her students what to do or "badger it into their heads." Both Karen and Reese began teaching a planned lesson and realized, while teaching it, that their plans did not sufficiently address students' needs. They each had a range of ways to address those needs including use of materials, connecting to prior models and knowledge, incorporation of student talk and class discussion, and adjustment of the amount of content and pace of instruction. Karen clearly felt frustrated by her lesson but has ways to deal with future ones, while

Reese felt empowered by her ability and willingness to reconsider what she had planned and bring in alternatives, ones that proved successful for student understanding. Karen was able to reflect on her teaching actions and pose logical and suitable alternatives to what she had done and Reese was able to change plans in the course of implementing them, reflecting-in-action. Both behaviors—reflecting on action and reflecting-in-action—are signs of reflective practice that demonstrate progress toward acquiring the behaviors of expert teachers.

During teaching and learning events, Education 201 and 301 participants used questions and prompts that were qualitatively different from the Education 101 group's strategies. In the following interaction, Karen, an Education 201 focus participant, was working with a child on the following problem: Arthur baked 15 muffins. How many more muffins does Arthur have to bake to have 43 muffins? After some silent time, Karen began talking.

KE: What are you thinking about?

Student: I am thinking of what I could add to 15 to get 43...

KE: Wow....So you know that you are going to need...you have 15 but you need a lot more to get to 43. And you are thinking of that number? And you are just thinking in your head? You don't need to do anything on the paper?

S: Hmm... (student began recording...)

KE: Show me what you have so far....okay...what number do you have?

S: 45.

KE: And what number do you have to get to?

S: 43...

KE: Oh, so you're already over the amount....so what could we do about that?

(Education 201 Karen teaching and learning event, May 31, 2006)

Her first question was an authentic one; she did not know the answer to it. The student's answer not only told her what numbers she was using, but how she used them,

the operation used, and the strategy to find the answer. While the statement, “And you are thinking in your head” was assumptive, the follow up question “You don’t need to do anything on paper?” led the student to represent her own solution on paper. The teacher took the student’s solution strategy—determining what is needed to add to 15 to ‘get to’ 43—and prompted the student to keep track of this process by recording it on paper. This support was very different from what Dalene and Tiffany did. The Education 101 teachers suggested to students that they draw, but the suggestion alone was insufficient. Beginning teachers told the students what to draw or how to change their drawing. In the above example, Karen suggested to the student that he use his own strategy but keep track of it on paper, rather than mentally.

In this next segment from the teaching and learning event, Sarah, another Education 201 focus participant, gave her student the same ‘muffin’ problem. The child reported the answer to Sarah. The answer was correct but Sarah first asked a closed question, requiring only a one word or correct answer, then followed up with a more divergent question that prompted the child to explain his entire strategy:

SJ: Are you sure it's 28? (He nodded) How are you sure it's 28?
 Student: (He looked at his paper, then looked up, then talked). When I first...when I...before I wrote on the paper, I was thinking in my head, what's 15 plus 10 and that's 25...and I kept the ten (uh huh) and I added another ten, 35, I kept that ten (uh um) plus 5, I kept the 5 and that equals 40 and I...and I kept the 3 and that equals 43. And I added the ten and ten plus five together and that equals 25 and I added 3. (Education 201 Sarah teaching and learning event, May 20, 2006).

While this student may not have needed teacher support to find the correct answer, teachers who elicit student explanation about the problem solving process support students in important ways. Requiring the student to explain why his answer makes sense

helped the student to both clarify and deepen his own understanding of the process he used by articulating it aloud. At the same time, the teacher accumulated assessment information, including the student's ability to add ten to any number ($15 + 10$, $25 + 10$), his facility to mentally compute, his knowledge of benchmark numbers (decade numbers, in this case, 40), his ability to keep track of a total, and his ability to continually find the difference between a 'running total' and another number (43). This assessment information helps teachers to support individual students in the future.

Education 201 and Education 301 participants were aware of the questions they asked and statements they made while teaching. At times Education 201 and Education 301 participants reworded their statements to elicit a process from the student rather than tell the student what to do. In the following example, Karen worked with the student on the 'muffin problem.' The student started with 15, added 30 and then needed to compensate.

KE: Okay, let's look at this...we had 15 to begin with...and you added...you just guessed, is that what you did (S nods), and said, 'Oh, I'll just add 30?' Okay, so...and then what did we get when we added 30?
 Student: 45
 KE: and we said we wanted...
 S: 30...43...
 KE: okay, so then I wonder why we added 2 then...
 S: Oops...
 KE: What do you think we should do about that, then?
 S: Take away from 3?
 KE: Okay, so let's look at this part again...so we started...why don't you explain what we have...
 (Education 201 Karen teaching and learning event, May 31, 2006)

Karen explained ("We had 15 to begin with and...") and prompted ("and we said we wanted...") the student in an attempt to get the student to understand that she exceeded

43 and now must ‘do something.’ When the student did not seem confident about what she should do (“Take away three?”), Karen began to explain the process again. This time, she realized ‘telling’ the student was ineffective. Instead, she asked the student to explain what went awry (“Why don’t you explain...”). The student finally succeeded at solving the problem. But Karen was not satisfied that the student understood the problem itself or the process she used to solve it. Later, Karen asked the student to solve the problem in another way and the student successfully solved it by starting with 43, subtracting 15 and finding a difference of 28. Requiring the child to use two ways to solve the problem was an intervention that Karen decided to employ, not one that I asked her to use. It convinced Karen that the student not only understood the problem structure but understood that both addition and subtraction could be used to solve the same problem. This change to the plan was not used by any of the Education 101 participants to ensure the student understood the problem or the process used to solve it.

In the following segment from another Education 201 teaching and learning event, Sarah asked a student to explain a process he used to solve a problem. A discussion of place value ensued. Sarah began to explain what the student stated, then required the student to explain. Sarah also used a conversational marker—‘Okay...’—to let the student know she was listening to his explanation, so he could continue giving it. Sarah also used a sentence beginning, followed by a pause, so the student could ‘fill in’ an answer (“that means the four is really...and the three is really...”). She supported the student by prompting in ways to get him to explain his own method, rather than parrot back a method she had explained.

Student: Because I know that the tens come before the ones.
 SJ: Okay, so this is...which is the ones place and which is the tens place?
 S: This is the ones and this is the tens (he pointed to paper).
 SJ: Okay.
 S: This is the ones and that is the tens.
 SJ: Okay, so if this is the tens place then that means this four is really...
 Student: 40.
 SJ: 40...and this three is really...
 S: 30
 (Education 201 Sarah teaching and learning event, May 20, 2006)

When Education 101 participants reviewed their video recorded teaching and learning event during the subsequent video elicitation session, they described what they, the teachers, or the students did during the event. Education 201 and Education 301 provided different kinds of information during the video elicitation. They 1) talked about the thinking behind the support provided students that may have not been explicit in the teaching and learning event, 2) discussed alternatives they could have provided but did not, and 3) related their previous teaching experience to this particular teaching context. None of these behaviors were demonstrated by the Education 101 participants. In the teaching and learning situation, Dalene, an Education 101 participant, suggested a student draw. In her video elicitation, she explained that she suggested this method because it is one that works for her:

So I suggested for him to draw because it helps me when I have a problem that I have difficulty with and he just said right now that it's kind of hard, so like, okay, what helps me is that I draw what the word problem is saying, so...just hoping that would help him. (Education 101 Dalene video elicitation, June 1, 2006).

In the following teaching and learning segment, Brianna, an Education 301 participant, realized the student was struggling and also suggested drawing:

I don't know if I am allowed to do this but...but maybe you could use pictures...pictures could help...maybe you could draw the seven seats. So, why don't you draw... put seven seats, that those represent the seats on the bus, using a circle or a square or whatever you want to use. (Education 301 Brianna teaching and learning event, August 28, 2006).

While Brianna's intervention was similar to the support offered by Dalene, Brianna's explanation during her subsequent video elicitation reveals a very different interpretation:

Maybe I could have said, "You could use pictures. How would you start the problem if you were going to use pictures?" I didn't really give her that opportunity to see if she could figure it out on her own. Right away I said, "Why don't you use pictures for the seven seats?" Maybe I could have at least seen if she would have known to do that first, before I gave it to her. I think I gave it to her because she was frustrated at this point. (Education 301 Brianna, video elicitation interview, August 28 19, 2006)

Brianna reflected on her teaching practice. She questioned her practice, explained why it was a premature teaching move, and suggested what she might have done differently.

Field experiences helped preservice teachers establish pedagogical reference points to inform future instructional practices. These reference points were more numerous for Education 201 and 301 teachers and were either missing or limited for Education 101 teachers. Knowledge from these early experiences helped preservice teachers make decisions about later teaching dilemmas. In the following example, my notes from Sarah's video elicitation session describes how she was using knowledge from her student teaching in a first grade placement to make sense of what the second grader in this teaching and learning event was doing. In the teaching and learning segment, the student solved the following: There are 247 girls on the playground and 138 boys on the playground. How many kids are on the playground?

Sarah stated that she is now recalling experiences from her first grade placement and CGI (Cognitively Guided Instruction) work. One thing they as teachers would ask (students) was "Does this remind you of a problem you have done before?" SARAH talked about how the number parts "get changed around" (decomposed) and she thought it was interesting that he could verbalize the familiarity without her asking.(Education 201 Sarah video elicitation interview, June 16, 2006).

Sarah's comments indicate a number of understandings from her field experience that are important for supporting students. She has acquired a useful strategy to use when students are given a new word problem: asking them a question to help them connect to previous knowledge and prior experiences. Because she had previous field experience where students decomposed numbers in order to operate on them, she is able to understand the strategy when it is used in this context. At the same time, her own prior knowledge is questioned and adjusted with new information when she "finds it interesting" that the student behaves differently than what she has seen in the past ("...verbalize without her asking"). This new information enhances Sarah's prior knowledge about what students are capable of; when she sees this kind of behavior in the future, she can use this information to better understand what students know and can do and adjust her student support appropriately.

In sum, as preservice teachers progress through professional development, they increase their repertoire of teaching strategies. Education 101 participants largely rely on careful explanation, demonstration, and pointing out important information to students while teaching. Education 201 and 301 participants have a number of strategies that have grown out of professional development experiences in the university classroom and in the field. These strategies include use of multiple models and a variety of approaches at the

same time that they attend to issues of pacing, incorporating student talk, and assessing student understanding. When students struggle, Education 101 participants tend to lead students or suggest approaches, while Education 201 and 301 participants elicit misconceptions and then incorporate other approaches or models to explain concepts. Education 201 participants are able to reflect on lessons after they occur and consider alternatives to employ in the future. Education 301 participants are able to reflect while teaching, assessing student understanding, then adjusting lessons to more appropriately address student need. While Education 101 participants have limited professional development experiences, Education 201 and 301 participants having a growing body of prior knowledge from which to draw when making instructional decisions.

Finding 4: A change in focus from product to process occurs when comparing the Education 101 participants' teaching practices to those of the Education 201 and 301 participants.

When Education 101 participants worked with students to solve word problems, the ways that they elicited the product and/or process from students was different from Education 201 and 301 participants. Education 101 participants typically focused on getting the students to come up with a correct answer. While they elicited the process from students, their treatment was cursory, as something they ought to do. When they asked questions to get students to explain processes for solving problem, these questions came after the correct answer had been given and validated, when the answer was

incorrect and eliciting the process served to correct it, or not at all. Education 201 and 301 participants emphasized the process as much as or more so than the product when interacting with students in teaching and learning events. While methods courses experiences emphasize that inquiring about students' processes reveals far more information than just an answer, the Educating 201 and 301 participants conveyed in teaching and learning events and video elicitations on these events that their intention to elicit process was as much interest about understanding the student as it was something they 'ought' to do out of professional 'duty.' Education 301 participants not only elicited the process, they pressed students to account for steps in the process as well.

Education 101 participants. In this segment, Tiffany regularly stated, "That's right" or "Good..." before moving onto giving the student the next problem to solve.

TN: How many blocks did you draw out...or how many muffins?

Student: 23.

TN: 23? Oh...very good. I like that...are you ready for another one? (S nodded) Okay...

(Education 101 Tiffany teaching and learning event, June 5, 2006)

Later on, Tiffany similarly confirmed the correct answer, after first prompting the student where to look for it in the drawing. She then asked the student a question about the process. However, the 'process' was one she had earlier explained to the student rather than elicited from him.

TN: Right...so from there can you tell me how many children can sit 2 to a seat, just looking at your picture?

Student: four.

TN: That's right...and how many have to sit three to a seat, then, if you have four?

S: There's 15.

TN: Wow, that's great, Ss. And how did you figure that one out?

(Education 101 Tiffany teaching and learning event, June 5, 2006)

Hilary, another Education 101 participant, gave a student the following problem to solve: There are 10 stickers on a page. If you have four pages of stickers, how many stickers do you have? When the student was done, Hilary communicated her satisfaction with the correct answer, even when the student ‘asked’ if the answer is correct (“40?”):

Student: It's 40?

HR: very good...very good...

(Education 101 Hilary teaching and learning event, June 5, 2006)

Another Education 101 participant, Dalene, similarly confirmed a correct answer to the same problem:

DA: How many stickers on a page?...and if you have four pages...

Student: You'll have 40 stickers...

DA: Good...okay...That's right...so you want to go onto the next problem?

(Education 101 Dalene teaching and learning event, May 26, 2006)

Later on in the teaching and learning event, Dalene asked the student to explain what he did to solve this problem: There are 247 girls on the playground and 138 boys on the playground. How many kids are on the playground? Although there are a number of ways the student could have solved this problem, he used a ‘partial sums’ method, combining first the hundreds, then the tens, then the ones ($200 + 100$, $40 + 30$, $7 + 8$; $300 + 70 + 15 = 385$). While she elicited his process, she validated his explanation as if it were the only ‘product, before moving the next problem:

Student: Done (capped pen).

Dalene: So how did you...can you explain to me what you did?

S: I did...(and he read what he had written on paper).

Dalene: That's right...good job.

(Education 101 Dalene teaching and learning event, May 26, 2006)

Education 201 participants. These individuals routinely asked students to explain the processes they used, even when students reported a correct answer. They often elicited information beyond the process as well. In the following teaching and learning event, Belinda had read the sticker problem to the student (There are 10 stickers on a page. If you have four pages of stickers, how many stickers do you have?). She asked the student about his process, after asking if he understood the problem and hearing him report correct answer:

BA: And then, I was just wondering...do you know what this problem is asking you to find out? (Ss nodded) What is it asking you find out?

Student: How many stickers there is in total.

BA: Total...The total? Okay, good. Can you show me how you would solve that problem? Or do you know how to solve that problem.

S: I already know the answer.

BA: You already know? What is it?

S: 40.

BA: 40. How did you know that?

S: Because 10 times four is 40.

BA: Okay, can you show me on your paper with the numbers how you saw it in your head (The Ss recorded on paper). Good. And what does the ten mean...in that.... Do you know? Over here, what does the ten mean?

S: The ten stickers.

BA: The ten stickers? And what does the four mean?

S: four pages.

BA: Good... and so that's 40 altogether? You're so smart...I didn't know how smart you'd be.

(Education 201 Belinda, teaching and learning event, June 15, 2006)

The student mentally solved the problem, but Belinda required him to record his mental process on paper. The student recorded ' $10 \times 4 = 40$ ' and Belinda pressed him to explain how the numbers in his equation connected to the objects in the problem.

Education 301 participants. Education 301 participants routinely elicited students' processes. However, once students reported their process, these preservice teachers

pressed students to explain the specific steps in the process. In addition to eliciting the steps the student used in his strategy to the ‘muffin problem’ (Arthur baked 15 muffins. How many more muffins does Arthur have to bake to have 43 muffins?), Gabrielle also required the student to explain the reasoning behind the steps in his process (“How did you choose how to jump?”), as well as connect the quantities he is using to the objects in the problem that they represent (“28 what?”):

GM: So what's your answer?

Student: 28.

GM: 28 what?

S: Muffins.

GM: 28 muffins. Can you tell me how you solved that?

S: First I started by jumping to 43. So I jumped 15 to 20, 20 to 40, then 40 to 43.

GM: Okay.

S: Then I had to add up all my jumps to find my answer.

GM: Okay. So you started with how many muffins?

S: 15.

GM: 15...then you figured out...what? How many you need to get to...

S: 43.

GM: 43. So you jumped how many here?

S: 5

GM: 5...and then you jumped...

S: 20...

GM: 20...and then you jumped...

S: 3.

GM: How did you choose how to jump...what numbers to write here?

S: Since I am really smart, I know how to get to the perfect level really fast. So I know that going to 20 would be the perfect...would be a friendly number, which would level me perfectly. Then I could get into the target tens area. And then I could just get to the target.

GM: What was your target tens area?

S: The forties...

GM: Excellent. Would you be able to solve that problem in a different way? Can you think of a different way to solve that problem?

S: Yeah, I can even do the same thing backwards.

(Education 301 Gabrielle teaching and learning event, August 9, 2006)

Gabrielle employed several pedagogical strategies to draw out detailed information from this student. She inserted 1) conversation ‘markers’ (“Okay...”) to let the student know she heard each step, 2) repetition of the student’s answers when he completed her prompts (“20...and then you jumped...”) and 3) used of the student’s unique mathematical vocabulary (“target tens”). These teaching moves encouraged the student to tell her about his strategy (jumping by tens from 15 to 43), his use of decade and friendly numbers (first to get from 15 to 20, then 20 to 40), and his facility in keeping track of the difference between 15 and 43. His confidence convinced her that he could extend himself further to solve the problem in another way. He successfully did so (working backwards from 43 to 15), relaying additional information about his understanding of the problem, the relationship between addition and subtraction (adding onto 15 to ‘get to’ 43, then ‘moving backward’ from 43 to 15), and his comfort and ability to use multiple strategies. The extension—solving the problem in a second way—was a decision made by Gabrielle and was not required for the teaching and learning event. Her decision to deviate from the task directions not only shows she has the confidence to reflect-in-action and adjust her instructional plan, but provides her with additional knowledge about the student’s process and understandings, knowledge she would not have acquired if she followed the original teaching ‘plan.’

Conclusion

A number of patterns have emerged from analysis of the data that describe how professional development affect preservice elementary mathematics teachers’ beliefs, knowledge, and practice from the time they enter until the time they exit preservice professional development. The remainder of this chapter will focus on the trajectory I

have found to exist regarding preservice teachers' beliefs, their views of mathematics teaching and learning, the support they provide students, and the degree to which they shift their focus from product to process when teaching.

Beliefs about Learning and Teaching Mathematics

Beliefs about what mathematics is and how easy it is to learn yielded interesting trends among participants at various stages in the course of becoming elementary mathematics teachers. During the course of preservice professional development, the beliefs change to be more in line with reform-oriented approaches to mathematics teaching and learning. When teachers are asked to rank statements related to beliefs about how mathematics is most effectively taught, results tend to more conclusive and more closely relate to a progression in professional development. That is, as preservice teachers engage in more professional development, their responses are more closely aligned with a reform-oriented approach to mathematics teaching than those who are just beginning training in becoming elementary mathematics teachers.

Preservice Teachers' Views of Mathematics Teaching and Learning

As preservice elementary mathematics teachers progress through the course of professional development in mathematics, they name increasingly more factors when explaining the complexity of teaching mathematics. Education 101 participants name general aspects of teaching that affect the challenge of teaching students mathematics. These challenges address issues related to student motivation and ability, to difficulty with the content, and to teacher's interest in and knowledge of content and pedagogy.

Their early teaching methods are characteristic of those they experienced when they were students. They focus on careful explanation and demonstration.

Education 201 participants and Education 301 participants who are completing their professional development speak of the many demands that teachers need to take into account in order to affect student learning. In addition to general aspects such as classroom management, pacing of instruction, and scaffolding of lesson content, they bring new perspectives from professional development experiences that inform their experiences as students. They understand that elementary mathematics teachers need to have content knowledge of how mathematics procedures work, rather than the ability to just explain how to do them. They know that pedagogical strategies incorporate a variety of models, many approaches, and allow students to solve problems in multiple ways, rather than focus primarily on teaching by telling. They realize that knowledge of students is gained through listening to their explanations, observing them problem solving, and questioning their thinking, rather than seeing them quickly arrive at correct answers. They assess their own teaching as well. Education 201 participants regularly reflect on their practices, assessing what worked, what did not, and why. They are able to suggest reasonable adjustments to subsequent lessons. Education 301 participants employ reflective practices as well. They are more able to reflect-in-action, while teaching, gathering assessment information from students and making appropriate adjustments.

Early on, beginning teachers are largely descriptive about what they see when observing teacher practice, shifting their attention away from description to providing evidence to accompany description; likewise, attention to aspects of teaching shifts to

attention to aspects of student learning as teachers develop. Beginning teachers (Education 101 participants) gave general descriptions of what they observe, noticing levels of student engagement and interest and use of materials in particular. Teachers at the end of their preservice professional development (Education 201 participants) provide descriptions of what they notice and support these descriptions with some evidence to illustrate these descriptions. These teachers focus on aspects of learning in general and aspects of learning mathematics in particular; they “name” these aspects by using vocabulary and terminology specific to the teaching profession. Teachers who are involved in professional development beyond the standard methods sequence (Education 301 participants) provide specific evidence to support their descriptions; terminology employed by these teachers made note of evidence of both teachers teaching as well as students learning.

Preservice Teachers' Support of Students

Instructional interventions employed by preservice teachers when working with a student change according to where they are in professional development. Early on in professional development, preservice teachers use a more limited range of questions and comments primarily for when students struggle, while those at the ending stages of professional development draw on a broader repertoire of questions and comments and use these to serve various instructional purposes. “What did you do here?” and “How did you figure that out?” were questions typically asked by Education 101 participants and helped the student to progress in solving the problem and give the teacher some general sense of what the student was doing. Teacher suggestions often served to point the

student toward the teacher's approach to solving the problem, such as "Do you know what I do?", while teacher confirmations of a student's process or efforts by the teacher to get the student back on track were illustrated by leading questions or comments. In general, the types of questions and comments used by beginning preservice teachers are more leading, convergent questions and preservice teachers at this stage in their profession development are satisfied when students report a correct answer.

By contrast, participants at the end of their professional development (Education 201 participants) use a wider range of prompts when students struggle at problem solving; these prompts serve not only help students to move forward and clarify their reasoning, but also provide the teacher with important information not necessarily evident from observing the student. "What do you think we should do?" and "Can you tell me how you solved that?" are typical questions asked by these teachers to support students when they are uncertain about an approach to use when beginning a problem solving task; others are used when students are puzzled by a step in the process they are using (such as "What are you thinking about?" and "How did you get to...?"). Preservice teachers at this stage of professional development ask some leading questions but more divergent, open-ended ones; they often elicit the process from students when only an answer to a problem is stated by the student.

Those teachers who had received additional experience in courses meant to advance their content and pedagogical knowledge of mathematics teaching and learning (Education 301 participants) not only had a larger repertoire of prompts and questions to use with their students than either the Education 101 and 201 teachers, but their interventions served a wide range of instructional purposes. Some questions promoted

student understanding of the overall process used (such as “How can you check and make sure?” “How did you figure that out?” and “What did you do?”), specific steps in their process (such as “You counted by what, though... What did you count by here?” and “How many do you have there?”) attempts to help students understand the solution represented on paper and how representations connect to information in the word problem (such as “And what do these represent?”), alternative problem solving approaches (such as “How else could have done that?”), uncovering more evidence behind the approach used (such as “Why did they have to be even?” “You thought this was the best way to be fair? Why?”) and alternative problems to pursue (such as “What if you had 100 candies? Would you do that same method?”). Few leading questions are employed by preservice teachers at this stage in the course of professional development. Individuals with the most preservice professional development use few closed-ended, convergent questions and when they do, they often “catch themselves” and reword their questions; open-ended, divergent questions predominate the repertoire of Education 301 preservice teachers when supporting students. When students supply an answer during problem solving, these teachers elicit steps in the process of solving the problem, as often as they ask students questions about the more general process used.

Preservice Teachers’ Focus on Product and Process When Teaching

As preservice elementary mathematics teachers progress through professional development they focus less on students’ answers and more on the process students used to arrive at the answer. When Education 101 participants were working with students in teaching situations, they were usually satisfied when the student was able to get a correct

answer. These participants confirmed students' accuracy before moving onto the next task. If they asked students to explain their process for solving a problem, it was after they had told the student they were correct.

Neither Education 201 nor 301 participants were satisfied with a correct answer. Eliciting the process from students was routine for both of these participant groups. In addition, Education 301 asked students implicitly or explicitly to explain steps in the process as well. Both groups of participants often asked students to demonstrate another process after the first.

Hopefully the results of this study, this concluding trajectory of development, and a summary of these results (see Tables 15 – 18) provide helpful information to those who conduct preservice professional development in elementary mathematics and to those who will pursue future, related studies on the development of preservice professionals as well. The implications of this study on current preservice education and future research can be found in the section that follows.

Table 15 *Trajectory of development for Finding 1*

Finding 1: With more professional development, preservice teachers' beliefs become increasingly aligned with a reform perspective.

101 Participants	--Responses to statements about mathematics teaching and learning show that beliefs are least oriented toward a reform-oriented stance.
201 Participants	--Responses to statements about mathematics teaching and learning show that beliefs are more aligned that Education 101 participants and less aligned than Education 301 participants.
301 Participants	--Responses to statements about mathematics teaching and learning show that beliefs are most oriented toward a reform-oriented stance.

Table 16 *Trajectory of development for Finding 2*

Finding 2: With more professional development, preservice teachers' views of mathematics teaching and learning become more complex.

101 Participants	<ul style="list-style-type: none"> • Regard teaching as: <ul style="list-style-type: none"> --demonstration, showing students step-by-step, pointing out important information. • Observations of teaching practice characterized as: <ul style="list-style-type: none"> --using general descriptions • Own practices characterized as: <ul style="list-style-type: none"> --limited strategies to support students, satisfied when students get correct answer, use leading questions • Reflection is <ul style="list-style-type: none"> --largely absent, most description of what happened • Language they use to talk about learning and teaching mathematics: <ul style="list-style-type: none"> --general, layperson language that demonstrates understanding of some formative concepts later developed in preservice professional development
201 Participants	<ul style="list-style-type: none"> • Regard teaching as: <ul style="list-style-type: none"> --complex, includes knowledge of content, pedagogy and students -- helping students understand why procedures work, showing them many ways, letting them use many strategies • Observations of teaching practice characterized as: <ul style="list-style-type: none"> -- focus on both teaching as well as learning, give more specific evidence, using language of the profession. • Own practices characterized as: <ul style="list-style-type: none"> -- having prior knowledge with students that they use -- using additional knowledge from professional development to plan instruction, anticipate challenges -- assessing through questioning students with open-ended questions -- not satisfied with students answers, they elicit process students used

Table 16 *Trajectory of development for Finding 2, Continued*

<ul style="list-style-type: none"> -- gathering more information about students as teaching unfolds and are beginning to use what they learn during lesson to make adjustments while teaching -- aware of the affect of the teacher's behavior on student learning · Reflection is characterized as : <ul style="list-style-type: none"> --reflecting on teaching afterwards by considering alternative teaching practices. · Language they use to talk about learning and teaching mathematics: <ul style="list-style-type: none"> -- specific and technical language to talk about learning and teaching practice 	<p>Regard teaching as:</p> <ul style="list-style-type: none"> -- more complex than 201, uncovering students' misconceptions is at the core of good teaching --helping students understand why procedures work, showing them many ways, letting them use many strategies · Observations of teaching practice characterized as: <ul style="list-style-type: none"> -- focusing on teaching with greater emphasis on student learning than 201 · Own practices characterized as: <ul style="list-style-type: none"> -- having prior knowledge with students like 201, but have additional and more specific prior knowledge of mathematics teaching -- using additional knowledge from professional development and Math Emphasis to plan instruction, anticipate challenges -- assessing students while and after teaching is critical to good instruction -- not satisfied with students answers, they elicit steps in the process students used -- gathering more information about students as teaching unfolds and are using what they learn during lesson to make adjustments while teaching -- appreciating importance of question to a greater agree and attend to wording, when to insert them, and why (to advance and assess student thinking) · Reflection is characterized as : <ul style="list-style-type: none"> -- able to reflect-in-action, using student feedback to adjust lessons in progress · Language they use to talk about learning and teaching mathematics: <ul style="list-style-type: none"> --specific and technical language to talk about learning and teaching practice, more specialized use of language particular to teaching mathematics
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Table 17 *Trajectory of development for Finding 3*

Finding 3: There is a change in the support preservice teachers provide students, from using a few teacher strategies that lead students toward a correct answer to a wider range of strategies that elicit students' processes.

101 Participants	<p>When students are done solving problems, they typically:</p> <ul style="list-style-type: none"> -- confirm that the answer is correct -- elicitation of process seems cursory <p>When students struggle or arrive at an incorrect answer, they typically:</p> <ul style="list-style-type: none"> -- make suggestions -- use questions to lead students <p>When questioning students, they:</p> <ul style="list-style-type: none"> -- primarily use convergent, closed questions <p>When teaching students, they:</p> <ul style="list-style-type: none"> -- demonstrate, provide step-by-step directions, explain clearly, point out important information
201 Participants	<p>When students are done solving problems, they typically:</p> <ul style="list-style-type: none"> -- elicit process students used <p>When students struggle or arrive at an incorrect answer, they typically:</p> <ul style="list-style-type: none"> -- wait -- use a different model -- suggest they keep track of their process on paper <p>When questioning students, they:</p> <ul style="list-style-type: none"> -- use some divergent, closed questions but begin to notice the structure and reword them to be open-ended -- use more convergent, open-ended questions than closed <p>When teaching students, they:</p> <ul style="list-style-type: none"> -- pace lessons -- scaffold instruction -- use models, visuals, and manipulatives -- make sure students understand the problem and the process they used to solve it -- incorporate student talk and explanation -- allow students to use multiple strategies to solve problems -- may require students to solve problems in multiple ways

Table 17 Trajectory of development for Finding 3, Continued

<p>301 Participants</p>	<p>When students are done solving problems, they typically:</p> <ul style="list-style-type: none"> -- elicit process and steps in the process students used <p>When students struggle or arrive at an incorrect answer, they typically</p> <ul style="list-style-type: none"> -- wait -- use a different model -- draw from prior knowledge and experiences to help students see connections -- look at them as opportunities to address misconceptions that otherwise could remain hidden <p>When questioning students, they:</p> <ul style="list-style-type: none"> -- use mostly convergent/open-ended, authentic ones -- use categories of questions, including guiding, clarifying, questions to advance student thinking and questions to assess student thinking <p>When teaching students, they:</p> <ul style="list-style-type: none"> -- pace lessons -- scaffold instruction -- connect to previous lessons and related concepts -- use models, visuals, and manipulatives -- make sure students understand the problem and the process they used to solve it -- incorporate student talk and explanation -- use multiple teaching approaches -- allow students to use multiple strategies to solve problems -- may require students to solve problems in multiple ways -- adjust lessons in process by using student feedback (reflect-in-action)
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Table 18 *Trajectory of development for Finding 4*

Finding 4: A change in focus from product to process occurs when comparing the Education 101 participants' teaching practices to those of the Education 201 and 301 participants.

101 Participants	When working with students, these participants... -- were usually satisfied with the correct answer -- sometimes elicited a process in a cursory way but only after they confirmed the correct answer
201 Participants	When working with students, these participants... -- rarely were satisfied with only the correct answer -- usually asked students to explain their process of solving problem -- may asked students to record process on paper -- often asked students additional questions about the process (e.g., 'How did you know to...?')
301 Participants	When working with students, these participants... -- rarely were satisfied with only the correct answer -- routinely asked students to explain process -- required students to provide specific steps in the process -- often required to solve the problem in another way

CHAPTER 5: IMPLICATIONS FOR PRESERVICE ELEMENTARY MATHEMATICS EDUCATION

This study confirms results from other studies regarding how students enter preservice professional development programs with beliefs, knowledge, and teaching practices in teaching elementary mathematics. According to the work of Foss and Kleinsasser and others, they regard mathematical knowledge as a collection of static facts, skills and procedures and find it unreasonable to expect students to construct mathematical knowledge (Foss & Kleinsasser, 1996; Muis, 2004; Seaman et al., 2005). Thus, teachers are responsible for dispensing knowledge that students are responsible for remembering.

Although knowledge about learning and teaching mathematics also accompanies preservice teachers to the methods classroom, Pajares (1992) found that established beliefs serve as powerful filters through which subsequent knowledge and experiences pass. Knowledge from professional development at the university and in the field mix with filters through beliefs and combines with existing knowledge in complicated and unique ways (Shulman, 1987). Early practices emerge from complex interaction between persistent beliefs, changing content and pedagogical knowledge and formative ideas about what mathematics instruction should like. (Ball, 1990a; Graham & Fennell, 2001; Munby et al., 2001).

Preservice professional development programs face a great challenge. They must realize that each prospective teacher enters the teaching program with a unique

package of professional knowledge that contains existing beliefs, knowledge, and practices about mathematics learning and teaching. They must aid preservice teachers in examining who they are as professionals, and help them to consider ways of adjusting what they believe and know in order to accommodate new knowledge from professional development experiences. Results from this study indicate ways in which preservice professional development programs can affect students. Suggestions for preservice professional development programs and future research are considered below.

Beliefs and Attitudes about Teaching and Learning Mathematics

The goal of this study was to understand beliefs and attitudes that occur at various stages of professional development, rather than change beliefs and attitudes or understand the connection between beliefs and practices. Beliefs about mathematics teaching and learning tend to be quite stable (Linek et al., 2003; Stuart & Thurlow, 2000), yet are known to change, especially when a change of beliefs has been a stated goal of university methods courses (Stuart & Thurlow, 2000). Beliefs about and attitudes toward teaching and learning mathematics can have a powerful influence on the ways in which new knowledge is incorporated by preservice teachers (Gill, 2004; Vacc & Bright, 1999) of preservice professional development.

Changing beliefs can result in a change of practices in classroom teachers (Stipek et al., 2001), yet little is known about the effect of belief change on the practices of preservice teachers (Crespo, 2003). Research on the connection between the changed beliefs and resulting changing practices in preservice elementary mathematics is an important topic to pursue in future research.

Preservice Teaching Practices

Results from this study indicate ways in which aspects of preservice teaching practice change during the course of professional development. Some of these changes have been directly related to how they work with students, while others have been related to the ways they interpret and talk about their own practices and the practices of others. These changes can serve as important benchmarks as practitioners develop from novice teachers into expert ones.

The impact of teaching practice on students. This study has found that preservice teachers change in the ways they support student learning and emphasize student process over product as they experience more professional development in teaching and learning elementary mathematics. In this study, beginning preservice teachers supported students by asking them leading questions and settle for a correct answer without explanation. After more professional development, preservice teachers asked more divergent, open-ended questions and required students to explain the process used, as well as specific steps in their processes. Whereas the former practices emphasize following the teacher's method of problem solving and values product over process, the latter allow students to construct their own understanding and values the problem solving as much as if not more so than a correct answer. These findings are consistent with findings from the work of Carpenter (1999) and others who propose instructional models that focus on student understanding of the processes they use rather than recall of procedures they do not understand (Carpenter, 1999; Franke & Carey, 1997; Hufferd-Ackles et al., 2004; Vacc & Bright, 1999).

Modeling teacher support for learning processes is an important practice for preservice teachers to explicitly see and carefully discuss during preservice professional development experiences, both in the university methods classroom and in field service placements as well. Consistency between the university classrooms and field settings is an important aspect for this modeled behavior to be transferred into teaching practice (Hart, 2000; Putnam & Borko, 2000; Smagorinsky et al., 2004). Accordingly, additional research that focuses on consistency between settings and resulting teaching practices would make an important contribution to preservice professional development.

Interpretations of teaching practice. If teachers in general and preservice teachers in particular are to improve their teaching practices, they must have opportunities to reflect on their own teaching and that of others. This study has made use of instruments that asked teachers to examine the practices of another teacher as well as their own. Their noticing behaviors shifted from a focus on teaching practice to a focus on student learning. General descriptions of practice turned into ones supported by specific evidence. Teacher reflection has been shown to be an important behavior of expert teachers (Argyris & Schön, 1974; Schön, 1987; Shulman, 2000; Shulman & Shulman, 2004). Preservice teachers in this study demonstrated an ability to shift from general descriptions of teaching practice to more detailed descriptions of student learning, supported by specific evidence. To help promote teaching expertise in preservice professionals, it is important that they have varied opportunities to view teaching practice, discuss what they notice, and shift their noticing habits from descriptions of what teachers are doing to evidence of student learning.

Preservice and new teachers are challenged by many demands when they teach and this study confirms that they become more aware of these demands with more professional development. In this study, additional professional development at the university and in the field promoted the development of repertoires for teaching. Preservice teachers acquire routines for teaching through experience with teaching. Use of these routines makes it easier for them to shift their attention from issues that plagued them early on, such as management of students and materials to issues that center on adjusting instructional practices based on student feedback.

This study has shown how preservice teachers develop an ability to reflect on their practice. Beginning preservice teachers primarily described what the students or the teacher was doing. Those who were finishing preservice professional development were able to reflect on their own teaching or the teaching of others, give alternative practices to the ones they implemented, and explain the effects of those alternatives. Preservice teachers who had finished preservice professional development as well as additional coursework in elementary mathematics content and pedagogy were beginning to reflect-in-action by adjusting their practices while teaching.

Studies have shown how difficult it can be difficult for preservice and new teachers to reflect-in-action, an important skill of the expert teacher (Cooney, 1999; Mewborn, 1999; Russell & Munby, 1991; Sherin & Van Es, 2005). Because reflecting on action is prerequisite to reflection-in-action, video recordings of teaching practice can help preservice and new teachers notice aspects they may have missed. These opportunities to develop the important skill of reflecting support a novice teacher's ability to notice various aspects of teaching that simultaneously occur, while serving to bridge

the shift from reflection on action to reflection-in-action. Use of video as a reflection tool can also help teachers shift from noticing teaching behavior to including reflections on student learning as well (Franke et al., 1998; Stuart & Thurlow, 2000). Greater use of videos for teaching holds promise for the development of preservice teachers. Video that demonstrates expert practice is in wide use but studies show that videos that contain puzzling teaching dilemmas show promise as well (Sherin et al., 2006; Sherin & Van Es, 2005; Van Es & Sherin, 2002). Clear and explicit use of video-recorded accounts of teaching—students' own and that of other teachers--would provide meaningful and appropriate contexts in which to develop reflective practice and the ability to notice the effect of teaching practice on student learning. Future research into the use of video to promote preservice teacher reflection and its subsequent effect on student learning holds promise as well.

Use of video recordings has been shown to be a promising tool to enhance reflection on teaching practice in this study as well as numerous others (Carboni & Friel, 2005; Franke et al., 1998; Hufferd-Ackles et al., 2004; Sherin et al., 2006; Sherin & Van Es, 2005).

A Final Thought

Mathematics has been and mostly likely will continue to be a content area that is puzzling and intimidating to many. While mathematical tasks present positive, yet conquerable challenges for some students, they are difficult, if not impossible for far too many others. Elementary preservice professional development programs have an overwhelming task. They must prepare their students to effectively teach a number of content areas to students of all elementary grades. While this study focused on preservice

professional development in only one content area, it reports only some of what is entailed in advancing the beliefs and attitudes, knowledge, and practices of preservice elementary mathematics teachers.

I have temporarily abandoned, rather than completed, many aspects of this study. I have begun to answer some of the questions with which I began, but I end this study with many more I can pursue and answer. I better understand some of the ways in which prospective elementary mathematics teachers develop over time and I hope that sharing these understandings will be helpful to those charged with preservice professional development in this content area.

APPENDICES

Appendix A: Math Experiences and Beliefs Questionnaire What do you know and think about mathematics teaching and learning?

This questionnaire was designed to learn about your experiences as a mathematics student to find out what you think about mathematics and mathematics instruction.

Elementary school experiences

1. What is most memorable about your elementary school mathematics?

High school experiences

2. Did you take mathematics during these years of high school?

	Yes (√ if taken)	No	Circle course length	
9 th grade			year	semester
10 th grade			year	semester
11 th grade			year	semester
12 th grade			year	semester

3. Make a check mark next to the mathematics courses you took in high school. For each checked course, circle the word that characterizes your experience of the content and your experience of the instruction. If you cannot remember if you took a particular course, put a question mark next to that level course.

Course	√ if taken	Overall experience: What you learned			Overall experience: How it was taught		
		positive	neutral	negative	positive	neutral	negative
General math		positive	neutral	negative	positive	neutral	negative
Algebra I		positive	neutral	negative	positive	neutral	negative
Algebra II		positive	neutral	negative	positive	neutral	negative
Geometry		positive	neutral	negative	positive	neutral	negative
Trigonometry		positive	neutral	negative	positive	neutral	negative
Pre-calculus		positive	neutral	negative	positive	neutral	negative
Calculus		positive	neutral	negative	positive	neutral	negative

4. List any additional high school mathematics courses you took.

What courses you took

When you took them

5. What is most memorable about your high school mathematics classes?

College experiences

6. What is/was your college major? _____

What is/was your college minor(s)? _____

7. Make a check mark next to the mathematics courses you took in college. For each checked course, circle the word that characterizes your experience of the content and your experience of the instruction. If you cannot remember if you took a particular course, put a question mark next to that level course.

Course	√ if taken	Overall experience: What you learned			Overall experience: How it was taught			length		
		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
General math		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
Algebra I		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
Algebra II		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
Geometry		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
Trigonometry		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
Pre-calculus		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem
Calculus		positive	neutral	negative	positive	neutral	negative	yr	qtr	sem

8. List any additional college courses you took.

What courses you took

When you took them

9. What is most memorable about your college mathematics classes?

All levels of mathematics

10. What qualities do you think are important for mathematics teachers to have? Why?

Appendix B: Quickwrite

How hard is it to learn mathematics?

Tell about an experience that led you to believe this way.

How hard is it to teach mathematics?

Tell about an experience that led you to believe this way.

Appendix C: Reflection on Teaching

1. Everyone views instructional situations differently. What three aspects of this video did you find noteworthy?

1)

2)

3)

2. You saw only a brief snapshot of a math lesson. Did you expect to see something that did not appear in the lesson?

_____ No

_____ Yes

Please identify anything that you expected to see.

3. Teachers are constantly faced with the pressure of deciding what to do next, and there is never a single best move. Assume that you were the teacher of these students. What problem/experiences might you provide next, and why?

Problem:

Rationale:

4. Please describe in detail what students said and did in response to this lesson.

(I recognize that you had the opportunity to view this video clip only one time, so please just do the best you can.)

5. Please explain what you learned about student understandings.

6. You have just spent time thinking and writing about this video clip, but you have not had an opportunity to talk about it with others. If you could talk about the clip with others, what, if anything, would you be most interested in discussing?

Appendix D: Sample Interview Protocol

Introduction: “I am going to ask you some questions about your “mathematics autobiography” to learn more about you as both a student and a teacher of mathematics. If I ask a question and you cannot think of situation or person to describe, or the situation or person has already been described, just let me know.”

1. Tell me a story about your life as mathematics student that most effects how you learn math today. It may have taken place in school or out of school, with peers or with adults. What happened?
2.
 - a. Was it a positive experience or a negative one?
 - i. (If negative) Do you have a more positive experience?
 1. What did that experience convey to you about teaching mathematics?
 2. What did that experience convey to you about learning mathematics?
 - ii. (If positive) Do you have an experience that was negative/less positive?
 1. What did that experience convey to you about teaching mathematics?
 2. What did that experience convey to you about learning mathematics?
3. Tell me a story about your life as mathematics teacher that most effects how you teach/ might teach mathematics today. It may have taken place in school or out of school, with peers or with adults. What happened?
 - a. What it a positive experience or a negative one?

- i. (If negative) Do you have a more positive experience?
 1. What did that experience convey to you about teaching mathematics?
 2. What did that experience convey to you about learning mathematics?
 - ii. (If positive) Do you have an experience that was negative/less positive?
 1. What did that experience convey to you about teaching mathematics?
 2. What did that experience convey to you about learning mathematics?
4. Tell me about a “good” math teacher.
 5. Tell me about a “bad” math teacher.
 6. Here are some word problems you will be giving a second grade student. Choose one you would like to talk about.
 - a. What do you think of this problem?
 - b. What do you think is not important?
 - c. What do you think a second grade student might do when given this problem?
 - d. How would you support them the student in solving the problem?

Appendix E

Video Elicitation on the Teaching and Learning Event
Sample Interview Protocol

BEFORE

1. In getting ready for the teaching and learning situation, what kinds of things did you do to plan for that situation?

2. What kinds of experiences led you to plan in this way?

DURING

#1: Sticker Problem (4x10) (start→3:25)

“Let’s watch this clip together. I will then rewind it back to the beginning of the clip and you can stop it when you would like to tell me what you were thinking at any point along the clip.”

#6: Playground (247+138) (25:42-30:55)

“Let’s watch this clip together. I will then rewind it back to the beginning of the clip and you can stop it when you would like to tell me what you were thinking at any point along the clip.”

#5: Mini Bus (7 seats, 19 children, 2 or 3 to a seat) (14:17-22:36)

“Let’s watch this clip together. I will then rewind it back to the beginning of the clip and you can stop it when you would like to tell me what you were thinking at any point along the clip.”

AFTER

As a result of this experience, what new insights do you have about learning mathematics?

As a result of this experience, what new insights do you have about teaching mathematics?

STUDENT WORK

In looking through this student work, what do you notice?

Is there anything thing here that you expected to see?

What surprises are there?

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