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Generalized Criteria of Characteristics Nonlinearity of Phase-Sensitive Detection Systems*

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Recent investigations [1,2] have shown that in the instrumentation of experimental research the total nonlinearity of characteristics of phase-sensitive detection systems is of prime importance. The idealized phase-sensitive detection system to be considered is shown in Fig. 1. The system input consists of an input signal $v^*_s(t)$ superimposed on a broad-band Gaussian noise $v^*_s(t)$. After time-invariant linear narrow-band filtering the sum $v_s(t) + v_n(t)$ is applied to the balanced phase-sensitive detector. Detector inputs are subtracted in a differential circuit. The dc output of varying amplitude represents the output signal V_o. In most cases of practical interest, the total nonlinearity of system characteristics is determined by the essential nonlinearity of characteristics of the phase-sensitive detector used, and is described by generalized equations $N_B = 1 - (\Delta_1)/\left\{ x\mu \; {}_1F_1 \, [1/2;2; - (\mu^2 + x^2)/2] \, (\pi/2 - \psi)\right\}$ and $N_e = 1 - (\Delta_1)/(\Delta_2)$.

The term $x = V_s/V_\sigma$ is the detector input signal-to-noise ratio; $\mu = V_c/V_\sigma$ is the system reference wave-to-noise ratio; V_s is the amplitude of the detector input sine signal; V_c is the amplitude of the system reference wave; V_σ is the root-mean square value of the detector input narrow-band noise; ψ is the phase angle between the detector input signal and the system reference wave; ${}_1F_1$ denotes the confluent hypergeometric function [3,4]; Δ_1 and Δ_2 signify the difference of hypergeometric functions ${}_1F_1 \, [-1/2; 1; - (\mu^2+x^2 + 2x\mu\cos\psi)/2] - {}_1F_1 \, [-1/2;1; (\mu^2 + x^2 - 2\mu x \cos\psi)/2]$ and ${}_1F_1 \, [-1/2;1; - (\mu+x)^2/2] - {}_1F_1 \, [-1/2;1; - (\mu - x)^2/2]$, respectively. Essential nonlinearity results from the inherent behaviour of the detector used in the system when signal is being detected in the presence of noise. Based on previous work [2] and comments made by A. R. Johnson [5], careful investigations show that the necessary and sufficient condition for nonlinearity minimum N_{BMIN} is given by a transcendental generalized criterion in the form

$$w[f(x_B)]\left\{ [x_B{}^2/2 + (\mu \, x_B \cos\psi)/2] \, . \, s[v(x_B)] - [x_B{}^2/2 - (\mu \, x_B \cos\psi)/2] \, m[t(x)] - \Delta_{1B}\right\} + \Delta_{1B}(x_B/2)^2 \, K[f(x_B)] = 0, \quad (1)$$

where functions $w[f(x_B)]$, $s[v(x_B)]$, $m[t(x_B)]$, and $K[f(x_B)]$ are given by

$$w[f(x_B)] = {}_1F_1 \, [1/2;2; - (\mu^2 + x_B{}^2)/2], \quad (2)$$

$$s[v(x_B)] = {}_1F_1 \, [1/2;2; - (\mu^2 + x_B{}^2 + 2\,\mu\, x_B \cos\psi)/2], \quad (3)$$

$$m[t(x_B)] = {}_1F_1 \, [1/2;2; - (\mu^2 + x_B{}^2 - 2\,\mu\, x_B \cos\psi)/2], \quad (4)$$

$$K[f(x_B)] = {}_1F_1 \, [3/2;3; - (\mu^2 + x_B{}^2)/2]. \quad (5)$$

Terms Δ_{1B} and Δ_{2B} signify the previously defined difference of hypergeometric functions, only x should be changed into x_B.

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By means of computer-aided analysis, using numerical solutions of Eq. (1) and high-density discrete-value calculations, the minimum nonlinearity expressed $N_{BMIN} = f^{*}(x_B)_{\mu,\psi}$ is calculated and plotted in Fig. 2. From curves it can be seen that N_{BMIN} is a monotonically decreasing function of x_B having a fast rate of decrease of almost a half order of magnitude for $x_B \leq 10$. N_{BMIN} varies less than 16% for $x_B \geq 10$ and $\psi \leq \pi/6$. For $x_B \geq 10$ and $\psi \geq \pi/6$, N_{BMIN} has approximately a constant value with variation of x_B. Furthermore, there are N_{BMIN} accumulation points at $x_B = 2.37295$ for $\mu \leq 0.1$ and for any value of ψ. The N_{BMIN} accumulation points are maximum values of N_{BMIN} for a given value of ψ.

Similarly, a generalized criterion for the maximum nonlinearity N_{CMAX} is given by

$$\begin{aligned}&\{\phi[g(x_C)] - z[h(x_C)]\}\{[x_C/2 - (\mu \cos \psi)/2]\, m[t(x_C)] \\ &- [x_C/2 + (\mu \cos \psi)/2]\, s[v(x_C)]\} \\ &+ \Delta_{1C}\{[(x_C + \mu)/2]\, \rho[g(x_C)] - [(x_C - \mu)/2]\, \ell[h(x_C)]\} = 0, \end{aligned} \tag{6}$$

where functions $m[(x_C)]$ and $s[v(x_C)]$ are given by relations (2) and (3), respectively. Other functions are defined by

$$\phi[g(x_C)] = {}_1F_1\,[-1/2;1;\, -(\mu + x_C)^2/2], \tag{7}$$

$$z[h(x_C)] = {}_1F_1\,[-1/2;1;\, -(\mu - x_C)^2/2], \tag{8}$$

$$\rho[g(x_C)] = {}_1F_1\,[1/2;2;\, -(\mu + x_C)^2/2], \tag{9}$$

$$\ell[h(x_C)] = {}_1F_1\,[1/2;2;\, -(\mu - x_C)^2/2]. \tag{10}$$

Terms Δ_{1C} signifies the previously defined difference of hypergeometric functions, only x should be changed into x_C. The maximum nonlinearity expressed as $N_{CMAX} = \varphi^{*}(x_C)_{\mu,\psi}$ is calculated by use of a high-density discrete-value approach and plotted in Fig. 3. From the curves in Fig. 3 we see that N_{CMAX} is a monotonically increasing function of x_C, having a fast rate of increase depending upon ψ. N_{CMAX} accumulation points are again at $x_C = 2.37295$ for $\mu \leq 0.1$ and for any value of ψ. Generally N_{CMAX} accumulation points are minimum values of N_{CMAX} for a given value of ψ.

Furthermore, applying the same method as in previous considerations, it is also of interest to calculate over a wide dynamic range of operating conditions the normalized form of characteristics of the phase-sensitive system as a function of ψ, for various values of μ and calculated values of x_B and x_C, considering above criteria. According to [2], normalized forms of the detector characteristics as a function of ψ, with μ, x_B, and x_C as parameters, are given by

$$(V_o/\eta_d V_\sigma)_B = (\pi/2)^{1/2}\, \Delta_{1B} \tag{11}$$

and

$$(V_o/\eta_d V_\sigma)_C = (\pi/2)^{1/2}\, \Delta_{1C} \tag{12}$$

where V_o and η_d are the system output signal and detector efficiency.

Calculations show that the numerical values of x_B and x_C are very close over a wide range of μ and ψ, although x_B gives the condition for minimum nonlinearity, and x_C for maximum nonlinearity. Consequently, both functions (11) and (12) can be represented by one curve for a set of values of μ, ψ, and x_B or x_C. Calculations show that the normalized output signal is almost independent of the phase angle for a ratio $\psi \leq 0.2$. For a $\psi \geq 0.2$ ratio, the normalized output signal considerably decreases its value, achieving $V_o/\eta_d V_\sigma = 0$ for $\psi = \pi/2$.

FOOTNOTE AND REFERENCES

The author wishes to express appreciation to E. Schroeder for writing the computer program.

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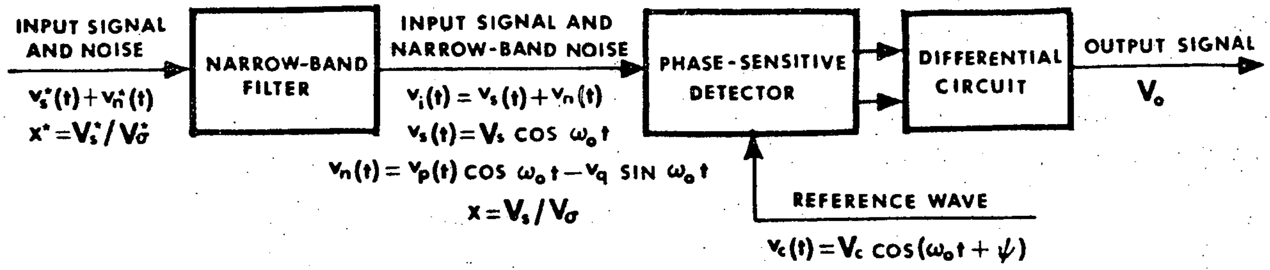

Fig. 1. Idealized phase-sensitive detection system.

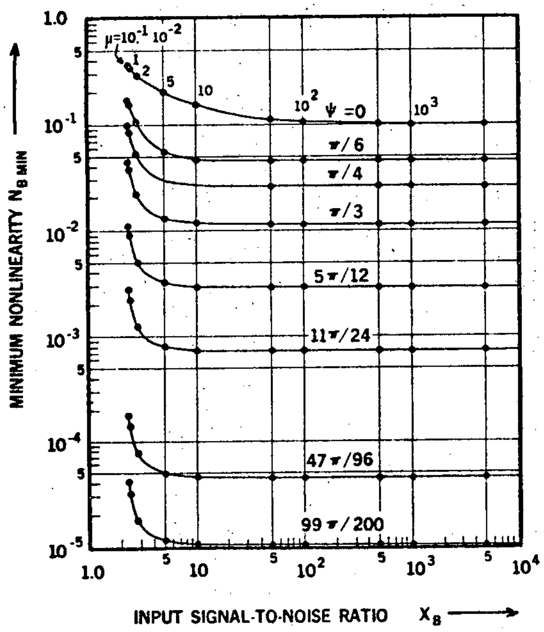

Fig. 2. Minimum nonlinearity N_{BMIN} as a function of the optimum value of the input signal-to-noise ratio x_B, with the phase angle ψ and the reference wave-to-input noise ratio as parameters.

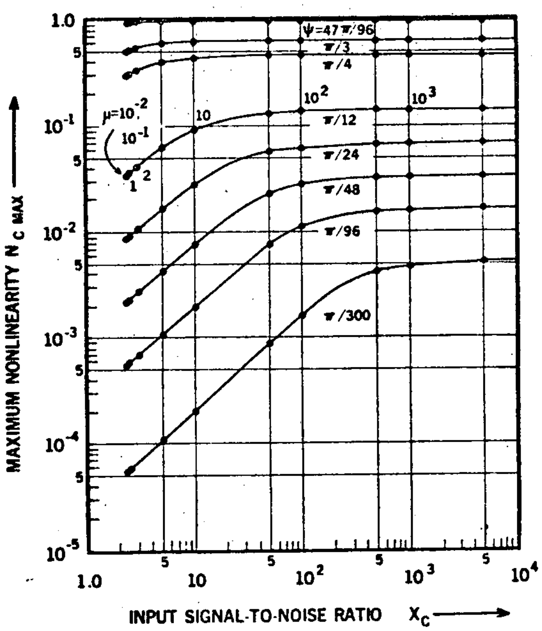

Fig. 3. Maximum nonlinearity N_{CMAX} as a function of the nonoptimum value of the input signal-to-noise ratio x_C, with the phase angle ψ and the reference wave-to-input noise ratio as parameters.

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