### Title
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# Doomsday: Friday, 13 November, A.D. 2026.

# Demographic-Technological Positive Feedback, Hyperbolic Acceleration of the Global Complexity Growth, and the 21st Century Singularity

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This paper examines the striking mathematical correspondence between two independently derived hyperbolic equations describing global evolutionary dynamics: Panov's time series of "biospheric revolutions" spanning 4 billion years of biological and cultural evolution, and von Foerster's famous "Doomsday" equation for world population growth (1960). Both phenomena are governed by equations of the form $y = C/(2027 - t)$, with singularity dates converging on the third decade of the 21st century ($R^2 = 0.9991$ and 0.996 respectively). The analysis demonstrates that this correspondence is not coincidental but reflects the operation of a nonlinear second-order positive feedback mechanism between demographic growth and technological development, formalized through the coupled differential equations of the Verhulst-Pearl logistic model and the Taagepera-Kremer technological growth equation. The mathematical singularity – the point at which equations project infinite values – should not be interpreted literally. Within social system, just as in physical and chemical systems exhibiting "blow-up" regimes (autocatalytic reactions, thermal explosions, gelation), other mechanisms intervene before infinity is reached. For world population, this transition began in

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the early 1970s with the fertility decline phase of the global demographic transition; evidence suggests a similar deceleration has commenced in global techno-scientific and economic growth rates. Thus, the 21st century can be called "singular" not because it will witness infinite growth, but because it marks the transition of global complexity evolution from the hyperbolic trajectory that has characterized it since the emergence of life on Earth to a qualitatively different evolutionary regime. The enduring significance of von Foerster's discovery lies in demonstrating that the most complex social system known to humanity can be described with extraordinary accuracy by astonishingly simple mathematical equations, revealing deep structural patterns beneath the apparent chaos of historical events.

## 1. Introduction: Panov's Time Series and the Hyperbolic Pattern

Some time ago I tried to analyze a time series compiled by Alexander Panov of Skobeltsyn Institute of Nuclear Physics at Moscow State University and presented by him in his article "Scaling law of the biological evolution and the hypothesis of the self-consistent Galaxy origin of life" (Panov 2005a: 221):

This time series looked as follows:

"0. The origin of life – **$4 \cdot 10^9$ years ago**. The biosphere after its appearance was represented by nucleusless procaryotes and existed the first 2–2.5 billion years without any great shocks.

1. Neoproterozoic revolution (Oxygen crisis) – **$1.5 \cdot 10^9$ years ago**. Cyanobacteria had enriched the atmosphere by oxygen that was a strong poison for anaerobic procaryotes. Anaerobic procaryotes started to die out and anaerobic procaryote fauna was changed by an aerobic eucaryote and multicellular one.

2. Cambrian explosion (The beginning of Paleozoic era) – **590–510 $\cdot 10^6$ years ago**[1]. All the modern phyla of metazoa (including vertebrates) appeared during a few of tens of million years. During the Paleozoic era the terra firma was populated by life.

3. Reptiles revolution (The beginning of Mesozoic era) – **$235 \cdot 10^6$ years ago**. Almost all paleozoic Amphibia died out. Reptiles became the leader of the evolution on the terra firma.

4. Mammalia revolution (The beginning of the Cenozoic era) – **$66 \cdot 10^6$ years ago**. Dinosaurs died out. Mammalia animals became the leader of the evolution on the terra firma.

[1] $570 \cdot 10^6$ years ago according to Panov 2005b.

5. Hominoid revolution (The beginning of the Neogene period) – **25–20 · $10^6$ years ago**[2]. A big evolution explosion of Hominoidae (apes). There were 14 genera of hominoidae between 22 and 17 millions years ago – much more than now. The flora and fauna became contemporary.
6. The beginning of Quaternary period (Anthropogene) – **4.4 · $10^6$ years ago**[3]. The first primitive Homo genus (hominidae) separated from hominoidae.
7. Palaeolithic revolution – **2.0–1.6 · $10^6$ years ago**[4]. *Homo habilis*, the first stone implements.
8. The beginning of Chelles period – **0.7–0.6 · $10^6$ years ago**[5]. Fire, *Homo erectus*.
9. The beginning of Acheulean period – **0.4 · $10^6$ years ago**. Standardized symmetric stone implements.
10. The culture revolution of neanderthaler (Mustier culture) – **150–100 · $10^3$ years ago**. *Homo sapiens neandertalensis*. Fine stone implements, burial of deadmen (a sign of primitive religions).
11. The Upper Palaeolithic revolution – **40 · $10^3$ years ago**. *Homo sapiens sapiens* became the leader of cultural evolution. Development of advanced hunter instruments – spears, snares. Imitative art is widespread.
12. Neolithic revolution – **12–9 · $10^3$ years ago**. Appropriative economy [foraging] had been replaced by productive economy [food production].
13. Urban revolution (the beginning of the Ancient world) – **4000–3000 B.C.** Appearance of state formations, written language and the first legal documents.
14. Imperial antiquity, Iron age, the revolution of the Axial time – **800–500 B.C.**[6]. The appearance of a new type of state formations – empires, and a culture revolution. New kinds of thinkers such as Zaratushtra, Socrates, Budda, and others.
15. The beginning of the Middle Ages – **400–630 CE.**[7] Disintegration of Western Roman Empire, widespread Christianity and Islam, domination of feudal economy.
16. The beginning of the New Time [Modern Period], the first industrial revolution – **1450–1550 CE**[8] Appearing of manufacture, printing of books, the New time culture revolution etc.
17. The second industrial revolution (steam and electricity) – **1830–1840**[9]. Appearance of mec hanized industry, the beginning of globalization in the information field (telegraph was invented in 1831), etc.
18. Information revolution, the beginning of the postindustrial epoch – **1950**. The main part of population of industrial countries work in the field of information production and utilization or in the service field, not in the material production".

[2] 24· $10^6$ years ago according to Panov 2005b.
[3] 4–5 · $10^6$ years ago according to Panov 2005b.
[4] 2–1.5 · $10^6$ years ago according to Panov 2005b.
[5] 0.7 · $10^6$ years ago according to Panov 2005b.
[6] 750 B.C. according to Panov 2005b.
[7] A.D. 500 according to Panov 2005b.
[8] A.D. 1500 according to Panov 2005b.
[9] 1830 according to Panov 2005b.

Usually, such time series are presented in the following way (see Fig. 1):

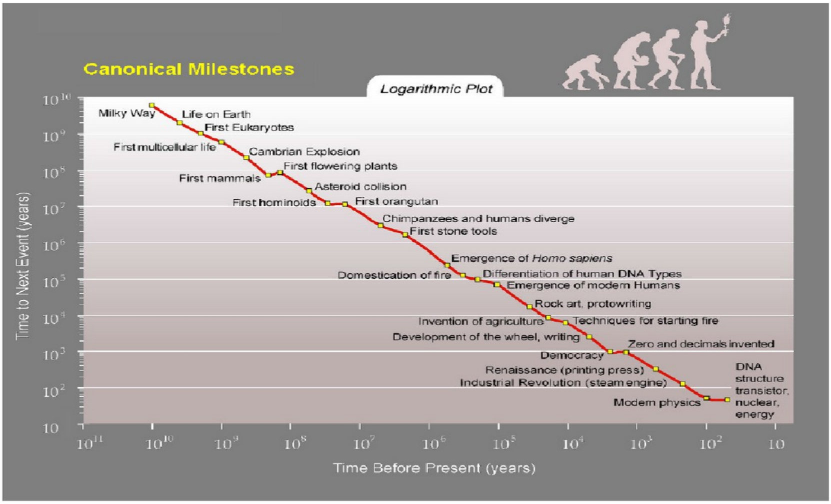



**Figure 1**. A log-log version of Kurzweil's "Countdown to Singularity" graph (= "Canonical Milestones"). *Source*: Kurzweil 2005: 20 (reproduced with permission of Raymond Kurzweil).

At this diagram, the X-axis denotes time before present (with a logarithmic scale), whereas the Y-axis indicates time to next event (in years; also, with a logarithmic scale).

However, Panov suggested that such a time series can be represented in a different way (e.g., Panov 2025b: 128–129). He divided one year by the time to the next event, and arrived at a variable that he himself called "frequency of phase transitions" (per a unit of time), but which could be also called (following Kurzweil 2001: 5) "paradigm shift rate", or "global complexity growth rate" (e.g., Korotayev 2024, 2025). The result of such a transformation looked as follows (see Fig. 2):

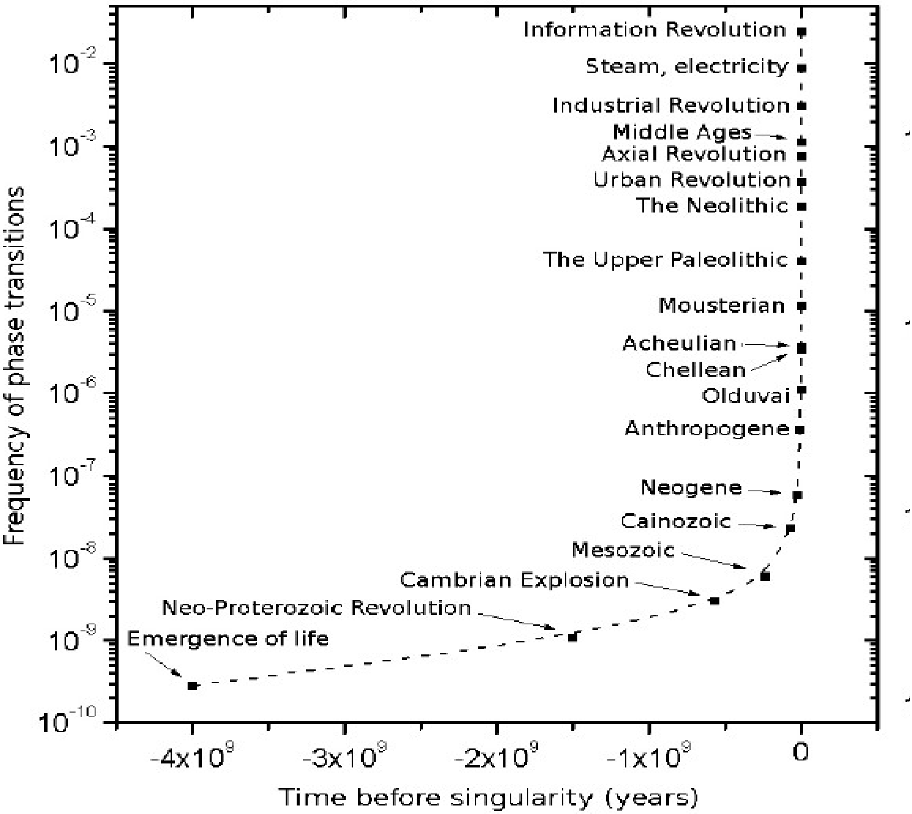



**Figure 2.** The dynamics of the global "frequency of phase transition" according to Panov (*source*: Nazaretyan 2018: 31, Fig. 3)

Of course, the shape of the curve at this graph is clearly hyperbolic, which, naturally, suggests that we are dealing here with hyperbolic acceleration.

## 2. Fitting the Hyperbolic Equation to Panov's Data

However, Panov, to a sort of my surprise, abstained from approximating his time series with a hyperbolic function, and preferred some other type of mathematical analysis.

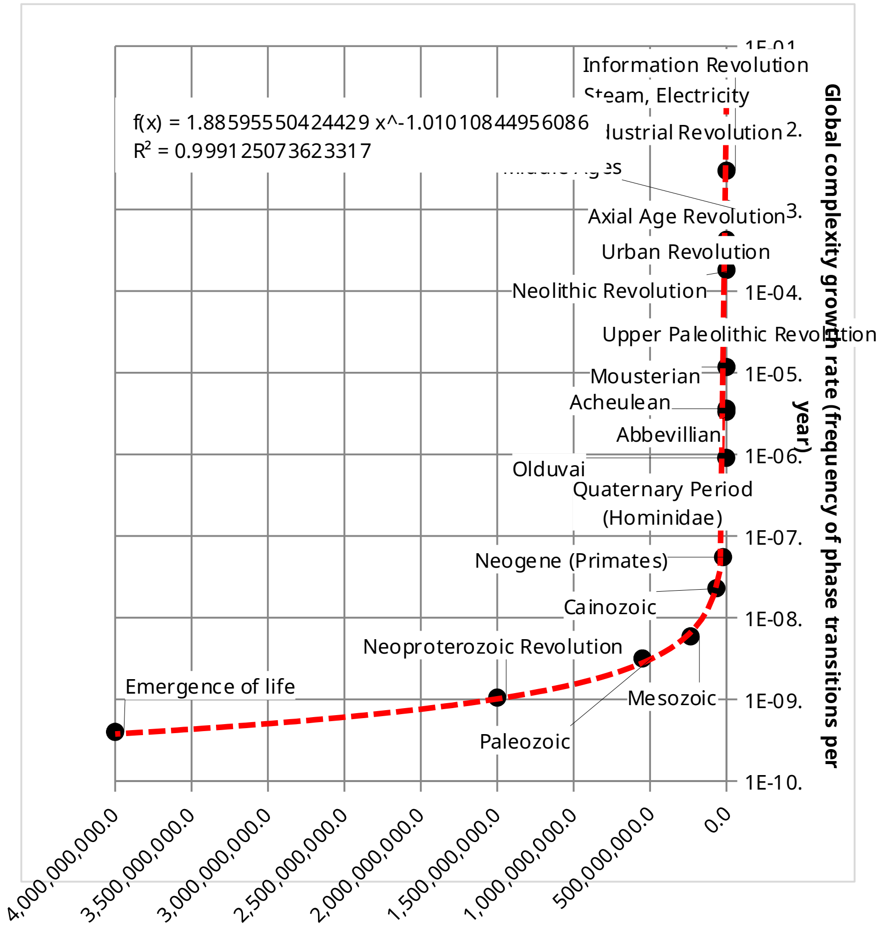



**Figure 3a.** Scatterplot of the phase transitions from Panov's list with the fitted power-law regression line (with a logarithmic scale for the Y-axis) – for the Singularity date identified as **2027** CE with the least squares method

In double logarithmic scale this diagram looks as follows (see Fig. 3b):

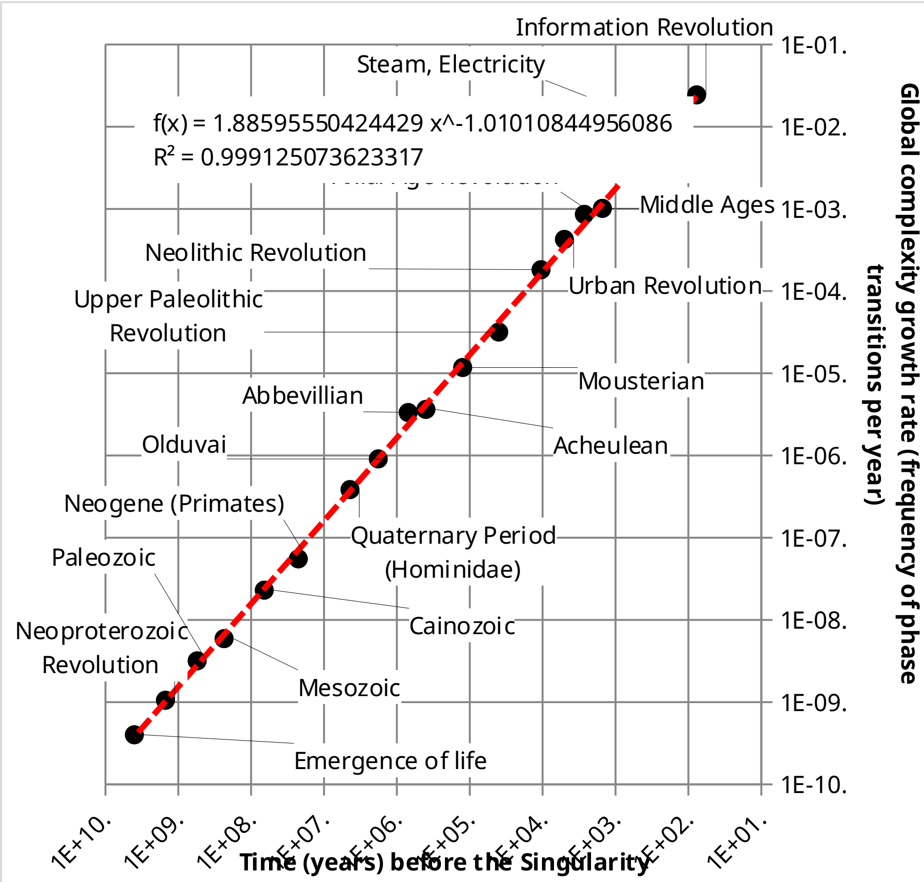



**Figure 3b.** Scatterplot of the phase transitions from Panov's list with the fitted power-law regression line (with double logarithmic scale) – for the Singularity date identified as **2027** CE with the least squares method.

As we see, Panov's time series is almost ideally ($R^2$ = 0.9991) described by the following power-law equation:

$$y=\frac{1.886}{x^{1.01}}, \qquad (1)$$

where $y$ is the global complexity growth rate / frequency of phase transitions / paradigm shift rate, $x$ is the time remaining till the singularity, and 2.054 and 1.003 are constants.

As $x$ denotes the time till the singularity, whereas the singularity date has been identified as 2027 CE with the least squares method, then

$x$ = 2027 – $t$ , and thus

$$y=\frac{1.886}{(2027-t)^{1.01}} . \quad (2)$$

Note that the denominator's exponent (1.01) turns out to be only negligibly different from 1 (well within the error margins); hence, there are all grounds to use this equation in the following simplified form:

$$y=\frac{1.886}{2027-t} . \quad (3)$$

And if we denote the constant in the numerator as $K$, then this equation can well be written as

$$y=\frac{K}{2027-t} . \quad (4)$$

Note that this equation has a singularity with $t$ = 2027 (we will discuss this mathematical singularity in more detail below).[10] Let us also recollect that the curve generated by this equation has a hyperbolic shape, and the respective explosively accelerating growth pattern is sometimes called hyperbolic.[11]

[10] Note that an almost identical equation (with a singularity date around 2029) turned out to be able to describe with a comparable extreme accuracy (Korotayev 2018, 2020b) the hyperbolic acceleration pattern that can be detected in another time series of global phase transitions that was compiled totally independently from Panov by Theodore Modis (2002, 2003) and later used by Raymond Kurzweil (2005: 20).

[11] An anonymous referee of this article rightly notes that a sequence of events has the $1/t$ hyperbolic form only if the sequence of events occurs in a geometric sequence, e.g., $t(n+1) = t(n)/F$ where $F$ is a constant. Then the ratio between two successive points should be $t(n)/t(n+1) = F$, i.e., constant over the very large time scale. Against this background, it is important to mention, that already Panov, during his initial mathematical analysis demonstrated that this is precisely the case, showing that the value of parameter $F$ (that Panov himself denotes as $\alpha$) for his series equals 2.67 ± 0.15. He also notes that $\alpha \approx e = 2.718\ldots$ (Panov 2005a: 222). On the possible implications of the latter point see Appendix 2 in Korotayev 2018 and 2020b.

## 3. The Von Foerster Discovery: "Doomsday: Friday, 13 November, A.D. 2026"

And here we are coming closer to Doomsday: Friday, 13 November, A.D. 2026. The point is that in 1960, Heinz von Foerster (see Fig. 4), Patricia Mora, and Lawrence Amiot published in one of the topmost scientific journals, *Science*, news about their striking discovery (Foerster et al., 1960). They showed that between 1 and 1958 CE the world's population ($N$) dynamics can be described in an extremely accurate way with the following astonishingly simple equation:

$$N_t = \frac{K}{(t_0 - t)^{0.99}}, \qquad (5)$$

where $N_t$ is the world population at time $t$, and $K$ and $t_0$ are constants, with $t_0$ corresponding to the so called "demographic singularity".[12]

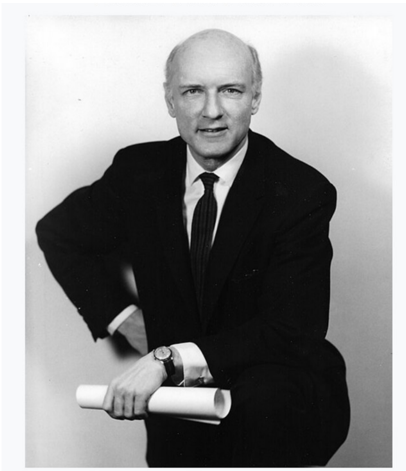
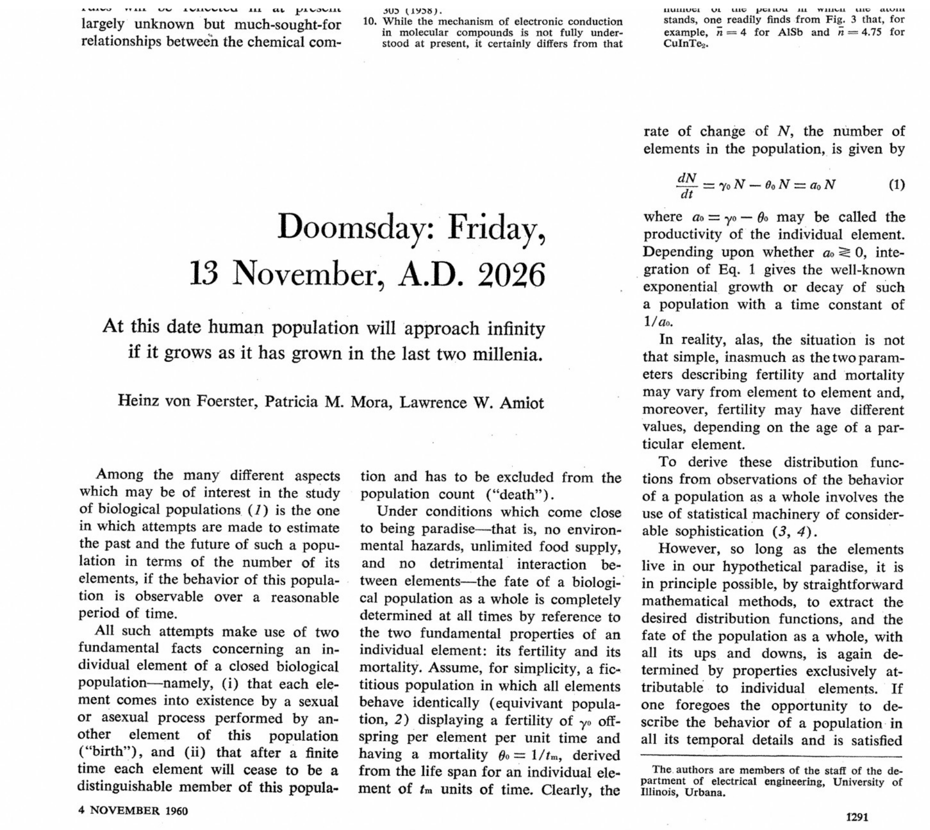


**Figure 4**. Heinz von Foerster at the Biological Computer Laboratory, the University of Illinois in 1963. *Source*: https://en.wikipedia.org/wiki/Heinz_von_Foerster#/media/File:HvF_01.jpg

[12] Note that von Foerster and his colleagues detected the hyperbolic pattern of world population growth for 1 CE – 1958 CE; later it was shown that this pattern continued for a few years after 1958, and also that it can be traced for many millennia BCE (Kapitza 1992, 1996; Kremer 1993; Tsirel 2004; Korotayev, Malkov, Khaltourina 2006a, 2006b; Podlazov, 2017). In fact, Kremer (1993) claims that this pattern is traced since 1 000 000 BP, whereas Kapitza (1996, 2003, 2006, 2010) even insists that it can be found since 4 000 000 BP.

Parameter $t_0$ was estimated by von Foerster and his colleagues as 2026.87, which corresponds to Friday, November 13, 2026; this made it possible for them to supply their article with a scientific public-relations masterpiece title – "Doomsday: Friday, 13 November, A.D. 2026" (von Foerster et al. 1960; see also Fig. 5).



largely unknown but much-sought-for relationships between the chemical com-

10. While the mechanism of electronic conduction in molecular compounds is not fully understood at present, it certainly differs from that

stands, one readily finds from Fig. 3 that, for example, $\bar{n} = 4$ for AlSb and $\bar{n} = 4.75$ for $CuInTe_2$.

# Doomsday: Friday, 13 November, A.D. 2026

At this date human population will approach infinity if it grows as it has grown in the last two millenia.

Heinz von Foerster, Patricia M. Mora, Lawrence W. Amiot

Among the many different aspects which may be of interest in the study of biological populations (*1*) is the one in which attempts are made to estimate the past and the future of such a population in terms of the number of its elements, if the behavior of this population is observable over a reasonable period of time.

All such attempts make use of two fundamental facts concerning an individual element of a closed biological population—namely, (i) that each element comes into existence by a sexual or asexual process performed by another element of this population ("birth"), and (ii) that after a finite time each element will cease to be a distinguishable member of this population and has to be excluded from the population count ("death").

Under conditions which come close to being paradise—that is, no environmental hazards, unlimited food supply, and no detrimental interaction between elements—the fate of a biological population as a whole is completely determined at all times by reference to the two fundamental properties of an individual element: its fertility and its mortality. Assume, for simplicity, a fictitious population in which all elements behave identically (equivivant population, *2*) displaying a fertility of $\gamma_0$ offspring per element per unit time and having a mortality $\theta_0 = 1/t_m$, derived from the life span for an individual element of $t_m$ units of time. Clearly, the rate of change of $N$, the number of elements in the population, is given by

$$\frac{dN}{dt} = \gamma_0 N - \theta_0 N = a_0 N \quad (1)$$

where $a_0 = \gamma_0 - \theta_0$ may be called the productivity of the individual element. Depending upon whether $a_0 \gtrless 0$, integration of Eq. 1 gives the well-known exponential growth or decay of such a population with a time constant of $1/a_0$.

In reality, alas, the situation is not that simple, inasmuch as the two parameters describing fertility and mortality may vary from element to element and, moreover, fertility may have different values, depending on the age of a particular element.

To derive these distribution functions from observations of the behavior of a population as a whole involves the use of statistical machinery of considerable sophistication (*3*, *4*).

However, so long as the elements live in our hypothetical paradise, it is in principle possible, by straightforward mathematical methods, to extract the desired distribution functions, and the fate of the population as a whole, with all its ups and downs, is again determined by properties exclusively attributable to individual elements. If one foregoes the opportunity to describe the behavior of a population in all its temporal details and is satisfied

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4 NOVEMBER 1960

1291

**Figure 5.** First page of von Foerster et al., 1960.

Note that parameter $t_0$ (2026.87) represents a *singularity* in the strict mathematical sense of this word. I would cite here quite an appropriate explanation of mathematical singularity by Raymond Kurzweil:

> 'Singularity' is an English word meaning a unique event with, well, singular implications. The word was adopted by mathematicians to

denote a value that transcends any finite limitation, such as the explosion of magnitude that results when dividing a constant by a number that gets closer and closer to zero. Such a mathematical function never actually achieves an infinite value, since dividing by zero is mathematically 'undefined' (impossible to calculate). But the value of $y$ exceeds any possible finite limit (approaches infinity) as the divisor $x$ approaches zero (Kurzweil 2005: 22–23).

In fact, with respect to Eq. (5), all the numbers can be divided into two unequal sets: (1) all the numbers except 2026.87 (~ Friday, November 13, 2026), and (2) 2026.87. The point is that for all the numbers except 2026.87 Eq. (5) has solutions; 2026.87 represents here a really singular point, as with $t$ = 2026.87, we have 0 in Eq. (5) denominator, and $t$ = 2026.87 turns out to be truly *singular*, exceptional, as it is the only point for which it is impossible to calculate the value of $N_t$.

Let us return now to Eq. (5). As was the case with Eq. (2) above, in von Foerster's Eq. (5) the denominator's exponent (0.99) turns out to be only negligibly different from 1, and as was already suggested by von Hoerner (1975) and Kapitza (1992, 1996), it can be written more succinctly as

$$N_t = \frac{K}{t_0 - t}. \tag{6}$$

As has been mentioned above, parameter $t_0$ was estimated by von Foerster and his colleagues as 2026.87, which gives us

$$N_t = \frac{K}{2026.87 - t}. \tag{7}$$

In fact, there are sufficient grounds to round the singularity date ($t_0$) to 2027. Indeed, von Foerster and his colleagues themselves identified the standard error margins for $t_0$ as ± 5.50 years (von Foerster et al. 1960: 1293). Actually, this implies that they had no serious grounds to identify the singularity date with such an accuracy, up to the exact day; in fact, the data they had at their disposal did not allow for such a precision. So, their decision to provide an exact ("doomsday") day for the singularity in their model may well be regarded as a sort of "scientific hooliganism" (or, if you like "scientific humor", or even "scientific trolling") aimed at attracting attention to their really outstanding discovery. Anyway, after the rounding of the singularity date we arrive at the following equation:

$$N_t = \frac{K}{2027 - t}. \qquad (8)$$

Quite predictably, even after the rounding of the singularity date, Eq. 8 continues describing the world population dynamics until the early 1970s in an almost perfect way (see Fig. 6):

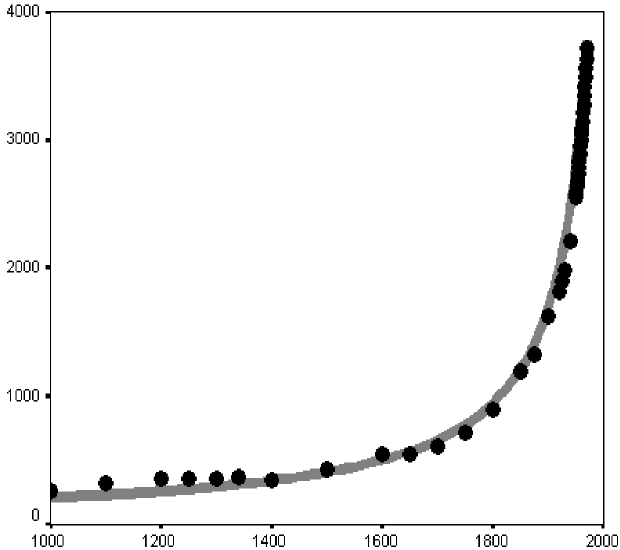


**Figure 6.** Correlation between empirical estimates of world population (in millions, 1000 – 1970) and the curve generated by von Foerster's Equation (8). *Note:* black markers correspond to empirical estimates of the world population by McEvedy and Jones (1978) for 1000–1950 and UN Population Division (2024) for 1950–1970. The grey curve has been generated by Eq. (8) with $K$ = 215 billion. $R^2$ = 0.996.

## 4. The Striking Similarity Between Two Equations

At this point, it is difficult not to see a striking similarity between Eq. (4)

$$y = \frac{K_1}{2027 - t} \qquad (4)$$

that describes so precisely ($R^2$ = 0.9991) the dynamics of the global complexity growth rate since the origins of the life on the Earth until the most recent decades, on the one hand, and, on the other hand, Eq. (8)

$$N_t = \frac{K_2}{2027 - t} \tag{8}$$

that describes almost as accurately ($R^2$ = 0.996) the dynamics of the world population growth up to the early 1970s. Indeed, the only difference between Eq. (4) and Eq. (8) is the difference of the values of the constant *K*.

It appears necessary to note that those two extremely similar equations describe the dynamics of very different variables – the global complexity growth rate for Eq. (4) and the world population size for Eq. (8).

In this article, I will try to show that this extreme similarity is in no way a coincidence, as those equations are very closely linked describing two sides of the same process.

We will start our account with the explanation how demographic-technological positive feedback produced the above-mentioned hyperbolic growth throughout most of the human history.

To start with, as has been shown by Sergey Kapitza (1992, 1996) and Michael Kremer (1993), the algebraic hyperbolic equation Eq. (6a)

$$N_t = \frac{C}{t_0 - t}. \qquad \text{(6a[13])}$$

can be regarded as a solution of the following differential equation:

$$\frac{dN}{dt} = \frac{N^2}{C} \tag{9}$$

where *C* is a constant (for an analytical solution of Eq. 9 see Appendix).

[13] Note that, before this point, we denoted, after von Foerster, the constant at the enumerator of the hyperbolic equation with letter *K*. However, starting from this point, we will switch (after Kapitza) to *C*, as we will need a bit later the Verhulst – Pearl logistic equation where letter *K* is traditionally reserved for carrying capacity.

This equation can be also written as $$\frac{dN}{dt} = aN^2$$ where $$a = \frac{1}{C}$$ .

In our context *dN/dt* denotes the absolute population growth rate at some moment of time. Hence, this equation states that the absolute population growth rate at any moment of time should be proportional to the square of population at this moment.

Note that this significantly demystifies von Foerster’s “enigma” of the world population hyperbolic growth. Now to explain this hyperbolic growth we should just explain why for many millennia the world population's absolute growth rate tended to be proportional to the square of the population.

## 5. The Demographic-Technological Positive Feedback Mechanism

Most mathematical models of the world population hyperbolic growth (Taagepera 1976, 1979; Kremer 1993; Podlazov 2000, 2017; Tsirel 2004; Korotayev 2005, 2008; Korotayev et al. 2006a) are based on the following assumptions:

**1) The first one is "the Malthusian (1978 [1798]) assumption” that up until recently “population was limited by the available technology, so that the growth rate of population” tended to be “proportional to the growth rate of technology"** (e.g., Kremer 1993: 681–682).

This statement looks quite convincing. Indeed, throughout most of human history the world population was limited by the technologically determined ceiling of the carrying capacity of land. E.g., with foraging subsistence technologies, the Earth could not support more than 10 million people, because the amount of naturally available useful biomass on this planet is limited, and the world population could only grow over this limit when the people started to apply various technological means to artificially increase the amount of available biomass, that is with the transition from foraging to food production. However, the extensive agriculture also can only support a limited number of people, and further growth of the world population only became possible with the intensification of agriculture, other technological improvements and so on.

Perhaps, the best way to model this assumption has been proposed by Sergey Tsirel (2004) who suggests to use for this purpose just the famous Verhulst – Pearl logistic equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{10}$$

where $N$ represent population size (number of individuals), $r$ is the intrinsic growth rate, and $K$ is the carrying capacity (the maximum number of individuals that the environment can support).

This equation generates a rather characteristic "logistic" curve (see Fig. 7):

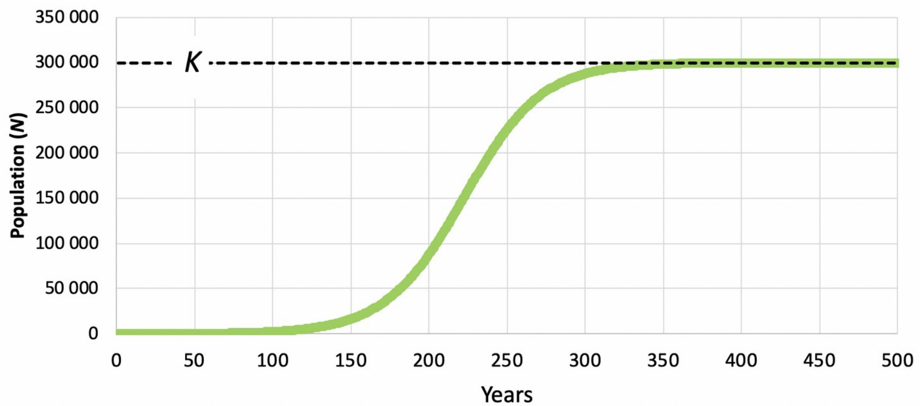


**Figure 7.** Logistic curve generated be Eq. (12) with $r$ = 0.04; $K$ = 300 000 and $N_0$ = 50. This could be regarded as a simulation of, e.g., the population dynamics after a group of settlers consisting of 50 persons gets to an island that (with the given level of technology) can support a population of 300 thousand (in the absence of further migration to / from this island and significant technological growth).

Note that within this model population ($N$) tends to carrying capacity ($K$). Note that, with respect to the human populations, the value of $K$ heavily depends on the level of available technology – an island that, with a foraging technology, could hardly support a population of 100 hunter-gatherers, may well be able to support a couple orders of magnitude higher population of intensive agriculturalists. In fact, human carrying capacity ($K$) can well be regarded as a measure of the level of technological development. The level of the development of life-support technologies can well be measured through the number of people whose life can be supported by those technologies. Hence, below we will use human carrying capacity (*K*) as a proxy of the level of life-support technology development.

It is also important to note that Verhulst – Pearl equation provides a rather convincing mathematical description how population can be limited by available technology.

However, as is well known, the technological level is not a constant, but a variable (e.g., Grinin et al. 2020; Turchin 2025). And, in order to describe its dynamics, the second basic assumption is employed (it is given below in Michael Kremer's formulation):

**2) "High population spurs technological change because it increases the number of potential inventors… All else equal, each person's chance of inventing something is independent of population. Thus, in a larger population there will be proportionally more people lucky or smart enough to come up with new ideas"** (Kremer 1993: 685), thus, **"the growth rate of technology is proportional to total population"** (Kremer 1993: 681).

In general, we find this assumption rather plausible – in fact, it is quite probable that, other things being equal, within a given period of time, one billion people will make approximately one thousand times more inventions than one million people.

This assumption is expressed mathematically in the following way:

$$\frac{dT}{dt} = aNT, \qquad (11)$$

where *T* is the level of technological development, and *N* is the population.

Actually, this equation says just that the absolute technological growth rate at a given moment of time is proportional to the technological level observed at this moment (the wider is the technological base, the more inventions could be made on its basis), and, on the other hand, it is proportional to the population (the larger the population, the higher the number of potential inventors).

As has been mentioned above, the level of the development of life-support technologies can well be measured through the number of people whose life can be supported by those technologies, human carrying capacity (*K*). And within the present analysis it appears appropriate to use the human carrying capacity (*K*) as a proxy of the level of life-support technology development:

$$\frac{dK}{dt}=aNK. \qquad (11a)$$

Thus, we arrive at the following mathematical model of global demographic and technological growth, consisting of two differential equations:

$$\frac{dN}{dt}=rN\left(1-\frac{N}{K}\right), \qquad (10)$$

$$\frac{dK}{dt}=aNK. \qquad (11a)$$

Note that this model turns to be capable to describe rather accurately the world population dynamics up to the end of the 20th century (see Fig. 8):

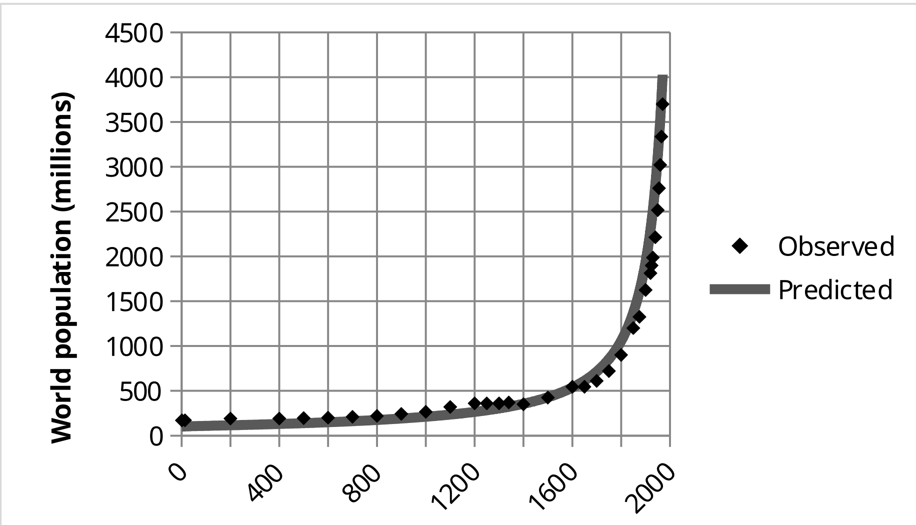


**Figure 8.** Correlation between empirical estimates of world population (in millions, 1 – 1970) and the curve generated by mathematical model (10)-(11a). *Note:* black markers correspond to empirical estimates of the world

population by McEvedy and Jones (1978) for 1000–1950 and UN Population Division (2024) for 1950–1970. The grey curve has been generated by mathematical model (10)-(11a) with $r = 0.04$; $b = 5\cdot10^6$; $N_0$ = 100 million; $K_0$ = 105 million. $R^2 = 0.99$.

Quite a reasonable question might appear at this point. As has been mentioned above, till the early 1970s the world population grew along a hyperbolic curve described be von Foerster's equation

$$N_t = \frac{C}{2027 - t}. \tag{8a}$$

However, at the first glance, one could hardly find anything particularly hyperbolic about mathematical model (10)-(11a). So, how has it managed to generate a hyperbolic curve so perfectly?

The answer to this question looks as follows.

As can be seen above in Fig. 5 above, within the logistic model (10) the human population ($N$) tends to $K$:

$$N \rightarrow K$$

Hence,

$$as\frac{dK}{dt} = aNK \wedge N \rightarrow K\,, \frac{dK}{dt} \rightarrow aK \bullet K = aK^2 = \frac{K^2}{C}, \text{ with } a = \frac{1}{C}\,.$$

As we remember, the solution of differential equation $dX/dt = X^2/C$ looks like $X_t = C/(t_0 - t)$.

Hence, within mathematical model (10)-(11a) $K$ would tend to a hyperbolic trajectory $C/(t_0 - t)$:

$$K_t \rightarrow \frac{C}{t_0 - t}.$$

And as world population ($N$) tends to $K$ it would also tend to a hyperbolic trajectory:

$$N_t \rightarrow K_t \rightarrow \frac{C}{t_0 - t}.$$

Such models provide a rather convincing explanation of why throughout most of human history the world population followed the hyperbolic pattern with the absolute population growth rate tending to be proportional to $N^2$. For example, why will the growth of population from, say, 10 million to 100 million, result in the growth of *dN/dt* 100 times? Model (10)-(11a) explains this rather convincingly. The point that the world population has grown 10-fold from 10 to 100 million implies that human technology has also grown approximately 10 times (given that it will have proven, after all, to be able to support ten times larger population). On the other hand, the growth of a population 10 times also implies a 10-fold growth of the number of potential inventors, and, hence, a 10-fold increase in the relative technological growth rate. Hence, the absolute technological growth rate will grow $10 \times 10 = 100$ times (as, in accordance with Taagepera – Kremer technological growth equation, an order of magnitude higher number of people having at their disposal an order of magnitude wider technological basis would tend to make two orders of magnitude more inventions). And as $N$ tends to the technologically determined carrying capacity ceiling, we have good reason to expect that *dN/dt* will also increase just by about 100 times.

In fact, our model suggests that the hyperbolic pattern of the world's population growth could be accounted for by the nonlinear second order positive feedback mechanism that was shown long ago to generate just the hyperbolic growth, known also as the "blow-up regime" (e.g., Samarskii et al. 2011).

In our case this nonlinear second order positive feedback looks as follows (see Fig. 9a):

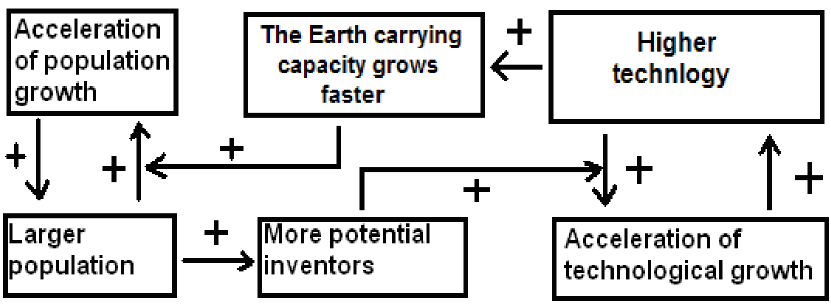


**Figure 9a**. Cognitive scheme of the nonlinear second-order positive feedback between technological development and demographic growth.

In fact, this positive feedback can be graphed even more succinctly (see Fig. 9b):

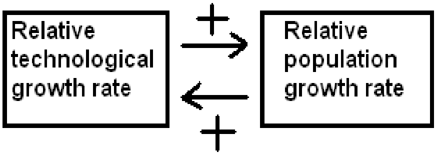


**Figure 9b.** Cognitive scheme of the nonlinear second-order positive feedback between technological development and demographic growth (version 2).

Note that the relationship between technological development and demographic growth cannot be analyzed through any simple cause-and-effect model, as we observe a true dynamic relationship between those two processes – each of them is both the cause and the effect of the other.

## 6. Why the Two Equations are Almost Identical

Now let us return to the question that we posed above – why Eq. (4a)

$$y = \frac{C_1}{2027 - t} \qquad (4a)$$

that describes so precisely ($R^2$ = 0.9991) the dynamics of the global complexity growth rate since the origins of the life on the Earth until the most recent decades is almost identical to Eq. (8a)

$$N_t = \frac{C_2}{2027 - t} \qquad (8a)$$

that describes almost as accurately ($R^2$ = 0.996) the dynamics of the world population growth up to the early 1970s.

Actually, model (10)-(11a) explaining the mechanism of the hyperbolic growth of the world population (that was observed until the early 1970s) suggests the answer to this question. Indeed, it relies among other things on the following assumption: “In a larger population there will be proportionally more people lucky or smart enough to come up with new ideas" (Kremer 1993: 685), thus, "the growth rate of technology is proportional to total population" (Kremer 1993: 681).

Now, have a new look at the human section of Panov’s series (Panov 2005a: 221):

11. The Upper Palaeolithic revolution – **40 · $10^3$ years ago**. *Homo sapiens sapiens* became the leader of cultural evolution. Development of advanced hunter instruments – spears, snares. Imitative art is widespread.

12. Neolithic revolution – **12–9 · $10^3$ years ago**. Appropriative economy [foraging] had been replaced by productive economy [food production].

13. Urban revolution (the beginning of the Ancient world) – **4000–3000 B.C.** Appearance of state formations, written language and the first legal documents.

14. Imperial antiquity, Iron age, the revolution of the Axial time – **800–500 B.C.**[14]. The appearance of a new type of state formations – empires, and a culture revolution. New kinds of thinkers such as Zaratushtra, Socrates, Budda, and others.

15. The beginning of the Middle Ages – **400–630 CE.**[15] Disintegration of Western Roman Empire, widespread Christianity and Islam, domination of feudal economy.

16. The beginning of the New Time [Modern Period], the first industrial revolution – **1450–1550 CE**[16] Appearing of manufacture, printing of books, the New time culture revolution etc.

17. The second industrial revolution (steam and electricity) – **1830–1840**[17]. Appearance of mechanized industry, the beginning of globalization in the information field (telegraph was invented in 1831), etc.

18. Information revolution, the beginning of the postindustrial epoch – **1950**. The main part of population of industrial countries work in the field of information production and utilization or in the service field, not in the material production".

It is easy to see that, as regards the human part of the Big History, Panov's list consists almost exclusively of the most important technological revolutions in the human history, such as the Neolithic (Agrarian) Revolution, the so called "Urban Revolution" of around 4,000–3,000 BCE[18], Industrial Revolution etc. Thus, what Panov

[14] 750 B.C. according to Panov 2005b.

[15] A.D. 500 according to Panov 2005b.

[16] A.D. 1500 according to Panov 2005b.

[17] 1830 according to Panov 2005b.

[18] It involved such breakthrough technological innovations as the invention of metallurgy, plow, pottery wheel, writing and so on.

himself calls “frequency of phase transitions”[19], with respect to the human part of the Panov series, turns out to be “frequency of technological revolutions” per a unit of time, that is nothing else than the *technological growth rate*.

Hence, the point that von Foerster – Kapitza hyperbolic equation (8) (that describes with an unusual precision the dynamics of the world population growth up to the early 1970s) is almost identical to equation (4) above (that describes so precisely the dynamics of the global complexity growth rate – or global technological growth rate – until the most recent decades) is in a very good agreement with the above discussed mathematical models of hyperbolic growth of the world population, as such models tend to consider the hyperbolic growth of the world population as a result of the functioning of the positive feedback mechanism of the second order between demographic growth and technological development, when technological development (most vividly manifested precisely as “phase transitions” – e.g., the Neolithic Revolution, or the Industrial Revolution) significantly accelerated the growth rate of the population, which (by virtue of the principle "the more people, the more inventors”) through collective learning mechanisms accelerated onset of each successive “phase transition” / “biospheric revolution” (that usually corresponded to a new major technological breakthrough). In particular, this correlates very well with an idea that was developed long ago by Taagepera (1976, 1979), Kremer (1993), Podlazov (2000, 2017), and Tsirel (2004) who demonstrated that the global technological growth rate is to be proportional to the global population, and who explained the hyperbolic growth of the world population through this point.

Let us recollect a rather apt formulation of the second assumption of the abovementioned models suggested by Michael Kremer: the “high population spurs technological change because it increases the number of potential inventors… All else equal, each person's chance of inventing something is independent of population. Thus, in a larger population there will be proportionally more people lucky or smart enough to come up with new ideas" (Kremer 1993: 685); hence, "the growth rate of technology is proportional to total population" (Kremer 1993: 681).

We have already mentioned that for the human part of the Big History the “phase transitions” identified by Panov correspond rather closely to the major technological breakthroughs in the human

[19] And what we called “global complexity growth rate” (e.g., Korotayev 2024, 2025).

history; hence, for the human part of the Big History the global complexity growth rate detected in this series can well be regarded as a proxy for the global technological development rate. Thus, already the theory developed by Taagepera, Kremer, Podlazov, and Tsirel would allow to expect that for the human part of the Big History we should find a rather high correlation between the size of the world population and the global complexity growth rate (≈ global technological development rate). However, I am sure that Taagepera, Kremer, Podlazov, and Tsirel themselves will be a bit surprised to see that their theoretical expectation finds such a strong support ($r$ = 0.997, $p$ < 0.001) for their hypothesis in a test employing an apparently rather imperfect proxy of the global technological growth rate (see Figs. 10 and 11):

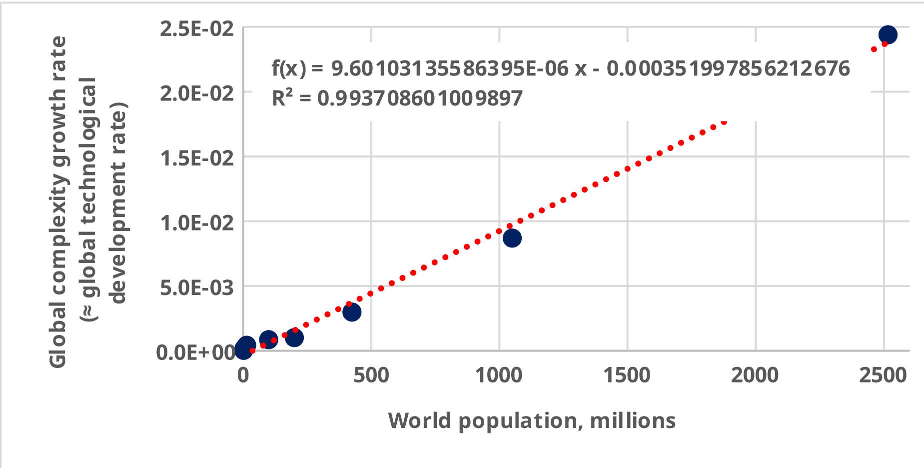


**Figure 10**. Correlation between the world population and the global complexity growth rate (≈ global macrotechnological development rate) in the Panov series, scatterplot with a fitted regression line, **natural scales** (*source of data on the historical world population estimates*: Kremer 1993: 683).

In the double logarithmic scale this correlation looks as follows (see Fig. 11):

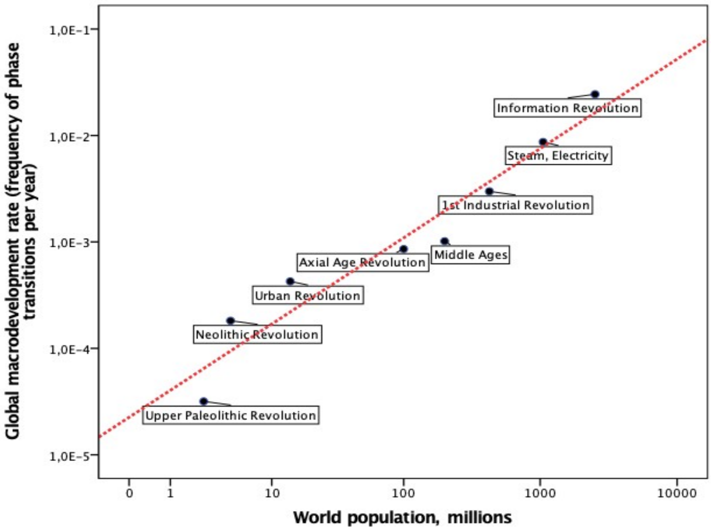


**Figure 11**. Correlation between the world population and the global complexity growth rate (≈ global technological development rate) in the Panov series, scatterplot with a fitted regression line, **double logarithmic scale** (*source of data on the historical world population estimates*: Kremer 1993: 683).

It appears that it is this very high correlation predicted by the abovementioned models that explains the point that von Foerster – Kapitza hyperbolic equation (8) (that describes the dynamics of the world population growth) is almost identical to equation (4) above (that describes the dynamics of the global complexity growth rate / global technological growth rate). Indeed, if, throughout most of the human history, the global growth rate of technology was proportional to total population of the world, the equations describing dynamics of both variables should be almost identical (differing only by the values of constants). And, as we see, this was just the case.[20]

[20] The point that the growth of the global complexity throughout biological and sociocultural eras of the Big History can be described by a single equation of (4) type seems to be accounted for by the point that the hyperbolic character of global biological complexity growth (measured through the biodiversity proxy) can be similarly accounted for by a non-linear second-order positive feedback between the diversity growth and community structure complexity (which suggests the presence within the biosphere of a certain analogue of the collective learning mechanism) (Markov and Korotayev 2007, 2008; Korotayev and Markov 2014, 2015; Grinin et al. 2013, 2025, 2026). This feedback appears to work via two parallel mechanisms: 1) decreasing extinction rate (more taxa – higher is the

Note also that we have shown analytically that equation (4) of the hyperbolic acceleration of the global complexity growth rate / world technological growth rate can be derived from the von Foerster – Kapitza equation (8) of the world population growth (Korotayev 2020a: 581–582; 2020b: 59–60).

## 7. The Singularity as a Transition Point, Not an Infinite Value

But how seriously should we take the prediction of "singularity" contained in mathematical models such as Eq. (4) ? Should we really expect with Kurzweil (2001, 2005) that around 2029[21] we should deal with a few orders of magnitude acceleration of the technological growth – indeed, predicted by equation y = C/(2029–t)[22] describing the hyperbolic acceleration in the time series published at page 20 of *Singularity is Near* (see Fig. 1 above) if we take this equation literally[23]?

I do not think so. This is suggested, for example, by the empirical data on the world population dynamics. As we remember, the global population growth acceleration pattern discovered by Heinz von Foerster is identical with planetary macroevolutionary acceleration patterns of Panov and Modis – Kurzweil, and it is characterized by the singularity parameter (2027 CE) that is simply identical for Panov and has just 2-year difference with Modis – Kurzweil. However, what

alpha diversity, or mean number of taxa in a community – communities become more complex and stable – extinction rate decreases – more taxa, and so on), and 2) increasing origination rate (new taxa facilitate niche construction; newly formed niches can be occupied by the next 'generation' of taxa). The latter makes the mechanisms underlying the hyperbolic growth of global biological complexity / biodiversity and human population / global sociocultural complexity even more similar, because the total ecospace of the biota is analogous to the 'carrying capacity of the Earth' in demography. As far as new species can increase ecospace and facilitate opportunities for additional species entering the community, they are analogous to the 'inventors' in the macrohistorical demographic models whose inventions increase the carrying capacity of the Earth. The hyperbolic growth of the Phanerozoic biodiversity suggests that 'cooperative' interactions between taxa can play an important role in evolution, along with generally accepted competitive interactions. Due to this 'cooperation' (~ 'collective learning'?), the evolution of biodiversity acquires some features of a hyperbolically self-accelerating process. The same naturally refers to cooperation/collective learning as regards the global sociocultural evolution. The discussion above suggests that we can trace rather similar macropatterns within both the biological and social phases of Big History which allows to describe the hyperbolic growth of complexity through biological and social phases of the Big History with a single mathematical model.

21 See Ranj 2016.

22 See, e.g., Korotayev 2018, 2020a, 2020b.

23 This is done, for example, by Nazaretyan (2017a, 2017b, 2018, 2020).

are the grounds to expect that by Friday, November 13, A.D. 2026 the world population growth rate will increase by a few orders of magnitude as is implied by von Foerster equation?[24] The answer to this question is very clear. There are no grounds to expect this at all. Indeed, as we showed quite time ago, “von Foerster and his colleagues did not imply that the world population on [November 13, A.D. 2026] could actually become infinite. The real implication was that the world population growth pattern that was followed for many centuries prior to 1960 was about to come to an end and be transformed into a radically different pattern. Note that this prediction began to be fulfilled only in a few years after the "Doomsday" paper was published” (Korotayev 2008: 154).

Indeed, starting from the early 1970s the world population growth curve began to diverge more and more from the almost ideal hyperbolic shape it had before (compare Fig. 4 and Fig. 11) (see, e.g., Kapitza 2003, 2006, 2010; Livi-Bacci 2012; Korotayev, Malkov, and Khaltourina 2006a, 2006b; Korotayev, Goldstone, and Zinkina 2015; Grinin and Korotayev 2015; UN Population Division 2024), and in recent decades it has been taken more and more clearly decelerating shape – the trend towards hyperbolic acceleration has been clearly replaced with the quasi-logistic slow-down, with a clear perspective of transition to a negative population growth rate (see Fig. 12):

[24] Note that these lines are written in September 2026.

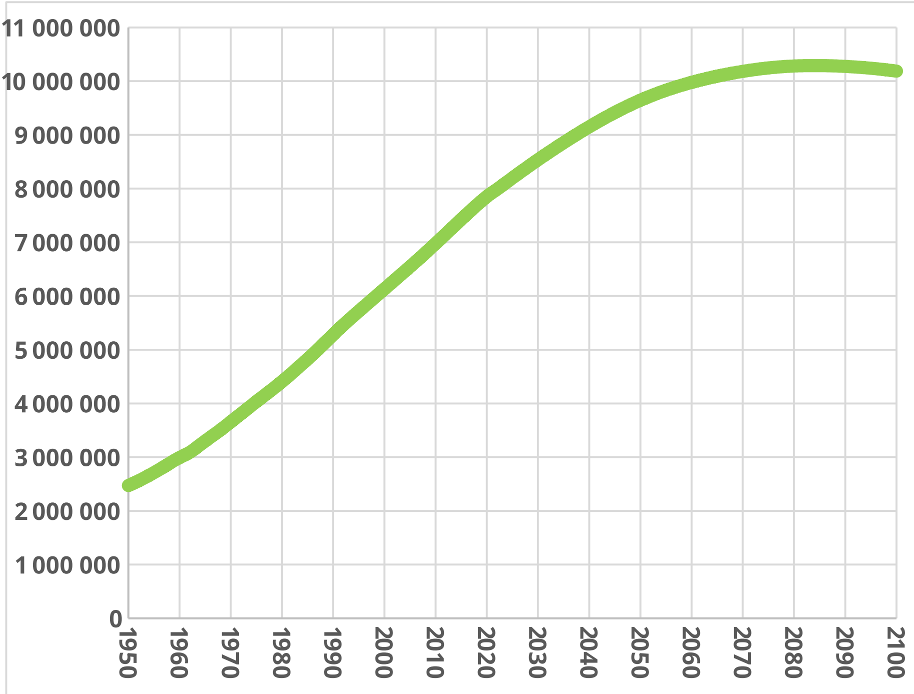


**Figure 12.** World population dynamics (billions), empirical estimates of the UN Population Division for 1950–2023 with its middle forecast to 2100. *Data source*: UN Population Division 2024.

In some respect, it may be said that von Foerster did discover the singularity of the human demographic history; it may be said that he detected that the human World System was approaching the singular period in its history when the hyperbolic accelerating trend that it had been following for a few millennia (and even a few millions of years according to some) would be replaced with an opposite decelerating trend. The process of this trend reversal has been studied very thoroughly by now (see, e.g., Vishnevsky 1976, 2005; Chesnais 1992; Caldwell et al. 2006; Khaltourina and Korotayev 2007; Korotayev, Malkov, and Khaltourina 2006a, 2006b; Korotayev 2009; Gould 2009; Dyson 2010; Reher 2011; Livi-Bacci 2012; Choi 2016; Podlazov 2017; Adams 2022; Bao 2022) and is known as the "global demographic transition" (Kapitza 2003, 2006, 2010; Podlazov 2017). Note that, in case of global demographic evolution, the transition from the hyperbolic acceleration to a quasi-logistic deceleration started a few decades before the singularity point mathematically detected by von Foerster.

There are all grounds to maintain that the deceleration of planetary macroevolutionary development / global complexity growth rate has also already begun – and it started a few decades before the singularity time points detected both in Panov and Modis – Kurzweil time series.[25] This is well supported by the growing body of evidence suggesting the start of the long term deceleration of the global techo-scientific and economic growth rates in the recent decades[26] (see, e.g., Krylov 1999, 2002, 2007; Huebner 2005; Khaltourina and Korotayev 2007; Maddison 2007; Modis 2002, 2005, 2012, 2020; Akaev 2010; Gordon 2012; Teulings and Baldwin 2014; Piketty 2014; LePoire 2005, 2009, 2013, 2015a, 2015b, 2020a, 2020b).[27]

## 8. The 21st Century Singularity in Big History Perspective

So, in some respects, one can well talk about the 21st century singularity, as our century can be called really singular in the sense that in our century we see the transition of the global complexity evolution from the hyperbolic trajectory that it followed for billions of years since the emergence of life on our planet to a qualitatively different pattern whose trajectory is not clear yet, but about which

25 As has been rightly noted by an anonymous referee of this paper, beyond population growth changing, technology might have continued acceleration for a longer period. The slowing of technology (or speeding up of social response, e.g., Harold Linstone's choice), might have very important implications – no longer unlimited exponential growth, increased diffusion of innovative capacity, and convergence of world economies.

26 An anonymous reviewer of this article justly notes that it is interesting that we are just discovering this (billions of years-old) trend just as it must come to an end. Just like falling from a tall building with rapid continuous acceleration toward the ground, the person (at least in a vacuum) would not notice the fall until they approach the "end" of the pattern, i.e., hit the ground.

27 However, it should be noted that the point that in the “singularity” / inflection point zone we observe the highest macroevolutionary development rates suggests that around the 21st century “singularity” we could expect a “singular” phase transition, a new major Big History Threshold that might be comparable with, say, Big History Threshold 6 (“Emergence of Humans and Collective Learning” [Christian 2008]) rather than with an ordinary phase transition from the Panov or Modis – Kurzweil lists (e.g., Korotayev 2020a; Akaev et al. 2025). What could this be? A substantial change of the human biological nature / transhuman transition? Strong AI? Emergence of the global brain? All these (alone or together) would mean that the human history as we know it would be replaced with some other history (though in no way can this be predicted by calculating mathematic singularities). As has been mentioned, this seems to be connected with the point that around the 21st century “singularity” / inflection point the rate of global macrodevelopment has reached its maximum values, which seems to be able to produce in the forthcoming decades a singular global phase transition even against the declining (but still extremely high) technological growth rates.

we can be perfectly sure that it will not be hyperbolic anymore (see, e.g., LePoire 2013, 2014, 2015a, 2015b, 2016, 2020a, 2020b).

Still, moving on to the conclusion, I would like to emphasize that I have no doubt the Heinz von Foerster and his colleagues made in 1960 a really outstanding discovery. Indeed, they showed for the first time that the long-term dynamics of at least one of the most important characteristics (population $N$) of the most complex social system (the humankind) can be described mathematically with an astonishingly high accuracy by extremely simple equations. Later it has been shown that equally simple equations can describe many other characteristics of global development including the world GDP, urbanization, literacy, rate of global complexity growth, in general, world technological growth rate, in particular, and so on (Taagepera 1976, 1979; Johansen and Sornette 2001; Modis 2002; Panov 2005a, 2005b; Korotayev et al. 2006a, 2006b, 2015, 2016; Ekstig 2010a, 2010b, 2012; Korotayev and Grinin 2013; LePoire 2013, 2016, 2020b; Korotayev and Malkov 2016; Korotayev 2018, 2020b, 2024, 2025; Fomin 2020; Bellina et al. 2025).

In no way the point that von Foerster and his colleagues chose the "doomsday" metaphor to attract attention to their really important discovery does invalidate this discovery. This was first of all a piece of scientific humor, a joke; and I don't think von Foerster and his colleagues should be blamed for the point that some people took this joke too seriously.

The "doomsday" metaphor stemmed from the point that the equation that was found by von Foerster to describe so perfectly the dynamics of the world population possessed a mathematical singularity. However, as we could see above, there is nothing miraculous about this point. As we could see above, the presence of the second order positive feedback between population growth and technological development (discovered by Taagepera [1976, 1979] and Kremer [1993]) led to the fact that throughout most of the human history the absolute population growth rate was in long term proportional to population squared. Above we could also see that if the absolute population growth rate is proportional to population squared, the population growth follows a trajectory best described by an equation that includes a singularity time/date ($t_0$) as a parameter, that is it follows a trajectory that tends toward infinity within a finite time ("goes to infinity at $t_0$").

In no way, such a phenomenon is restricted to the world population, or similar characteristics of the global system. In fact, it is very well known, e.g., in chemistry and physics where it is usually denoted as “blow-up regimes”. Thus, blow up regimes are found: (1) in autocatalytic reactions; for example, in the simplest autocatalytic scheme, $A + 2B \rightarrow 3B$, the rate of production of the autocatalyst $B$ is proportional to the square of its concentration: $d[B]/dt = k \cdot [B]^2$. The solution, $[B](t) = 1/(t_0 - t)$, diverges to infinity at a finite time $t_0$; (2) in high-order autocatalysis with diffusion ("blow-up"); (3) in thermal explosion (S-mode / Frank-Kamenetskii Theory); (4) in chemotaxis (Keller–Segel model); (5) in polymerization and gelation – in branching polymerization reactions, the number of active centers can grow hyperbolically, leading to a sudden divergence of viscosity at the gel point – a finite-time singularity in the sol–gel transition; (6) in non-linear heat conduction – in media where thermal conductivity depends strongly on temperature, the heat equation can yield solutions where temperature becomes infinite in a finite time – a regime of thermal blow-up relevant to plasma physics and laser heating; (7) in magnetic X-point collapse – in 2D incompressible dissipationless Hall magnetohydrodynamics (MHD), solutions exhibit finite-time singularities associated with magnetic X-point collapse. The Hall effect leads to the formation of finite-time singularities of the velocity derivatives and the electric current density at the magnetic neutral line; and so on (Kumar and Mondal 2026; Litvinenko and McMahon 2015; Marras et al. 2025; Olwoch and Needham 2000; Samarskii et al. 2011; Stadler et al. 2000; Taira 2001).

Of course, in all real physical and chemical systems, the singularity (infinity) is never actually reached. Before that point, other mechanisms – such as changes in state, radiative losses, relativistic corrections, or dissipative effects – intervene to "cut off" the growth and transition the system into a new qualitative state. As we see, in this respect “blow-up regimes” found in the global development (including the hyperbolic pattern of the world population growth discovered by von Foerster and his colleagues in 1960) are not so much different from the blow-up regimes found in physical and chemical systems.

## 9. Conclusion: The Scientific Significance of von Foerster's Discovery

The analysis presented in this article reveals a remarkable and robust pattern: the dynamics of global complexity growth across billions of years of Earth's biological and cultural evolution follows a hyperbolic trajectory that is mathematically indistinguishable from the hyperbolic growth pattern of world population discovered by von

Foerster and his colleagues in 1960. Both phenomena are governed by equations of the form $y = C/(2027 - t)$, with singularity dates that converge on the third decade of the 21st century.

The explanation for this striking correspondence lies in the operation of a nonlinear second-order positive feedback mechanism between demographic growth and technological development. Throughout most of human history, population growth and technological innovation have mutually reinforced each other through a dynamic relationship where each serves simultaneously as cause and effect of the other. As Taagepera – Kremer principle articulates, larger populations generate proportionally more inventors, accelerating technological change, which in turn raises carrying capacity and enables further population growth. This positive feedback loop, formalized through the coupled differential equations of the Verhulst-Pearl logistic model and Taagepera – Kremer technological growth equation, naturally generates hyperbolic "blow-up" regimes – the same mathematical phenomenon observed in autocatalytic chemical reactions, thermal explosions, and other physical systems of this kind.

However, the mathematical singularity – the point at which the equations project infinite values – should not be interpreted literally. Just as in physical and chemical systems, other mechanisms intervene before infinity is reached, cutting off the growth and transitioning the system into a qualitatively new state. For the world population, this transition began in the early 1970s, when the hyperbolic acceleration gave way to logistic deceleration – the well-documented global demographic transition. There is growing evidence that a similar deceleration has already commenced in global techno-scientific and economic growth rates, suggesting that planetary macroevolutionary development is undergoing a transition analogous to, though not identical with, the demographic transition.

Thus, the 21st century can indeed be called "singular" – not because it will witness infinite growth or technological transcendence of biological limits, but because it marks the transition of global complexity evolution from the hyperbolic trajectory that has characterized it since the emergence of life on Earth to a qualitatively different evolutionary regime whose contours remain uncertain but whose departure from hyperbolicity is increasingly evident.

The enduring significance of von Foerster's "Doomsday" paper lies not in its apocalyptic metaphor but in its demonstration that the most complex social system known to humanity – the global human population – could be described with extraordinary accuracy by an astonishingly simple mathematical equation. This discovery, later extended to world GDP, urbanization, literacy, the global complexity growth rate and so on, revealed that beneath the apparent chaos of historical events there exist deep structural patterns amenable to rigorous quantitative analysis. The "doomsday" framing was scientific humor, a rhetorical device to draw attention to a genuinely important insight; it should not detract from the scientific achievement it accompanied.

The convergence of independently compiled time series – Panov's phase transitions, Modis-Kurzweil's canonical milestones, and von Foerster's population data – all pointing toward the same mathematical structure and the same temporal horizon, suggests that we are observing a real phenomenon: the culmination of a macroevolutionary trend that has been accelerating for billions of years. Whether the transition that lies ahead will be smooth or turbulent, gradual or abrupt, is a question that remains open. But that such a transition is underway, and that our century is uniquely positioned at this inflection point, is a conclusion that the evidence supports with remarkable consistency.

In this sense, the study of hyperbolic acceleration in global complexity growth does not foretell doom – it illuminates the extraordinary dynamic that has shaped life, humanity, and civilization, and it challenges us to understand the new trajectory upon which our species now embarks.

## Appendix: Mathematical Derivation

Let us show analytically that the algebraic hyperbolic equation Eq. (6a)

$$N_t = \frac{C}{t_0 - t} \qquad (6a)$$

can be regarded as a solution of the following differential equation:

$$\frac{dN}{dt} = \frac{N^2}{C} \qquad (9)$$

where $C$ is a constant.

Take equation (9). Separate the variables (that is, move everything containing $N$ to one side of the equation, everything containing $t$ to the other) to get:

$$\frac{dN}{N^2}=\frac{dt}{C} \tag{A}$$

Through integration get:

$$\int\frac{dN}{N^2}=\int\frac{dt}{C}+const \; . \tag{B}$$

As is known,

$$\int\frac{dx}{x^n}=\frac{x^{1-n}}{1-n}+some\,constant \text{ if } n\neq 1 \; .$$

Thus (omitting the constant),

$$\int\frac{dN}{N^2}=\frac{N^{1-2}}{1-2}=\frac{N^{-1}}{-1}=\frac{-1}{N} \; ,$$

hence,

$$-\frac{1}{N}=\frac{t}{C}+const$$

where *const* is an arbitrary constant of integration (which absorbs constant from Eq. (C)).

Thus,

$$\frac{1}{N}=\frac{-t}{C}-const \; .$$

$$N=\frac{1}{\frac{-t}{C}-const} \; .$$

Let $const_1$ = – *const* (another arbitrary constant). Then:

$$N_{(t)} = \frac{1}{const_1 - \frac{t}{C}}$$

and

$$N_{(t)} = \frac{C}{C \bullet const_1 - t}.$$

Finally, let $C \bullet const_1 =$ $t_0$ (a new constant). Then the general solution is just:

$$N_t = \frac{C}{t_0 - t} \tag{6a}$$

where $t_0$ is a constant dependent on the initial conditions.

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