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Contribution of heavy bosons and fermions to the action for a Robertson-Walker metric

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The contributions of heavy, spatially homogeneous, boson and fermion fields to the effective action for a Robertson-Walker space-time are calculated. For scale factors larger than the Compton wavelengths of the particles associated with these matter fields the equations of motion for this scale factor are the same as those for a matter-dominated universe. Some speculations about the forces driving the expansion of the very early Universe are presented.

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I. INTRODUCTION

When studying physical phenomena at scales much larger than the Compton wavelength of some particle, we expect these particles to manifest themselves in that the part of the original action, not involving these particles, is modified into an effective one. In this work we would like to study the effects of very heavy particles on the gravitational action and on the subsequent evolution of the Universe. We shall study such a quantum effect on cosmology in the so-called "minisuperspace" model, where the large number of degrees of freedom are collapsed into a few effective ones. The Robertson-Walker space-time (RWST) is taken to describe the underlying geometry and it provides only one dynamical variable, the scale factor $R(t)$ or equivalently a spatial volume $V(t) = 2\pi^2 R^3(t)$. Matter fields are lumped into ones that are spatially constant. Contributions of such truncated bosonic fields have been previously studied [1]. The inclusion of fermions into general relativity has a long history. Application to cosmology goes back to Isham and Nelson [2], where certain inappropriate constraints were put on the spinors; the quantization of the combined Einstein-Dirac system in a Hamiltonian formulation was done by Nelson and Teitelboim [3]. In the above works the Dirac field was treated either classically or as a commuting first-quantized wave function. The full quantum mechanical problem has been considered from the point of view of the Wheeler-DeWitt equation [4, 5]. We shall discuss the effects of quantum fluctuations of both fermionic and bosonic matter fields on a classical RWST by calculating the one-loop contributions of massive, spatially constant, Dirac and scalar fields to the effective action. The combined problem has also received attention [6] in the Wheeler-DeWitt formalism. It should be pointed out that integrating out degrees of freedom directly in a RWST may not yield the same results as first freezing them out around an arbitrary space and then setting the metric to the RW form [7].

The total action for a spinor field of mass m, a boson field of mass M and the vierbein field e^μ_a is given by

$$
\begin{aligned}
S &= S_{EH} + S_D + S_B \,,\\
S_{EH} &= -\int d^4x \det e \frac{{}^{(4)}\mathcal{R}(e)}{\kappa^2}\,,\\
S_D &= \int d^4x \det e \frac{e^{\mu a}}{2}\Big[i\bar\psi\gamma_a\partial_\mu\psi - i\partial_\mu\bar\psi\gamma_a\psi \\
&\qquad -\bar\psi\{\gamma_a, B_\mu\}\psi\Big] - m\bar\psi\psi\,,\\
S_B &= \int d^4x \det e \frac{1}{2}\Big[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - M^2\phi^2 \\
&\qquad -\frac{\xi}{6}{}^{(4)}\mathcal{R}(e)\phi^2\Big]\,,
\end{aligned}
\tag{1}
$$

where

$$
B_\mu = \frac{1}{2}\omega^{ab}_\mu \sigma_{ab}\,. \tag{2}
$$

κ is related to Newton's coupling constant, $\kappa^2 = 16\pi G$ and ξ represents the strength of direct coupling of the boson fields to the Riemann curvature.

In the case of an *adiabatically varying* or slowly changing scale factor we will evaluate the Grassmann path integrals over the matter variables. The resulting effective action will again depend only on the $R(t)$. The adiabaticity condition will hold whenever $R(t) \geq 1/m$ or $R(t) \geq 1/M$; thus as R increases more and more matter fields will satisfy this condition and will leave a residual contribution to the effective action. For large R the form of the effective action is of the same as that for a matter-dominated Friedmann universe with bosons contributing a positive energy density and fermions a negative one.

As the evaluation of bosonic contributions is less subtle we shall concentrate first on the contributions of Dirac fields and in Sec. II we calculate the contribution of a fermion to the effective action and; based on the experience gained in that section, we do the same for bosons

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in Sec. III. In Sec. IV the equations of the scale factor $R(t)$ are analyzed and compared to those of a matter-dominated universe and speculations on the the reason for expansion in the *very* early Universe are presented.

II. FERMION EFFECTIVE ACTION

For a RWST the Dirac action in Eq. (1) takes the form

$$S_D = \int dt\, V(t)\bigg(\frac{i}{2}(\psi^\dagger\dot{\psi} - \dot{\psi}^\dagger\psi) - mN(t)\psi^\dagger\gamma^0\psi -N(t)\frac{\mu}{4R}\psi^\dagger\gamma^5\psi\bigg)\,, \tag{3}$$

where use has been made of

$$e^{\nu a}\{\gamma_a, B_\nu\} = \frac{\mu}{4R}\gamma^0\gamma^5\,, \tag{4}$$

which is valid for a RWST and $\mu = 3/\sqrt{2}$ for a closed geometry and $\mu = 0$ for an open or flat geometry [4]. $N(t)$ is the lapse function and in the present context serves as Lagrange multiplier; it may be set equal to one once the equations of motion have been obtained.

Care must be exercised in the choice of variables over which the path integrations are performed; this problem arises as $V(t)$ multiplies the kinetic energy part of the Dirac Lagrangian resulting in $V(t)\psi^\dagger$ being the momentum conjugate to ψ rather than $\psi^\dagger$ itself. The correct path integration is over canonical positions and momenta of

$$\exp\left(i\int dt[p\dot{q} - H(p,q)]\right)\;;$$

for the present case we find

$$\exp(i\Delta S_D) = \int\prod_t[d(V\psi^\dagger)d\psi]\exp\left\{i\left[\int dt\, V\psi^\dagger\left(i\dot{\psi} + \frac{i\dot{V}}{2V}\psi - mN\gamma^0\psi - N\frac{\mu}{4R}\gamma^5\psi\right)\right]\right\}\,. \tag{5}$$

In order to evaluate this functional integral we need the solutions to the eigenvalue equation

$$i\dot{\psi} + \frac{i\dot{V}}{V}\psi - mN\gamma^0\psi - N\frac{\mu}{4R}\gamma^5\psi = N(t)\lambda\psi\,, \tag{6}$$

subject to the normalization condition

$$\int dtN(t)V(t){\psi^\dagger}_\lambda\psi_{\lambda'} = \delta(\lambda-\lambda')\,. \tag{7}$$

Let $\psi = \eta/\sqrt{V}$; then η satisfies

$$i\dot{\eta} + mN\gamma^0\eta - N\frac{\mu}{4R}\gamma^5\eta = N(t)\lambda\eta\,, \tag{8}$$

with the normalization condition

$$\int dtN(t){\eta^\dagger}_\lambda\eta_{\lambda'} = \delta(\lambda-\lambda')\,. \tag{9}$$

We may solve these equations *only* under the assumption that the scale factor $R(t)$ is slowly varying [8]. It is then straightforward to find ΔS_D of Eq. (5),

$$\Delta S_D = \frac{1}{\pi}\int dt\, d\omega \ln\left(\omega^2 + \frac{\mu^2N^2}{16R(t)^2} + m^2N^2\right)\,, \tag{10}$$

and the result is

$$\Delta S_D = 2\int dtN(t)\sqrt{\frac{\mu^2}{16R(t)^2} + m^2} + C\,, \tag{11}$$

where C is an infinite constant that is independent of $R(t)$ and $N(t)$. This is the expected result as

$$-2\sqrt{\frac{\mu^2}{16R(t)^2} + m^2}$$

is the energy of the filled Dirac sea for this single-mode problem [4].

III. BOSON EFFECTIVE ACTION

Ignoring the self-couplings of the bosons, the matter action for an isotropic boson field coupled to a RWST is

$$S_B = \int dt\frac{V(t)}{2}\left(\frac{\dot{\phi}^2}{N(t)} - M^2N(t)\phi^2 - \frac{\xi N(t)\phi^2}{R(t)^2}\right)\,. \tag{12}$$

Again we rewrite the above in terms of the canonical momentum $\pi(t) = V(t)\dot{\phi}(t)/N(t)$:

$$S_B = \int dt\left[\pi\dot{\phi} - \frac{V(t)}{2}\bigg(\frac{N(t)\pi^2}{V(t)^2} - M^2N(t)\phi^2 - \frac{\xi N(t)\phi^2}{R(t)^2}\bigg)\right]\,. \tag{13}$$

We now seek the eigenvalues of the pair

$$N(t)\lambda\pi = \dot{\phi} - \frac{N(t)}{V(t)}\pi\,,$$
$$N(t)\lambda\phi = -\dot{\pi} + \left(V(t)N(t)M^2 + \frac{\xi N(t)}{R(t)^2}\right)\phi\,, \tag{14}$$

subject to the normalization condition

$$\int dtN(t)\left(\pi_\lambda\pi_{\lambda'} + \phi_\lambda\phi_{\lambda'}\right) = 2\delta(\lambda-\lambda')\,. \tag{15}$$

Again, under the assumptions of an adiabatically varying scale factor we may solve these equations and find, as in the fermion case,

$$\Delta S_B = -\frac{1}{2}\int dtN(t)\sqrt{\frac{\xi}{R(t)^2} + M^2} + C'\;; \tag{16}$$

C' is, as before, an infinite constant that is independent of the geometric parameters. $-\frac{1}{2}\sqrt{\xi/R(t)^2 + M^2}$ is the zero-point energy of the single-boson mode.

IV. EQUATIONS OF MOTION AND CONCLUSIONS

The vierbein action for a RWST is

$$S_{EH} = \int dt \frac{-12\pi^2}{\kappa^2} \left(\frac{R(t)\dot{R}(t)^2}{N(t)} - N(t)R(t) \right) . \quad (17)$$

Varying the total action, $S_{\rm EH} + \Delta S_{\rm F} + \Delta S_{\rm B}$, with the latter two terms given by Eq. (11) and Eq. (16), respectively, with respect to $N(t)$ and setting this quantity equal to one yields an equation of motion

$$\frac{12\pi^2}{\kappa^2} R(t)[\dot{R}(t)^2 + 1] - \frac{1}{2} \int dt \sqrt{\frac{\xi}{R(t)^2} + M^2} + 2 \int dt \sqrt{\frac{\mu^2}{16R(t)^2} + m^2} = 0 . \quad (18)$$

For $R(t) \gg 1/(m, M)$ these equations have the same form as those for a matter-dominated epoch [9] with a negative matter density coming from the fermions and a positive one from the bosons. We might be tempted to conclude that for small $R(t)$ we mimic a radiation-dominated era; however, for $R(t) \ll 1/(m, M)$ our assumption of adiabaticity fails as at those times one can show, by solving Eq. (18), that $\dot{R}/R \sim 1/t^{\frac{1}{2}}$. We plan to return to this point and study the effective action for a small, rapidly varying R [10].

We find that for any scale factor R the effects of fermions with masses larger than $1/R$ are such as to take away from the total energy of the Universe a factor of twice the mass of a fermion and that neutral boson add one-half the mass of a particular boson. Of course, at the present size of the Universe the contribution of any familiar field is totally negligible. However, we can imagine that prior to the Planck epoch stringlike excitations with masses much larger than the Planck mass started decoupling as the scale factor of the RWST kept increasing. As long as the contribution of the boson was greater than that of the fermions, we would simulate a matter-dominated universe. This would require that any supersymmetry be broken prior to the Planck era. Should supersymmetry hold in that the number of bosonic degrees of freedom with a given mass is four times that of Dirac ones with the same mass and with $\xi = \mu^2$, the freezing out of the degrees of freedom would have no effect.

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