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CALCULATIONS OF NEUTRON EVAPORATION FROM ^{252}Cf FISSION FRAGMENTS BASED ON THE SHELL MODEL*

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The average neutron kinetic energies and the residual total gamma ray energies as a function of fragment mass are calculated for the spontaneous fission of ^{252}Cf on the basis of the Nilsson model and of the B.C.S. Hamiltonian.

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The emission of neutrons in the spontaneous fission of ^{252}Cf has been the subject of a detailed experimental investigation by Bowman et al. [1]. Among the main experimental features reported by these authors were the average number of neutrons $\bar{\nu}$, and the average center of mass kinetic energy of the neutrons $\bar{\eta}$, both as a function of the fission fragment mass A. The authors pointed out the surprising fact that, whereas the saw-tooth function $\bar{\nu}(A)$ is very asymmetric with respect to mass 126, the kinetic energy $\bar{\eta}(A)$ is nearly symmetric with respect to this mass.

A very detailed analysis performed by Lang [2] shows that, while the neutrons are indeed statistically emitted by the excited fragments, it is necessary to assume the presence of strong shell effects in order to account for the quantitative features of the experimental data. In fact, the level density parameters extracted by Lang in his analysis show a strong dip in the fragment mass region about $A = 132$ ($N \simeq 82$, $Z \simeq 50$).

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At present it is possible to calculate realistic level densities on the basis of the shell model and the pairing Hamiltonian. Therefore, it is interesting to explore whether the features in the experimental data associated with the statistical emission of neutrons can be quantitatively accounted for on the basis of the present day theoretical knowledge.

This note presents the results of neutron evaporation calculations which were carried out for various fission fragment masses on the basis of realistic level densities [3,4]. In these calculations the values of $\bar{\nu}(A)$ were used as input data in order to obtain the initial excitation energies of the fragments, while the values of $\bar{\eta}$ were calculated and compared to experimental data. In addition, the residual excitation energy emitted in the form of gamma rays was obtained from the calculations and also compared to experimental data.

The neutron evaporation from fission fragments was carried out using the Monte-Carlo method. The probability P, of emitting a neutron of kinetic energy ε from a nucleus of excitation energy E is given by [5]

$$P(\varepsilon)d\varepsilon \propto \sigma_{inv}(\varepsilon)\varepsilon\rho(x,0)d\varepsilon \quad . \tag{1}$$

In this expression $x = E - B_N - \varepsilon$, B_N is the neutron binding energy, $\rho(x,0)$ is the density of the zero angular momentum states of the residual nucleus and $\sigma_{inv}(\varepsilon)$ is the inverse cross section for a neutron of kinetic energy ε. The last quantity was obtained using a square well optical potential. Angular momentum effects associated with the neutron evaporation are approximately accounted for by such a formula. This approximation is justified if the level density is assumed to be proportional to 2I + 1 where I is the angular momentum of the fission fragments [6]. Such a proportionality occurs if, as in this case, the average square angular momentum of the fission fragments is somewhat smaller than the spin cutoff parameter σ^2.

The crucial quantity in the calculation is the level density ρ. In the present calculation the level densities were obtained from the Nilsson model by generating the grand partition function and using the saddle point approximation method [3,4]. The appropriate deformation parameters ε_2 and ε_4 and the parameters κ and μ for each nucleus under investigation were obtained from the work of Ragnarsson [7]. The level densities were calculated for a paired system of nucleons. The residual interaction was introduced in the grand partition function by means of the B.C.S. Hamiltonian. A more traditional calculation was also performed on the basis of the Nilsson model but without inclusion of the pairing Hamiltonian. The necessary even-odd correction was performed by defining an effective excitation energy E^* given by the relation

$$E^* = E + n\Delta$$

where E is the excitation energy, Δ is the even-odd mass difference given by $\Delta = 11/(A)^{1/2}$ and n = 0 for even-even nuclei, n = 1 for odd nuclei and n = 2 for odd-odd nuclei.

The calculations of $\bar{\eta}$ were performed for selected masses A, of the fission fragments. The charge of each fragment was assumed equal to the most probable charge for that fragment in the fission of ^{252}Cf. The experimentally determined values of $\bar{\eta}$ for a given A are in fact averages over a few mass units due to the finite mass resolution in the experiment. On the other hand, it was found that the calculated values of $\bar{\eta}$ vary very slightly within the experimental mass resolution. Therefore no average over mass was performed in the calculation.

The average excitation energy of each fission fragment was adjusted so that the calculated average number of evaporated neutrons agreed with the experimental value of $\overline{\nu}$ to within less than 5%. The initial distributions of excitation energies were assumed to be gaussian. Their widths were taken from the experimental second moments of the neutron distributions as a function of fragment mass measured by Gavron and Fraenkel [8]. The neutron binding energies were taken from the Myers and Swiatecki mass formula [9].

The experimental and calculated values of the average kinetic energy of the neutrons in the center of mass system $\overline{\eta}$, are plotted in fig. 1. In this calculation the level densities were derived without including pairing. In the heavy fragment region, the calculated results are seen to be in good agreement with experimental data. In particular the minimum in the value of $\overline{\eta}$ in the vicinity of mass 145 is reproduced. It should be mentioned that the finite resolution in the measurement of the fission fragment masses introduces a dispersion in the measured $\overline{\eta}$ curve around mass 144. Hence, the true experimental $\overline{\eta}$ distribution should present a sharper minimum, thus bringing the calculated data in closer agreement with experiment. On the other hand, the calculated values of $\overline{\eta}$ for light fission fragments are somewhat larger than the experimental values.

The values of $\overline{\eta}$ obtained by using level densities with the inclusion of pairing are presented in fig. 2. The comparison with the experimental data shows that good agreement in the light fragment region is obtained while in the heavy fragment region the agreement is slightly worse.

The residual energies of the fragments emitted in the form of gamma rays is given by the initial excitation energy of the fragment minus the energy carried away by the evaporated neutrons. The residual gamma energy is strongly dependent upon the neutron binding energy of the last nucleus in the evaporation cascade. In particular, the binding energy is strongly dependent upon the odd-even nature of the nucleus. The measurements of the total gamma energy as a function of fragment mass [10] include a large number of different initial nuclei in the evaporation cascades because of the finite mass resolution and the charge dispersion. In order to account approximately for this effect in calculating the residual gamma energy, the residual gamma energy of a nucleus consisting of N neutrons and Z protons (N, Z), was averaged over four different initial cascade nuclei. These nuclei were (N, Z), (N + 1, Z), (N, Z + 1), and (N + 1, Z + 1). Such a procedure does average out the odd-even effects in the binding energies of the residual nuclei of the evaporation cascade.

The experimental gamma energies vs. fragment mass, shown both in fig. 3 and fig. 4 indicate a slight saw-tooth behavior, following the trend of the average number of neutrons $\overline{\nu}$ as a function of mass A. One reason for the increase in the residual gamma energy as a function of $\overline{\nu}$ is the following. The higher the value of $\overline{\nu}$, the closer is the residual nucleus to the line of β-stability. Hence, the neutron binding energy becomes larger and so does the residual gamma energy. The calculated residual gamma ray energies for ^{252}Cf, as a function of fragment mass, can be compared to experimental data [10]. Again the calculations were performed using level densities derived with and without pairing. The calculations performed without pairing are observed to be in good agreement with experiment (see fig. 3).

When pairing is included in the calculation (see fig. 4) the experimental saw-tooth behavior is not reproduced. In the light fragment region the agreement between calculations and experimental results is not as good as when pairing is left out.

The overall agreement between theoretical calculations and experimental data is gratifying. However, it may be somewhat puzzling to observe that no clear cut decision can be made in favor of either form of calculation. The calculation on the basis of the pairing Hamiltonian has a much better theoretical justification. Furthermore, the difference between the level densities calculated in the two ways is substantial. The reason for such a lack of sensitivity lies in the fact that the calculated quantities reflect more the energy dependence of the level density than the absolute magnitude of the level density itself. This behavior depends mainly upon the magnitude of the ground state shell correction and to some extent upon pairing. The somewhat better agreement obtained by leaving pairing out may very well be due to some slight systematic inadequacy of the shell model employed in the calculation.

As a conclusion, the present calculations show that it is feasible to describe quantitatively the statistical decay of fission fragments on the basis of our knowledge about nuclear structure without the introduction of empirical parameters and without any attempt to fit the data.

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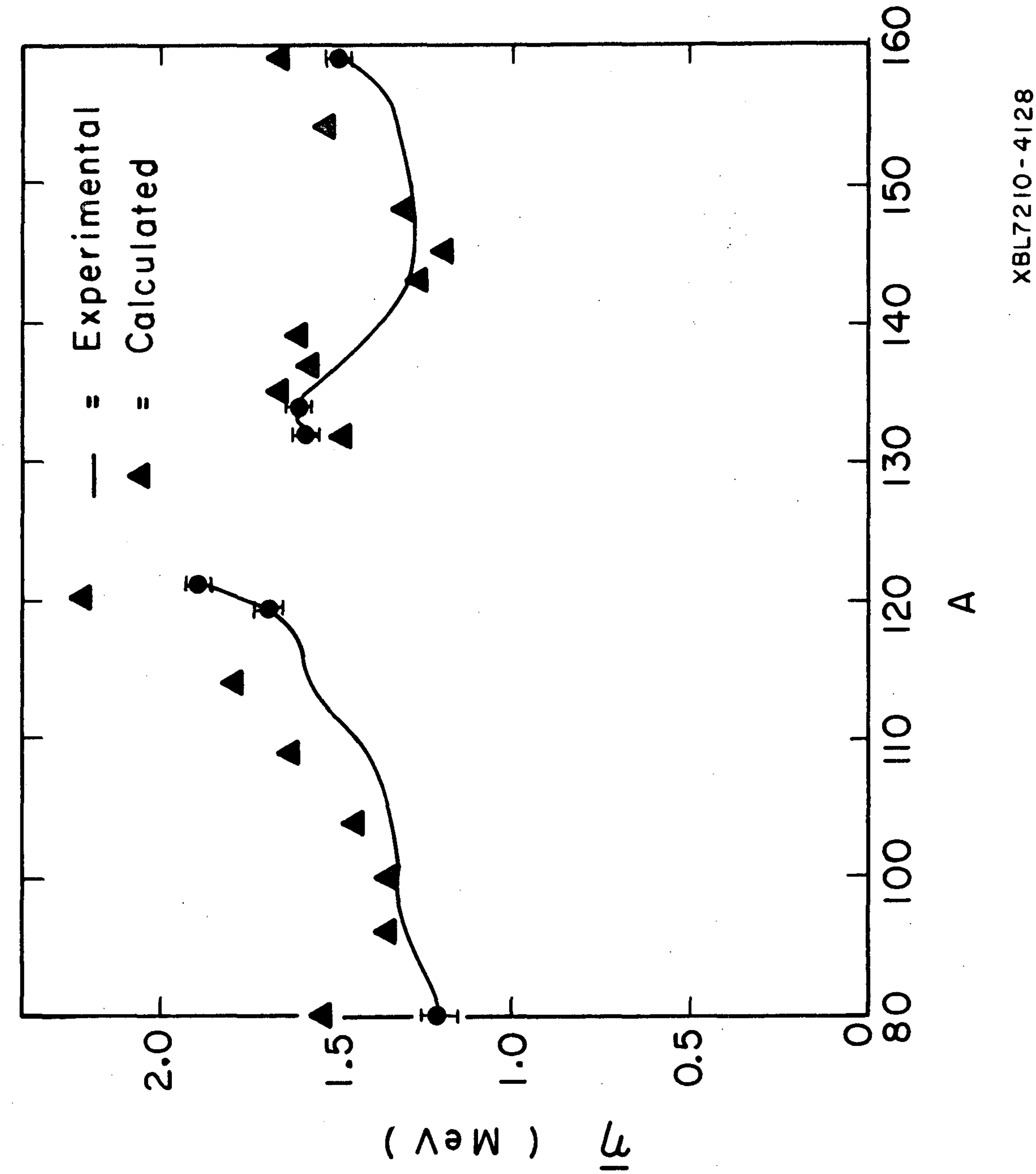
Figure Captions

Fig. 1. Experimental and calculated values of the average center of mass kinetic energy of the neutrons $\overline{\eta}$. The experimental results are represented by a solid line. Typical experimental errors are shown by full dots with error bars. The theoretical values are presented by means of triangles. The calculated values are obtained by using level densities without pairing.

Fig. 2. Same as in fig. 1. The theoretical values are obtained by including pairing in the level density calculations.

Fig. 3. Experimental and calculated values of the average energy emitted in the form of gamma rays $\overline{E_\gamma}(A)$ as a function of A. The results are obtained by using level densities without pairing.

Fig. 4. Same as in fig. 3. The results are obtained by including pairing in the level density calculations.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▲ = Calculated
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1.5
2.0
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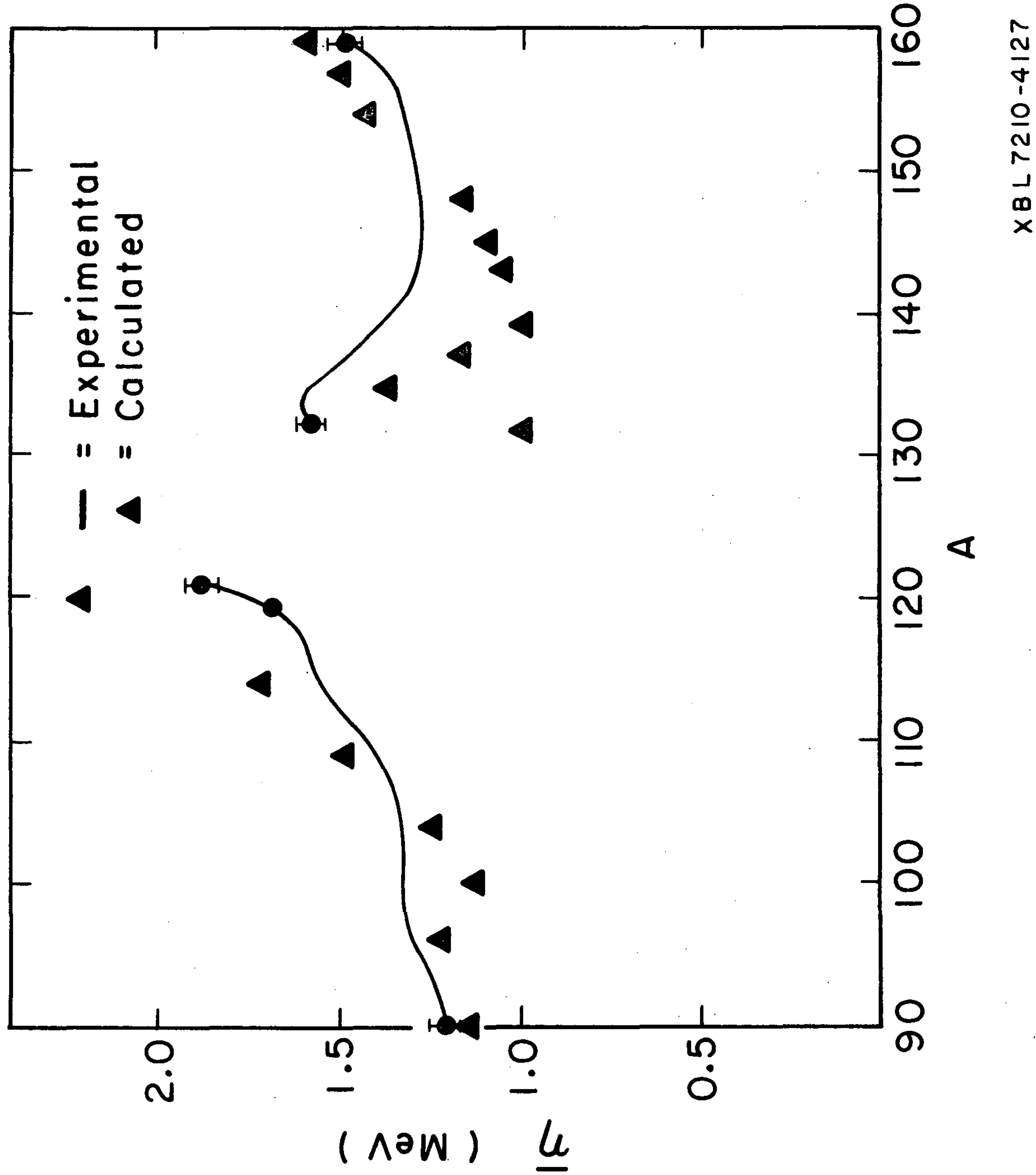
100
110
120
130
140
150
160
XBL7210-4128

Fig. 1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▲ = Calculated
A
η (MeV)
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110
120
130
140
150
160
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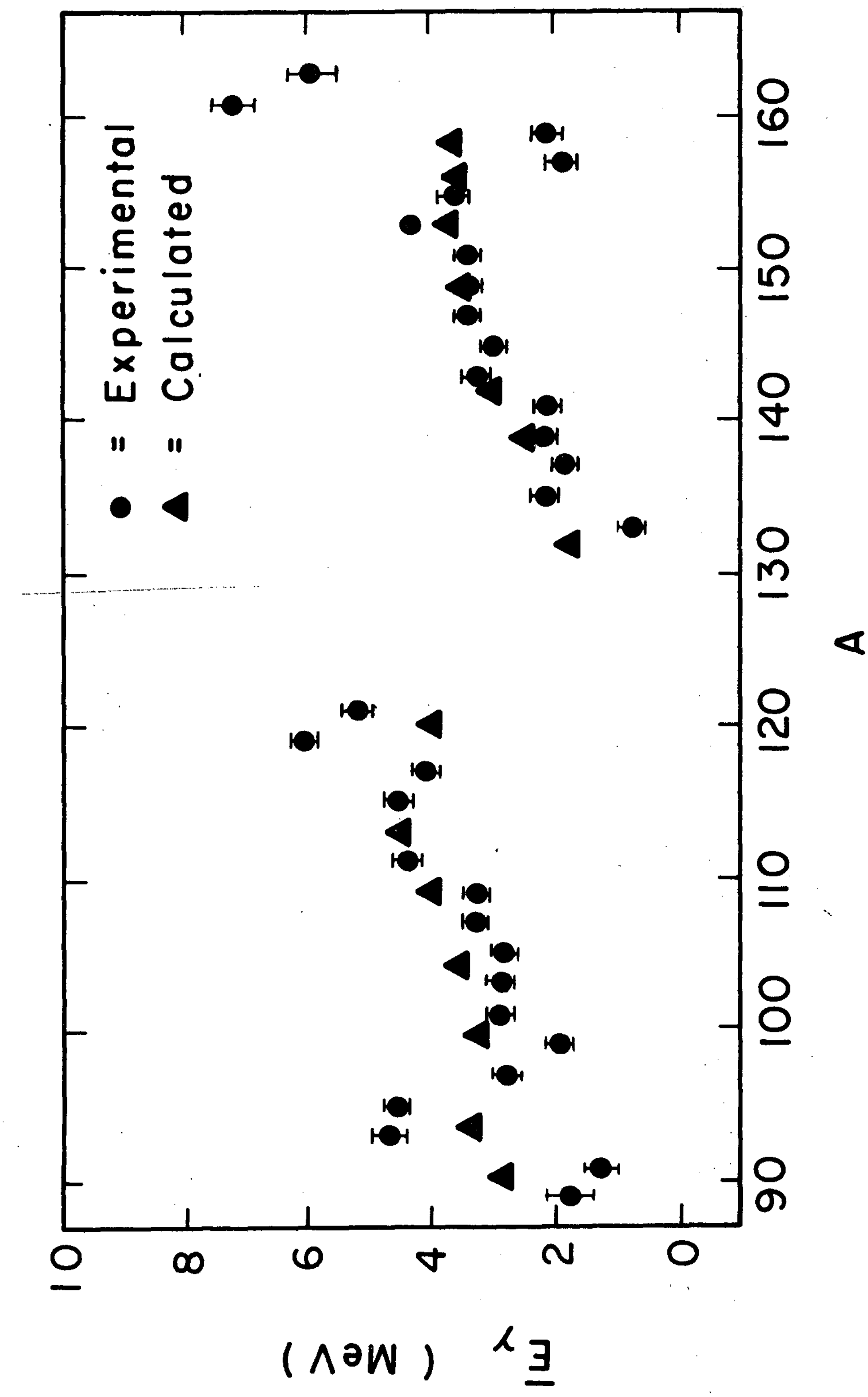
1.0
0.5
XBL 7210-4127

Fig. 2



XBL 7210-4129

Fig. 3

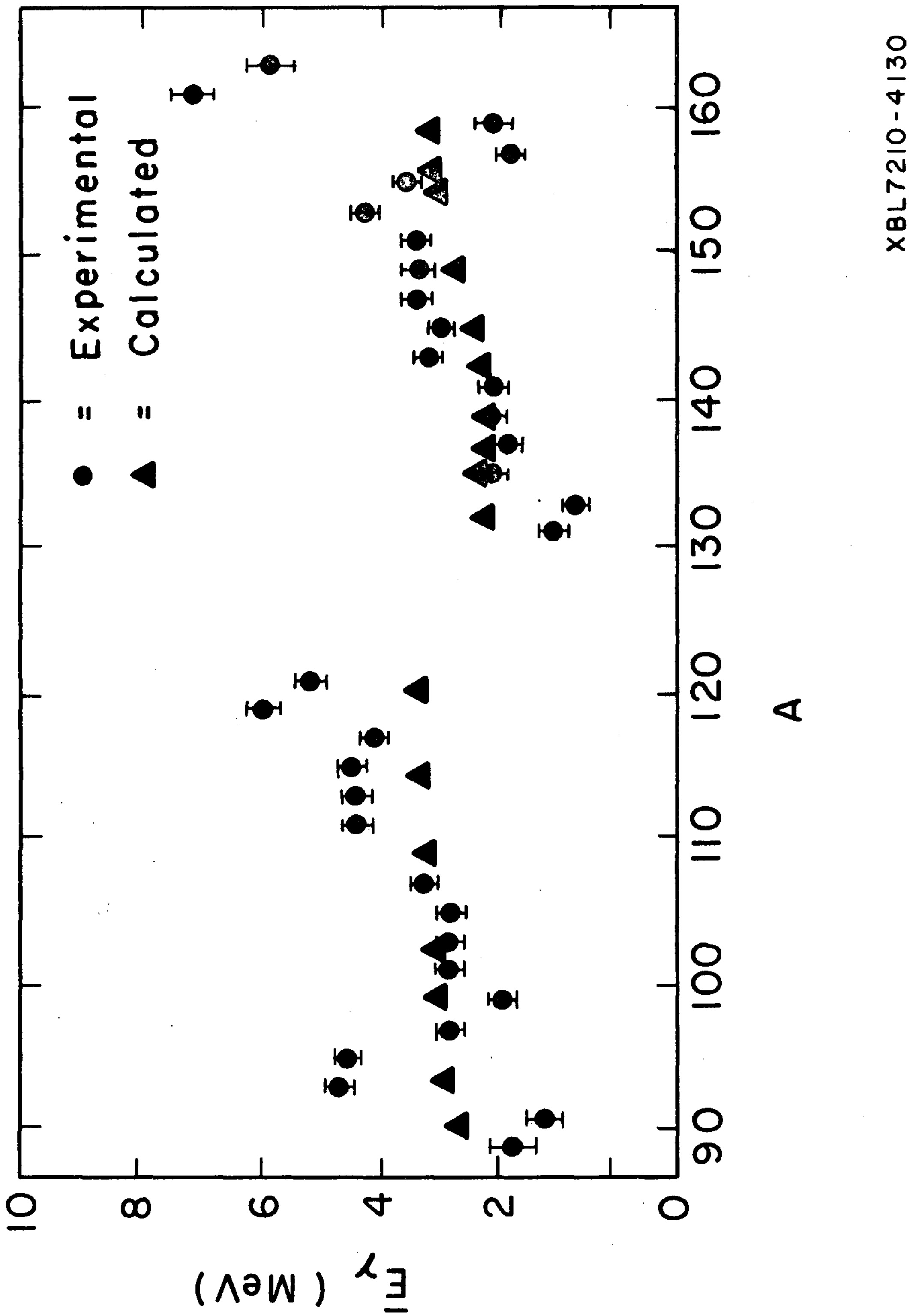

● = Experimental
▲ = Calculated
E_γ (MeV)
0
2
4
6
8
10
A
90
100
110
120
130
140
150
160
XBL7210-4130

Fig. 4

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