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Radiation Laboratory

A COMPUTER ANALYSIS FOR COMPLEX SODIUM IODIDE GAMMA SPECTRA

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ABSTRACT

A method of analysis for complex or continuous gamma spectra obtained from a 3-in. NaI(Tl) crystal detector is presented. The crystal response was calibrated with a number of sources. A method of interpolation for intermediate energies was then devised. An IBM 704 computer program was written to perform this interpolation and to obtain the incident gamma-ray spectrum by applying a matrix method. An option is provided in the program to allow correction for small gain-drift effects on the spectrum. Though written for a particular crystal and shield configuration, the program can easily be modified to accommodate other shielding arrangements and crystal sizes.

A. Introduction

For the detection of gamma radiation, a single NaI(Tl) crystal has advantages of high efficiency and simplicity; yet its response to radiation is complicated by the presence of the Compton tail on the spectrum and by other nonlinear effects. Discrete gamma lines can be resolved fairly quantitatively by inspection, if they are not too numerous; but more formal analysis is required for spectra containing large numbers of lines or for continuous spectra. This report describes a simple and flexible method of analysis in which the IBM 704 computer is used.

The Fortran listing of the program written for the spectrum analysis (or unfolding) is reproduced at the end of this report. The program can be modified quite easily to accommodate varying experimental conditions. A change in shield configuration from that used here may produce a different response in the detector, especially as the radiation scattered into the crystal is affected. Also, the accuracy of the program may be improved if necessary by fitting more complex functions to the parameters describing the detector response.

The method described here was developed for an experiment measuring gamma-ray yields in nuclear reactions. It was felt that a separate and slightly more extensive description of the spectrum analysis would be useful.

B. Calibration of the Crystal Response

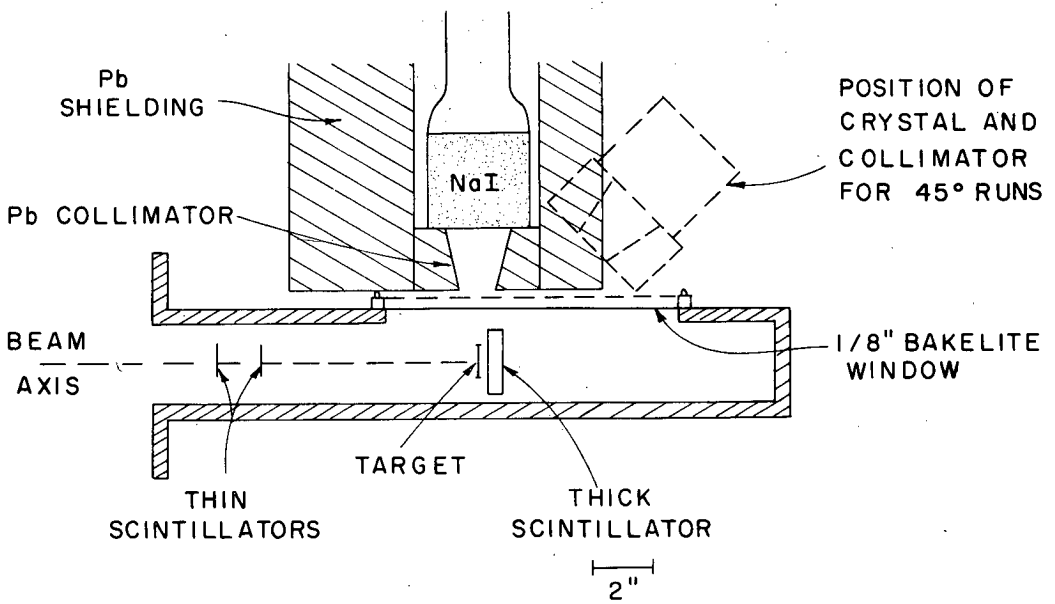
In order to correct for the non-unique response of the NaI(Tl) crystal to gamma radiation, its output pulse-height distribution for radiation of any energy must be known. Not only the photopeak efficiency but also the whole spectrum from a monoenergetic gamma line must be obtained. For arbitrary energies, the response may be found by interpolation between known responses

of standard sources. The over-all response is a function of the shield configuration. Our arrangement is illustrated in Fig. 1. To reduce the large backscattering peak resulting from the close proximity of shield and crystal, a 2-in.-thick collimator was placed in front of the crystal. The collimator also served to position the crystal at a reproducible distance from the source holder. A conical hole, tapering from a 2-in. diameter at the face of the crystal to a 1 1/4-in. diameter at the side near the source, minimized scattering at the inner edges of the collimator. To duplicate the conditions under which the bombardment spectra were obtained, the sources were located in the target holder, which was kept at the same position relative to the shield for each source.

The electronic equipment used in the calibration was relatively simple. The NaI(Tl) pulses were amplified in a DD2 linear amplifier and recorded on a Penco 100-channel pulse-height analyzer. Background spectra were taken after each source run. The sources used for calibration are listed in Table I.

The components into which the spectra were resolved included the photopeak, two escape peaks, the Compton distribution, the scattering peak, and the annihilation peak. The position, height (or area), and width of these components were fitted with functions of the gamma energy.

While the response of the crystal may be calculated theoretically, a strictly empirical approach was preferred because of possible effects of the shield and target block on the spectrum. The predicted proportionality of the half-width of the photopeak to the square root of the gamma energy was found to be a good approximation and was retained. The functional dependences were otherwise derived empirically. The components of the spectra and the dependence of their parameters on the gamma energy E are found in Table II.



MU-24507

1. Shield configuration: (a) perspective view of shield blocks; and (b) relation of shield to target block, shown from above.

Table I

Sources used in crystal calibration

Source	E_γ (Mev)
Am^{241}	0.060
Co^{57}	0.122
Na^{22}	0.51 (1.28 Mev not used)
Rb^{86}	1.08
Na^{24}	1.38
	2.75
Po - Be	4.45

Table II

Dependence of spectrum components on gamma energy E and pulse height x: E and x in Mev

Component	Condition		Height of component	Half-width
Photopeak height $P_1(E)$	$E < 0.65$		$P_1 = 4.59\times10^{-3}E^{-0.033}$	$0.0815\sqrt{E}$
	$E > 0.65$		$P_1 = 2.53\times10^{-3}E^{-1.46}$	(10% of E at E=0.66)
1st escape peak height $P_2(E)$	$E < 1.75$		$P_2 = 0$	Same as photopeak
	$E > 1.75$		$P_2 = 0.623 P_1 \ln(E/1.75)$	
2nd escape peak height $P_3(E)$	$E < 1.75$		$P_3 = 0$	Same as photopeak
	$E > 1.75$		$P_3 = 0.247\ P_1 \ln(E/1.75)$	
Annihilation peak height A(E)	$E < 1.75$		$A = 0$	
	$E > 1.75$		$A = 1.2\ [P_2(E)+P_3(E)]$	
Position of Compton edge c(E)	$E < 0.5$		$c = 0.6E$	
	$0.5 < E < 1.0$		$c = 0.9E-0.15$	
	$E > 1.0$		$c = E-0.25$	
Basic Compton height H(E)			$H = \frac{7.7\times10^{-3}(E+0.235)-1.806\times10^{-3}}{10(E+c)\ (E+0.235)}$	
Compton distribution height D(E,x)	$E < 1.75$	$x < c$	$D = H$	
		$c < x < E$	$D = H(E-x)/(E-c)$	
		$x > E$	$D = 0$	

Table II (continued)

Dependence of spectrum components on gamma energy E and pulse height x: E and x in Mev

Component	Condition		Height of component	Half-width
Compton distribution height D(E,x)	E> 1.75; 1.22< E-x		D=x	
	0.87< E-x< 1.22;	P_2< H	D=x	
		P_2> H	D=H+(P_2-H)(x-E+1.22)/0.35	
	0.51< E-x< 0.87		D=P_2	
	E-c< E-x< 0.51		D=P_2+0.1H(x-E+0.51)/(c-E+0.51)	
	0< E-x< E-c		D=(P_2+0.1H)(E-x)/(E-c)	
	E-x< 0		D=0	

Since accuracy of the response functions on the order of 10% was adequate for purposes of the calculation, no effort was made to derive more accurate parameters. It should be relatively easy to modify the program to incorporate more accurate functional dependences, should greater accuracy be desired, since this step of the calculation is handled separately in function subroutine CURVE.

C. Method of Analysis

The non-unique response of the NaI(Tl) crystal complicates the analysis of a complex spectrum. An efficiency correction cannot be applied separately to each interval of the spectrum because each contains Compton events from higher-energy gamma rays. It is necessary in any method of analysis to have a spectrum going nearly to zero at the high-energy end. Otherwise there will be contributions of uncertain magnitude from higher-energy photons to the portion of the spectrum being analyzed. Extrapolating the spectrum to one additional channel can provide an approximate correction for a small number of counts above the upper energy limit. The number of counts in this channel is then subtracted from all the channels being analyzed.

A stripping technique is the simplest method for unfolding a complex spectrum. Pulses in the highest energy interval are assumed to constitute a photopeak. Then the remainder of the pulse-height distribution associated with a photopeak of that size and energy is calculated. This distribution is subtracted from the initial spectrum and the process is repeated with the remainder until all the channels have been treated. A disadvantage of this method is that errors accumulate in the lower-energy channels.

Accordingly, matrix methods have been developed to unfold, or correct, NaI(Tl) gamma spectra.[2,3] In such procedures, the crystal response to gamma

radiation is represented by a matrix. One dimension provides the energy intervals and the other the pulse-height intervals. The rows then denote pulse-height distributions corresponding to the incident energies. This response matrix is inverted and multiplied by the vector representing the observed pulse-height distribution, to obtain the incident-energy distribution. Such a method was found satisfactory by Hubbell and Scofield in analyzing continuous bremsstrahlung spectra.[3]

The matrix may be derived from the response functions as follows:[4] We denote the response of the counter in counts per photon by $R(V,E)$, a function of both gamma energy E and pulse-height voltage V; if the incident spectrum of the photons is $N(E)$, the observed pulse-height distribution is

$$C(V) = \int_0^\infty R(V,E)\, N(E)\, dE.$$

In the matrix formulation, this equation appears as

$$\overline{C} = \overline{R} \times \overline{N}$$

where

$$R_{ij} = \int_{V_{i-1}}^{V_i} R(V, \overline{E}_j)\, dV,$$

$$R(V,\overline{E}_j) \approx \int_{E_{j-1}}^{E_j} R(V,E)dE \;/\; E_j - E_{j-1},$$

$$C_i = \int_{V_{i-1}}^{V_i} C(V)dV,$$

and

$$N_j = \int_{E_{j-1}}^{E_j} N(E)\, dE.$$

In theory, inversion of the matrix R leads to the initial spectrum:

$$\overline{N} = \overline{R^{-1}} \times \overline{C}.$$

While mathematically exact, this method is unsuitable where there are statistical fluctuations in the data and inaccuracy in the response function. It was found to fail completely with the experimental spectra. Presumably because of poor statistics at the high-energy end of the spectrum and the presence of discrete peaks there, large fluctuations between positive and negative values were seen in the unfolded spectrum.

In order to avoid such difficulties, an iterative method had been developed by Scofied, who replaced the matrix inversion by an iterative procedure.[4] Even in the face of the unfavorable conditions mentioned above, it was found to work well on experimental spectra.

In the iterative method, the vector representing the observed spectrum is multiplied by the response matrix to obtain a doubly folded spectrum. Each channel or element of the observed spectrum is individually corrected by a factor equal to the ratio of the original to the doubly folded value of that element. The corrected spectrum vector is multiplied by the response matrix and the ratios are again found, element by element. These ratios are applied as corrections to the originally observed spectrum (not to the first corrected spectrum). The process is repeated, applying correction factors each time to the original spectrum, until the desired degree of convergence is obtained.

If we denote the original observed spectrum by $\overline{C}_o$, the corrected observed spectra by $\overline{C}_i$, and the approximations to the incident gamma spectrum by $\overline{N}_i$, then

$$\overline{N}_1 = \overline{C}_o .$$

$$\overline{C}_1 = \overline{R} \times \overline{N}_1$$

$$\left(\overline{N}_2\right)_i = \frac{\left(\overline{N}_1\right)_i}{\left(\overline{C}_1\right)_i} \times \left(\overline{C}_o\right)_i$$

$$\overline{C}_2 = \overline{R} \times \overline{N}_2$$

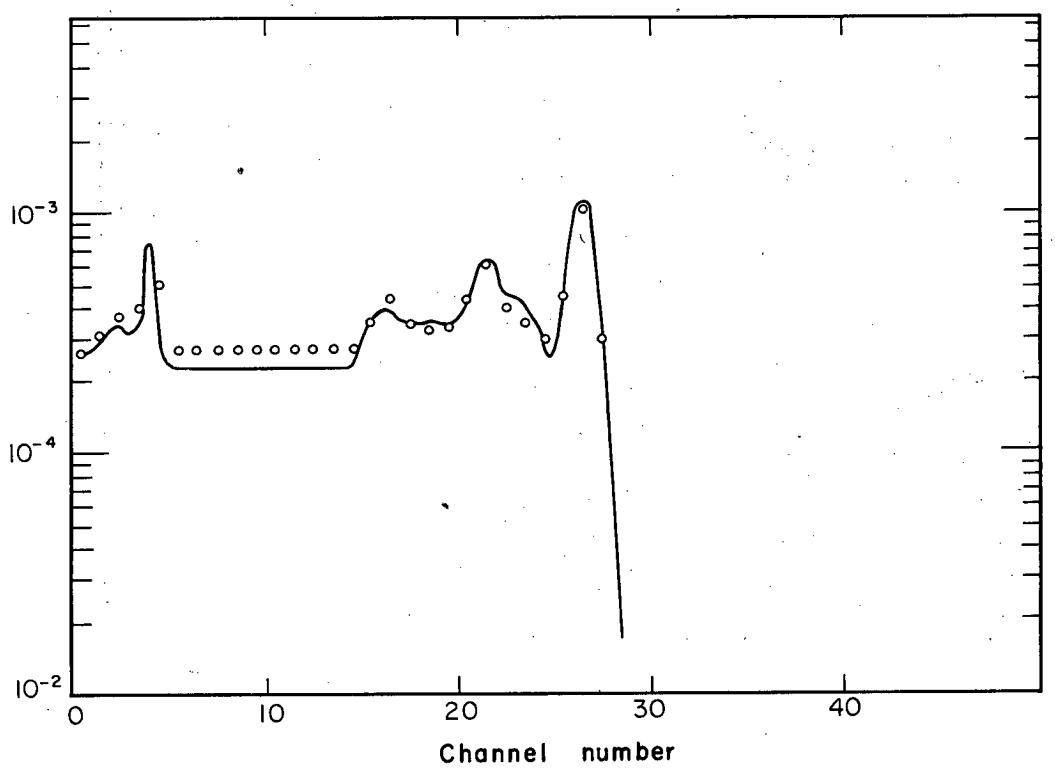
$$\left(\overline{N}_3\right)_i = \frac{\left(N_2\right)_i}{\left(C_2\right)_i} \times \left(\overline{C}_o\right)_i$$

In practice, 50 iterations give convergence within one part in 10^4.

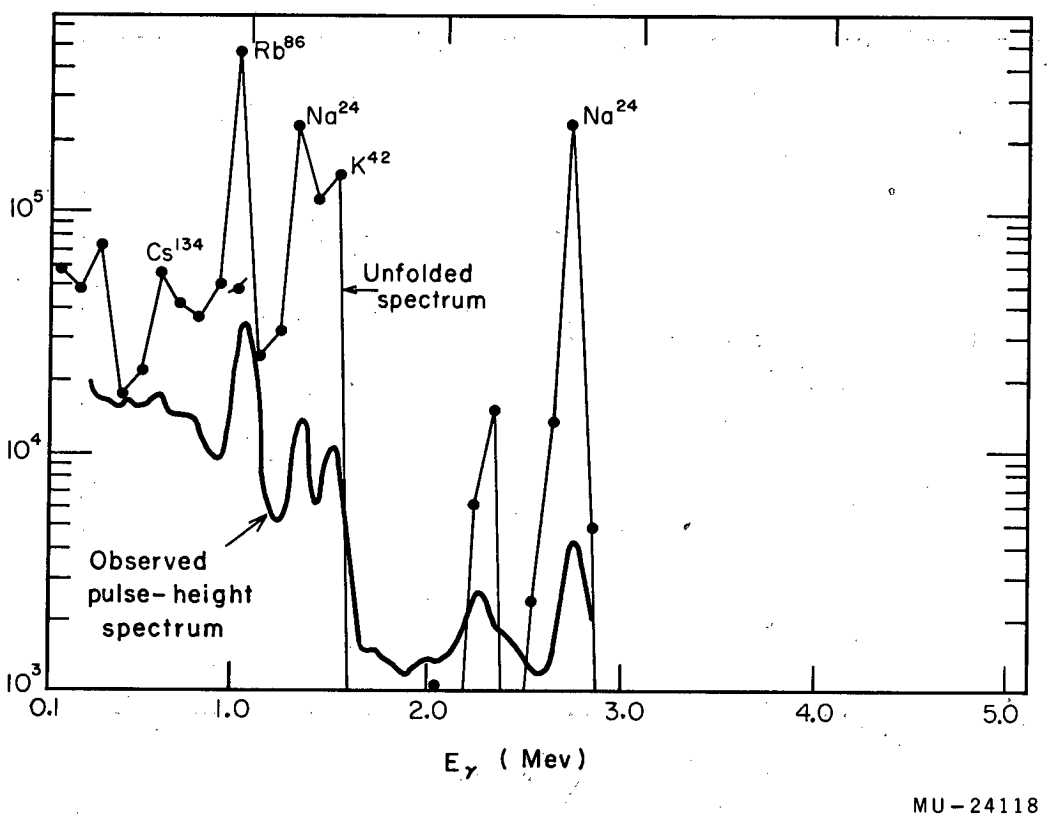
A comparison of the spectrum of the 2.75-Mev gamma ray of Na^{24} with the corresponding row of the generated response matrix is provided in Fig. 2. The lower-energy gamma ray of Na^{24} at 1.38 Mev is subtracted from the experimental spectrum for purposes of the comparison. The peaks visible are, in order of increasing energy or channel number, the scattering peak, annihilation peak, two escape peaks, and the photopeak.

The analysis of a complex spectrum by the program is demonstrated in Fig. 3. A spectrum of several sources is shown before and after unfolding. All the lines except those at 1.38 and 1.55 Mev are well resolved with 50 channels. The accuracy of the matrix is fair; the areas of all the unfolded peaks are 10% higher than the source strengths as measured individually with a standardized crystal. The matrix fails to allow sufficiently for the first escape peak of Na^{24}, with a remainder of 5% of the unfolded photopeak. Also, the valley between the Cs^{134} and Rb^{86} peaks does not fall as low as would be desirable. The peak at 0.35 Mev may result from contamination originating in the reactor production of the Rb^{86} and Na^{24}.

Another check on the validity of the unfolding procedures was made by unfolding a known spectrum. A spectrum of gamma rays in fast coincidence with the fission of Cf^{252} was unfolded, and the results compared with those of Smith, Fields, and Friedman;[5] the average gamma energy was found to be



2. Comparison of the spectrum of the 2.75-Mev gamma ray of Na^{24} with the appropriate row of the 50-channel response matrix. The spectrum of the 1.38-Mev gamma ray has been subtracted from the experimental spectrum shown.



3. Comparison of observed and unfolded spectra for a mixture of sources.

0.78 Mev, in excellent agreement with the published value of 0.80 Mev. Both spectra showed a peak at about 0.3 Mev, with the main difference being a greater yield in the published spectrum at higher energies. The two are compared in Fig. 4.

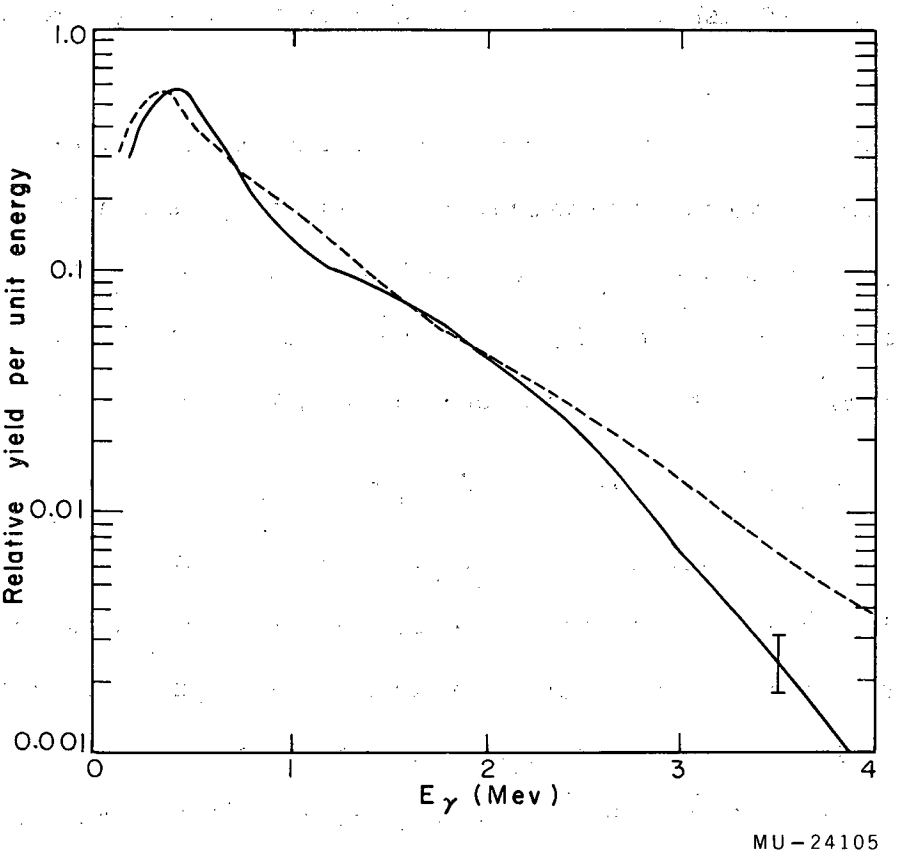
Another possible source of error in the analysis of spectra lies in the statistical fluctuations. The procedure of unfolding is similar to differentiation, and fluctuations in the input spectrum are magnified in the product. To reduce such random effects, various types of smoothing were employed. One was smoothing by the computer according to the formula

$$Y_i' = 1/4\ Y_{i-1} + 1/2\ Y_i + 1/4\ Y_{i+1}$$

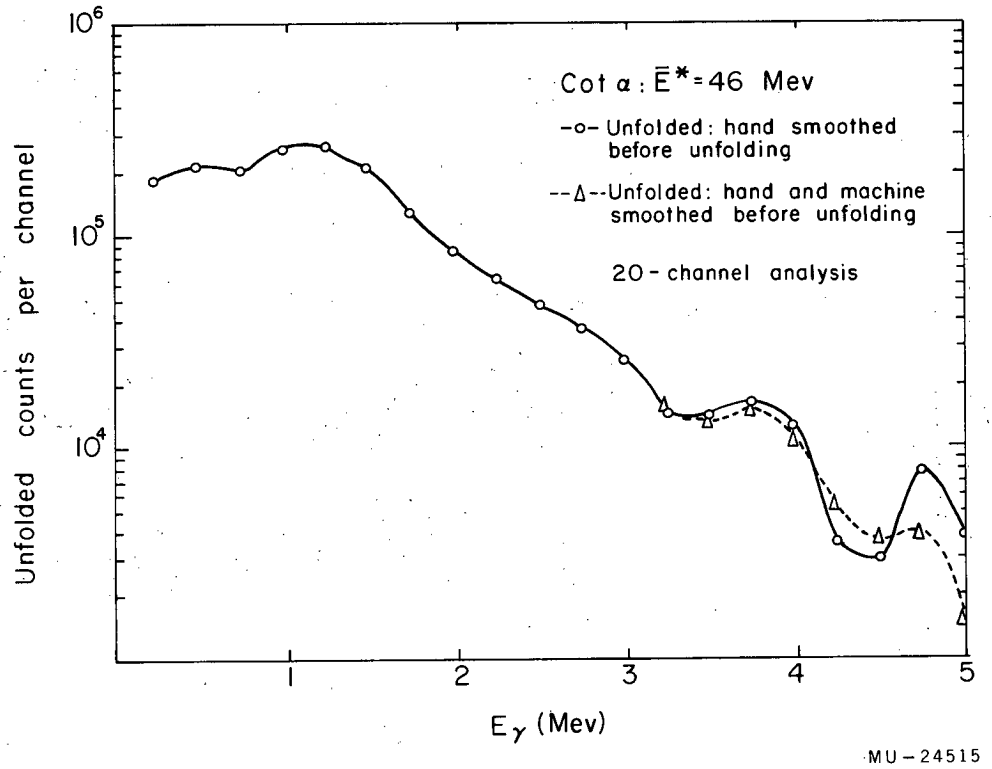
either before or after unfolding, or both. Since this formula does not take the better statistics at the low-energy end of the spectrum into account, hand smoothing was tried on all input channels with fewer than 100 counts. All combinations of the three types of smoothing were tried on a test spectrum from the bombardment of Co^{59} with α particles, for both fifty and twenty channels. The results from certain of the smoothing combinations are shown in Figs. 5 and 6. In all cases the average energies were within 3% of each other. Statistical fluctuations caused the unsmoothed 50-channel analysis to generate spurious peaks, whereas a smoothed spectrum appeared to be handled adequately by 20 channels. Hand smoothing alone was selected for use in our experiment, because it did not eliminate structure in the spectrum where the statistics were good. Options remain in the program to cover all combinations of smoothing and number of channels.

D. Computer Program GAMSPEC

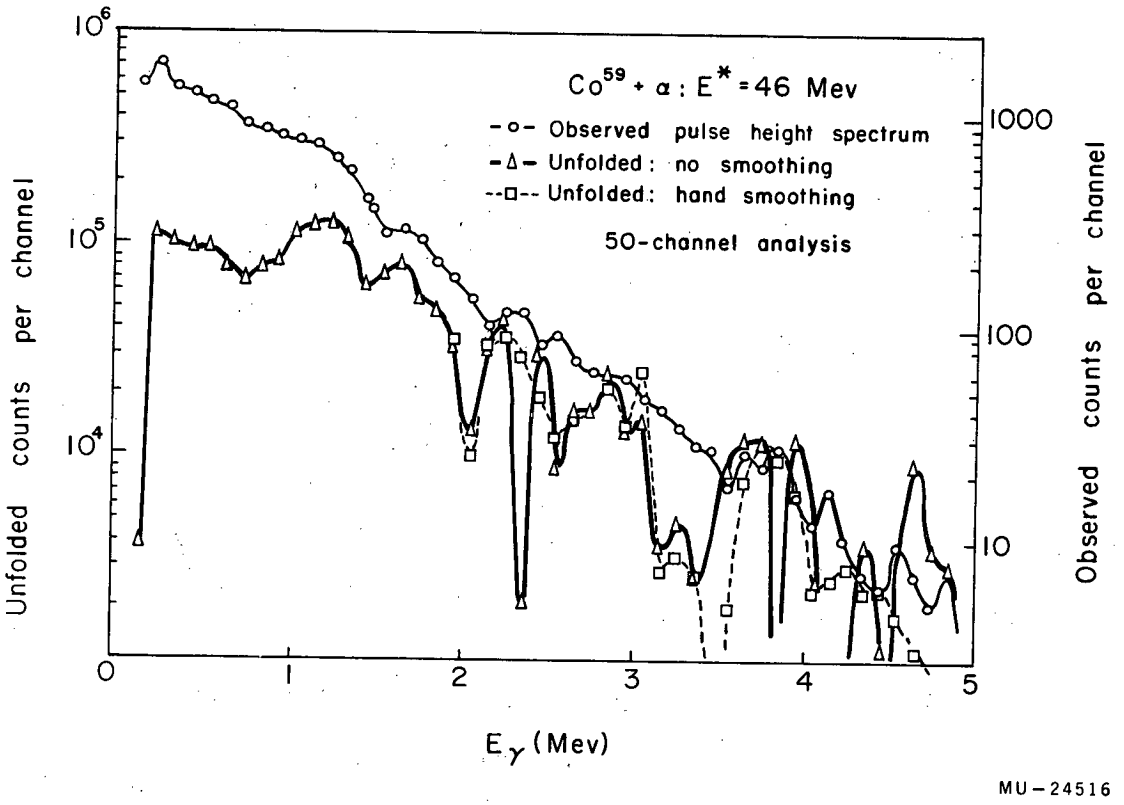
The IBM program that performs the gamma spectral analysis is written in Fortran as a main program and three subprograms. The computation



4. Comparison of unfolded fission gamma-ray spectrum of Cf^{252} with that obtained by Smith, Fields, and Friedman (Ref. 5).



5. Spectrum unfolded with various smoothing options; 20 channels.



6. Spectrum unfolded with various smoothing options; 50 channels.

proceeds as follows:

To correct for gain drift observed in a spectrum, subroutine GAINAJ may be called in to apply a correction. The response matrix may then be read from a tape or generated for the required energy range by subroutine RESPON, using the values of the response function R(V,E) generated in function subroutine CURVE.

Spectra may be processed by the program in batches having the same energy limits and number of pulse-height intervals or channels. A gain adjustment factor is specified for each spectrum individually. The data cards for the spectrum are preceded by a card specifying the following constants:

(a) MESH — the number of channels in the spectrum and the dimension of the response matrix;

(b) NRUNS— the number of spectra in the batch;

(c) THRES— the lower gamma-energy limit; and

(d) EMAX — the upper gamma-energy limit.

The data cards then follow, with each spectrum preceded by the factor GAIN by which the energy scale is to be multiplied. For example, to move a peak from 25 volts to 26 volts pulse height, GAIN is set equal to 1.04. Formats for the cards are given in the listing reproduced in the appendix.

If the value of GAIN is not unity, the program initially calls in subroutine GAINAJ to apply the correction. This subroutine finds the corrected spectrum by interpolating linearly between the points of the input spectrum and correcting for changed channel width. In extrapolating at either end when necessary, it assumes an exponential dependence on pulse height. This approximation was reasonable for the spectra studied. The corrected input spectrum is written on tape 3.

The response matrix may be generated in subroutine RESPON or read from tape 5. The use of the sense switches in exercising the various options is summarized in Table III. After the response matrix has been delivered to the main program, the gain-adjusted input spectrum is smoothed if required; then the spectrum is unfolded according to the iterative scheme. The successive approximations are optionally listed on tape 3. The spectrum may be smoothed after unfolding, if desired.

As a check after the analysis has been completed, the resultant incident spectrum vector is multiplied by the response matrix, and the product written on tape 3. If the number of iterations is sufficient, the product is equal to the input spectrum corrected for gain. The unfolded spectra are written, smoothed and unsmoothed, on tape 3.

In the unfolding, 50 iterations are specified, but the successive approximations to the incident spectrum are checked for divergence after each iteration. However, in our calculation, no early exits from the iteration loop because of divergence were observed.

In writing the response matrix, subroutine RESPON selects the energy and pulse-height intervals according to the specified values of MESH, THRES, and EMAX. The pulse-height intervals are divided into 10 subintervals, the response of the crystal calculated in function subroutine CURVE, and an average taken over the 10 values.

The running time for the evaluation of the response matrix is about 3 minutes for 20 channels and 18 minutes for 50 channels. After generation of the matrix, or if it is already on the tape, the running time per spectrum is less than 30 seconds for 20 channels and slightly over 1 minute for 50 channels.

Table III.

Sense-switch options.

Switch	Position	Result
1	Up	No smoothing before unfolding.
	Down	Machine smoothing applied before unfolding.
2	Up	No smoothing after unfolding.
	Down	Machine smoothing applied after unfolding.
3	Up	Successive approximations are not listed.
	Down	Successive approximations listed on tape 3.
4	Up	No correction to geometry.
	Down	Correction factor of 2.25 applied to product spectrum for second experimental source position.
5	Up	Response matrix read from tape 5.
	Down	Response matrix generated and written on tape 5.

ACKNOWLEDGMENTS

The author would like to express his gratitude to Dr. Bernard G. Harvey for his advice and encouragement. This work was greatly facilitated by discussions with Dr. N.E. Scofield, Mr. Harry Bowman, and Dr. Lloyd Mann.

Thanks are due to Mr. Douglas Brainard for assistance with the coding in times of difficulty.

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APPENDIX

Fortran Listings of GAMSPEC and Subroutines

```
C     PROGRAM BLOB FOR ENERGIES OF DEFORMED ROTATING  LIQUID DROP

    1 DIMENSION BSTEP(40),INTGAM(40), SURFE(40,40), ECOUL(40,40),
     XEROT(40,40),ETOTAL(40,40),W(6), Z(6), PSI(6)
    5 READ 10, XSTART, XSTOP, RSTART, RSTOP, ISTEPS, JSTEPS, KSTEPS,
     XLSTEPS, XFIXED, RFIXED
   10 FORMAT (4F8.4 ,4I4,2F8.4)
   12 READ 13, (W(I),I=1,6),(Z(J),J=1,6)
   13 FORMAT (6F12.9 )
   30 DO 200 I=1,ISTEPS
      B = I
      BSTEPS = ISTEPS
      BETA = (B*1.0)/BSTEPS
      BSTEP(I) = BETA
   35 DO 200 J=1,JSTEPS
      G=J
      GSTEPS=JSTEPS
      GAMMA      = 120.0+((G-1.0)*60.0)/(GSTEPS-1.0)
      INTGAM(J) = GAMMA
      PI= 3.14159265
      GAMMA = GAMMA/57.2957795
   50 CANG=COSF(GAMMA)
      BANG=COSF(GAMMA+(2.0*PI)/3.0)
      AANG=COSF(GAMMA -(2.0*PI)/3.0)
   60 ASQ= EXPF ((2.0*BETA)*AANG)
      BSQ= EXPF ((2.0*BETA)*BANG)
      CSQ= EXPF ((2.0*BETA)*CANG)
   70 ALPASQ= (ASQ-CSQ)/ASQ
      DLTASQ= (BSQ-CSQ)/BSQ
      TSQ= DLTASQ/ALPASQ
      T= SQRTF(TSQ)
      ALPHA= SQRTF(ALPASQ)
      CHI= SQRTF(1.0-ALPASQ)
      PHI = ATANF(ALPHA/CHI)
   80 RHO=SQRTF(1.0-DLTASQ)
      CINVER= EXPF(-BETA*CANG)

   83 CALL ELLIP (PHI, T, W, Z, E, F)
   85 BSURF= 0.5*CINVER*(RHO*CHI+(1.0/ALPHA-ALPHA)*F+ (ALPHA*E))

C     NOW CALCULATE BCOUL AND BROT

  100 EPSLON = SQRTF(1.0/(ASQ-CSQ))
      TAU = EPSLON * SQRTF(ASQ-BSQ)
      CALL ELLIP(PHI, TAU, W, Z, E, F)
  120 BCOUL = EPSLON*F
      BROT = 2.0/(ASQ+BSQ)
      SURFE(I,J) = BSURF-1.0
      ECOUL(I,J) = BCOUL-1.0
  200 EROT(I,J) = BROT

C     WRITE TABLES OF SURFACE AND COULOMB ENERGY AND BROT

      WRITE OUTPUT TAPE 1,210
  210  FORMAT(22H1SURFACE ENERGY CHANGE)
  211 WRITE OUTPUT TAPE 1,212, (INTGAM(J),J=1,JSTEPS)
  212 FORMAT (7H0GAMMA= 14I8)
  213 WRITE OUTPUT TAPE 1,214
  214 FORMAT(5H BETA)
      DO 215 I=1, ISTEPS
  215 WRITE OUTPUT TAPE 1,216, BSTEP(I), (SURFE(I,J), J=1, JSTEPS)
  216  FORMAT (1H F5.3, F10.5, 13F8.5)
      WRITE OUTPUT TAPE 1,230
```

```
  230 FORMAT(22H1COULOMB ENERGY CHANGE)
  231 WRITE OUTPUT TAPE 1,212, (INTGAM(J),J=1,JSTEPS)
  232 WRITE OUTPUT TAPE 1,214
  234 DO 236 I=1,ISTEPS
  236 WRITE OUTPUT TAPE 1,216, BSTEP(I), (ECOUL(I,J), J=1, JSWEPS)
      WRITE OUTPUT TAPE 1,240
  240 FORMAT(27H1ROTATIONAL PARAMETER BROT    )
  241 WRITE OUTPUT TAPE 1,212, (INTGAM(J),J=1,JSTEPS)
  242 WRITE OUTPUT TAPE 1,214
  244 DO 246 I=1,ISTEPS
  246 WRITE OUTPUT TAPE 1,216, BSTEP(I), (EROT (I,J), J=1, JSTEPS)

C     OPTION FOR SINGLE X AND R VALUE

  247 IF (SENSE SWITCH 3) 248,255
  248 DO 250 I=1, ISTEPS
      DO 250 J=1, JSTEPS
  250 ETOTAL(I,J)=SURFE(I,J)+2.0*XFIXED*ECOUL(I,J)+RFIXED*EROT(I,J)
  252 GO TO 287

C     COMPUTE ENERGY TABLES FOR MESH OF X AND R VALUES

  255 DO 285 K=1,KSTEPS
      XSTEP = K-1
      XSTEPS = KSTEPS-1
      X = XSTART + XSTEP*(XSTOP-XSTART)/XSTEPS
  260 DO 285 L=1,LSTEPS
      RSTEP = L-1
      RSTEPS = LSTEPS-1
      R = RSTART + RSTEP*(RSTOP-RSTART)/RSTEPS
  270 DO 275 I=1,ISTEPS
      DO 275 J=1,JSTEPS
  275 ETOTAL(I,J)=SURFE(I,J)+2.0*X    *ECOUL(I,J)+R      *EROT(I,J)
  278 WRITE OUTPUT TAPE 1, 280
  279 WRITE OUTPUT TAPE 1, 277, X, R
  277 FORMAT (28H0FISSIONABILITY PARAMETER X=  F5.3,
     X16HLAMBDA SQUARED = F5.3)
  280 FORMAT (51H1TOTAL ENERGY CHANGE AS FRACTION OF SURFACE ENERGY )
  281 WRITE OUTPUT TAPE 1,212, (INTGAM(J),J=1,JSTEPS)
  282 WRITE OUTPUT TAPE 1,214
  284 DO 285 I=1,ISTEPS
  285 WRITE OUTPUT TAPE 1,216, BSTEP(I),(ETOTAL(I,J), J=1, JSTEPS)
  286 GO TO 299
  287 CONTINUE
  288 WRITE OUTPUT TAPE 1, 280
  290 FORMAT (51H1TOTAL ENERGY CHANGE AS FRACTION OF SURFACE ENERGY )
  291 WRITE OUTPUT TAPE 1,212, (INTGAM(J),J=1,JSTEPS)
  292 WRITE OUTPUT TAPE 1,214
  294 DO 295 I=1,ISTEPS
  295 WRITE OUTPUT TAPE 1,216, BSTEP(I),(ETOTAL(I,J), J=1, JSTEPS)
  299 CONTINUE
  300 END (0,1,0,0,1)
```

```
      SUBROUTINE ELLIP (PHI, T, W, Z, E, F)
      DIMENSION W(6), Z(6), PSI(6)

C     DETERMINE THE NUMBER OF ITERATIONS

    5 IF(T-0.55) 10,10,15
   10 N=1
   11 GO TO 30
   15 IF(T-0.85) 17,17,20
   17 N=2
   18 GO TO 30
   20 IF(T-0.95) 22,22,25
   22 N=3
   23 GO TO 30
   25 IF (T-0.99) 27 ,27,29
   27 N = 5
   28 GO TO 30
   29 E = SINF (PHI)
      DIFFE = (1.0+E)/(1.0-E)
      F = 0.5*LOGF (DIFFE)
      GO TO 91
   30 PSI(1) = 0.0

C     DO SUMMATIONS TO FIND E AND F

      DO 35 I=1,N
      FLI = I
      FLN = N
   35 PSI(I+1) = FLI*PHI/FLN
      F = 0.0
      E = 0.0
   40 DO 90 I=1,N
      FSUMJ = 0.0
      ESUMJ = 0.0
   50 DO 75 J = 1,6
      OMEGA = (PSI(I+1) - PSI(I))*W(J) + PSI(I)
      SINOM = SINF(OMEGA)
   60 CN = SQRTF(1.0-T*T*SINOM*SINOM)
      FTERMJ = Z(J)/CN
   70 ETERMJ = Z(J)*CN
      FSUMJ = FSUMJ + FTERMJ
      ESUMJ = ESUMJ + ETERMJ
   75 CONTINUE
      FTERMI =FSUMJ*(PSI(I+1) - PSI(I))
      ETERMI =ESUMJ*(PSI(I+1) - PSI(I))
      F = FTERMI + F
      E = ETERMI + E
   90 CONTINUE
   98 RETURN
  100 END(0,1,0,0,1)
```

```
C      GAMMA SPECTRUM ANALYSIS - PROGRAM GAMSPEC

       DIMENSION R(50,50), F(50),H(50), B(50,50), X(50,50),D(50,50),
      XW(50,50), SUM(50), ESUM(50), E(50), RESID(50), DIF(50,50),
      X Y(50,50)
       READ 5, MESH, NRUNS, THRES, EMAX
     5 FORMAT (2I5, 2F10.3 )
       WRITE OUTPUT TAPE 3,13
    13 FORMAT     (14H1INPUT SPECTRA     )
    10 DO 35 JS=1, NRUNS
       READ 12, GAIN, (F(I), I=1,MESH     )
    12 FORMAT (7F10.3)
       WRITE OUTPUT TAPE 3, 93, JS, (F(J), J=1,MESH)
       CALL EFM (0,0)

C      ADJUST GAIN IF REQUIRED

       IF (GAIN-1.0) 20,30,20
    20 CALL GAINAJ (F, H, MESH, GAIN, THRES, EMAX)
       DO 25 I=1, MESH
    25 B(I,JS) = H(I)
       GO TO 35
    30 DO 34 K=1, MESH
    34 B(K,JS) = F(K)
    35 CONTINUE
       FLMESH = MESH
       DO 36 J=1, MESH
       EINDEX = J
    36 E(J) = (( EMAX-THRES)/FLMESH) * (EINDEX-0.5) + THRES

C      GENERATE RESPONSE MATRIX OR READ IT FROM TAPE

       IF (SENSE SWITCH 5) 40,47
    40 CALL RESPON ( MESH, EMAX, THRES, R )
       WRITE OUTPUT TAPE 5, 45, ((R(I,J), I=1,MESH), J=1, MESH)
    45 FORMAT (1P5E18.9)
       END FILE 5
       REWIND 5
       GO TO 48
    47 READ INPUT TAPE   5, 45, ((R(I,J), I=1,MESH), J=1, MESH)
       REWIND 5
    48 CONTINUE

C      OPTIONAL SMOOTHING BEFORE UNFOLDING

       IF (SENSE SWITCH 1) 50,58
    50 DO 55 N=1,NRUNS
       MINUS = MESH - 1
       DO 53  L=2, MINUS
    53 X(L,N) = 0.25*B(L-1,N) + 0.5*B(L,N) + 0.25*B(L+1,N)
       X(MESH,N) = 0.25* B(MINUS,N) + 0.75* B(MESH,N)
       X(1,N) = 0.75*B(1,N) + 0.25* B(2,N)
       DO 55 L=1, MESH
    55 B(L,N) = X(L,N)

C      UNFOLD BY SUCCESSIVE APPROXIMATIONS

    58 DO 140 N=1,NRUNS
       DO 110 I=1, MESH
       DO 110 J=1,50
       DIF (I,J) = 0.0
       D(I,J) = 0.0
   110 W(I,J) = 0.0
```

```
      DO 115 I=1,MESH
  115 W(I,1) = B(I,N)
      DO 120 I = 1, MESH
      DO 120 J = 1, MESH
  120 D(I,1) = R(I,J) * W(J,1) + D(I,1)
      DO 130 L= 1,49
      LL= L+1
      DO 122 I=1, MESH
  122 W(I,L+1) =(W(I,L) / D(I,L)) * W(I,1)
      RESID(1) = 0.0
      RESID (LL) = 0.0
      DO 125 J=1, MESH
      DIF(J,L) = W(J,L+1) - W(J,L)
  125 RESID (LL) = RESID (LL) + DIF(J,L) * DIF(J,L)
      RESID (LL) = SQRTF (RESID(LL))
      IF (L-2) 127,127,126

C     CHECK FOR DIVERGENCE OF APPROXIMATIONS

  126  IF (RESID(LL) -RESID(LL-1) ) 127,230,230
  127 DO 130 I=1, MESH
      DO 130 J=1, MESH
  130 D(I,L+1) = R(I,J)*W(J,L+1) + D(I,L+1)
      GO TO 131
  230 LL=LL-1
  131 DO 132 I=1, MESH
  132 X(I,N) = W(I,LL)

C     OPTIONAL WRITING OF APPROXIMATIONS

      IF (SENSE SWITCH 3) 133,140
  133 WRITE OUTPUT TAPE 3, 134
  134 FORMAT (28H1 SUCCESSIVE APPROXIMATIONS  )
      DO 135 J=1,25
  135 WRITE OUTPUT TAPE 3 , 136, N,J,(W(I,J), I=1,MESH), RESID(J)
  136 FORMAT (4HORUN I2, 3H J=I2,1P10E10.3/(1PE21.3,1P9E10.3 ))
  140 CONTINUE

C     GEOMETRY FACTOR FOR 45 DEGREE RUNS

      IF (SENSE SWITCH 4) 142,150
  142 DO 146 N=1, NRUNS
      DO 146 I=1, MESH
  146 X(I,N) = X(I,N) * 2.25
  150 DO 69 J=1,NRUNS

C     COMPUTE SUM OF COUNTS AND AVERAGE ENERGY

      SUM (J) = 0.0
      ESUM (J) = 0.0
      DO 68 L = 1, MESH
      ESUM (J) = ESUM (J) + E (L)*X(L,J)
   68 SUM (J) = SUM (J) + X (L,J)
   69 ESUM (J) = ESUM (J) / SUM (J)

C     WRITE SMOOTHED INPUT SPECTRUM AND UNFOLDED SPECTRUM

   90 WRITE OUTPUT TAPE 3, 92
      DO 91 J =1,NRUNS
```

```
   91 WRITE OUTPUT TAPE 3, 93, J, ( B (I,J), I = 1, MESH )
   92 FORMAT ( 41 HO SPECTRA ADJUSTED FOR GAIN AND SMOOTHED    )
   93 FORMAT (  11HORUN NUMBER I2,1P10E10.3/(13H                1P10E10.3)
   94 FORMAT (26H              TOTAL COUNTS  F10.1 ,  15H AVERAGE ENERGY
     X F8.3  )
      WRITE OUTPUT TAPE 3, 96
   96 FORMAT (25HOUNFOLDED GAMMA SPECTRA    )
      DO 95 J = 1, NRUNS
      WRITE OUTPUT TAPE 3, 93, J, (X (I,J), I=1, MESH )
   95 WRITE OUTPUT TAPE 3, 94, SUM (J)    , ESUM (J)

C     MULTIPLY UNFOLDED SPECTRUM BY RESPONSE MATRIX TO CHECK

      WRITE OUTPUT TAPE 3,320
  320 FORMAT (14HOCHECK SPECTRA )
      DO 325 N=1,NRUNS
      DO 324 I=1, MESH
      DO 324 J=1, MESH
  324 Y(I,N) = Y(I,N) + R(I,J) *X(J,N)
  325 WRITE OUTPUT TAPE 3, 93, N, (Y(I,N), I=1, MESH)

C     OPTIONAL SMOOTHING AFTER UNFOLDING

      IF (SENSE SWITCH 2)  240, 97
  240 WRITE OUTPUT TAPE 3, 250
  250 FORMAT (26H SMOOTHED UNFOLDED SPECTRA  )
      DO 350 N=1,NRUNS
      MINUS = MESH - 1
      DO 340 L=2, MINUS
  340 Y(L,N) = 0.25*X(L-1,N) + 0.5*X(L,N) + 0.25*X(L+1,N)
      Y(MESH,N) = 0.25* X(MINUS,N) + 0.75* X(MESH,N)
      Y(1,N) = 0.75*X(1,N) + 0.25* X(2,N)
      DO 345 L=1,MESH
  345 X(L,N) = Y(L,N)
  350 WRITE OUTPUT TAPE 3, 93, N, (X(I,N), I=1,MESH)
   97 CONTINUE
      END FILE 3
      REWIND 3
  100 END ( 0,1,0,0,0 )


      SUBROUTINE GAINAJ (F,H,NBOXES,GAIN, THRES, EMAX)

      DIMENSION E(50), EA(50), H(50), G(50), F(50)
      BOTE = THRES * GAIN
      TOPE = EMAX * GAIN
      DO 10 N=1, NBOXES
      BOXES = NBOXES
      P=N
      E(N) =(((TOPE-BOTE )/BOXES) * (P-0.5))+ BOTE
      G(N) = F(N)/GAIN
   10 EA(N)=(((EMAX-THRES)/BOXES) * (P-0.5))/     + THRES
      IF ( GAIN - 1.0 ) 60,60,14

C     GAIN TO BE INCREASED

   14 H(1) = G(1) - (G(2)-G(1))*(E(1)-EA(1))/(E(2)-E(1))
   15 DO 50 N=2,NBOXES
      DO 30 L=2,N
```

```
      M=N-L+1
      IF (EA(N)-E(M)) 30,30,45
   30 CONTINUE
   45 H(N) =G(M) + (G(M+1)-G(M))*(EA(N)-E(M))/(E(M+1)-E(M))
   50 CONTINUE
   55 GO TO 90

C     GAIN TO BE DECREASED

   60 DO 80 N=1,NBOXES
      NLESS = NBOXES - N
      DO 70 L=1,NLESS
      M=N+L
      IF (EA(N) -E(M)) 75,75,70
   70 CONTINUE
   72 H(N) = G(NBOXES) *(G(NBOXES)/G(NBOXES-1))**((EA(N)-E(NBOXES))/
     X (E(NBOXES)-E(NBOXES-1))  )
      GO TO 80
   75 H(N) = G(M) -(G(M)-G(M-1)) * (E(M)-EA(N))/(E(M)-E(M-1))
   80 CONTINUE
   90 RETURN
   95 END (0,1,0,0,1)


      SUBROUTINE RESPON (MESH, EMAX, THRES,R)

      DIMENSION R(50,50) , E(50)
      FLMESH = MESH

C     LOCATE ENERGY INTERVALS

      DO 12 J=1, MESH
      EINDEX = J
   10 E(J) = (( EMAX-THRES ) /FLMESH ) * ( EINDEX- 0.5 ) + THRES
      DO 12 K = 1, MESH
   12 R (J,K ) = 0.0

C     LOCATE PULSE HEIGHT INTERVALS

   15 DO 60 J = 1, MESH
      DO 50 I = 1, MESH
      ELEM = 0.0

C     AVERAGE RESPONSE OVER 10 PULSE HEIGHT INTERVALS

      DO 25 L = 1,10
      XINDEX = 10* ( I-1 ) + L
      X = (( EMAX-THRES) / (FLMESH*10.0)) * (XINDEX-0.5) + THRES
      ELL = L
      IF (X- 1.25*E(J)) 25,25,55
   25 ELEM = ELEM    + CURVE(E(J),X)
   50 R (I,J) = R (I,J) + ELEM   /10.0
   55 R (I,J) = R (I,J) + ELEM / ELL
   60 CONTINUE
      DO 70 I=1,MESH
      DO 70 J=1,MESH
   70 R(I,J) = R(I,J) * 100.0/ FLMESH
      RETURN
      END (0,1,0,0,1)
```

```
      FUNCTION CURVE (E,X)
C     PHOTO AND ESCAPE PEAKS
      PHOT1 = 0.0
      PHOT2 = 0.0
      PHOT3 = 0.0
      C = 0.361
      U = 0.0815* ((E)**0.5)
C     CONSTANTS FOR 10 PERCENT RESOLUTION AT 0.66 MEV
      IF (E-0.65) 5,7,7
    5 HITE1 = 0.00459 * E**(-0.033)
      GO TO 8
    7 HITE1 = 0.00253 * E**(    -1.46)
    8 PHOT1 = HITE1 * EXPF(-(X-E       )**2.0/ (C*U**2.0))
      IF (E-1.75)  11,9,9
    9 HITE2 = 1.437*HITE1*LOGF(E/1.75) /2.303
      HITE3 = 0.569*HITE1*LOGF(E/1.75) /2.303
      PHOT2 = HITE2 * EXPF(-(X-E+0.51  )**2.0/ (C*U**2.0))
      PHOT3 = HITE3 * EXPF(-(X-E+1.02  )**2.0/ (C*U**2.0))
      IF (X-0.4) 10,102,102
  102 IF (X-0.6) 103,103,10
C     ANNIHILATION PEAK
  103 ANNHT = 1.2*  (HITE2 + HITE3)
      ANNIH = ANNHT * EXPF(-(X-0.51) **2.0 / 0.00122 )
      GO TO 11
   10 ANNIH = 0.0
   11 IF (E-1.0) 14,12,12
C     COMPTON DISTRIBUTION
   12 COMPEJ = E-0.25
   13 GO TO 20
   14 IF (E-0.5) 17,15,15
   15 COMPEJ = 0.9*E - 0.15
   16 GO TO 20
   17 COMPEJ = 0.6 * E
   20 CONTINUE
      COMAR = 0.0077 - 0.001806 / (E+0.235)
      COMPHT = COMAR / ((E+COMPEJ     ) * 10.0 )
C     SCATTER PEAK
  121 IF (X-0.5) 22,21,21
   21 SCATR = 0.0
      GO TO 24
   22 SCATHT = 0.455*COMPHT
      SCATPS = 0.145 + 0.0675*E
      SCWID = 0.08
      SCATR = SCATHT * EXPF (-(X-SCATPS)**2./(C*SCWID**2.0))
C     COMPTON DISTRIBUTION WITH ESCAPE PEAKS
```

```
 24 IF (E-1.75) 25,25,30
 25 IF ( X-COMPEJ) 26,26,27
 26 COMPT = COMPHT
    GO TO 50
 27 IF (X-E ) 28,28,29
 28 COMPT = COMPHT * (E-X) / (E-COMPEJ)
    GO TO 50
 29 COMPT = 0.0
    GO TO 50
 30 IF (X-E+1.22) 31,31,34
 31 COMPT = COMPHT
 32 GO TO 50
 34 IF (X-E+0.87) 35,38,38
 35 IF (HITE2 - COMPHT) 31,31,135
135 COMPT = COMPHT+ (HITE2-COMPHT)*((X-E+1.22)/0.35)
 36 GO TO 50
 38 IF (X-E+0.51) 39,42,42
 39 COMPT = HITE2
 40 GO TO 50
 42 IF (X-COMPEJ) 43,45,45
 43 COMPT = HITE2  + (0.1*COMPHT) *((X-E+0.51)/(COMPEJ-E+0.51))
 44 GO TO 50
 45 IF (X-E)46,48,48
 46 COMPT = (HITE2 + 0.1 * COMPHT) * (E-X) / (E-COMPEJ)
 47 GO TO 50
 48 COMPT = 0.0
 50 CONTINUE
 55 CURVE = PHOT1 + PHOT2 + PHOT3 + SCATR + COMPT  + ANNIH
    RETURN
    END (0,1,0,0,1)
```

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