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Ernest O. Lawrence

Radiation Laboratory

PION-MOMENTUM SPECTRUM FROM K^- ABSORPTION IN HELIUM

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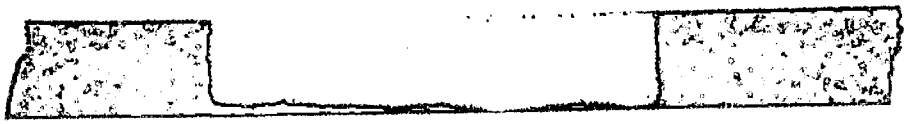

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Berkeley, California

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PION-MOMENTUM SPECTRUM FROM K^- ABSORPTION IN HELIUM

Jack Leitner

May 6, 1959

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ABSTRACT

The pion-momentum spectrum from K^--absorption stars in helium is calculated in the impulse approximation. The kaon is assumed to be pseudoscalar and to be captured from the 1s state. A final state consisting of free particles is used. The effect of final bound states is considered and taken into account phenomenologically. All final-state interactions are neglected.

It is shown that the shape of the momentum spectrum is determined mainly by kinematics--i.e., the shapes and positions of the spectral peaks are quite insensitive to detailed dynamical assumptions such as the assumed K parity and capture orbit. The effect of final-state interactions is discussed.

PION-MOMENTUM SPECTRUM FROM K^- ABSORPTION IN HELIUM

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INTRODUCTION

It has recently been found that Λ^o's emerge from K^- absorption stars in helium in great profusion.[1] It is possible to estimate the expected rate of Λ^o production from the reaction

(a) $$K^- + He^4 \rightarrow \Lambda^o + \pi + 3N$$

on the basis of the K^--p absorption results[2] and charge independence. This expected rate is ~ 8%; the observed rate is larger, by a factor of roughly 4. It would seem then that a large proportion of the emergent Λ^o's originates from K^- interactions which produce a Σ, with a subsequent Σ-nucleon interaction giving rise to a Λ^o,[3] according to the reaction

(b) $$K^- + He^4 \rightarrow \Sigma + \pi + 3N \; ; \; \Sigma + N \rightarrow \Lambda^o + N.$$

In order to estimate the relative contributions of (a) and (b) to the total Λ^o production, the pion-momentum spectrum from K absorptions yielding neutral hyperons is studied. In the absence of internal motion, the momentum spectrum from (a) and (b) would show sharp spikes at 244 Mev/c and 160 Mev/c respectively. The initial-state internal motion, detailed dynamics of the process, etc. broaden the spectrum into two peaks of finite width. The purpose of this study is to calculate the spectrum explicitly and investigate its dependence on detailed dynamical assumptions.

The emergent pion spectrum of Reactions (a) and (b) is calculated in the impulse approximate.[4] The kaon is taken to be spin O and pseudoscalar.[5] All final-state interactions are neglected; 1s-state K capture is assumed.[6] A final state consisting of free particles is used. The effect of final bound nucleon states is considered and taken into account phenomenologically.

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* On leave from Syracuse University

It is shown that the shape of the momentum spectrum is essentially determined by kinematics. The spectral peak widths are quite insensitive in shape and position to dynamical assumptions, i.e., they are little affected by the relative importance of terms in the transition matrix or by details of the internal motion. The spectrum is likewise insensitive to the assumed K parity and capture orbit. The effect of final-state interactions is discussed.

DESCRIPTION OF CALCULATION

In the laboratory system, the matrix elements M, for both processes (a) and (b),

$$K^- + He^4 \rightarrow \pi + \left(\begin{matrix}\Lambda^o \\ \Sigma\end{matrix} + 3N\right),$$

is $M = \langle \chi_f \int\int \psi_f^* (r_j, r_\pi)\, T(\vec{p}_j, \vec{k}, \vec{q}, \vec{\sigma}_1)\, \phi_{n\ell}(|\vec{r}_k - R|)\, \psi_i(r_j, r_k) d\vec{r}_j \,|\, \chi_i \rangle$,

where ψ_i = initial helium wave function,

ψ_f = final-state wave function,

$\phi_{n\ell}$ = K meson bound-state wave function,

T = a transition operator, the form of which depends upon the dynamics of the process,

χ_i, χ_f^o = appropriate spinors,

$\vec{r}_j$ = baryon lab coordinate j = 1, 2, 3, 4,

$\vec{r}_\pi$ = pion lab coordinate j = 1, 2, 3, 4,

$\vec{r}_k$ = kaon lab coordinate j = 1, 2, 3, 4,

$\vec{k}$ = pion lab momentum,

$\vec{q}$ = kaon lab momentum,

$\vec{p}_j$ = center-of-mass momentum of baryon j in the final-state four-baryon system (4 BS).

See Fig. 1 for a diagram of the coordinate systems.

For the purposes of calculation, it is convenient to transform from $\vec{r}_j$ to a particular set of relative coordinates $\vec{\rho}_2, \vec{\rho}_3, \vec{\rho}_4, \vec{R}$ (where, in accordance with the impulse approximation, the K capture is presumed to take place on nucleon No. 1).

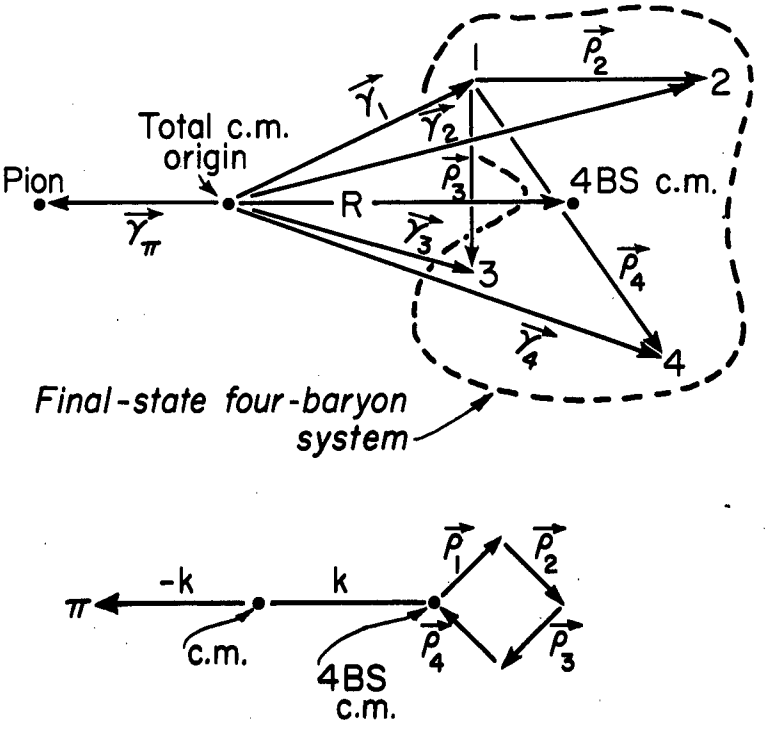

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Fig. 1

This transformation is defined by:

$$\begin{aligned}
\vec{\rho}_2 &\equiv \vec{r}_2 - \vec{r}_1, \\
\vec{\rho}_3 &\equiv \vec{r}_3 - \vec{r}_1, \\
\vec{\rho}_4 &\equiv \vec{r}_4 - \vec{r}_1, \\
\vec{R} &= \frac{m_Y + m(\vec{r}_2 + \vec{r}_3 + \vec{r}_4)}{M},
\end{aligned} \tag{1}$$

where m = nucleon mass,

m_Y = hyperon mass, and

$M = 3m + m_Y$.

In terms of these variables, M becomes

$$M = \left\langle \chi_f \left| \int \psi_f^*(\rho_j)\, T(\vec{\rho}_j, \vec{q}, \vec{k}, \vec{\sigma},)\, \phi_{n\ell}\, \psi_i(\rho_j) d\vec{\rho}_1 d\vec{\rho}_2 d\vec{\rho}_3 \right| \chi_f \right\rangle . \tag{2}$$

The ingredients of M are now considered in the order in which they appear above.

A. The Helium Wave Function

The helium wave function ψ_i is given by Dalitz;[7] ψ_i is taken as a Gaussian function of the relative coordinates $|\vec{r}_i - r_j|$, $\psi_i = e^{-\bar{a}\Sigma |\vec{r}_i - \vec{r}_j|^2}$, where $\bar{a}$ is determined by the condition that the nucleon density fit the data of Hofstader's electron-helium scattering experiment,[8] yielding $\bar{a} = \frac{9}{32R^2}$ $_{He}$, (R_{He} = 1.44 fermi). With the transformation (1), ψ_i becomes

$$\psi_i = e^{-\bar{a}\sum_j \rho_j^2}, \text{ for } j = 2, 3, 4. \tag{3}$$

B. The Transition Operator and K Wave Function

The most general form of the transition operator ρ can be written in terms of vector products created from all vectors entering the problem; see, for example, the work of Pais and Trieman.[9] The transformation properties of the vector products contained in T are determined by the condition that M be scalar. Since the pion is pseudoscalar, if the relative $K\Lambda$ parity is odd, T must transform as a scalar. We assume here that this is indeed the case, although the final results are insensitive to this assumption.

The most general scalar form of T is rather complex:

$$T = A + B\Sigma p_j^2 + C k^2 + D\Sigma\vec{p}_j \cdot \vec{k} + E\,\Sigma\vec{\sigma}\cdot\vec{p}_j X\vec{k}$$
$$+ H\,\Sigma\vec{p}_j \cdot \vec{q} + J\,\vec{k}\cdot\vec{q} + K\Sigma\vec{\sigma}\cdot\vec{p}_j X\vec{q} + L\vec{\sigma}\cdot\vec{k}X\vec{q}.$$

This expression becomes considerably less formidable as a result of our physical approximations.

T is an operator acting on the K wave function. Since in the impulse approximation T is different from 0 only at $\vec{r}_1$ (the position of the nucleon on which the process is taking place), $T\phi_{ne}$ must be evaluated at r_1. Physically, since the kaon coordinate corresponds to a Bohr orbit of $a_0 \approx \frac{\hbar^2}{\left(\frac{mm_k}{m+m_k}\right)e^2} \approx 1510^{-13}$, which is ~ 10 times the helium radius, this is equivalent to evaluating $T\phi_{n\ell}(|r_k - r_1|)$ at 0.

If we consider 1s-state capture only, terms in T independent of $\vec{q}$ give $T\psi_{1s}(0) \sim 1$, and terms in T proportional to q give $T\nabla\psi_{1s}(0) \sim 0$. We therefore drop all terms proportional to $\vec{q}$ in considering s-state absorption.

Calculation of the rate of pion emission requires $|\langle \chi_f | [T\phi_{1s}]_0 |\chi_i\rangle|^2$. Expanding $(T\phi_{1s})_0^2$ and dropping higher-order terms ($\gtrsim k^3$), we get

$$|\langle T\phi_{1s}{}^0\rangle|^2 \sim a^2 + 2R_e\, a^*b\,(\Sigma p_j)^2 + R_e\, a^*c\, k^2$$
$$+ 2R_e\, a^*d\,\Sigma\vec{p}\cdot\vec{R} + 2R_e\, a^*f\,\Sigma\vec{p}_j\cdot\vec{p}_i. \qquad (4)$$

In the spirit of the impulse approximation, we further drop the terms proportional to $\vec{p}_i \cdot \vec{p}_j$, representing internal correlations within the 4 BS, and obtain the relatively simple expression

$$|\langle T\phi_{1s}{}^0\rangle|^2 \sim 1 + a k^2 + b\,\Sigma p_j \mu_j, \qquad (5)$$

where $\mu_i = \cos\theta_j$ and θ_j is the angle between $\vec{p}_j$ and $\vec{k}$.
The undetermined constants a, b, c representing detailed dynamics can be calculated only with further assumption. No attempt to calculate a, b, and c is made here in view of the lack of a reliable dynamical theory of strong interactions.[10]

C. The Final-State Wave Function

Neglecting final-state interactions (see p. 13), the final-state wave function can be written as $\psi_f = e^{-i \frac{\vec{k}}{\hbar} \cdot \vec{r}_\pi} \psi_{4BS}(r_j, p_j)$. It has been shown that the hyperon emerges as a free particle in ~99% of all K interactions.[5] The extent to which bound-nucleon states emerge from K stars is not well known yet, but is estimated to be small; ψ_f is therefore taken to be a free-particle-product wave function:

$$\psi_f^* = e^{\frac{i\vec{k} \cdot \vec{r}_\pi}{\hbar}} \quad e^{\frac{i\vec{k} \cdot \vec{R}}{\hbar}} \quad e^{-i \sum_j \vec{p}_j \cdot \vec{\rho}_j}.$$

In accordance with the impulse approximation, we evaluate

$$\vec{r}_\pi \text{ at } \vec{r}_1 = R - \frac{m}{M} \sum_j \vec{\rho}_j ;$$

then
$$\psi_f^* = e^{\frac{-i}{\hbar} \sum_j \vec{\rho}_j \cdot (\vec{p}_j + \frac{m}{M} \vec{k})}. \tag{6}$$

THE GENERAL FORM OF THE RATE

Evaluation of the matrix element is straightforward. From (3) and (6) and (2), we obtain

$$M = (2\pi)^3 \prod_j \int e^{-\frac{\vec{p}}{h} j \cdot (\vec{p}_j + \frac{m}{M} \vec{k})} \times \left| < T_j \phi_{1s} > \right|_0 e^{-\bar{a} p_j^2} p_j^2 \, dp_j \, d\mu_j^1 ,$$

where $\mu_j^1 = \cos \theta_j^1$,

$\theta_j^1 =$ angle between $\vec{\rho}_j$, $\vec{p}_j^0$,

$p_j^0 = \vec{p}_j + \frac{m}{M} \vec{k}$,

$\frac{m}{M} \vec{k} = \vec{k}_0$;

then
$$M = (2\pi)^3 < T \phi_{1s}^{(0)} > \prod_j V_j ,$$

where
$$V_j = \int_0^\infty \left[\int_{-1}^{1} e^{-i\rho_j |p_j^0| \mu_j^1} \, d\mu_j^1 \right] \exp\left(- \bar{a} \rho_j^2 \right) \rho_j^2 \, dp_j .$$

Letting $a \equiv \bar{a}\hbar^2 \cong .5410^4$, we have

$$M = \left(\frac{\pi}{a}\right)^9 e^{\left[-\sum_j (p_j^o)^2\right]} \left\langle T\, \phi_{1s}(0) \right\rangle ,$$

or, from (5),

$$M \sim \left[1 + a k^2 + b \sum_j p_j^2 + ck \sum p_j \mu_j\right] \exp -\sum_j \frac{\vec{p}_j + \vec{k}_o^{\,2}}{2a} \qquad (7)$$

Since only relative rates are calculated here, all multiplicative normalization factors will be dropped throughout the calculation.

The total rate for processes (a) and (b) is

$$R_T \sim \int |M(p_j)|^2 d\vec{p}_1\, d\vec{p}_2\, d\vec{p}_3\, \delta(E-E_f)\, d\vec{k},$$

where E is the initial energy and E_f is the final energy of the system. The relative rate $R(k)\, d\vec{k}$ for producing a pion with momentum $\vec{k}$ is then

$$R(k)\, d\vec{k} \sim d\vec{k} \int |M|^2\, \delta(E-E_f)\, d\vec{p}_1\, d\vec{p}_2\, d\vec{p}_3 \qquad (8)$$

Using (7) and (8), one can write the desired spectrum in the form

$$R(k)\, dk \sim \left[R_o(k) + aR_1(k) + bR_2(k) + cR_3(k)\right] dk, \qquad (9)$$

where

$$R_o = k^2 \prod_j \left[\int e^{-\frac{1}{2a}(\vec{p}_j + \vec{k}_o)^2} p_j^2\, dp_j\, d\mu_j\, \delta(E-E_f)\right]$$

$$R_1 = k^2 R_o$$

$$R_2 = k^2 \int \prod_j \left[e^{-\frac{1}{2a}(\vec{p}_j + \vec{k}_o)^2} p_j^2\, dp_j\, d\mu_j\right] \sum_e p_e^2\, \delta(E-E_f)$$

$$R_3 = k^3 \int e^{-\frac{1}{2a}(\vec{p}_j + k_o)^2} \sum p_j \mu_j \prod_j (d\mu_j\, p_j^2\, dp_j)\, \delta(E-E_f) .$$

(10)

The intergration of $R_j(k)$ is straightforward and the results are summarized as shown on the next page. The general form of $R_j(k)$ is

$$R_j(k) \sim k^r f(k)^{s/2} e^{-\frac{m}{a}\left[f(k) + \Delta\right]}$$

where r and s are integers and

$$f(k) \equiv m_k + m - my - B - \sqrt{k^2 + \mu^2} - \frac{k^2}{2M},$$

$$\Delta \equiv \frac{3m}{M}\frac{k^2}{2M}.$$

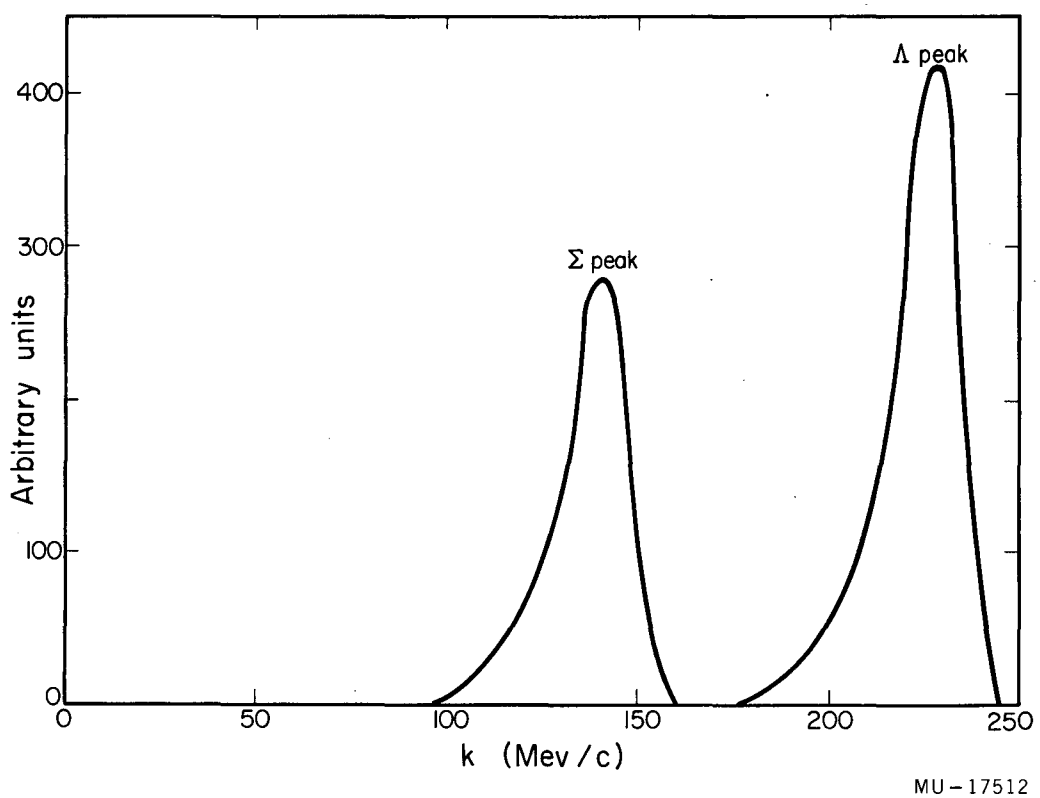
The zeroth order rate, $R_o(k)$, is evaluated explicitly in the Appendix; it is shown in Fig. 2. For comparison, the Λ peaks of both $R_o(k)$ and $R_1(k)$ are shown in Fig. 3. Mathematically, the similarity of all terms $R_j(k)$ is evident from (11). For low values of k, $R_j(k)$ is supressed by the factor k^r; $R_j(k)$ increases monotically up to the region of the maximum, where its behaviour is primarily determined by the exponential $\exp\left[\frac{-m}{a} f(k)\right]$. Note that all $R_j(k)$ are proportional to this term. For very large values of k -- i, e., $k \sim 0.95\, k_{max}$ -- the factor $f(k)^{s/2}$ rapidly damps the spectrum. Since all $R_j(k)$ are similar functions of k, the relative importance of each term -- i. e., the relative magnitude of a, b, and c -- does not seriously affect the total spectrum (9). R(k) can thus be represented to a good approximation by the function $R_o(k)$ alone.

DISCUSSION OF RESULTS

The insensitivity of the spectrum R(k), as calculated in the impulse approximation, to details of dynamics of both the initial state and the interaction can be easily understood on physical grounds. The effect of the Fermi-motion dynamics is apparent from the appropriate Fourier transform of the helium wave function:

$$|\phi(\theta_j p_j)|^2 \sim \prod_j e^{-\frac{p_j k_o}{a}} \cos\theta_j \prod_j e^{-\frac{p_j^{k_o}}{2a}}$$

$$\sim \prod_j e^{-510^{-3} p_j} \cos\theta_j \prod_j e^{-10^{-4} p_j^2}, \qquad (12)$$


Λ peak
Σ peak
400
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Arbitrary units
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200
250
k (Mev/c)
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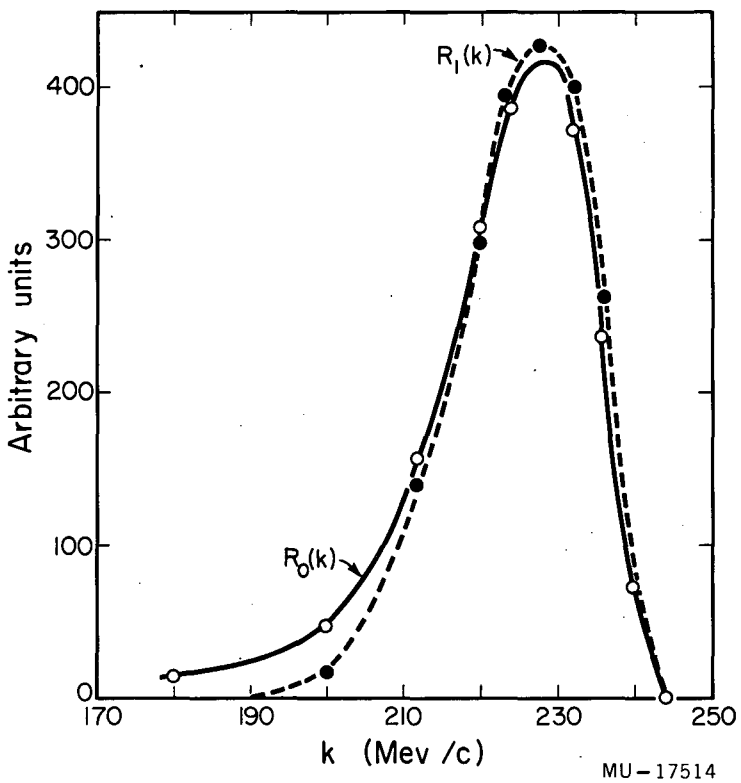
Fig. 2


$R_1(k)$
$R_0(k)$
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Arbitrary units
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190
210
230
250
k (Mev /c)
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Fig. 3

General Form of the Spectrum

$R_o(k)$	$= k^9 \, f(k)^{7/2} \, e^{-m/a}\left[f(k) + \Delta\right]$,
$R_1(k)$	$= k^{11} \, f(k)^{7/2} \, e^{-m/a}\left[f(k) + \Delta\right]$,
$R_2(k)$	$= k^{11}\left[f(k)^{9/2} + \frac{2}{33}(m/a2)^2 \left(\frac{m}{M}\right)^4 f(k)^{11/2} \, k^4\right] e^{-m/a}\left[f(k) + \Delta\right]$,
$R_3(k)$	$= k^9 \, f(k)^{7/2}\left[1 + \frac{\sqrt{2m^3}/2}{Ma} \, k + 1/3 \, \frac{m^{11}}{3M^5 a^5} \, k^5 \, f(k)\right] e^{-m/a(f+\Delta)}$,

where $f(k) \equiv m_k + m - m_y - B - \sqrt{k^2+\mu^2} - \frac{k^2}{2M}$,

$\Delta \equiv \frac{3m}{M} \frac{k^2}{2M}$.

For example, for $a = b = c = 1$, one has

$$R(k) \sim e^{-m/a}\left[f(k) + \Delta\right] k^9 \, f^{7/2} \left[2 + k^2 f + \frac{\sqrt{2m^3}/2}{Ma} \, k + \frac{1m^{11}}{3M^5 a^5} \, k^5 f^2\right] + \frac{2m^6}{33 a^2 M^4} \, k^6 \, f^2$$

where $|\phi|^2$ represents the distribution of internal momenta p_j and their c.m. angles θ_j which enter the problem. It is clear from the first exponential that $|\phi|^2$ is insensitive to the magnitude of the angles θ_j. Thus the pion spectrum is insensitive to the probability distribution of θ_j, i.e., to the detailed dynamics of the initial state. The second exponential in (12) shows that internal momenta ~ 200 Mev/c are of importance -- this corresponds to an average nucleon velocity $\beta_N \lesssim 0.2$, which in the impulse approximation is also the effective c.m. transformation velocity. Since the pion velocity ($\beta_\pi \sim 0.8$) is much larger than β_N, the pion-mementum spectrum R(k) is not seriously spread by the internal motion.

It should be stressed that this insensitivity persists no matter what the form of the transition matrix, since the spectrum is mainly determined by the impulse-approximation kinematics in which the K^- capture is considered a K^--N interaction.

It is not difficult to show explicitly that the assumptions of s-state capture and pseudoscalar kaon can be relinquished without any qualitative change in the pion spectrum.

We consider, firstly, the possibility of p-state capture. In this case, only terms in the transition matrix proportional to $\vec{q}$ will contribute to $R_{2p}(k)$, since

$$\phi_{2p}(0) \sim 0 \quad \text{and} \quad (\nabla\phi_{2p})(0) \sim \text{const.}$$

From (4) and (7), it follows that the 2p-state matrix element is

$$|M_{2p}|^2 \sim \left[dk^2 + e \Sigma p_j^2\right] e^{-\frac{\Sigma_j (p_j + k_o)^2}{2a}},$$

leading to essentially the same results as (9)

$$R_{2p}(k) \sim d\, R_1(k) + e\, R_2(k).$$

Secondly, we consider the possibility of a scalar K^-. In this case, the T matrix must transform as a pseudoscalar. The most general form for T is then

$$T\,(\text{scalar K}) \sim g\vec{\sigma} \cdot \sum_j \vec{p}_j + h\vec{\sigma} \cdot \vec{k} + \ell\vec{\sigma} \cdot \vec{q}.$$

From (8), this leads directly to a rate

$$R(k)_{\text{scalar } K} \approx g^2 R_2(k) + h^2 R_1(k) + 2\text{Re } g^* h R_3(k),$$

again, extremely similar to the spectrum (9).

It is clear from the above consideration that the pion-momentum spectrum is essentially determined by the kinematics of the K-N interaction and that its shape is insensitive to detailed dynamical assumptions within the framework of the impulse approximation.

The effect of bound-nucleon final-state production on R(k) is essentially a kinematical one. The only kinematic difference between the free-particle and bound-state problem is the available kinetic energy in the final states. This difference appears in the energy δ function (see Appendix). In the bound-state problem $\delta(E-E_f)$ becomes $\delta(f(k) + V - \epsilon_2 - \epsilon_3 - \epsilon_4)$, where V = final-state potential energy, $0 \leqslant V \leqslant 27$ Mev. To the extent, then, that internal correlations in the final bound states are ignored, the problem reduces exactly to the free-particle problem, with the exception that f(k) be replaced by f(k) + V.

$$\text{Therefore } R_o^{\text{bound}}(k) \approx k^9 \left[f(k) + V\right]^{7/2} \exp\left[\frac{-m}{a}(f(k) + V + \Delta)\right].$$

It is easily seen that in the production of bound final states, the peak position differs by only ~ 0.6 V Mev/c from its value in free-nucleon final-state production. Thus the qualitative features of the spectrum are not changed by the presence of bound final states.

The impulse-approximation theory of $\Lambda + \pi$ production described above is, without doubt, a severe oversimplification. The neglect of final-state interactions is, in particular, its most serious defect. This statement should be qualified—the omission of final-state pion interactions is not unreasonable, while a neglect of final-state Σ interactions is seriously inadequate. This is so simply because pion-nucleon interactions are important only near resonance, i.e., at a c.m. energy of ~200 Mev in the I = 3/2 state. Since energy conservation requires that the π-N c.m. energy be $\lesssim$ 70 Mev in both reactions (a) and (b), pion scattering is likely to be negligible. On the other hand, the Σ is known to interact quite strongly, as evidenced by the importance of the Σ-Λ conversion process. Further, Σ interactions could take place off the energy shell, which

of course would lead to a significant broadening of the emergent pion-momentum spectrum. An estimate of the importance of this effect has been made recently by Karplus and Rodberg[11] for K^--deuterium interactions. They find that widely different pion spectral shapes are indeed possible. Since Σ-Λ conversion is at least as important in helium as it is in deuterium, it seems likely that this spectral broadening should also appear in K^--He absorption. The significant question whether the broadening is sufficient to vitiate the usefulness of the spectrum in distinguishing between direct and indirect Λ production cannot at this time be answered by further calculation, but must be referred to experiment.

It can be hoped, in spite of its naivete, that the simple impulse-approximation theory is sufficient to describe Λ production at least qualitatively (this is the case in K^--D absorption). The fact that the calculated pion spectral shape is insensitive to all dynamical assumptions, excepting final-state Σ interactions, is an encouraging aspect of the theory, so far as it concerns the separation of direct and indirect Λ production.

The impulse-approximation spectrum can be adequately represented by the function $R_0(k)$ shown in Fig. 1. In order to more readily compare with experiment, where momentum measurement is limited to an accuracy of $\sim \pm$ 20 Mev/c, we have folded R(k) into an experimental resolution function,

$$\exp\left[-1/2\left(\frac{p_\pi - k}{20}\right)^2\right].$$

The resulting spectrum, $F(p_\pi)$, is shown in Fig. 4. Preliminary experimental results[1] are in reasonable agreement with $F(p_\pi)$.

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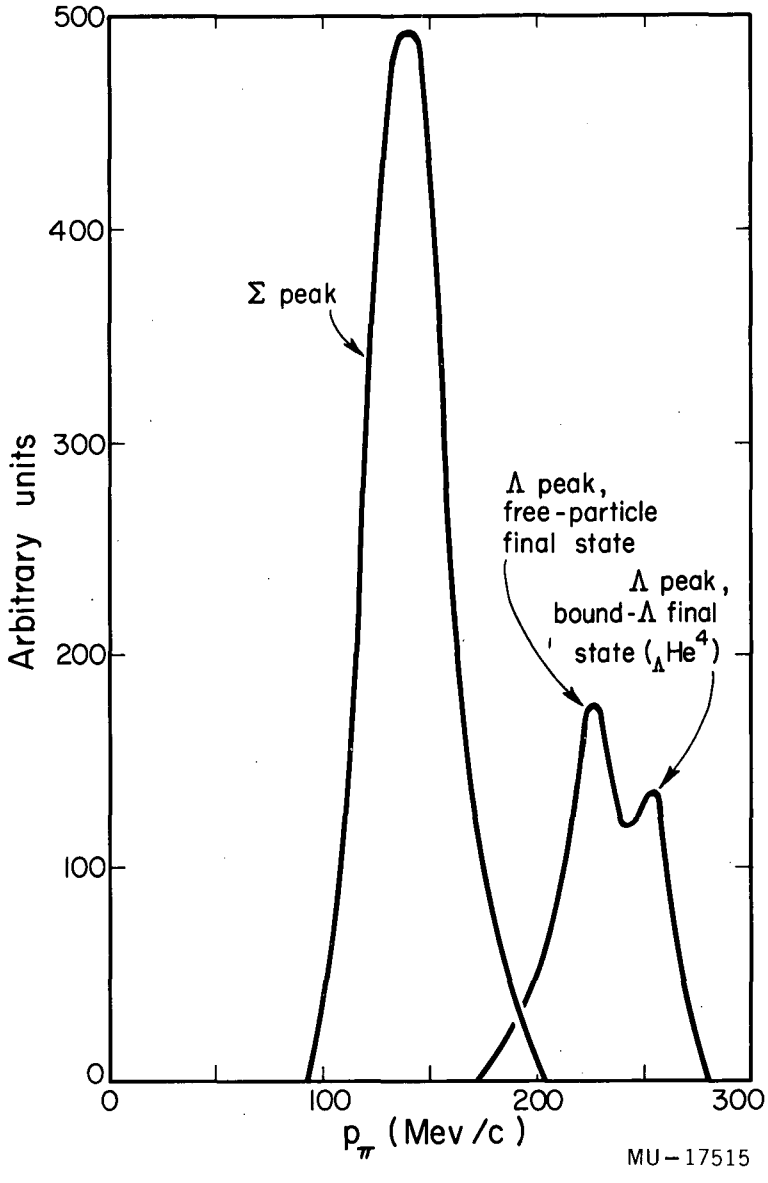
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state ($_{\Lambda}He^{4}$)
0
100
200
300
p_{π} (Mev/c)
MU-17515

Fig. 4

Appendix

Calculation of the Zeroth Order Spectrum

Letting $X_j^2 = \dfrac{\rho_j^2 + 2 k_o p_j \mu_j + k_o^2}{2a}$, we see, from (8),

$$R_o(k) \sim \frac{k}{k_o^3} \int_0^\infty \prod_j \left[\int_{p_j - k_o}^{p_j + k_o} e^{-x_j^2} X_j dX_j \, p_j \, dp_j \right] \delta(E - E_f)$$

$$\sim \frac{1}{k} \int \prod_j \left[e^{\frac{-\rho_j^2 + k_o^2}{2a}} \sinh\left(\frac{\rho_j k_o}{a}\right) p_j \, dp_j \right] \delta(E - E_f) .$$

Energy conservation gives

$$E - E_f \approx mk + m - m_y - B - \sqrt{k^2 + \mu^2} - \frac{k^2}{2M} - \sum_j \frac{p_j^2}{2M} ,$$

where B - He^4 binding = 27 Mev and μ = π mass.
Defining

$$f(k) = m_k + m - m_y - B - \sqrt{k^2 + \mu^2} - \frac{k^2}{2M} ,$$

and writing R_o in terms of the variables $\epsilon_j = \dfrac{p_j^2}{2M}$,

we have $R_o \sim \dfrac{\exp\left(-3/2 \dfrac{k_o^2}{a}\right)}{k} \int_0^\infty \prod_j \exp\left(-\dfrac{m}{a} \epsilon_j\right) \sinh\left(\dfrac{k_o}{a} 2m \epsilon_j\right)$

$$\delta (f - \epsilon_2 - \epsilon_3 - \epsilon_4) \, d\epsilon_j .$$

Writing the δ function in terms of a dummy variable ω, we have

$$\delta (f - \epsilon_2 - \epsilon_3 - \epsilon_4) = \int_{-\infty}^{\infty} e^{i\omega(f - \epsilon_2 - \epsilon_3 - \epsilon_4)} \, d\omega ;$$

R_o becomes

$$R_o \sim \frac{e^{-3/2\ \frac{k_o^2}{a}}}{k} \int e^{i\omega f} \prod_j \underline{\Phi}(\omega)\, d\omega,$$

where $\underline{\Phi}_j(\omega) = \int_0^\infty \exp\left[-\epsilon_j\left(\frac{m}{a} + i\omega\right)\right] \sinh\left(\sqrt{\frac{2m}{a}} k_o \sqrt{\epsilon_j}\right) d\epsilon_j$.

Letting $\tau = \frac{m}{a} + i\omega$, we find

$$\underline{\Phi}_i(\tau) \sim \frac{k}{\tau^{3/2}}\ e^{\left(\frac{mk_o^2}{2a^2}\right)}\ \frac{1}{\tau}, \quad \text{thus}$$

$$R_o \sim k^2\ e^{-\frac{m}{a}} \left[f + \left(\frac{3m}{M}\right)\frac{k^2}{2M}\right] \frac{1}{2\pi i} \int_{\beta-i\infty}^{\beta+i\infty} e^{ft}\ \frac{e^{\left(\frac{3k_o^2 m}{2a^2}\right)}}{\tau^{9/2}}\ \frac{1}{\tau}\, d\tau$$

This integral is, except for numerical factors, a well-known integral representation of a Bessel function[12] of order $\gamma = 7/2$. Let $\Delta = \frac{3m}{M}\ \frac{k^2}{2M}$ (note $\frac{\Delta}{f} << 1$ except for very high k).

Then $R_o \sim e^{-\frac{m}{a}(f+\Delta)}\ f(k)^{7/4}\ k^{11/2}\ i^{-7/2} J_{7/2}\left(i\sqrt{\frac{3k_o^2}{2a^2}}\ mf\right)$.

Noting that the argument of $J_{7/2}$ is $\lesssim 0.07$, we may take, to a very good approximation[13],

$i^{-7/2} J_{7/2}\left(i\sqrt{\frac{3k_o^2 mf}{2a^2}}\right) \sim \left(k\sqrt{f}\right)^{7/2}$. Then the rate becomes:

$$R_o(k) \sim k^9\ f(k)^{7/2}\ e^{-\frac{m}{a}} \left[f(k) + \Delta\right]$$

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