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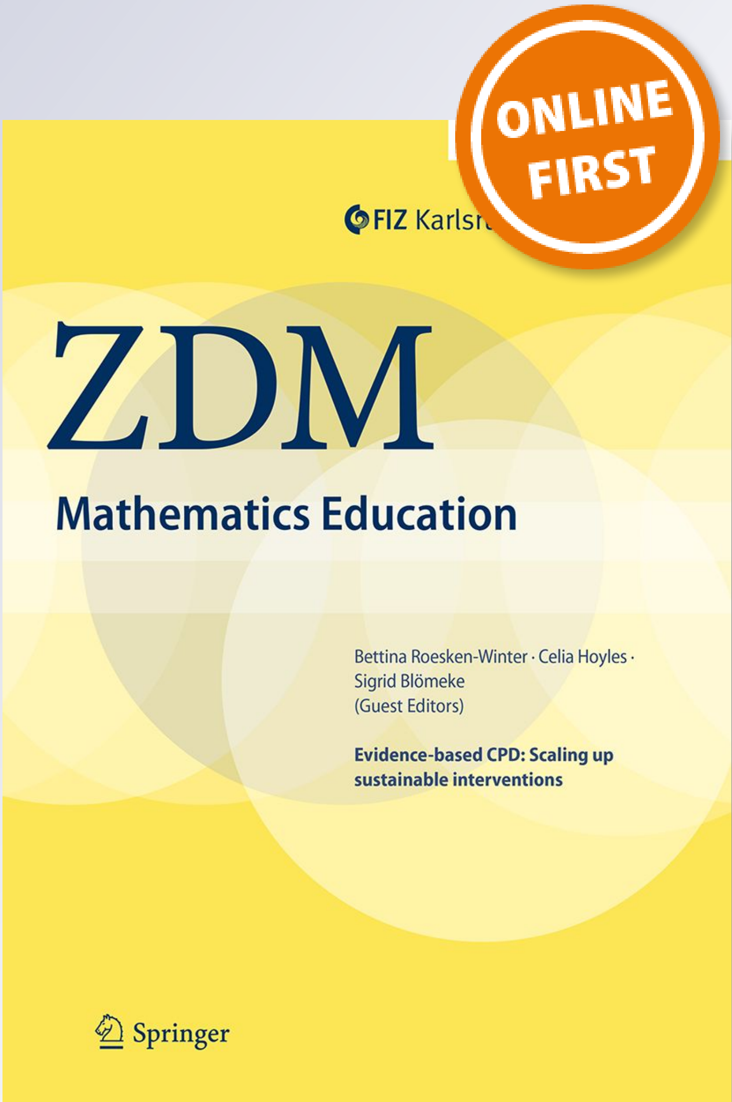

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ORIGINAL ARTICLE

Using the academic literacy in mathematics framework to uncover multiple aspects of activity during peer mathematical discussions

Judit Moschkovich[1] · William Zahner[2]

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Abstract
This paper illustrates how the academic literacy in mathematics framework (Moschkovich, J Math Behav 40:43–62, 2015) can be used to uncover the multiple layers of work bilingual learners accomplish during mathematical discussions. Using this framework allows researchers to examine students' joint mathematical activity in terms of mathematical proficiency, mathematical practices, and mathematical discourse. The use of the framework is illustrated through analysis of two mathematical discussions among middle school students. We conclude with reflections on the utility of the framework and consider possible pedagogical implications of this work.

Keywords Mathematical discourse · Peer discussions · Academic literacy in mathematics · Multilingual mathematics classrooms

1 Introduction

This paper illustrates how we used the academic literacy in mathematics (ALM) framework (Moschkovich 2015), a situated and sociocultural theoretical framework (Moschkovich 2002), to uncover how bilingual learners engaged in mathematical reasoning during two peer discussions. Both discussions involved middle school students working on a mathematics task and a moment of disagreement followed by eventual agreement. In one case, four-eighth graders negotiated how to create a distance-time graph representing a story of a bicycle trip. In a contrasting case, a group of sixth graders debated their answers to a computational exercise after they had used a division algorithm to change a fraction to a percent. While the discussions shared some features, the discussions also differed along important dimensions including the conceptual focus of the talk.

We use the dimensions of the ALM framework to analyze both interactions and to consider the affordances of each peer discussion for promoting mathematics learning, specifically the appropriation of academic mathematical language, for linguistically diverse students. The ALM framework highlights three dimensions—mathematical proficiency, practices, and discourse—and can thus be used for analyzing either procedurally focused or conceptually focused discussions. Since discussions in real classrooms are likely to include both of these emphases, the ALM framework can be used to make sense of both types of discussions.

The first example (Sect. 3.1) is from a study in an eighth-grade bilingual mathematics classroom. Classroom observations and videotaping were conducted during a unit from *Connected Mathematics Project* titled Moving Straight Ahead (Lappan et al. 1998). The first excerpt illustrates how mathematical discourse, talk, and text, are connected to participation in mathematical practices. Specifically, the example shows how participants engaged in a mathematical discussion using hybrid resources—multiple modes of communication, multiple sign systems, and multiple registers (everyday and academic). The second example (Sect. 3.2) is from a study that used a sociolinguistic approach to analyze bilingual sixth grade students' participation in mathematical discussions (Zahner and Moschkovich 2010). We re-analyze

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an excerpt from this group's discussions to illustrate how the ALM framework can also be used to examine the interaction of the aspects of mathematical proficiency, practices, and discourse during a procedurally focused discussion. We begin by clarifying key aspects of the ALM framework and connecting this framework to the theoretical assumptions of the broader situated and sociocultural perspective on learning mathematics.

2 The ALM framework

Academic literacy in mathematics is defined as three integrated aspects: mathematical proficiency, mathematical practices, and mathematical discourse (Moschkovich 2015). The view of ALM presented here is different from previous approaches to academic language in several ways. First, the ALM framework includes cognitive aspects of mathematical activity such as mathematical reasoning, thinking, conceptual development, and metacognition—the traditional cognitive aspects of mathematical proficiency. Additionally, the ALM framework includes sociocultural aspects of mathematical activity—participation in mathematical practices—and discursive aspects of mathematical activity—participation in mathematical discourse. This integrated view, rather than separating mathematical proficiency, mathematical practices, and mathematical discourse, assumes the aspects work together.

The view of language(s) and discourse in the ALM framework not only connects mathematical cognition to sociocultural practices, it also assumes that meanings for academic mathematical language are socioculturally situated in mathematical practices and the classroom setting, and dynamic rather than static or given by definitions (Gee 1999). This complex view of mathematical discourse also assumes that mathematical discourse draws on hybrid resources (Gee 1999; Gutierrez et al. 1999) and involves not only oral and written text, but also multiple modes, representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and registers (school mathematical language, home languages and the everyday register). In the following we expand on the three interrelated aspects of ALM.

2.1 Mathematical proficiency

Mathematical proficiency (Kilpatrick et al. 2001) consists of five interwoven strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The five strands of mathematical proficiency in Kilpatrick et al. (2001) provide a cognitive account of mathematical activity focused on knowledge, metacognition, and beliefs. Mathematical proficiency cannot be reduced to procedural fluency. Conceptual understanding, strategic competence, and adaptive reasoning are as important, if not more important, than fluent arithmetic computation (Hiebert and Carpenter 1992). Conceptual understanding is fundamentally about the meanings that learners construct for mathematical solutions: knowing the meaning of a result (what the number, solution, or result represents), knowing why a procedure works, and/or explaining why a particular result is the right answer. Other aspects of conceptual understanding are connecting procedures to concepts and connecting procedures to multiple representations such as words, drawings, symbols, diagrams, tables, graphs, or equations (Hiebert and Carpenter 1992). Reasoning, logical thought, explanation, and justification are closely related to conceptual understanding. Student reasoning can provide evidence of conceptual understanding when a student explains why a particular result is the right answer or justifies a conclusion. Conceptual understanding and procedural fluency are related and often develop in tandem (Star 2005). Within the ALM framework, we highlight the reciprocal relationship between conceptual understanding and procedural fluency.

2.2 Mathematical practices

From a sociocultural perspective, mathematics students are not only acquiring mathematical knowledge, they are also learning to participate in valued mathematical practices (Moschkovich 2013). Some of these practices include problem solving, sense-making, modeling, and looking for patterns, structure, or regularity. Research and development efforts in mathematics education have assumed that mathematics instruction in schools should parallel, at least in some ways, the practices of mathematicians (for example Cobb et al. 1993; Lampert 1990). In our own research, we have used a Vygotskian theoretical framing (Vygotsky 1978) to describe how students participate in mathematical practices during tutoring (Moschkovich 2004) or classroom discussions (Moschkovich 1999; Zahner 2015; Zahner et al. 2012). We use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to "… highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems …" (p. 13). Using the term *practice*[1] shifts from purely cognitive accounts of mathematical activity to accounts that assume the social, cultural, and discursive nature of mathematical activity. From this perspective,

[1] In using the terms *practice* and *practices* in the sense used by Scribner (1984), we make a distinction between the concept of *practices* and other common uses, for example practice as repetition or rehearsal, or practice as in "my teaching practice."

mathematical practices are not only cognitive—i.e., involve mathematical thinking and reasoning as described in the strands of mathematical proficiency—but also social and cultural—they arise from communities and mark membership in communities—and semiotic—they involve semiotic systems (signs, tools, and their meanings).

While many researchers have used the concept of mathematical practices (e.g., Rasmussen et al. 2005), and current U.S. mathematics standards include eight standards for mathematical practice, a distinguishing feature of the ALM framework is that this perspective assumes that participation in mathematical practices includes setting or using goals (even when these are implicit), discourse, and situated meanings for words, symbols, and other tools.

2.3 Mathematical discourse

We use the phrase mathematical discourse, rather than mathematical language, to refer to the *communicative competence* (Hymes 1972) necessary and sufficient for competent participation in mathematical practices. The phrases *mathematical language* or *academic language* can have multiple meanings. The phrase *academic language* can be interpreted in reductionist ways to mean to vocabulary or grammar. In contrast, the ALM framework uses a more complex view of what constitutes mathematical discourse. Studies examining the language of the discipline of mathematics (e.g., Pimm 1987; Schleppegrell 2007) provide a complex view of mathematical language as not only specialized vocabulary—new words and new meanings for familiar words—but also as extended discourse that includes other symbolic systems as well as artifacts (Moschkovich 2002), syntax and organization (Crowhurst 1994), the mathematics register (Halliday 1978), and discourse practices (Moschkovich 2007). Overall, the view used here is that mathematical discourse is more than language (Moschkovich 2007), it involves other symbolic systems as well as artifacts. Discourse is embedded in mathematical practices, and meanings are situated and develop through participation in mathematical practices.

While numerous discourse practices might be called mathematical, academic mathematical discourse has been described as having some general characteristics. In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued in mathematics (Forman 1996). Abstracting, generalizing, and searching for certainty are also highly valued. Generalizing is reflected in common mathematical statements, such as "The angles of any triangle add up to 180°", "Parallel lines never meet", or "a + b (always) equals b + a". What makes a claim mathematical is, in part, the attention paid to describing in detail precisely when the claim applies and when it does not. Mathematical claims are also often tied to mathematical representations (symbols, graphs, tables, or diagrams).

3 Illustrating applications of the ALM framework

In the following two sections, we analyze two cases using the ALM framework. For a summary of how to use this framework to analyze student activity, see Table 1. To illustrate this framework, the analyses presented here use the following questions: (1) What strands of mathematical proficiency were evident in student activity? (2) How were students participating in mathematical practices? And (3) How were students participating in mathematical discourse, specifically, what texts, modes, representations, purposes, or language resources were evident in student activity? These questions were selected from a larger set of analysis questions designed following Gee's (1999) questions for Discourse analysis.[2]

A previous analysis (Moschkovich 2002) used the following questions, which loosely followed Gee's (1999) questions for Discourse analysis, to examine mathematical discussions: (a) What are the *situated meanings* of some of the words and phrases that seem important in the situation? (b) What are the multiple *resources* and *sign systems* (speech, writing, images, and gestures) students use to communicate mathematically? In particular, how is "stuff" other than language relevant? and (c) What Discourses are involved, being produced, relevant (or irrelevant)? In particular, what discourse practices are student participating in that are relevant in mathematically educated communities or that reflect mathematical competence? For this article, the questions are similarly framed by a sociocultural perspective but focus more specifically on the three aspects of ALM.

The main goal in selecting these excerpts from larger data sets is to highlight the utility of the ALM framework for examining two contrasting cases of school mathematics discussions. The two excerpts we examine include bilingual (Spanish–English) discussions because we both conduct research in bilingual/multilingual settings where some students are learning the language of instruction. In such settings, the mediation of language and discourse in students' mathematics learning is often highly visible. However, we note that the ALM framework is not limited to analysis of multilingual mathematical discussions, and the framework could be used to examine discussions that are not explicitly bilingual. Based on our ethnographic observations, each excerpt is fairly typical of the larger corpus of data from each project. However, the purpose of using these excerpts

[2] The relationship of Gee's questions to the ALM framework is specific to Moschkovich's work analyzing classroom discourse. We note, however, that there also exist several other approaches for the analysis of classroom discourse, for example O'Connor and Michaels (1996) or Sfard (2008) among others.

Table 1 Sample questions for using the ALM framework for analyzing student activity

	Mathematical proficiency	Mathematical practices	Mathematical discourse
For analyzing student activity	Which strands of math proficiency were evident in student activity?	Which math practices were evident in student activity?	How did students participate in mathematical discourse, specifically, what texts, modes, representations, purposes, or language resources were evident in student activity?

is to illustrate theory, not theory confirmation. Therefore, we did not select these discussions based on how well they represent the data corpus, the frequency of such discussions in this classroom, or other criteria that would be relevant for selection of samples in theory confirmation (Moschkovich and Brenner 2000).

3.1 Case 1: co-constructing a graph of motion

Our first analysis examines a discussion among four girls and their teacher as the students created a graph to represent a story of a bicycle trip. The discussion involved using multiple representations, modes of communication, sign systems, languages, and texts. This discussion stood out in part because two students discussed at length whether the horizontal axis on a distance time graph should be time or distance, in spite of repeated suggestions by the teacher and another student in the group to use the convention that time is on the *x*-axis.

The students (Maria, Iris, Francis and Kristina), were emergent bilinguals in Spanish/English and all had been in a two-way bilingual immersion program at a school in the Northeast U.S. for several years. Three of the students, Maria, Iris, and Kristina, spoke Spanish at home and also primarily in class. The class typically followed a launch-explore-summarize format in which the students worked on a single problem over an extended period of time. A fuller description of the research setting is in Moschkovich (2008).

The discussion in Excerpt 1 occurred towards the middle of a classroom period, after the teacher had launched the lesson and before the whole-class summarizing discussion of the students' solutions. During the previous lessons, students had worked on a series of problems involving a bicycle tour. On this particular day, the students worked on the 5th segment of the tour. The problem required students to read a text describing an 80-mile trip that took 7½ h to complete. The students' task was to create a set of distance-time data that met the criteria in the description of the trip from the text, and students needed to decide how to represent this information in a table of distance time data and a corresponding graph (Fig. 1).

For this class period, students worked in groups of 4 and used their notebooks and the problem statement as resources to work on their solutions. After finishing, groups were to transfer their solutions to a large chart paper. The activity culminated with groups presenting their work to the whole class. During presentations, the teacher and the class would evaluate the quality of each solution. Because the problem involved some estimation and guesswork, the teacher told students that they should expect differences amongst the final solutions of each group.

Excerpt 1 starts after the group had already made a table of data and they had created a graph with the *x*-axis labeled

Malcom and Sarah's Notes

- We started at 8:30 am and rode into a strong wind until our midmorning break.
- About midmorning, the wind shifted to our backs.
- We stopped for lunch at a barbecue stand and rested for about an hour. By this time, we had traveled about halfway to Norfolk.
- At around 2:00 p.m., we stopped for a brief swim in the ocean.
- At around 3:30 p.m., we had reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing by. Since bikes are prohibited on the bridge, the riders put their bikes in the van, and we drove across the bridge.
- We took 7½ hours to complete today's 80-mile trip.

Fig. 1 The text students used for the graphing problem in Case 1 (Lappan et al. 1998, p. 23)

from 0 to 16 and the *y*-axis labeled from 0 to 7.5. The teacher approached the students as they finished labeling the *y*-axis. She watched the group for a short amount of time and then leaned into the group. The following exchange took place.

3.1.1 Excerpt 1[3]

1. Teacher: ((pointing to *x*-axis)) This is time, from… what do these numbers mean? Up to sixteen?
2. Maria: The time. ((rising intonation, possibly signaling doubt))
3. Kristina: ((looking at Iris)) Explicale tú porque… ((“You explain it to her because…”))
4. Iris: Fué porque nos equivocamos y no dejamos espacio, nosotros ibamos a dejar espacio pero se iba a ver feo solo dejar espacio. ((“We made a mistake and did not leave space, we were going to leave space but it was going to look ugly to only leave space”))
5. Teacher: OK, and then the miles ((pointing at the *y*-axis)) from zero point zero to seven point five? Miles?
6. Maria: Yeah
7. Teacher: Miles. How many miles did they go?
8. Maria: Eighty. ((Exhales and looks up in a gesture that possibly indicates she realized they had made a mistake.))
9. Kristina: Oh my…
10. Teacher: Where did you get those numbers from?
11. Maria: From here ((pointing to table of numbers in notebook)) those are supposed to be here ((pointing to *x*-axis on graph))
12. Teacher: uh-huh
13. Maria: Now we have to flip it over and do it again? ((turns chart paper over)).

Based on Kristina's response (line 9) to the teacher's question “how many miles did they go?” (line 8), the students appeared to recognize that they had done something wrong. However, it was not clear that all of the students in the group recognized *what* went wrong, that is, that they had put the wrong quantities on each axis. It is possible some group members thought they had put the wrong numbers for each quantity. Maria's statement in line 11 left open both interpretations of this situation. After the discussion in Excerpt 1, the group turned the chart paper over to start their graph a second time on the opposite side of the paper.

Prior to the group's second attempt to create the graph, Maria, Iris and the teacher were intently focused on how many grid spaces or marked intervals were needed to label each axis. The teacher asked “how are you going to do your intervals?” and the students proceeded to discuss the intervals on each scale in Spanish, referring to the intervals as “do it ten by ten” (“hazlo de diez en diez”). The teacher referred to the intervals saying (“cada diez”) “every ten” and (“por diez”) “by tens,” or (“por cinco”) “by fives.” These phrases echo and parallel how the teacher and other students in this class were talking about the intervals on the scale on their graphs (Moschkovich 2008). This talk and activity involved the important concept of unitizing and student discussions focused on that concept (Moschkovich 2008). Furthermore, the meaning of these phrases, whether in English or Spanish, were not given by any definition provided by the teacher or a textbook, but instead were situated in the classroom history, grounded in the inscriptions, and negotiated by the participants.

As the students created their graph a second time, the group attempted to work quickly. The students labeled their axes, and again put time on the *y*-axis and distance on the *x*-axis. Then they started to plot points from their table of data. Since time was on the *y*-axis, as they placed points for the “midmorning break” and the “lunch break” in the story, the plotted points created a vertical line segment. When Maria plotted the points (40, 4.5), (40, 5.0), and (40, 5.5), there was a momentary breakdown in the group's activity as Iris suggested plotting a horizontal set of points, rather than a vertical set of points. Once again the teacher joined the group. However, this time the teacher did not intervene to tell the students that the quantities on each axis were reversed. Maria erased their points and started plotting each point from their list a third time, leaving the axes the same.

3.1.2 Excerpt 2

1. Maria: We have to start ((plotting the points)) over again.

[3] Clarifying comments and gestures are indicated with double parenthesis (()). Translations appear in comments in quotation marks: ((“translation”)).

2. Teacher: The points were not on the right place?
3. Iris: Dos punto cinco es veintidos. (("two point five is twenty two"))
4. Teacher: One more minute. ((Teacher walks away from group))
5. Iris: Dos punto cinco es veintidos, es como por aquí. Veintiocho por aquí. (("Two point five is twenty two, it's like around here. Twenty eight around here"))
6. Maria: Treinta y cuatro acá. Es en tres punto cero… No es malo, tres punto cero es veintiocho, en dos punto cinco. (("Thirty four is here, it's in three point zero… it's not wrong, three point zero is twenty eight, in two point five"))
7. Iris: Bueno, dos punto cinco en dos punto cinco esta buena, tres punto cero… (("OK, two point five in two point five it's good, three point zero…"))
8. Maria: Es veintiocho, te digo, está mala, una de estas está mala. (("It's twenty eight, I'm telling you, it's wrong, one of these is wrong"))
9. Maria: Uno punto cinco es quince, acá. (("One point five is fifteen, here"))
10. Maria and Iris: Dos punto cero es quince, también. (("Two point zero is fifteen also"))
11. Iris: … 'ta buena, aquí está, dos punto cero es quince. (("it's good, here it's two point zero is fifteen"))
12. Maria: Dos punto cero es quince, y por qué? ((Two point zero is fifteen, and why?"))
13. Iris: Porque sí. (("Because it is…"))
14. Maria: Ya vamos a terminar el tiempo. (("We are going to run out of time"))
15. Kristina: Esto iba acá y esto iba acá ((repeatedly pointing at the two axes "This went here and this went here"))
16. Iris: Oh, no!
17. Francis: It's upside down.
18. Iris: Está bien así, no importa. (("It's good like this, it doesn't matter"))
19. Kristina: No, porque tú tienes que poner quince acá y si tu pones aquí el siguiente… (("No because you have to put fifteen here and if you put the next one here…"))
20. Iris: Mira, uno punto cinco es quince y dos punto cero es quince también. Entonces vamos a poner el punto acá arriba (("look one point five is fifteen and two point five is fifteen too. So we are going to put the point here above"))
21. Teacher: OK, y vamos a verlo, vamos a ver si es diferente o es lo mismo que las otras. (("OK, and we will see it, we will see if it is different or is the same as the others")).

It is worth noting that this was not the first bicycle trip that these students had graphed or the first distance-time graph they had discussed. All of the distance-time graphs they had seen so far in this class had distance on the y-axis and time on the x-axis, following the typical convention. The teacher had also briefly introduced the concept of dependent and independent variables during a whole class discussion. Thus, it is not clear why the students in this group had chosen to put distance on the x-axis. What is clear is that this was a very compelling view of the graph for them since they returned to putting distance on the x-axis twice during their group work. They returned to that view of the graph in spite of two interventions by the teacher and questioning by a group member as they set up the graph the second time (between Excerpt 1 and Excerpt 2 Francis asked "Isn't that what we did last time?" but her question was not taken up). With this summary of the discussion in place, we now consider this discussion in terms of the questions framing this paper.

What mathematical proficiency was evident in the discussion? The task itself required connecting three representations (a text, a table, and a graph). The mathematical proficiency required for this task certainly involved conceptual understanding, since connecting and making sense of three symbol systems (text, table, and graph) is a typical way for a task to involve conceptual understanding (Leinhardt et al. 1990). First, students needed to read and understand the text that describes the situation. The genre of the text in this problem is not a traditional word problem, but rather a narrative description of a situation to be mathematized. The purpose of the text, in contrast to text in, for example, language arts or social studies class, is not to tell a story, make an argument, or persuade the reader but to provide a situation to be represented using mathematical inscriptions. Students need to read and understand not only the text but also create and interpret a mathematical representation, the table. These multiple coordinations make this task complex for a school mathematics problem.

At first glance it may seem that the discussion in Excerpt 1 and 2 focused on a simple procedural fact: choosing which variable goes on which axis and plotting points. However, using the ALM framing, we can see that this discussion was not focused on procedural fluency. The students were focused on several conceptual aspects of the graphs. First, they focused on the intervals on the scales, reflecting the concept of unitizing (Lamon 1996). They were also focused on where the dependent and independent variables go on each of the axes in a graph. The many times that the students repeatedly return to their first perspective, putting the time on the y-axis, signals that labeling the axes was much more than a simple procedure for them, it involved an important and compelling way to view the graph that they were confused by and revisited more than once.

How were the students participating in mathematical practices? The task was posed in a way that required the students to engage in the practice of quantitative reasoning.

Specifically, the students had to quantify information presented in a text and create representations of data corresponding to the situation presented in the text. Opportunities for other mathematical practices were also present as a result of the task, activity structure framing the task, and the norms in the classroom.[4] For example, because the activity structure provided by the classroom norms required that students discuss their responses in small groups, arrive at joint group solutions, and present a group solution to the whole class, this task and its activity structure provided opportunities for students to engage in other valued mathematical practices such as constructing arguments and critiquing the reasoning of others.

We note that the students made many claims (e.g., Maria's assertion "these are supposed to be here" when she initially discovered their error), but they rarely provided support or evidence for any of their claims. Given the disconnect between claims and justifications, it would be hard to describe this group discussion as constructing or critiquing arguments. However, we also note that the practice of argumentation emerged later in the class discussion facilitated by the teacher.

How were students participating in mathematical discourse? The task was designed and enacted in this classroom to require that students use, interpret, and create several types of texts (a story, a graph, talk), modes (reading, listening, writing, drawing), and multiple representations (words, tables, symbols). During the discussion excerpts presented here, students focused on interpreting the meaning of the graph, at times removed from a connection to the other texts and representations. This analysis highlights that students may need support not only in reading (a story or graph), but also in connecting texts to each other.

The students and the teacher used both Spanish and English, as well as multiple registers as resources to participate in this discussion. First, they used everyday ways of talking as a resource, drawing on the phrases "do it ten by ten" and "by five" to describe the units on the scales. The meanings of these phrases were situated in this classroom, and, as shown in Moschkovich (2008), these everyday ways of talking about scales and intervals served as a resource for joint mathematical reasoning in this classroom. Second, the members of this group also used their home language as a resource for reasoning about a task posed in English. Again, this language resource depended on the norms of this particular classroom, where the teacher was bilingual and the students were at liberty to choose the language they used to discuss mathematics (we note that language policies promoting bilingualism are not the norm in U.S. schools). We close this section by noting that students in Case 1 used their home language to focus on conceptual aspects of the graphs, not procedures. In contrast, although the norms in the classroom in case also permitted the use of home language, we will see next how the discussion in Case 2 that was in Spanish was focused completely on procedures. This contrast in how bilingual students used their primary language as a resource to do mathematics together suggests that we should be cautious and not over-generalize about bilingual students' use of home language(s) to do mathematics.

3.2 Case 2: discussing a procedural calculation

Excerpt 2 was recorded as part of a study that examined sixth-graders' peer interactions during mathematics group work (Zahner and Moschkovich 2010, 2011). This study used discourse analysis to explore how one group of bilingual middle school students simultaneously engaged in interactional positioning and joint mathematical reasoning. The participating students were sixth graders (ages 11–12) from one class in a "dual-immersion" bilingual school in California. Over 90% of the students in this school identified as Latino/a and all students spoke both Spanish and English.

This study was conducted in Ms. B's sixth grade mathematics class taught in English. Ms. B. was bilingual and she interacted with her students in both Spanish and English during individual work time and small group consultations. One group of students was video recorded across a week, and Ms. B selected a focal group with students who represented the range of prior mathematics grades in her class. Adopting a naturalistic approach to these classroom discussions (Moschkovich and Brenner 2000), we did not dictate group size and composition, and we did not provide coaching for the students in how to discuss mathematics with peers.

The routines in this classroom were different from the routines in the eighth grade classroom in Case 1. In this classroom, daily lessons typically followed a sequence of teacher explanation followed by individual and small group work time. During the whole class portion of each class, the teacher usually modeled how to solve a particular type of problem. During individual and group work time, students worked on practice exercises related to the lesson. These exercises usually required applying a known procedure.

The primary data were video recordings of 5 h of classroom interaction, recorded across 1 week, supplemented with ethnographic field notes. The topic for the week was converting rational numbers between decimal, percent, and fraction forms and solving equations involving proportional relationships. We created video logs of the full set of video recordings and we then selected excerpts when the students engaged in sustained mathematical discussions (Pirie and Schwarzenberger 1988) for further analysis. In total, we

[4] Claims about the classroom norms are the result of extended ethnographic observations and are described in other studies (for example in Moschkovich 2008).

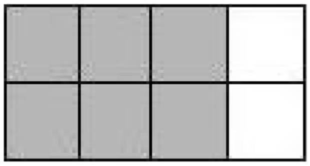

Fig. 2 The task discussed in Excerpt 3 included a rectangle like this with the instructions "Find the percent that is shaded."

transcribed 56 min of the students' interactions. In the initial study these transcripts were analyzed using tools from conversation analysis with the goal of understanding the interactional routines of these peer mathematical discussions (e.g., in Zahner and Moschkovich 2011 we examined code switching). Here we focus on one excerpt and use the three guiding questions from the ALM framework to examine a mathematics discussion qualitatively different from that in Case 1.

In Excerpt 3, Claudia, Amber, Francisco, Diego, and Joaquin participated in a 2-min discussion negotiating conflicting answers to an exercise about percent calculations. At the start of the excerpt Claudia and Diego were working on their papers independently. Amber was not writing and appeared to be looking at another group of students. Francisco had his head down on the table on top of his worksheet and Joaquin was away from the group sharpening his pencil. In the first move of the discussion Claudia attempted to verify her answer for this exercise with Amber and Francisco (line 1). Amber and Francisco had an answer different from Claudia's (lines 2 and 5), and in the ensuing discussion the group engaged in a rapid back-and-forth debate over which answer was correct. Ultimately the students appealed to their teacher, Ms. B, to settle the disagreement. The teacher, in turn, nominated Joaquin (who was away from the group sharpening his pencil for the entire discussion) to provide a final answer (Fig. 2).

3.2.1 Excerpt 3

1. Claudia: Is it 75% for number two?
2. Amber: Point thirty-three.
3. Claudia: No!
4. Amber: uh-huh!
5. Francisco: Point thirty-three is the same thing as 33%.
6. Claudia: No xxx porque fíjate (("No xxx because look")), six over eight huh (get) the percent that is shaded one two three four five six.
7. Amber: (my bad).
8. Claudia: Six.
9. Francisco: Isn't it shaded to unshaded?
10. Claudia: uh-huh.
11. Amber: Write the percent that is shaded.
12. Claudia: One two three four five six.
13. Francisco: Six ta two, y allí va el point (("and the point goes there")).
14. Claudia: ((looks away, shaking head)) eh?
15. Francisco: Oh my god.
16. Amber: Es- esa, se divide seis afuera y dos adentro. (("it's that one, one divides six outside and two inside"))
17. Claudia: No but I'm just telling you that, um, it's 75%, it's six over eight.
18. Francisco: 75%?
19. Claudia: 'ira teacher, (("Look, teacher")) ((looks back at teacher who has moved to back of classroom)) maestra maestro. (("teacher, teacher"))
20. ((Teacher is talking to Joaquin away from the group by the pencil sharpener))
21. Teacher: ((turning to Claudia)) Yes, did you ask your table?
22. Claudia: Yeah.
23. Teacher: Ask your, ask your table. Did you ask Joaquin?
24. Claudia: Qué? (("what?"))
25. Teacher: Ask Joaquin. Is it about the math?
26. Joaquin: Ask me
27. Claudia: ((still addressing teacher)) Qué? No, le iba a decir que, por eso, pero esto um it's its six over eight. (("What, no I was going to say, for this one, it's six over eight."))
28. Teacher: Did you ask Joaquin?
29. Claudia: ((turns to look at Joaquin))
30. Joaquin: ((looks at Claudia's paper nodding)) yeah.
31. Teacher: ((to Joaquin)) Si- sit down over there, I'll get your pencil ready Go ask them over there.
32. Claudia: Told you ((looking at Amber)) 'ira, viste Joaquin sí sabe (("Look, see Joaquin does know")) it's six over eight.
33. Amber: Well I didn't do this one, he did it ((gesturing toward Francisco)).

Note that mathematically, although Claudia was correct, it is possible to see how the two competing answers that the students considered, 75% (Claudia's answer) and 0.33 (Amber and Francisco's initial answer), were both related to the diagram. Claudia's answer followed the standard solution method of setting up a fraction, 6/8, and then converting that fraction to decimal form using a long division procedure. Amber and Francisco's competing answers appeared to be the result of considering the ratio 2:6 (corresponding with the number of unshaded squares to shaded squares) and converting that ratio to decimal form.

What mathematical proficiency was evident in the discussion? Given the plausibility of both answers considered in this discussion, it is possible to imagine how comparing these competing answers might have afforded an opportunity for the students to engage in a conceptually-focused discussion comparing their solution methods. However, in this group, the discussion of the differing answers to this exercise primarily focused on the procedural fluency component of mathematical proficiency. While there was potential for the students to demonstrate other forms of mathematical proficiency, especially conceptual understanding and adaptive reasoning, this potential was not realized in this discussion.

The teacher's redirection of Claudia's question back to the group (lines 21 and 28) appeared to be an attempt by the teacher to develop habits among the students consistent with the dimension of proficiency of a productive disposition. That is, one potential interpretation of the teacher's move in lines 21 and 28 is the teacher was asking the students to make sense of mathematics together, rather than to rely on the teacher as the arbiter of mathematical correctness (Lampert 1990). However, given the brevity of Joaquin's response in line 30, it is not clear whether this move from the teacher necessarily developed students' habit of seeing mathematics as sensible, or helped the students to see themselves as able to reason through a mathematical debate. Instead, Joaquin's brief answer may have inadvertently reinforced the idea that mathematics is an activity focused around finding correct answers, rather than making sense of concepts and relationships.

How were students participating in mathematical practices? This extended discussion focused primarily on the procedure for setting up a fraction and converting it into its equivalent decimal and percent forms. With such a focus, the students' discussion can be characterized as reflecting what Thompson et al. (1994) termed a calculational discussion. That is, the students discussed the calculations necessary to make the conversion among representations of rational numbers, but they did not address why using the calculation $6 \div 8$ was correct or why $2 \div 6$ was not correct for this problem. It is possible to imagine how Francisco's initial answer might have been a springboard for a discussion of how to define the parts and the whole in problems involving ratios, fractions, and percent calculations. Unfortunately, Francisco's idea did not receive much careful consideration from the group or the teacher, so the discussion remained at the procedural level, rather than delving into argumentation for or against the claim that the percent shaded is different from the ratio of unshaded to shaded squares.

Other mathematical practices had minor importance in this discussion. The students paid some attention to precision, but primarily in relation to setting up and correctly executing long division. For example, in line 6 Claudia clarified that she used $6 \div 8$ because the problem asked for the percent of shaded squares and she proceeded to count the six shaded squares aloud to model her actions. Had the task and classroom norms had required justification, the discussion could have included more mathematical practices. Following this observation, there was little evidence in this discussion of the students engaging in disciplinary forms of justification or argumentation.

How were students participating in mathematical discourse? Students were certainly participating in a discussion, but what kind of mathematical discourse was evident? First, as described above, the primary focus of this discussion was on calculations and procedures, not concepts or reasoning. With regard to texts, the students primarily referred to the diagram and the calculations they added to their papers. They did not use other texts as resources, but they did seem to have a shared understanding of how to set up ratios and percent using a long division algorithm.

There were several situated meanings that provided language resources. In Excerpt 3, we see evidence of locally situated meanings such as "inside" and "outside" for parts of the long division algorithm. These colloquial terms drew upon "everyday" words and meanings for the purpose of communicating about a mathematical procedure. The students used several informal terms to describe the process of setting up and executing their calculations. For example, Claudia used the terms "adentro"/"inside" and "afuera"/"outside" for the position of the dividend and divisor in the standard US division algorithm. Francisco used the word "point" as a shortened version of "decimal point." Given the ways the students oriented to these words, it appears that these terms were shared among the group members (e.g., we did not see evidence of communication breaking down). Although the norms in this classroom permitted the use of home language (a similarity to the bilingual setting in Case 1), the discussion in Excerpt 3 that was in Spanish was focused completely on procedures, not concepts or reasoning. This fact indicates analysts must use caution to avoid overgeneralizing about how bilingual students use their linguistic resources to do mathematics.

4 Discussion and conclusion

In the foregoing analyses, we examined two discussions among bilingual middle school students in US schools using the ALM framework (Moschkovich 2015). That framework grew out of situated and sociocultural perspectives on mathematics learning (Forman 1996; Moschkovich 2002), and incorporates the three interconnected dimensions of mathematical proficiency, mathematical practices, and mathematical discourse. The two discussions we examined reflect a common divide in school mathematics instruction: In some classrooms, discussions are about non-procedural tasks designed to support students' conceptual understanding of key mathematical concepts. In other classrooms, discussions focus on routine exercises and the procedures or computations used to solve those exercises. The ALM framework provided us with analytical questions that allowed us to examine each discussion along three dimensions and to compare and contrast the discussions. The ALM framework guided our multilevel analysis, considering, for example, how the tasks created or constrained opportunities for these students to engage in different strands of mathematical proficiency.

Mathematics educators argue that students need opportunities to develop all of the strands of mathematical proficiency outlined in Kilpatrick et al. (2001). While the discussion in Case 1 involved some procedural knowledge (e.g., graphing ordered pairs), it also involved conceptual understanding (e.g., unitizing when deciding the scale on the axes, defining the independent and dependent variables). In contrast, the discussion in Case 2 focused on the procedural fluency strand of mathematical proficiency. Supporting conceptual understanding requires students to learn more than definitions or "shortcuts" related to mathematical concepts. For example, memorizing the "fact" that time is always the independent variable when modeling motion situations would not lead to conceptual understanding of the concepts of dependent and independent variable. Additionally, such memorization may lead to confusion when that "fact" needs to be revised (e.g., in the case of modeling the period of a pendulum, time depends on length). We highlight that classroom discussions—in pairs, among small groups, and among a whole class—have been shown to support the development of both procedural fluency and conceptual understanding (Hiebert and Grouws 2007; Zahner et al. 2012).

The first discussion in Sect. 3.1 incorporated multiple mathematical practices. First, the discussion aligned with disciplinary literacy practices in that it focused on a problem that required the students to generate and connect multiple representations including the given text, a numerical table, and a graph (Leinhardt et al. 1990). The problem and the classroom norms included opportunities for the students to engage in additional mathematical practices such as constructing arguments and critiquing the reasoning of others and modeling with mathematics. In many ways, the struggles these students experienced when setting up their graph reflect struggles that mathematical modelers face when making sense of unfamiliar information using mathematical tools.

In contrast, the discussion in Sect. 3.2 did not afford as many opportunities for students to engage in mathematical practices. The discussion focused on a procedural task, and it occurred only because Claudia's answer was different from the answer Amber and Francisco had found earlier. If the students had agreed on their answers initially, it is likely that this discussion would not have occurred. Given the procedural focus of the discussion in Case 2, it included few, if any, of the mathematical practices typical of academic mathematics. Instructional support for student engagement in mathematical practices such as argumentation requires opportunities for students to think about and negotiate the mathematics under discussion. This is supported when students have access to tasks and classroom norms that create opportunities to share and justify their claims in a variety of participation structures (teacher-led, small group, pairs, student presentations, etc.).

The contrast in the quality of the discussions in Sects. 3.1 and 3.2 highlights how the quality of mathematical discourse depends on the purposes of that discourse and resources available. In order to support students in participating in productive mathematical discourse, instruction needs to include time and support for mathematical discussions where students communicate their reasoning and use multiple representations, texts, and modes of communication. Drawing on the Vygotskian notion of mediation (Vygotksy 1978), we note that the quality of the mathematical discourse in these two cases was largely shaped by the tasks under discussion. Looking, for example, at the calculational discussion in Case 2, it was not surprising that a procedural task generated a mathematical discussion focused on procedures, the result, or the answer. Similarly, looking at the discussions in Case 1, it is not surprising that an open-ended task that required connecting multiple representations resulted in a mathematical discussion more focused on making sense of the representations.

However, we caution that an instructional task alone cannot ensure the quality of mathematical discourse. The quality of student discussions is also shaped by the norms established in a classroom (Cobb et al. 2001). In the classroom where Case 1 was recorded, we documented through extended ethnographic observations (Moschkovich 2008) how students were expected to make sense of a problem for themselves and in their group, discuss their choices, and then compare results collectively. These norms were reflected in the sustained joint interaction evident among the group of students as they persevered in attempting to create their graph. In contrast, in the classroom of Case 2, the norms for the students' mathematical discussions reflected the norms of a typical U.S. classroom where the goals of following procedures, executing calculations, and arriving at correct answers dominate mathematics teaching and learning interactions (Lampert 1990). Following the situated and sociocultural assumptions of the ALM framework, we note that these discussions were situated in particular sociocultural settings, and the norms of each setting influenced both discussions.

Mathematical argumentation is both a practice and a form of mathematical discourse. We note that these two cases share an important feature related to argumentation: students in both cases made many claims that were not backed

Table 2 Sample questions for using the ALM framework for designing lessons or tasks

	Mathematical proficiency	Mathematical practices	Mathematical discourse
For designing lessons	Which strands of math proficiency are possible with a task? Can a task be modified to include more strands or address one strand in more depth? Does the task require conceptual understanding and/or reasoning? Can the task be modified to require conceptual understanding and/or reasoning? What is necessary to maintain a focus on conceptual understanding and/or reasoning?	Which math practices are necessary (or possible) for solving the problem? Are additional math practices possible? What participation structures are necessary to engage students in those math practices?	What math texts are involved (or possible)? What modes, purposes, or representations are involved (or possible)? Are there any possible language resources to consider that may be specific to students' out of school experiences or their home communities?

up by evidence, argument, or justification. Making a claim may be the start for constructing arguments, but students need support, modeling and practice in learning to provide mathematical evidence, support, or justification for those claims. In whole class discussions, a teacher can moderate the discussion and act as a facilitator who models supporting claims with appropriate mathematical evidence (e.g., Lampert 1990). During peer discussions in small groups, however, students may make claims, but not support them with mathematical evidence. Further research is needed to consider this issue, perhaps contrasting discussions that have a procedural or conceptual focus.

In this paper, we have illustrated the utility of the ALM framework to analyze mathematical proficiency, practices, and discourse made evident in students' joint interactions. We focused on contrasting discussions (conceptual versus procedural) from bilingual settings to show the wide utility of the ALM framework. We note that the ALM framework explicitly highlights that academic literacy in mathematics is multidimensional and cannot be reduced to, for example, helping students acquire static meanings for words provided by the teacher or a textbook. As previous analyses show (Moschkovich 2008, 2015), mathematical meanings are situated in the history of a classroom, negotiated during discussions, and grounded in activity. The guiding questions framing this analysis are designed to bring focus on each of the aspects of ALM without destroying the interdependence of the aspects of the ALM framework. Additionally, the power of the ALM framework is made evident as the framework provides a series of guiding questions (see Table 1) for analyzing classroom interactions.

The ALM framework can also be useful for designing learning experiences targeting the development of ALM, as summarized in Table 2 below.

These guiding questions for ALM can be used to design mathematics learning experiences in multilingual classrooms. In addition to considering these questions for designing tasks, the design of instruction that provides opportunities for the development of ALM should consider tasks not in isolation but as they are framed by the norms in the classroom, the activity structure for each task, the texts involved, and other relevant contextual aspects of mathematical activity as it occurs in each classroom setting.

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References

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences, 10*, 113–164. https://doi.org/10.1207/S15327809JLS10-1-2_6.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. In E. Forman, N. Minick & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development* (pp. 91–119). New York: Oxford University Press.

Crowhurst, M. (1994). *Language and learning across the curriculum*. Scarborough: Allyn and Bacon.

Forman, E. (1996). Learning mathematics as participation in classroom practice: Implications of sociocultural theory. In L. Steffe, P. Nesher, P. Cobb, G. A. Goldin & B. Greer (Eds.), *Theories of mathematics learning* (pp. 115–130). Mahwah, NJ: Lawrence Erlbaum.

Gee, J. (1999). *An introduction to discourse analysis: Theory and method*. New York: Routledge.

Gutiérrez, K. D., Baquedano-López, P., & Tejeda, C. (1999). Rethinking diversity: Hybridity and hybrid language practices in the third space. *Mind, Culture, and Activity, 6*(4), 286–303.

Halliday, M. A. K. (1978). Sociolinguistic aspects of mathematical education. In M. A. K. Halliday (Ed.), *Language as a social semiotic: The social interpretation of language and meaning* (pp. 194–204). Baltimore: University Park Press.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Hanbook of research in mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Reston: National Council of Teachers of Mathematics.

Hymes, D. (1972). On communicative competence. In J. B. Pride & J. Holmes, (Eds.) *Sociolinguistics* (pp. 269–293). London: Penguin. **(Reprinted in Duranti, A. (Ed.). (2009). Linguistic anthropology: A reader (Vol. 1). John Wiley & Sons)**

Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding it up: Helping children learn mathematics.In J. Kilpatrick, J. Swafford & B. Findell (Eds.), National Research Council. Washington, DC: National Academy Press.

Lamon, S. (1996). The development of unitizing: Its role in children's partitioning strategies. *Journal for Research in Mathematics Education, 27*(2), 170–193.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29–64.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998). *Connected mathematics*. White Plains: Dale Seymour Publications.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs and graphing: Tasks learning and teaching. *Review of Educational Research, 60*(1), 1–64.

Moschkovich, J. N. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*, 11–19.

Moschkovich, J. N. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*, 189–212. https://doi.org/10.1207/S15327833MTL04023_5.

Moschkovich, J. N. (2004). Appropriating mathematical practices: A case study of learning to use and explore functions through interactions with a tutor. *Educational Studies in Mathematics, 55*, 49–80.

Moschkovich, J. N. (2007). Examining mathematical discourse practices. *For the Learning of Mathematics, 27*, 24–30.

Moschkovich, J. N. (2008). "I went by twos, he went by one:" Multiple interpretations of inscriptions as resources for mathematical discussions. *The Journal of the Learning Sciences, 17*(4), 551–587. https://doi.org/10.1080/10508400802395077.

Moschkovich, J. N. (2013). Issues regarding the concept of mathematical practices. In Y. Li & J. N. Moschkovich (Eds.), *Proficiency and beliefs in learning and teaching mathematics: Learning from Alan Schoenfeld and Günter Toerner* (pp. 257–275). Rotterdam: Sense Publishers.

Moschkovich, J. N. (2015). Academic literacy in mathematics for English learners. *The Journal of Mathematical Behavior, 40*, 43–62. https://doi.org/10.1016/j.jmathb.2015.01.005.

Moschkovich, J. N., & Brenner, M. E. (2000). Integrating a naturalistic paradigm into research on mathematics and science cognition and learning. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 457–486). Mahwah: Lawrence Erlbaum.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. *Discourse, Learning, and Schooling, 63*, 103.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. New York: Routledge.

Pirie, S., & Schwarzenberger, R. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in Mathematics, 19*, 459–470.

Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing mathematical activity: A practice-oriented view of advanced mathematical thinking. *Mathematical Thinking and Learning, 7*, 51–73. https://doi.org/10.1207/s15327833mtl0701_4.

Schleppegrell, M. (2007). The linguistic challenges of mathematics teaching and learning: A review. *Reading Writing Quarterly, 23*, 139–159. https://doi.org/10.1080/10573560601158461.

Scribner, S. (1984). Studying working intelligence. In B. Rogoff & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 9–40). Cambridge: Harvard University Press.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge: Cambridge University Press.

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36*(5), 404–411. https://doi.org/10.2307/30034943.

Thompson, A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A. (1994). Calculational and conceptual orientations in teaching mathematics. In A. Coxford (Ed.), *1994 yearbook of the NCTM* (pp. 79–92). Reston: NCTM.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge: Harvard University Press.

Zahner, W. (2015). The rise and run of a computational understanding of slope in a conceptually focused bilingual algebra class. *Educational Studies in Mathematics, 88*(1), 19–41. https://doi.org/10.1007/s10649-014-9575-x.

Zahner, W., & Moschkovich, J. N. (2010). Talking while computing in groups: The not-so private functions of computational private speech in mathematical discussions. *Mind Culture and Activity, 17*, 265–283.

Zahner, W., & Moschkovich, J. N. (2011). Bilingual students using two languages during peer mathematics discussions: ¿Qué significa? Estudiantes bilingües usando dos idomas en sus discusiones matemáticas: What does it mean? In K. Tellez, J. N. Moschkovich & M. Civil (Eds.), *Latinos and mathematics education: Research on learning and teaching in classrooms and communities* (pp. 37–62). Charlotte: Information Age Publishing.

Zahner, W., Velazquez, G., Moschkovich, J. N., Vahey, P., & Lara-Meloy, T. (2012). Mathematics teaching practices with technology that support conceptual understanding for Latino/a students. *Journal of Mathematical Behavior, 31*, 431–446. https://doi.org/10.1016/j.jmathb.2012.06.002.