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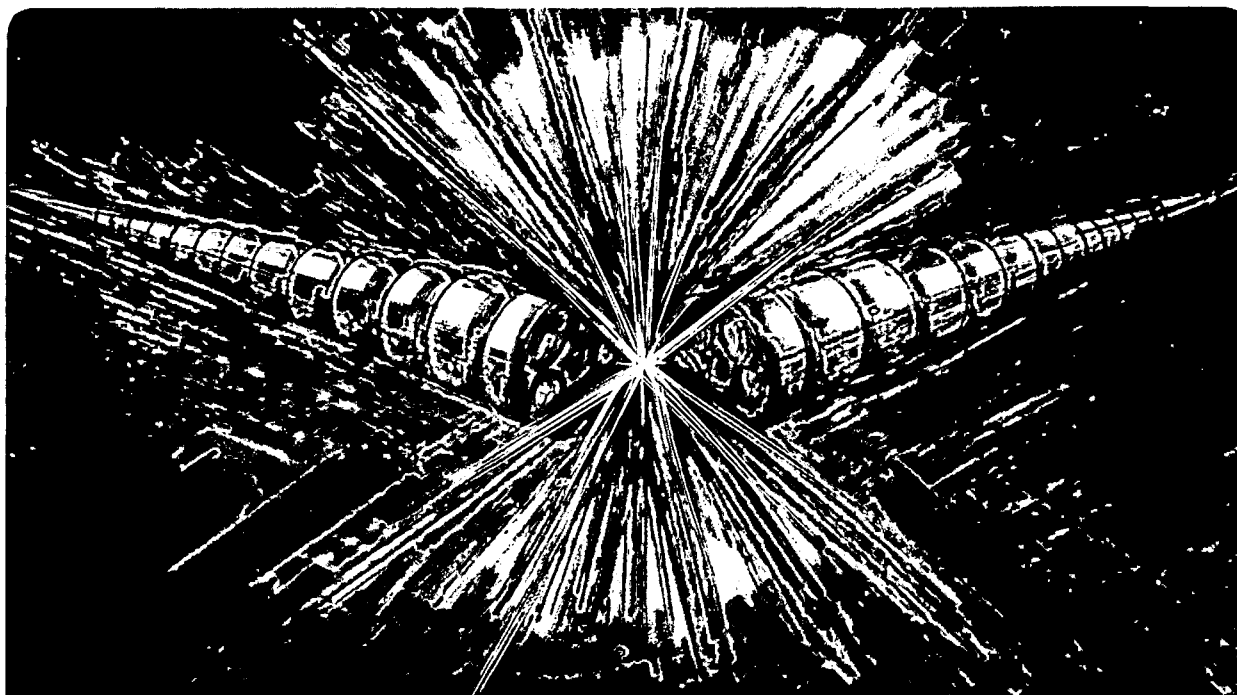
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M.R. Howells

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**THE WHISPERING GALLERY
AS AN OPTICAL COMPONENT IN THE X-RAY REGION***

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THE WHISPERING GALLERY AS AN OPTICAL COMPONENT IN THE X-RAY REGION

By Malcolm R. Howells

THIS PAPER WAS MOSTLY WRITTEN IN THE 1980'S AND LEFT. IT IS PUBLISHED NOW (1995) AS AN LBL REPORT WITH MINOR ADDITIONS SINCE THERE SEEMS TO BE SOME CURRENT INTEREST IN THE MATERIAL

ABSTRACT

The whispering gallery phenomenon in acoustics has been known and studied for more than a century, and the same effect has been observed to take place with waves other than sound waves. In this paper we review the theoretical basis and attractive features of the whispering gallery as a soft x-ray optical component and indicate some of its potential applications. We then describe what may be its most unique capability which, in favorable cases, is to provide a way to manipulate the phase difference between the s and p polarization components and thus to generate circularly or elliptically polarized soft x-rays.

INTRODUCTION

The first scientific explanation of the whispering gallery was provided by Lord Rayleigh in *The Theory of Sound* [Rayleigh, 1945] and in two papers in the early part of this century. His description was in terms of a wave field that was essentially guided by the curved interface between two different acoustical media. The media needed to be such that total reflection could take place. The problem attracted the attention of a number of famous investigators including two Nobel Laureates, Rayleigh and C.V. Raman [Raman, 1921, 1922a, 1922b], and the "father of modern acoustics," W. C. Sabine [Sabine, 1922].

Apart from acoustical phenomena in air, somewhat similar behavior is observed for waves inside solid materials, and surface acoustic waves [see for example Das et al., 1982] (also known as Rayleigh Waves [see for example Ash, 1985]) are important in ultrasonic devices. Analogous seismic waves are also found at the surface of the earth.

From the beginning Rayleigh recognized the possibility of observing the phenomenon in electromagnetic radiation, and the literature provides various examples in the radiowave [Budden and Martin, 1962], infra-red [Krammer, 1978] and visible [7] spectral regions. The operation of waveguides, light-pipes and optical fibers are evidently closely related processes. Similar optics, although quite different physics, is employed in the guide tubes used for the transport of low-energy neutrons. The wave theory of the whispering gallery is also closely related to a fundamental problem in optics: the variation of light intensity near the point of tangency of a ray to a caustic [Landau and Lifschitz, 1951].

The possibility of applying the whispering gallery in the soft x-ray region was first proposed by A.V. Vinogradov and collaborators in a series of papers beginning in 1982. These authors

recognized that total reflection is possible in this region and that in a few cases there were materials with optical constants which would give high efficiency for a whispering gallery. Since 1982 the theoretical analysis of whispering gallery waves guided by well-modelled realistic surfaces has been pursued by the Vinogradov group in a series of publications. More recently a particularly thorough and elegant treatment has been provided by Braud and Hagelstein. The latter authors added polarization properties to the list of issues discussed by the Vinogradov group. Sixteen papers by these two groups are cited at various points in this work.

Experimentation involving wavelengths shorter than that of visible light has been rather slight. A few experimenters have attempted to operate hollow tubes as x-ray light pipes [see for example Pantell and Chung, 1978]; and some success has been reported, particularly if the standard of comparison was the absence of the light pipe and thus an inverse square loss of x-rays with distance. The most important experimental results are recent ones by the Vinogradov group in which whispering-gallery reflectors performed at or near theoretical in the soft x-ray region [Aleksandrov, 1992; Vinogradov, 1987a, 1987b, 1990].

THEORETICAL MODELS

The basic ideas of both a geometrical-optics and a wave-optics picture of the whispering gallery were worked out by Rayleigh. The geometrical-optics model [Rayleigh, 1945] starts from the notion of an initial ray incident on a circular whispering gallery of radius R (Figure 1). The ray is a chord of the circle and the distance of closest approach of that chord to the center of the circle is R' . Suppose we define another circle concentric with the first and of radius R' . The initial ray then progresses around the circle by multiple reflection and is confined to the region between the two circles. Any other rays from the same source point or from other source points will be similarly confined provided that they initially form a chord of the whispering gallery lying between the two circles. The situation is reminiscent of a waveguide except that one of the surfaces is, at most, touched by the rays and therefore does not need to be a real guiding surface. According to this picture, the efficiency of a whispering gallery is the product of the efficiencies of whatever number of individual reflections are needed to get round. The description also allows a quantitative estimate of the accumulated phase difference between s and p polarized radiation and of the phase space acceptance of the system. We return to the matter of making such estimates later.

The question of a wave theory of the whispering gallery occupied Rayleigh for some time until, as he wrote in 1910 [Rayleigh, 1910], he "...recognised that most of what I sought lay, as it were, under my nose." The mathematical description of wave propagation around a whispering gallery was simply a limiting form of the well known solution to the wave equation with cylindrical boundary conditions such as could be applied, for example, to the circular membrane problem. The solution for the electric field perpendicular to the plane of the whispering gallery is correspondingly

$$E = J_n(k_0 r) \cos(\omega t - n\theta) \quad (1)$$

where k_0 is $2\pi/\lambda_0$, λ_0 is the free space wavelength at angular frequency ω , r and θ are the polar coordinates, t is the time and n an integer. For the moment we are thinking of the case of a perfectly-conducting wall, so we take $E = 0$ at $r = R$ in exact analogy to the membrane problem. We can see that when θ advances by 2π , the cosine term executes n oscillations, so that the condition $n\lambda_0 \sim 2\pi R$, must be satisfied to construct a standing wave mode. For the cases of interest to us, we

clearly need very large values of n , and the Bessel function approaches one of its limiting cases: the Airy function. One can readily see from tables of Bessel functions that for large n , the function is zero at the origin and remains essentially zero for a considerable range of $k_0 r$, just as the present problem requires. To achieve the simplest whispering-gallery waveform, we set the boundary at the first zero of the Bessel (Airy) function (Fig. 2). As we discuss more quantitatively later, the wave function is then non-zero only for a narrow region just inside the wall of width about $n^{1/3} \lambda_0 / 2\pi$, which is much smaller than R . The phenomenon of concentration of the wave in the immediate neighborhood of the wall, is thus explicable in conventional wave theory and for the case of soft x-rays should become extremely pronounced. The most lucid exposition of this theory is that of Braud and Hagelstein [Braud, 1991]. However, rather than reproduce their picture we attempt a more “hand-waving” type of treatment with the goal of providing a readable introduction to a review of experimental work, past and future.

We therefore arrive at the scheme shown in Fig 2. This general picture of the operation of a whispering gallery will be analyzed in more detail later and will be seen to confirm the efficiency estimates that can be obtained by the geometrical optics analysis. However, since the latter estimates are much easier to deal with, both analytically and computationally, we consider them first.

GEOMETRICAL OPTICS CALCULATIONS

We are interested in estimating the efficiency of a whispering gallery that turns an incoming beam through an angle α when the initial angle of grazing incidence is θ . The number of bounces is thus $N = \alpha/2\theta$. We wish to find the efficiency for both s and p polarization and the accumulated difference in phase, $\Delta\phi_{s-p}$, between the s and p components. The case of special interest in the problem at hand is the limit of these quantities as θ tends to zero. We see that the losses suffered by the beam are equal to the loss per bounce times the number of bounces, and as θ becomes small the former tends toward zero and the latter toward infinity. We will now show that the product of the two approaches a finite limit that we can calculate. We start with the expressions for the amplitude reflectances r_s and r_p in s and p polarization, as given, for example, by Born and Wolf [1965].

$$r_s = \frac{\sin \theta - \sqrt{\epsilon - \cos^2 \theta}}{\sin \theta + \sqrt{\epsilon - \cos^2 \theta}} \quad (2a)$$

$$r_p = \frac{\sqrt{\epsilon - \cos^2 \theta} - \epsilon \sin \theta}{\sqrt{\epsilon - \cos^2 \theta} + \epsilon \sin \theta} \quad (2b)$$

where ϵ is the (complex) dielectric constant of the reflecting medium. Now it is useful to consider the transmitted beam travelling at angle θ_T to the boundary inside the medium and to define $S = s + it = \tilde{n} \sin \theta_T$. Using Snell's law this gives $S = \sqrt{\epsilon - \cos^2 \theta}$, and as $\theta \rightarrow 0$, $S \rightarrow \sqrt{\epsilon - 1}$. We can now represent the wave inside the medium as $\exp(i\mathbf{k}\mathbf{h} \cdot \mathbf{r})$ where \mathbf{h} is a unit vector in the direction of propagation, $k (= \tilde{n}k_0)$ is the propagation constant inside the medium, and $\tilde{n} \equiv 1 - \delta - i\beta = \sqrt{\epsilon}$

$$\begin{aligned}
E &= E_0 e^{[-i\bar{n}k_0(xh_r + zh_r)]} \\
E &= E_0 e^{[-ik_0(x\bar{n}\cos\theta_r + z\bar{n}\sin\theta_r)]}
\end{aligned}
\tag{3}$$

Using Snell's law and the above definitions we obtain

$$E = E_0 e^{[-ik_0(x\cos\theta + z\sqrt{\epsilon - \cos^2\theta})]} \tag{4}$$

which shows that the wave in the medium is a non-homogeneous wave whose surfaces of constant phase make an angle $\tan^{-1}(\cos\theta/s)$ to the interface while its surfaces of constant amplitude are parallel to the interface. According to (4) the z -dependence is equivalent to a complex "skin depth" $\Delta = (-ik_0\sqrt{\epsilon - \cos^2\theta})^{-1}$. This is a generalization of the normal skin depth valid for any incidence angle and for materials where both the real and imaginary parts of the refractive index are non-negligible. It represents an oscillatory but decaying signal inside the medium. For whispering gallery applications we will be interested in the case $\theta \rightarrow 0$, so we define

$$\Delta_0 = (-ik_0\sqrt{\epsilon - 1})^{-1} \tag{5}$$

It is interesting to evaluate the attenuation depth given by Eq. (5). For many elements in the x-ray region, f_1 tends toward the atomic number, so we can often use the approximation $f_1/\text{atomic weight} \approx 1/2$. (f_1 is the real part of the atomic scattering factor.) Using this, (5) leads to a $1/e$ depth in Å of $68/\sqrt{\text{density (gm/cc)}}$. This is only weakly dependent on the wavelength and material and leads to the general conclusion that the penetration of the wave field, in grazing incidence x-ray and soft x-ray reflections is about 30 Å.

Returning to (2a), we note that for $\theta = 0$, $r_s(0) = 1$, and we wish to explore the behavior near $\theta = 0$. We therefore expand r_s as a Taylor series about $\theta = 0$ as follows

$$r_s = r_s(0) + \left(\frac{dr_s}{d\theta}\right)_{\theta=0} \theta + \dots \tag{6}$$

Evaluating the derivative using (2) we obtain

$$r_s = 1 - \frac{2\theta}{\sqrt{\epsilon - 1}} + \dots, \tag{7}$$

from which we conclude that the overall amplitude reflecting efficiency, R_s , of the whispering gallery is given by

$$R_s = \lim_{\theta \rightarrow 0} \left(1 - \frac{2\theta}{\sqrt{\epsilon - 1}}\right)^{\alpha/2\theta} \tag{8}$$

Now, using one of the definitions of the exponential function [Footnote 1] we obtain the final result for the efficiency

$$|R_s|^2 = \left| \exp \frac{-\alpha}{\sqrt{\varepsilon-1}} \right|^2 = \exp \left[-2\alpha \operatorname{Re} \left(\frac{1}{\sqrt{\varepsilon-1}} \right) \right] \quad (9)$$

In a similar fashion we have for r_p

$$r_p = 1 - \frac{2\theta\varepsilon}{\sqrt{\varepsilon-1}} + \dots, \quad (10)$$

and

$$|R_p|^2 = \exp \left[-2\alpha \operatorname{Re} \left(\frac{\varepsilon}{\sqrt{\varepsilon-1}} \right) \right] \quad (11)$$

To obtain the limit of $\delta\phi_{s-p}$ (the s to p phase difference) as $\theta \rightarrow 0$, we start from the single reflection expression [Born and Wolf, 1965]

$$\tan \delta\phi_{s-p} / 2 = \frac{\sin \theta \sqrt{\cos^2 \theta - \varepsilon}}{\cos^2 \theta} \quad (12)$$

which gives $\delta\phi_{s-p} = 0$ at $\theta = 0$ and leads to a Taylor series:

$$\tan \delta\phi_{s-p} / 2 = \sqrt{1-\varepsilon} \cdot \theta + \dots \quad (13)$$

Multiplying by the number of bounces $\alpha/2\theta$, we get an estimate for the accumulated phase difference $\Delta\phi_{s-p}$,

$$\lim_{\theta \rightarrow 0} \Delta\phi_{s-p} = \alpha \operatorname{Im}(\sqrt{\varepsilon-1}) \quad (14)$$

These relations can be used as a basis to evaluate materials to see if they offer promising possibilities for efficient whispering galleries. All the equations relating to the efficiency have been fully covered by Vinogradov et al. [1985b], and those relating to $\Delta\phi_{s-p}$ have been given by Braud [1990].

MATERIAL PROPERTIES NEEDED FOR AN EFFICIENT WHISPERING GALLERY

Using the definition for the complex refractive index \tilde{n} , we find that Eq.(9) becomes [Vinogradov et al., 1983]

$$|R_s|^2 = \exp \left\{ \frac{-2\alpha \sin \left[\frac{1}{2} \tan^{-1} \frac{\beta}{\delta} \right]}{\sqrt{2\delta} \left(1 + \frac{\beta^2}{\delta^2} \right)^{1/4}} \right\} \quad (15)$$

Making use of the approximations $\beta, \delta \ll 1$ and $\beta/\delta \ll 1$, we can approximate (15) as

$$|R_s|^2 \approx \exp\left[-\frac{\alpha\beta}{\sqrt{2}\delta^{3/2}}\right] \quad (16)$$

which allows the conclusion that for a reasonably efficient whispering gallery ($|R_s|^2 > 0.1$ with rotation angle π) it is necessary to have $\beta/\delta^{3/2} < 1$. If this condition is met, then the condition $\beta/\delta \ll 1$ will usually be met also. The condition $\beta, \delta \ll 1$ is always true in the soft x-ray region.

Some values of the efficiency calculated from (9) or (15) are plotted in Fig. 3 for a few materials that have suitable optical constants. We see that the useful cases fall into three main regions: greater than 250 Å, 50 – 150 Å, and less than about 10 Å. The cases in the middle region arise because the absorption crosssections of many materials have a “Cooper” minimum [see for example Manson and Cooper, 1968] in that region. Unfortunately, there are no good materials in the “water window” region in which x-ray microscopy of biological samples is often done. The values given are for a rotation angle of π . The values for other angles can be found from the argument that the efficiency of cascaded whispering galleries is the product of the individual efficiencies. For example, at 25 Å, the calculated efficiency of LiF is 0.04 for angle π or 0.20 for angle $\pi/2$.

Figure 4 shows some efficiency values for carbon which is a material of special interest on account of its high resistance to contamination and its promise of performance at 1-2 Å. It is noteworthy that the calculated efficiency values are sensitive to quite small errors in the optical constants, although an exception to the uncertainty is the light elements in the 1-2 Å region where the behavior is quite free-electron-like and predictable.

WAVE TREATMENT

The treatment provided by Rayleigh gives an excellent insight into the physics of whispering galleries in general, although it has certain differences from the x-ray case which we discuss later. We return now to the use of Eq. (1) to represent an *s*-polarized wave (that is, a transverse electric wave) progressing around a whispering gallery. To see how the Bessel function approaches the Airy function as a limiting case, consider the representation [Footnote 2]

$$J_n(z) = \frac{1}{\pi} \int_0^\pi \cos(z \sin \omega - n\omega) d\omega \quad (17)$$

In the present case of very large n , Rayleigh [1910] argues that the value of the integral is mainly determined by the region near the stationary point of the argument of the cosine at $\omega = 0$, (what we would now call the Principle of Stationary Phase). Therefore, approximating the sine to third order we obtain

$$J_n(z) = \frac{1}{\pi} \int_0^\pi \cos\left[(n-z)\omega - \frac{1}{6}z\omega^3\right] d\omega \quad (18)$$

Changing the variable to

$$u = \omega \left(\frac{z}{2} \right)^{1/3}, \quad (19)$$

gives the equivalence

$$J_n(z) \left(\frac{z}{2} \right)^{1/3} = \frac{1}{\pi} \int_0^\infty \cos \left(\rho u + \frac{u^3}{3} \right) du = \text{Ai}(\rho) \quad (20)$$

valid for n large and $(n - z)/n \ll 1$, where $\text{Ai}(\rho)$ is the Airy function as defined, for example, by Abramowitz and Stegun [1965], and ρ is defined by

$$\rho = (n - z) \left(\frac{2}{z} \right)^{1/3} \quad (21)$$

Using this and recalling that $z (= k_0 r) \simeq n \gg n^{1/3}$, we can write

$$k_0 r = n - 2^{-1/3} \rho n^{1/3} \quad (22)$$

Some relevant values of the Airy function are given in Table 1. If we consider that the wave is mainly located in the region of width Δr between the boundary and the radius at which it has fallen in amplitude to half of its peak value, then we have

$$\frac{\Delta r}{R} \simeq 2 \cdot 13 n^{-2/3} \simeq 2 \cdot 13 (k_0 R)^{-2/3} \quad (23)$$

Since $n \sim k_0 R$, we see that the localization in the region of the boundary will be extremely pronounced for short wavelengths.

The description so far defines a wave mode, and we need to ask how this can be coupled to a well-collimated incoming wave of the type one could get from a synchrotron radiation x-ray source. If we superimpose the two waves as in Fig. 5, we see that the maximum allowed angular mismatch is an angle ψ given by

$$\Delta r \simeq \frac{1}{2} r \psi^2 \quad (24)$$

Combining (23) and (24) we obtain

$$\Delta r \psi \simeq 0.48 \lambda \quad (25)$$

which has the form of a spatial coherence condition and defines the phase space of a beam that could couple to the whispering gallery. That is, the mode we have considered so far can only couple to a spatially-coherent incoming beam.

This is not so much of a limitation as it seems because in fact the mode considered is really only the fundamental of a whole family of modes which are generated by setting zeros of the Airy function other than the first one at the boundary. It would appear that any incident beam could be matched and can propagate by exciting a suitable linear combination of these modes. This is in accord with our geometrical optics picture of the phenomenon. Of course some incoming rays will be highly attenuated and this will be determined by their initial angle of incidence and the optical properties of the reflecting material.

We can gain a further understanding of the description of the whispering gallery using Airy functions, by formally solving the cylindrically symmetric boundary value problem for the electric field $E_z(r, \phi)$ – retaining for the moment the assumption of infinite conductivity and using the boundary condition $E_z = 0$ at $r = R$. We assume a time dependence $\exp(i\omega t)$ and an angular dependence $\exp(in\phi)$ where n is the phase change per unit angle of deflection, ω is the angular frequency. It is helpful, although not necessary for the present problem, to think of n as an integer, which would imply an angular function that was periodic with n nodes. Either way n will turn out to be the same as the n used earlier. With these assumptions the solution is $E_z = E(r) \exp(in\phi)$ where $E(r)$ must satisfy

$$\frac{d^2 E}{dr^2} + \frac{1}{r} \frac{dE}{dr} + \left(k_0^2 - \frac{n^2}{r^2} \right) E = 0. \quad (26)$$

This is Bessel's equation, of which the solution of interest to us would be $J_n(k_0 r)$.

To develop the approximate form of $J_n(k_0 r)$ needed for the whispering gallery (which we have already indicated will turn out to be an Airy function), we could use the method of series expansion as demonstrated by Vinogradov et al. [1983b]. However, an alternative approach proposed by H. Krammer [1978] provides an interesting insight into the problem. Consider the conformal transformation $W = R \ln(Z/R)$, where $Z = x + iy$ and $W = u + iv$. This maps the inner region of the circle $r = R$ into the left half of the u, v plane (Fig. 6). Equating real and imaginary parts yields

$$v = R\phi \quad \text{and} \quad u = R \ln(r/R) \quad (27)$$

The two-dimensional scalar wave equation $[\nabla_{x,y}^2 + k_0^2]E = 0$ is now transformed into

$$[\nabla_{u,v}^2 + k_0^2 \exp(2u/R)]E = 0 \quad (28)$$

while the boundary condition transforms to $E = 0$ at $u = 0$. We can now see that the condition $E \sim 0$, except in the narrow corridor close to the boundary, implies $r/R \sim 1$, so that $u = R \ln(r/R) \sim r - R$ and $u/R \ll 1$. This allows a linear approximation to the exponential function in (28) which now becomes:

$$\frac{d^2 E}{du^2} + \frac{d^2 E}{dv^2} + \left(1 + \frac{2u}{r} \right) k_0^2 E = 0 \quad (29)$$

We have $E(u, v) = E(u) \exp(ikv)$ where $v = R\phi$, $k = n/r$ ($k \neq k_0$). Substituting this in (29), we find for $E(u)$

$$\frac{d^2 E}{du^2} + [(k_0^2 - k^2) + (2k_0^2/R)u] E = 0 \quad (30)$$

Remembering that $u \sim r - R$, we introduce the new variables

$$\rho = \left(\frac{2k_0}{R} \right)^{1/3} (R - r) - \rho_i \quad (31)$$

and

$$\rho_i = \left(\frac{2k_0^2}{R} \right)^{-2/3} (k_0^2 - k^2) \quad (32)$$

Equation (30) now finally becomes

$$\frac{d^2 E}{d\rho^2} - \rho E = 0 \quad (33)$$

which is the differential equation for the Airy function $\text{Ai}(\rho)$ introduced earlier [Abramowitz and Stegen, 1965]. From Eqs. (31) and (32) we can see that ρ is a linear function of position in the radial direction with $\rho = \rho_i$ at the boundary. It is also evident that when $\rho = 0$, $k_0 r = n$, so that the free space value of the wavelength is exactly matched and this occurs at a distance $0.3\pi^{-1/3}\lambda_0$ inside the boundary, at $r = r_n$ (say). The wavelength at $r = R$ is slightly longer than the free space value.

Using (31) and (32) and the approximations $k_0^2 - k^2 \sim 2k_0(k_0 - k)$ and $k_0 r \sim n$, we recover equation (21), the original definition of ρ . The strength of the transverse electric field is sketched in Fig.(7) where, as discussed earlier, we have chosen to put the boundary at the first zero ρ_1 of $\text{Ai}(\rho)$. This choice sets $\rho_i = \rho_1$ and Eq.(32) then represents an eigenvalue condition for $k = (n/R)$, the propagation constant at the boundary.

EFFECT OF FINITE CONDUCTIVITY

We know from our discussion of grazing incidence reflection that the field does not really fall to zero exactly at the boundary but does so exponentially inside the reflecting medium over a distance of about 30 \AA and with complex skin depth Δ_0 [see, for example Jackson, 1962]. We must now make the slight adjustment to the radial dependence of the field so that both it and its normal derivative are continuous at the boundary [Footnote 3]. We know that inside the material, the field can be represented by

$$E_z = A \exp[-(r - R)/\Delta_0] \quad (r > R) \quad (34)$$

Thus at $r = R$

$$Ai(\rho_j + q) = A \quad \text{and} \quad ; \quad Ai'(\rho_j + q) = A / \Delta\rho , \quad (35)$$

where q is the small shift of the boundary away from ρ_j that we are trying to calculate, and $\Delta\rho$ is the ρ increment corresponding to Δ_0 . Since q is small we can see that $Ai(\rho_j + q) = q Ai'(\rho_j + q)$ so combining this with Eqs.(34) and (35) we get $q = \Delta\rho$. The spatial shift required to allow for finite conductivity in the soft x-ray range is therefore of the order of 30 Å.

We are now in a position to determine the attenuation of the j th mode of the whispering gallery as a function of ϕ . We know that in the infinite conducting case the condition $\rho = \rho_j$ at $r = R$ leads to the eigenvalue condition for k . We therefore expect that a correction δk in k will be created by the change from ρ_j to $\rho_j + \Delta\rho$ and that this will have an imaginary part that will account for the dissipation. From the definition of ρ we find that in fact $\delta k = k_0 \Delta_0 / R$, which implies that the original angular function $\exp(in\phi)$ now acquires an extra factor $\exp(iR\delta k\phi) = \exp(i\phi / \sqrt{\varepsilon - 1})$ which, upon taking the modulus squared, leads (as it should) to the same expression for the efficiency of the whispering gallery that we had before:

$$\lim_{\theta \rightarrow 0} (\text{efficiency}) = \exp\left[-2\phi Re\left(1/\sqrt{\varepsilon - 1}\right)\right] , \quad (36)$$

where ϕ is the same now as α . The sign of the square root is chosen so that the overall argument is negative. We note that, to the present order of approximation, the limit of the efficiency as $\theta \rightarrow 0$ depends only on ε and the angle turned and is independent of j , R and the path length of the beam along the mirror surface

RADIATIVE LOSSES

We have already indicated that there are at least two mechanisms to account for the losses of the whispering gallery. One is energy dissipation by the evanescent waves just inside the reflecting medium. The other is propagation of a non-evanescent wave into the medium. The theory of both these processes is described in detail by Vinogradov et al. [1983b, 1985c]. The treatment of dissipation given by these authors is based on expansion of the quantities involved in Bessel's equation in a power series in the small quantity $[2(k_0 R)^2]^{-1/3}$. The lowest order differential equations with the prevailing boundary conditions then give the Airy function variation of the field for $r < R$ and rapid exponential decay for $r > R$ as we have discussed. The possibility of a propagating wave in the medium only arises if the boundary is curved [Rayleigh, 1914]. If it is plane, the spatial period of the driving wave in free space is always too short to propagate in the medium $k > (1 - \delta)k_0$. However, when the boundary is sufficiently curved, the wave nodes of the driving wave will fan outward making the wavelength in the medium longer, eventually allowing a wave to propagate. This "radiative" loss of energy was investigated by Vinogradov et al. by solving Bessel's equation more exactly and deriving an efficiency comprising two terms:

$$\text{Efficiency} = R_0 \exp\left\{-2\alpha Re\left[\frac{1}{\sqrt{1 - \varepsilon}} e^{-\frac{2}{3}k_0 R(1 - \varepsilon)^{3/2}}\right]\right\} \quad (37)$$

where R_0 is the dissipative efficiency, and the second term is the radiative one. It depends strongly on R and tends toward unity for $R > R_{\min}$ [given by (40)] and becomes small for $R < R_{\min}$.

From the viewpoint of x-ray optics, this analysis is mainly important for understanding the degree to which a mode in the whispering gallery can “ride” over irregularities in the interface [see Eqs. (37, 38)]. However, it is interesting to note that radiative loss is the dominant one in acoustical whispering galleries and was the one analyzed by Rayleigh in his paper in 1914 [Rayleigh, 1914]. The paper considered the case of a “total reflection” whispering gallery, as we have, and developed the theory for the case prevailing in acoustics, namely, δ not constrained to be small but $\beta \ll \delta$. An analytical formula for the efficiency (equations 39–41 in [Rayleigh, 1914]) was derived, which, in our notation and after “translation” to the electromagnetic case could be written as follows:

$$\text{Efficiency} = \exp \left\{ -\frac{2\alpha}{\sinh 2\beta} e^{-2n(\beta - \tanh \beta)} \right\} \quad (38)$$

where $\cosh \beta = \epsilon^{1/2}$. In making this translation we have removed a factor n from Rayleigh's (41) and (42) so that his formula applies to a period of the whispering gallery (n waves) rather than a period of the wave. We then scaled to angle α instead of 2π and used the fact that the ratio of the densities of two gases is equal to the square of the refractive index at their interface. Rayleigh was satisfied that his theory explained the extreme persistence of the whispering gallery wave over long distances, remarking: “Calculation thus confirms the expectation that the whispering gallery effect does not require a perfectly reflecting wall, but that the main features are reproduced in transparent media provided that the velocity of waves is moderately larger outside than inside the surface of transition. And further, the less the curvature of this surface, the smaller is the refractive index (greater than unity) which suffices.”

To reconcile the two formulae, we subject the Rayleigh formula to the restriction: $|\epsilon - 1| \ll 1$, and note that $n \approx k_0 R$. The exponents on the right of our (40) and (41) are then seen to be equal up to third order in β and the formulae are reconciled.

BASIS FOR APPLICATIONS

The fact that in the limit $\theta \rightarrow 0$, the efficiency is independent of R is highly significant for applications. It implies that once a beam has satisfied the conditions to begin propagating in the fundamental mode of the whispering gallery, we may change the radius if we wish without losing the beam. By Liouville's Theorem and Eq. (25) for example, we can see that if we reduce R (tighten the curvature) the effect will be to reduce the width of the beam and to increase its divergence. A detailed geometric-optics calculation of these phase space properties of the whispering gallery has been given by Aleksandrov et al. [Aleksandrov, 1992]. The single-mode whispering gallery can thus function as both a relay optic and a concentrator. In what follows we explore some of the limitations governing the applications of whispering galleries.

First, suppose that we depart from the limit $\theta \rightarrow 0$ and allow θ to increase. This is difficult to study analytically but we have carried out a study based on repeated application of the Fresnel equations. The resulting calculated efficiencies do tend properly toward the limits calculated above for the case $\theta \rightarrow 0$. They also gradually diminish as θ increases, reaching half the maximum efficiency at a fraction f of the critical angle θ_c . For 180° deflection whispering galleries, f is usually in the range .5–.9 and increases for deflection angles less than 180° (Fig. 8).

Now that we have information about $\theta_{1/2} = f\theta_c$, we can use an argument similar to that used in deriving Eq.(24) to determine the maximum phase space acceptance, A , of the system when all the modes that can propagate efficiently are being used.

$$A = \frac{1}{2} R \theta_{1/2}^3 \quad (39)$$

This is generally much greater than the single-mode acceptance defined by Eq.(20).

Another important limit to understand is the minimum value of R for which the efficiency remains R -independent. We have seen that $r_n k_0 = Rk = n$ exactly, so that the wavelength at the boundary is slightly greater than the free space value. If it should ever exceed the free-space value by a fractional amount greater than δ the unit decrement of real part of the refractive index of the reflecting material, then a wave would begin to propagate in the medium and would represent an additional, radiative, form of loss which we considered in the previous section. We note in considering this that $R - r_n \gg |\Delta_0|$. The radius must therefore be greater than a minimum value R_{\min} given by the above refractive index condition

$$k_0 R_{\min} = \left(\frac{1.86}{\delta} \right)^{3/2} \quad (40)$$

The importance of R_{\min} is that it tells something about how well the wave can be guided over surface irregularities. A critical spatial frequency above which guiding will not occur is given by

$$f_{\text{crit}} = 2\pi / \sqrt{R_{\min}} \sigma \quad (41)$$

where σ is the root-mean-square roughness height.

POTENTIAL FOR STEERING HARD X-RAYS

It is of particular importance to understand the potential of the whispering gallery for steering beams of hard x-rays (1-2 Å) because the single bounce reflectors conventionally used can only deflect the beam by about $2\theta_c$ which is less than one degree in this spectral region. This has major implications for such matters as the architectural layout of synchrotron radiation laboratories. At $\lambda = 1.5$ Å, $\theta_{1/2}$ is about 3 milliradians and equation (37) gives a value of the phase space acceptance for a carbon whispering gallery of 1.5×10^{-7} meter.radians for a 10 m radius. This is greater than the emittance of many of the x-ray beams used in synchrotron radiation beamlines and much greater than that of the near-coherent beams expected from x-ray undulators at the new x-ray facilities at Argonne and Grenoble. If these beams were to be focused to a narrow width, say 50 microns, then the theory certainly predicts that they could be steered through angles of 5-6 degrees by the carbon whispering gallery. Such a focus could in fact be made, and this method of steering may very well be useful. However, it is not convenient. It would be much better to have a single reflector that could steer a natural undulator beam that had not been concentrated. Hard x-ray undulators actually give a near parallel beam of width say 1/2 mm and very small angular spread, and for such a beam one can collect the beam at an incidence angle less than $\theta_{1/2}$ only by using a large radius. For a mirror of manageable length, this implies a small steering angle and so the

possibility of deflections on the order of a right angle can be discounted for a circular reflector with a natural undulator beam. It appears, however, that there are opportunities for improved performance in this situation using shapes other than the circle, and this has been investigated by Braud and Hagelstein [Braud, 1992] and Vinogradov et al. [Artyukov, 1991].

SCATTERING AND CONTAMINATION

From the above discussions we can see that gross figure imperfections in the guiding surface will be tolerated very well but that microroughness may be more of a problem. Even here, however, there is some relief due the extremely small grazing angles involved. We have seen that the amplitude loss on reflection goes down linearly with angle at small θ . The fraction of the light intensity that is scattered, integrated over all angles, is given by $1 - \exp[-4\pi \sin \theta \sigma / \lambda]^2$, so that for small angles (linear approximation to the exponential) the amount of scattered light goes down like θ^2 . The scattered amplitude thus goes down at the same rate as the specular, so that the scattered-to-specular ratio is not expected to increase as the number of reflections increases. This suggests that the whispering gallery may not be so vulnerable to scatter as its surface-dependant nature would lead one to expect. A further reason for optimism is the fact that the wave extends to a depth greater than 30 Å in the reflector: a value far larger than the size of the irregularities in the superpolished surfaces that are produced today. In a later section we review the experimental evidence relating to the issue of scattering and conclude that while scattering is a significant issue in whispering gallery applications it can be sufficiently reduced by the use of modern supersmooth surfaces. Further evidence on this matter is provided by recent theoretical treatments by the Vinogradov group [Vinogradov, 1985a] and by Braud [Braud, 1992].

The most serious operational problem for whispering gallery mirrors is likely to be contamination, particularly by oxide layers. Since the wave only interacts with the top 30 Å or so of material, even a thin contamination layer would greatly alter the system properties. This suggests that either in-situ vacuum deposition would be indicated or the use of inert materials and special storage away from normal atmospheric pollutants. It would be interesting to consider one of the layered materials (transition metal dichalcogenides such as molybdenum disulfide, gallium selenide, etc.) in which all of the surface bonds are saturated making the surfaces inert with respect to most contamination processes.

The contamination question has recently been investigated experimentally by Scott et al. [1988]. These authors tested the reflection efficiency of aluminum surfaces in the spectral region 300–1000 Å. Their results are described more fully in a later section, but we point out here that, as expected from the above discussion, the high efficiencies predicted by the Fresnel equations near zero grazing-incidence angle are only achieved for material evaporated *in situ* at ultra-high vacuum and therefore free of oxide and other contaminant layers.

EXPERIMENTAL INVESTIGATIONS

The experimental investigation of acoustical whispering galleries was being reported even before the work of Rayleigh. At that time the prevailing view was that the effect was due to focusing, and this explanation is certainly true in some cases. However, the experimental work of Rayleigh [1904] in St. Pauls Cathedral in London using a bird call whistle as a source and a flame as a detector, established the extreme concentration of the wave in the neighborhood of the wall and led to Rayleigh's recognition of the role of wave guiding in this particular whispering gallery. Other

studies followed, notably those of Sabine [1922, 1964]. A particularly graphic one was conducted by Barton [1912], who photographed the patterns left in lycopodium powder by a high frequency sound wave guided by a circular wall in the laboratory. The imprint of a single-mode guided wave was clearly visible in the pictures.

It is difficult to trace the first studies using electromagnetic radiation, but presumably they used visible light. The guiding effect gained in importance with the advent of radio broadcasting and was found to be involved in the interaction of radio waves with the upper layers of the atmosphere [Das and Ayub, 1982]. In the infra-red spectral region, a spectacular demonstration of one-sided guiding was achieved by Krammer [1978] at a wavelength of 10.6 microns. This author used a reflector in the form of a helical strip to deliver a guided beam with 70% efficiency after three-and-one-half full rotations! The experimental studies using wavelengths shorter than those of visible light are our main interest, and they fall into three categories: first, "optical fiber" type experiments using hollow tubes as x-ray lightpipes; second, attempts to demonstrate a two-sided x-ray waveguide; and third, single-sided whispering-gallery experiments using sub-optical wavelengths.

The light pipe experiments have been carried out since at least as early as 1951 [Hirsch, 1951], and there are a considerable number [Marton, 1966; Mosher, 1976; Vetterling, 1976; Rinby, 1986] which essentially demonstrate by means of x-ray tube sources that various glass and metal tubes can be used as x-ray light pipes which gain intensity compared to an inverse-square fall-off. Watanabe et al. [1984] showed a similar result using synchrotron radiation. There has been at least one experiment (by Pantell et al. [1977, 1978]) giving more quantitative results and a meaningful theoretical interpretation as well. The main conclusion of these authors was that one can operate both straight and curved glass capillary tubes as light pipes at 1.54 \AA with $1/e$ attenuation lengths in the .5-1 m range. This performance was shown to be understandable in terms of ray propagation and Fresnel reflection efficiencies without allowance for roughness. Pantell and Chung [1979] subsequently provided a theoretical analysis explaining why roughness has relatively little effect on the transmission efficiency even up to a root-mean-square slope error of 100 seconds. A variant of the light-pipe experiments is to use a tapered guide so as to achieve condensation of the beam in either one [Livins et al., 1989] or both [Stern et al., 1988] directions.

Attempts to make x-ray waveguides have been less popular, presumably because of the apparent difficulties. Spiller and Segmuller [1974] showed a proof-of-principle that 1.54 \AA x-rays could propagate in a wave-guide consisting of a sandwich of a light, transparent material between two reflective metal layers. The size of the waveguide was not sufficient for beam transport applications, and there was no apparent potential for upscaling. Fischer and Ulrich [1980] reported observation of what are called "self-images" formed by interference between modes of low but different order in a two-sided planar waveguide. Images of a 2-micron-wide, slit object were produced in a 10-micron-wide, 90-mm-long guide using 44.8 \AA x-rays. The authors point out that the formation of images indicates that the phase relations between the waveguide modes are preserved over the entire length of the guide: a distance of 2×10^7 waves, and this in spite of a quoted surface roughness of 20 \AA rms. It appears that there is qualitative experimental evidence to match the also-qualitative theoretical arguments that roughness is less important than one might imagine for low-grazing-angle reflection.

The most successful x-ray-waveguide experiments have been those of Ceglio et al. [Ceglio, 1988]. These workers used microfabrication techniques to fabricate thick, transmission-diffraction gratings with open grooves consisting of slots, each of which could be considered as a waveguide. The structures are conceptually similar to amplitude, volume holograms, and waves propagate

through them by guiding even when there is no geometrical line of sight. The deepest slots reported were 0.1 micron wide by 0.7 micron deep and experiments were carried out at photon wavelengths of 50-300 Å. The components performed properly as gratings even when there was no line of sight through them (indicating guiding) and efficiencies up to 2% or so were obtained. However, the results depend on the goodness of the guiding process in a complicated way and the quality of the guiding surfaces was not known. Thus although the success of this type of experiment is broadly encouraging for future attempts at x-ray guiding, it does not provide quantitative evidence germane to the operation of x-ray whispering galleries.

Experiments in the VUV-soft-x-ray range involving multiple reflection are, of course, very numerous. However, there are only a few that have been directed toward exploring the optical properties of multiple reflection systems. An important series of experiments, for our purpose, was that of Scott et al. [1988] at Los Alamos National Laboratory. This group investigated the possibilities of multiple reflection for constructing a retroreflector for a VUV, free-electron laser. They first measured the reflection efficiency of various materials including aluminum both with and without contaminating layers. They found that the high values of efficiency yielded at small grazing angles were severely compromised by contamination. The efficiency of multiple reflection is determined by the *slope* of the reflectance against angle curve [see Eqs. (7-9)], and the data of Scott et al., reproduced in Fig. 10, show clearly that it is just this quantity that is sensitive to contamination. It is also significant that the clean-surface reflectance shown in Fig. 10 requires a value of the imaginary part of the refractive index of $.010 \pm .002$, less than half the published one to achieve the fit shown. In a later paper, the same group reports operation of a nine-bounce retroreflector with an efficiency of $89\% \pm 3\%$ at 584 Å. This is even higher than the value of 73% obtained (Fig. 3) using published optical-constants data. The number of reports of true (circular) whispering galleries in the VUV-soft-x-ray region seems to be only two. The first was that of Kaihola in 1981 [Kaihola, 1981] who, based on earlier calculations [Bremer and Kaihola, 1980], used a 14 mm-radius, cylindrical, polystyrene mirror to make an 80° deflection of radiation with an average of 15 bounces over the spectral range 40-180 Å. The achieved efficiency was over 10% from 80-180 Å with a peak of about 20% for 120-140 Å. The expected value at the peak based on the optical constants obtained from reflectance data, was 33%. This is fairly impressive agreement when the 10-15% uncertainty in the optical constants is considered. The experiment must have been a challenging one considering the tungsten bremsstrahlung source that was used. It represented the first indication that anything close to the performance predicted by simple theory (Eqs. (9) and (11)) could be achieved with a real mirror.

The most important piece of evidence that we have regarding the feasibility of x-ray whispering galleries, is the experiment of Vinogradov et al. [1987a] first reported in 1986. In this experiment a spherical mirror coated with carbon was used to deflect a 67 Å x-ray beam by 29°. The ingoing beam was collimated within 2 mr and of width 90 micron in the deflection plane, and the reflection efficiency was measured as a function of incidence angle. The results are shown in Fig. 11, from which the value of $\theta_{1/2}$ is seen to be $0.8 \theta_c$ while the limiting efficiency at zero grazing angle was 65%. The first important feature of the data is that they are in good agreement with the efficiency calculated from the optical constants of Henke et al. and the measured value (2.0 gm/cc) of the density of the carbon coating. The second point of significance is that the data agree better with the simple uncorrected calculated efficiency than with the scattering-corrected curve shown in Fig. 11. The latter is based on the measured value of 30 Å rms for the roughness of nominally identical but flat surfaces evaluated by integrated scatter of 67 Å radiation. This behaviour is attributed mainly to the fact that radiation scattered by small angles may still be guided by the

mirror and arrive at the detector, a fact not included in the calculation of the scattering-corrected curve. These results are strong evidence that soft x-ray whispering galleries can indeed deliver theoretical performance and add further weight to the view that the practical value of the whispering gallery effect may not be much compromised by the imperfectly smooth surfaces of real mirrors.

POLARIZATION EFFECTS

There are no difficulties in the soft x-ray region in producing plane-polarized radiation. Synchrotron radiation sources can deliver essentially 100%-polarized radiation under appropriate conditions and Bragg reflectors; both crystalline and artificial (multilayer reflective coatings) are efficient polarizers and analyzers. There are, however, serious limitations on present abilities to produce circularly- or elliptically-polarized soft x-rays. There are special magnet structures that, in combination with a suitable electron storage ring, can produce this type of radiation. However, these devices are major engineering projects involving a large commitment of resources. An alternative, although less efficient approach, is to use the off-plane radiation from a synchrotron radiation bending magnet. In light of the scarcity and high cost of all these types of sources, even a limited capability to produce circular or elliptically-polarized VUV or soft x-ray radiation by means of a simple device like the whispering gallery would be highly valuable. It would find application in the radiation scattering and absorption experiments (e.g. circular dichroism and magnetic circular dichroism) on chiral structures such as those found in biopolymers, in the study of magnetic effects in solids and other spin-dependant processes.

At the long wavelength end of the VUV region, circular- and elliptically-polarized radiation can be produced by a single reflection at a metal surface. The process begins with the use of a plane-polarized beam (making the s and p components initially in-phase) with its electric vector somewhere between the s and p position. The two components then suffer different phase changes on reflection and emerge with a phase difference $\delta\phi_{s-p}$ given by Eq. (12). If this is exactly $\pi/2$ and the emerging amplitudes are equal, then the light is circularly polarized, otherwise it is elliptically polarized. This procedure is useful as long as $\delta\phi_{s-p}$ is an appreciable fraction of $\pi/2$, which is true in the near UV and vacuum UV down to wavelengths of a few hundred Å. At shorter wavelengths it becomes small and the fraction of elliptically-polarized light resulting from a single reflection becomes too small to be useful. In this case the next thing to try is multiple reflections, the idea being that the s -to- p phase difference will accumulate with each reflection. The properties of the two and three reflection polarizers often used for producing plane from unpolarized radiation are of interest here and have been analysed by Schledermann and Skibowski [Schledermann, 1971] and Johnson and Smith [Johnson, 1983], respectively. The conclusion of the latter authors is that one can get circularly-polarized radiation with a zero-net-deviation device with throughput values in the range 4-19% for the spectral region 400-2500 Å using gold or platinum reflecting surfaces. Since the whispering gallery can be regarded as a limiting form of multiple reflection it should provide useful amounts of relative phase shift and good throughput for the cases identified earlier. The amount of phase shift is given by Eq. (14). We consider the example of silver at 95 Å, which gives the s -to- p phase difference and throughput values shown in Fig. 9 for a 180° whispering gallery. It will always be complicated to get exactly circularly polarized light by this method, but so long as the elliptically-polarized fraction is substantial, as it is in the example in Fig. 9, and the handedness can be chopped, one can usually still extract the required information.

It seems clear that if reflecting surfaces of sufficient quality are available, then whispering galleries have the potential to provide useful amounts of elliptically-polarized radiation down to at

least the carbon K edge in wavelength. The question of how to reverse the handedness arises and one can imagine various approaches. Reversal requires changing the sign of the s to p phase difference. In a reflecting system this can only be achieved by interchanging the s and p beams. One way to do this would be to rotate the entire reflector about the incoming beam: not a convenient thing to do in some experiments. Another interesting way to try would be to make the reflector from a flexible metal strip and twist the input end back and forth through 90° while keeping the output end (and thus the sample) stationary. Such methods would probably limit the chopping frequency to a few Hertz.

Table 1. Some useful values of the Airy function $Ai(\rho)$.

ρ	$Ai(\rho)$	Remarks
0	0.3550	$r = r_n, \lambda = \lambda_0$
-2.338	0	1st zero
-1.019	0.5356	1st maximum
0.345	0.2678	half maximum
1.840	0.05356	0.1 maximum
2.080	0.03100	
2.920	0.00756	

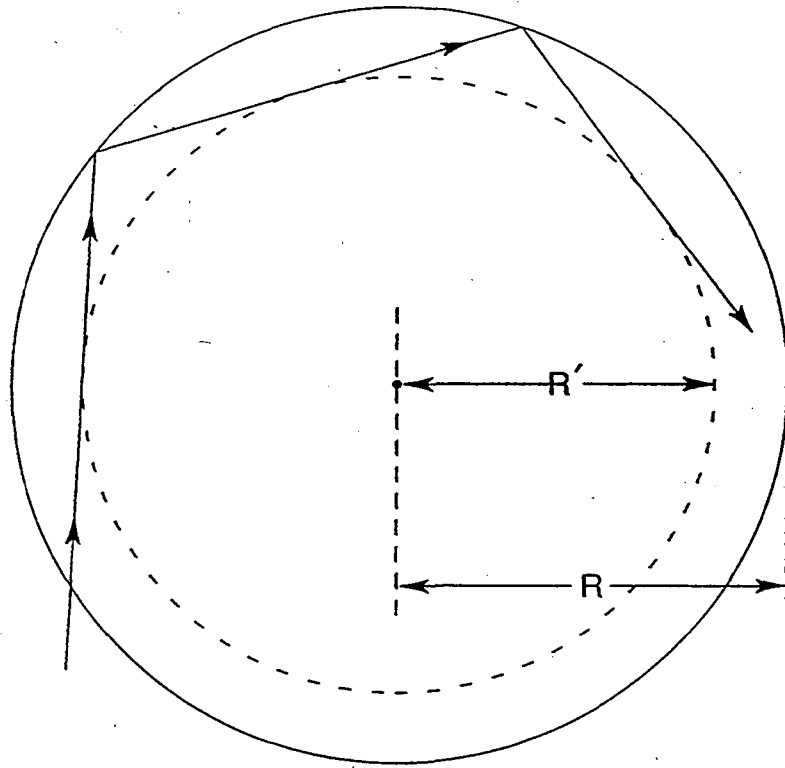
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- Footnote 1. e^x can be defined as $\lim_{m \rightarrow \infty} [1 + x/m]^m$.
- Footnote 2. Equation (17) is generally true only for integral n . However, for large n , this requirement can be relaxed. See, for example, Abramowitz and Stegen, 1965, Eq. 9.1.22.
- Footnote 3. To see that these are indeed the boundary conditions, consider a TE wave incident at a small but finite grazing angle against an interface represented by the x,y plane, with the E vector in the y direction. We thus have $E_x = E_z = 0$, $E_y \neq 0$ and $H_x =$ small, but finite, $H_y = 0$, $H_z \neq 0$. From the i component of the equation $\text{curl } \mathbf{E} = -\partial \mathbf{B} / \partial t$ we have, $-\partial E_y / \partial z = -\mu \mu_0 i \omega H_x$ and we know that one of the boundary conditions is H_x continuous across the boundary. Therefore, our boundary conditions are $\partial E_y / \partial z$ and E_y continuous.
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Fig 1

Fig 2

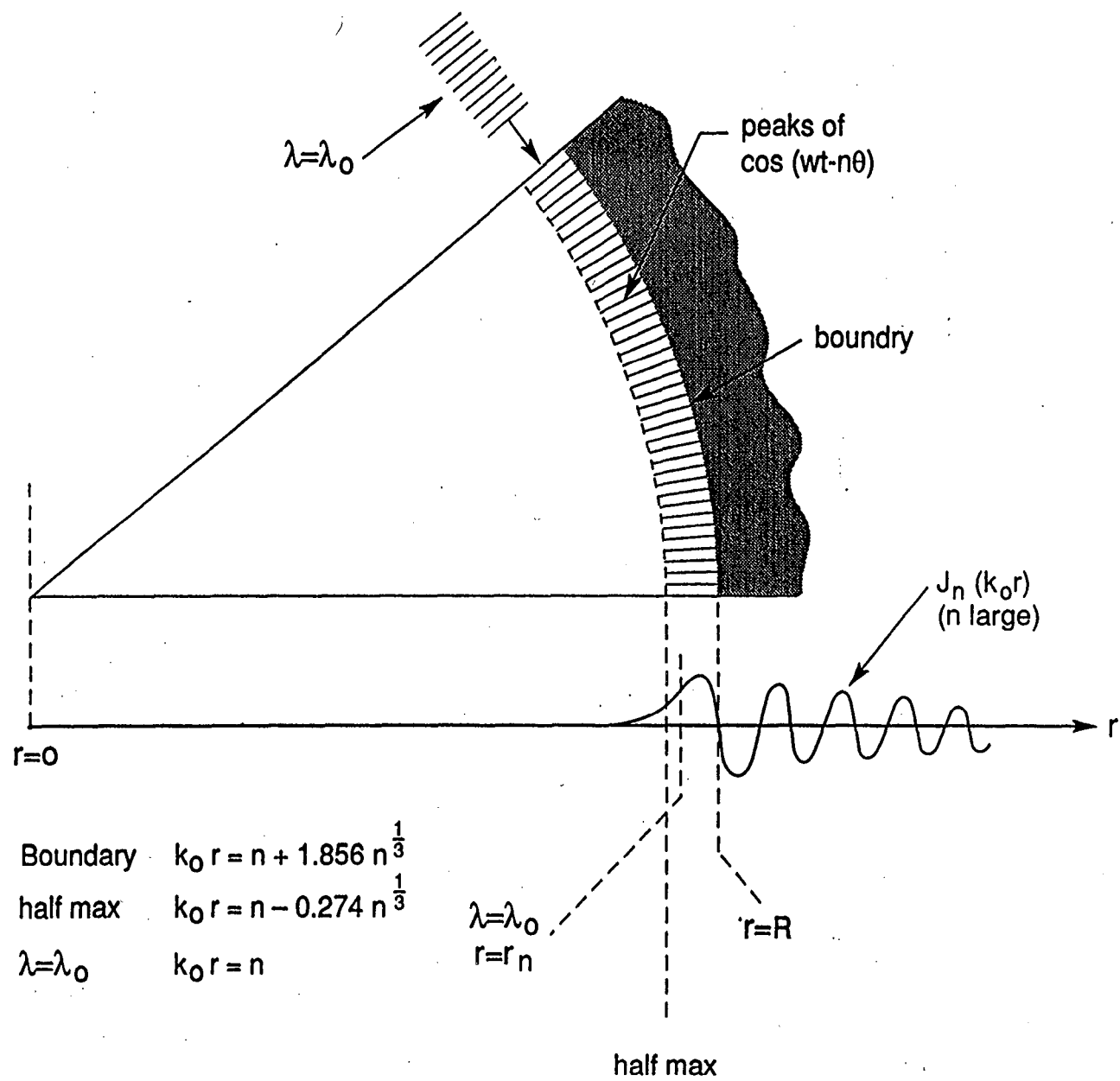
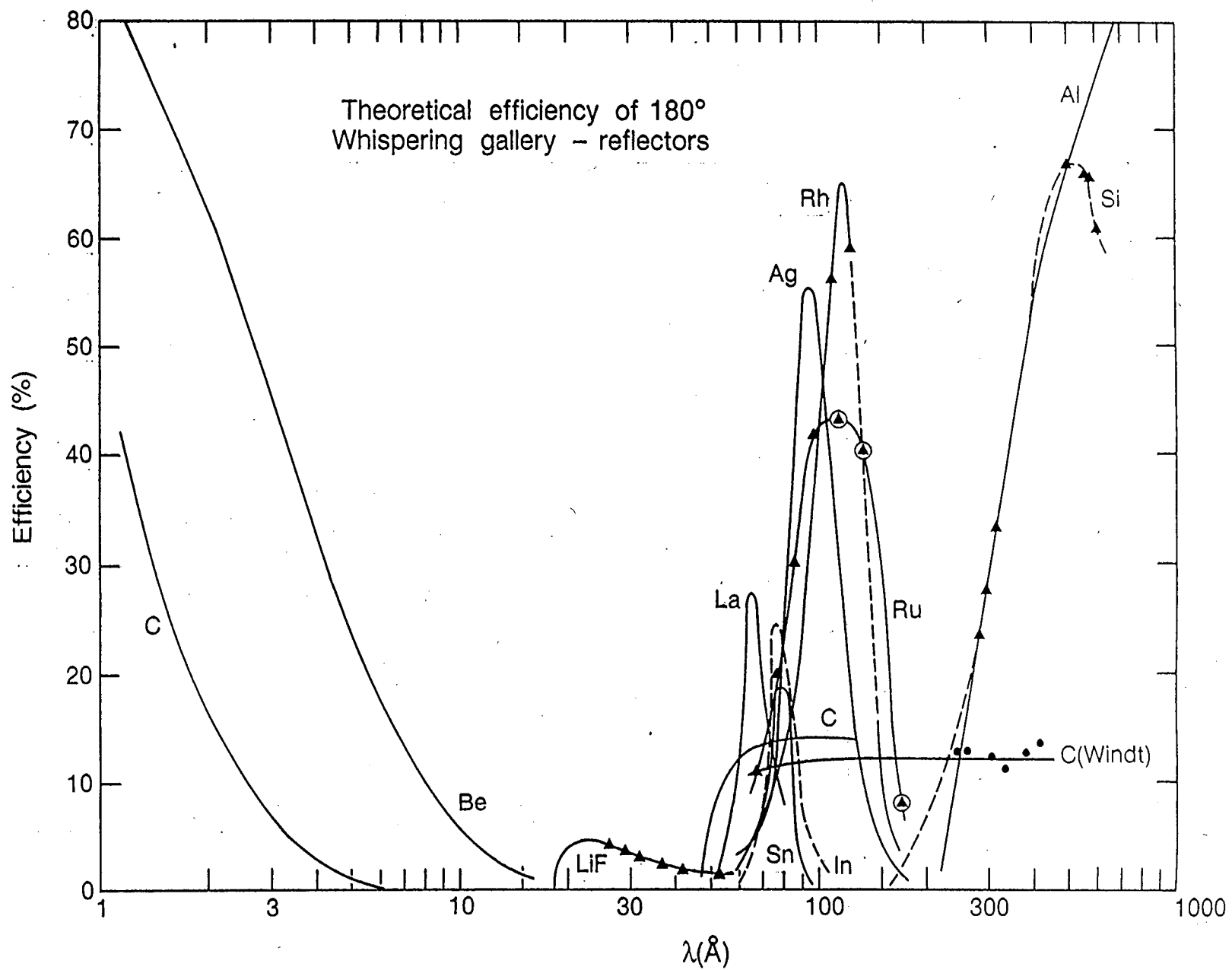
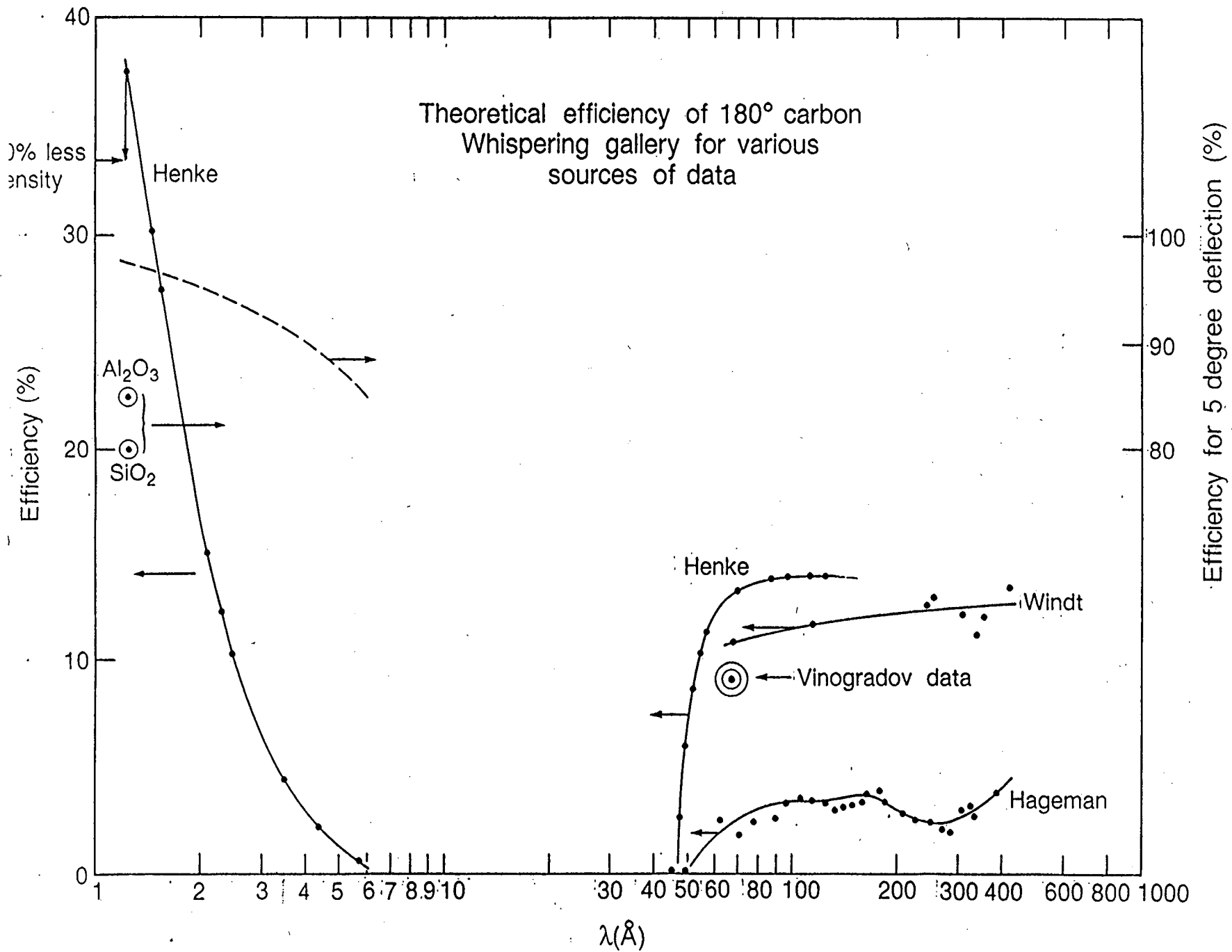
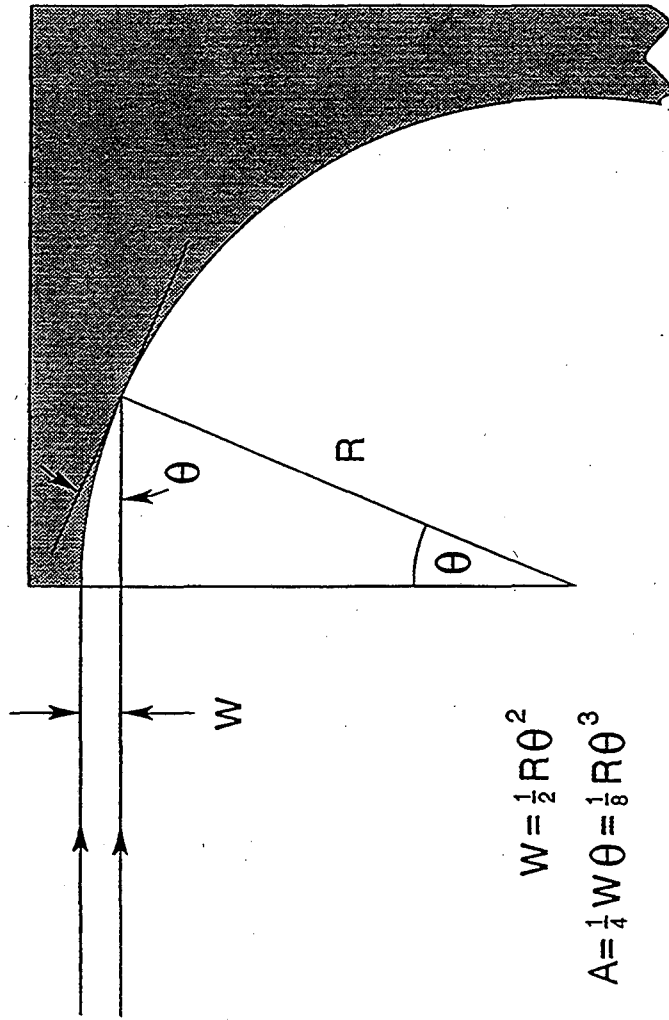


Fig 3



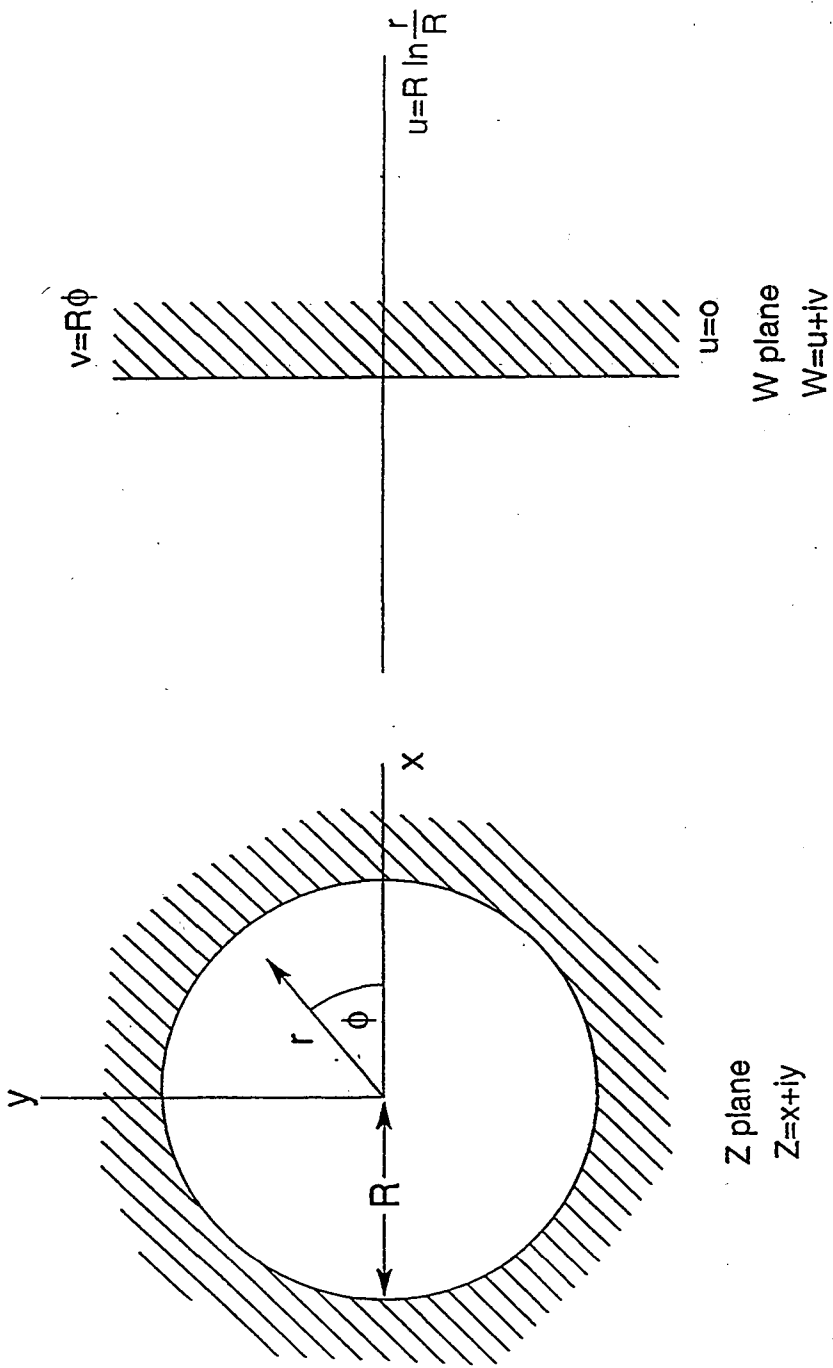




$$W = \frac{1}{2} R \theta^2$$

$$A = \frac{1}{4} W \theta = \frac{1}{8} R \theta^3$$

Fig 5



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Fig 6

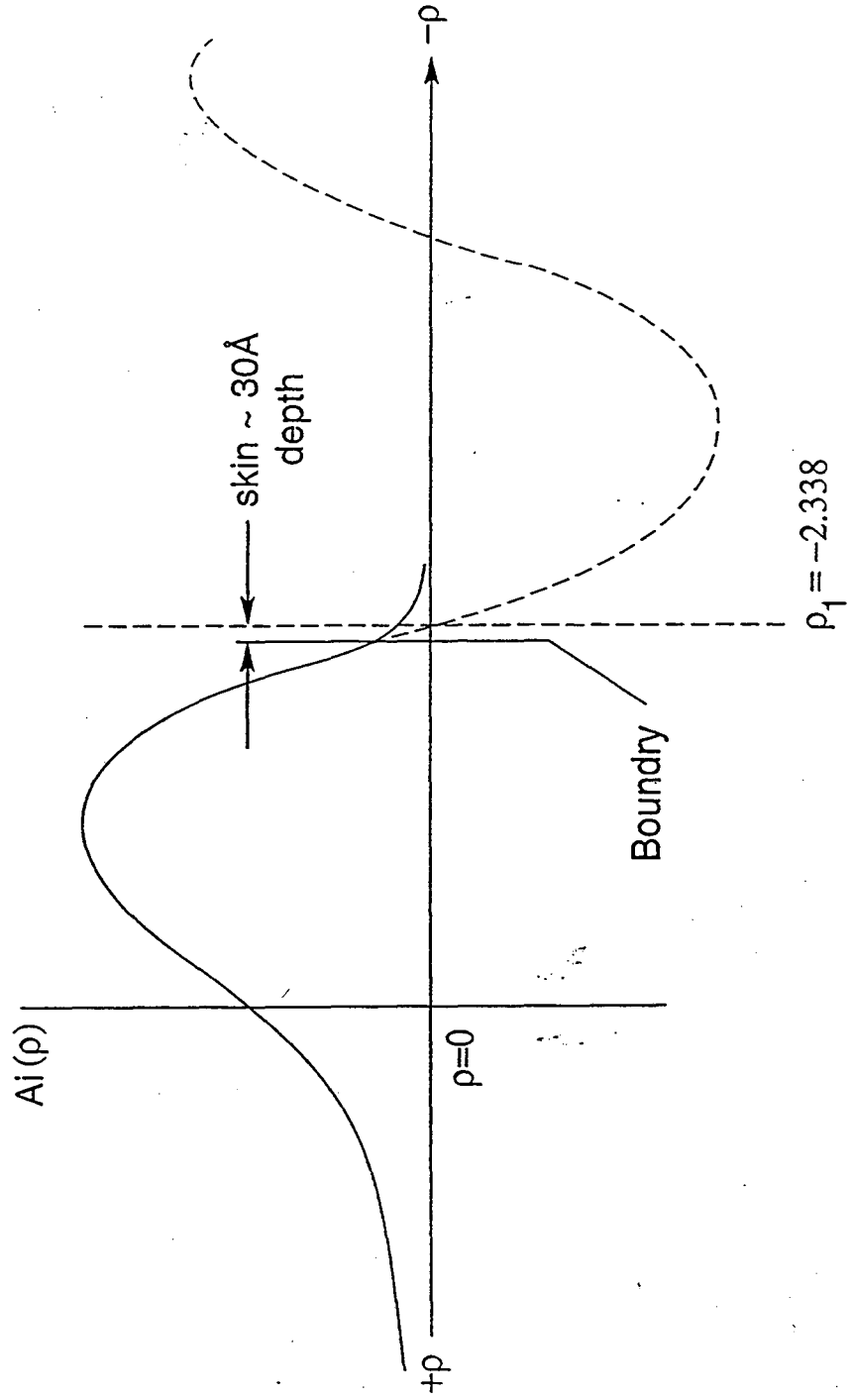


Fig 7

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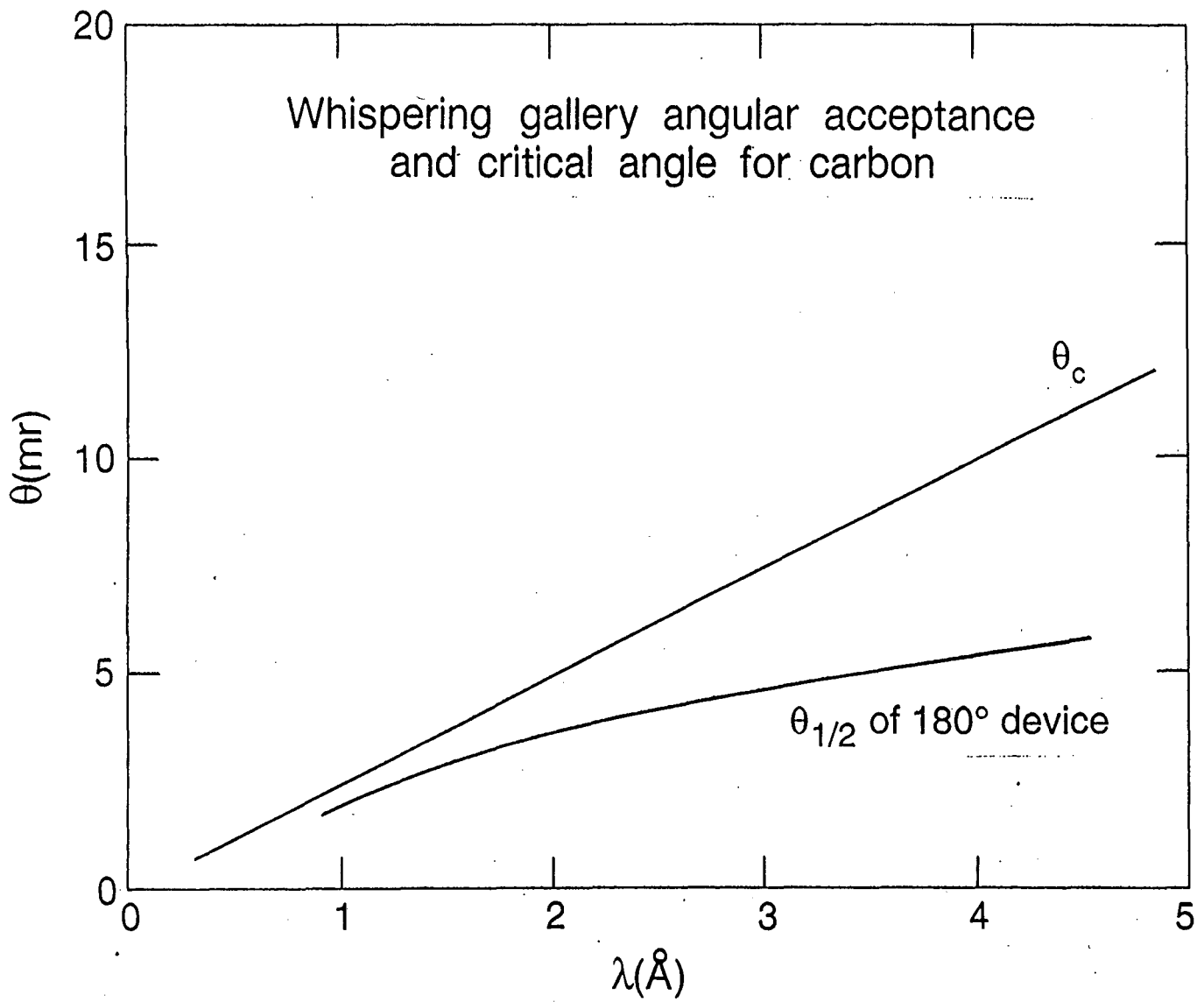


Fig 8

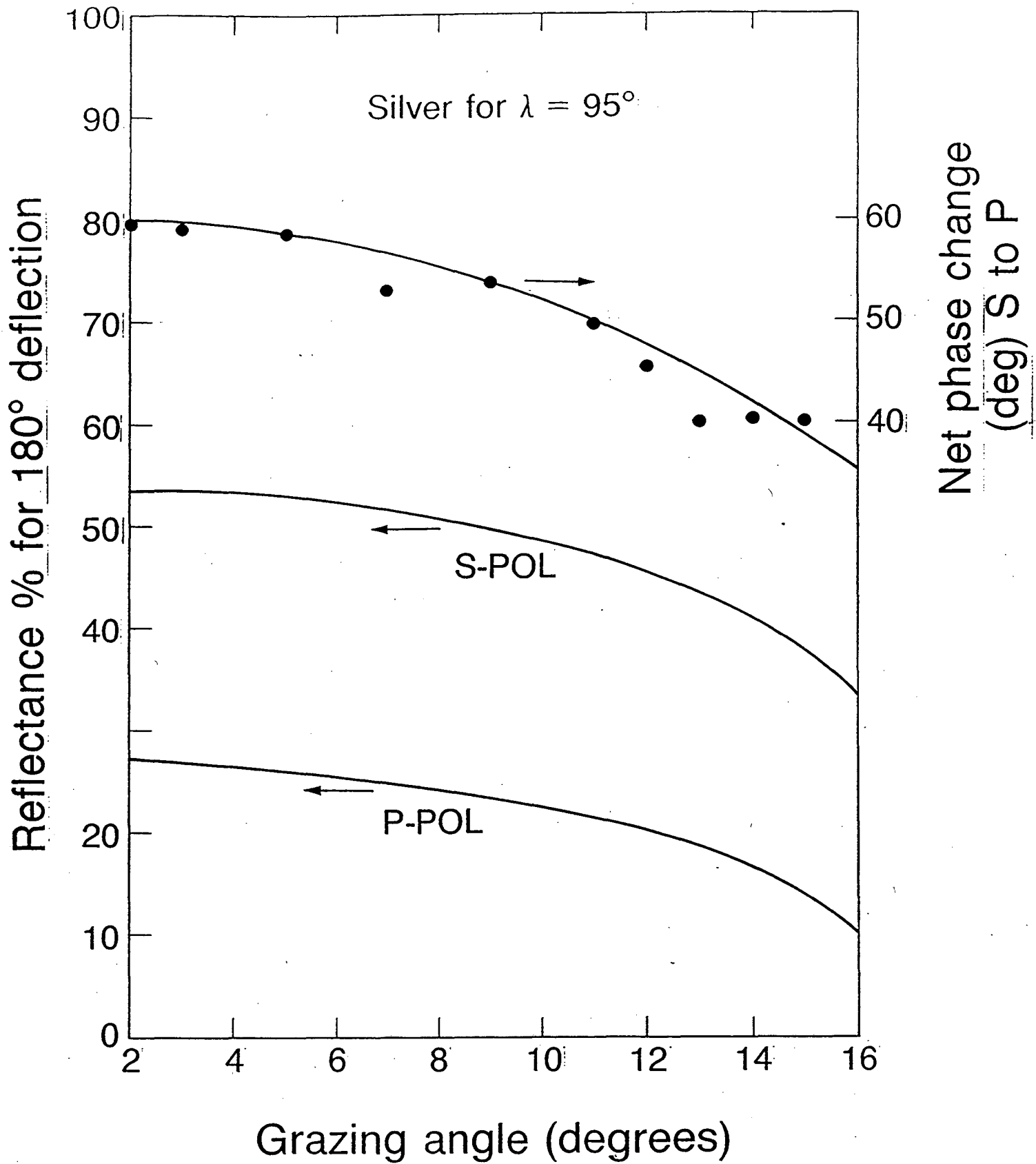


Fig 9

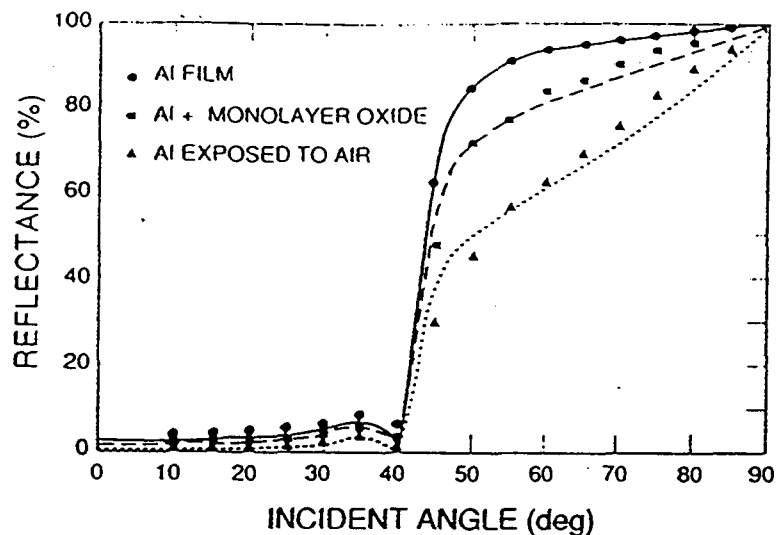
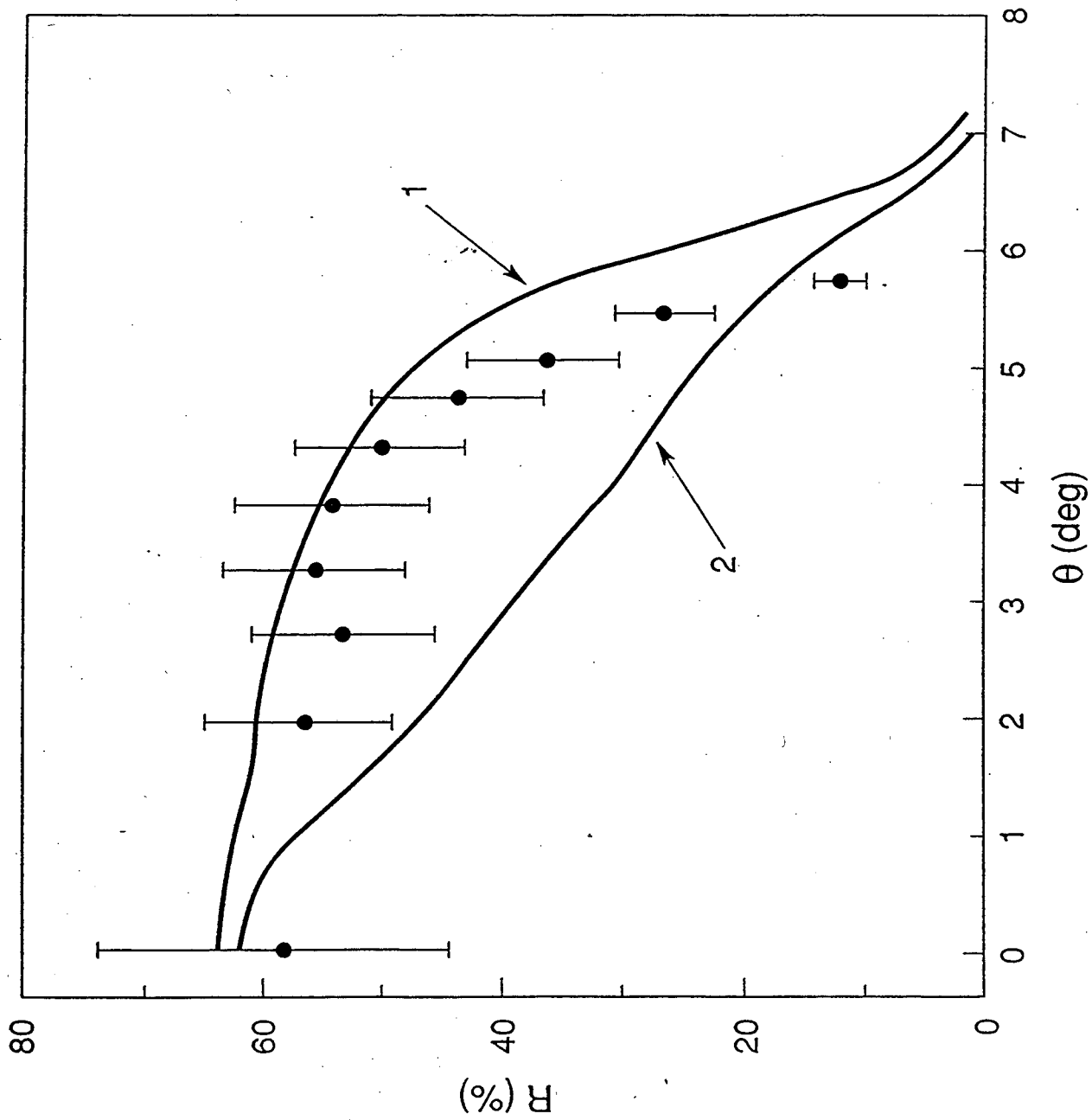


Fig. 4. Reflectance vs angle of incidence data for an aluminum film at 58.4-nm wavelength. The solid line is calculated for aluminum that is contamination free (corresponding to the ● data). The dashed line is calculated for aluminum with one surface monolayer of oxide (corresponding to the ■ data). The dotted line is calculated for aluminum with three surface monolayers of oxide (corresponding to the ▲ data).

Fig 10



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Fig 11

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