

Lawrence Berkeley National Laboratory

Recent Work

Title

GENERALIZED CRITERIA OF CHARACTERISTICS NONLINEARITY OF PHASE-SENSITIVE
DETECTION SYSTEMS

Permalink

<https://escholarship.org/uc/item/5gm9w7r2>

Author

Leskovar, Branko.

Publication Date

1969-10-01

For the 3rd Hawaii Intern. Conf. on
System Sciences, Honolulu, Jan. 14-16, 1970

UCRL-19332
Preprint

cy. 2

GENERALIZED CRITERIA OF
CHARACTERISTICS NONLINEARITY OF
PHASE-SENSITIVE DETECTION SYSTEMS

Branko Leskovar

NOV 25 1969
LIBRARY AND
DOCUMENTS SECTION

October 1969

AEC Contract No. W-7405-eng-48

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

UCRL-19332

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Generalized Criteria of Characteristics Nonlinearity of
Phase-Sensitive Detection Systems*

Branko Leskovar
Lawrence Radiation Laboratory
University of California
Berkeley, California

Recent investigations [1,2] have shown that in the instrumentation of experimental research the total nonlinearity of characteristics of phase-sensitive detection systems is of prime importance. The idealized phase-sensitive detection system to be considered is shown in Fig. 1. The system input consists of an input signal $v_s^*(t)$ superimposed on a broad-band Gaussian noise $v_n^*(t)$. After time-invariant linear narrow-band filtering the sum $v_s(t) + v_n(t)$ is applied to the balanced phase-sensitive detector. Detector inputs are subtracted in a differential circuit. The dc output of varying amplitude represents the output signal V_o . In most cases of practical interest, the total nonlinearity of system characteristics is determined by the essential nonlinearity of characteristics of the phase-sensitive detector used, and is described by generalized equations

$$N_B = 1 - (\Delta_1) / \left\{ x \mu {}_1F_1 \left[1/2; 2; -(\mu^2 + x^2)/2 \right] (\pi/2 - \psi) \right\} \text{ and } N_e = 1 - (\Delta_1) / (\Delta_2).$$

The term $x = V_s/V_o$ is the detector input signal-to-noise ratio; $\mu = V_c/V_o$ is the system reference wave-to-noise ratio; V_s is the amplitude of the detector input sine signal; V_c is the amplitude of the system reference wave; V_o is the root-mean square value of the detector input narrow-band noise; ψ is the phase angle between the detector input signal and the system reference wave; ${}_1F_1$ denotes the confluent hypergeometric function [3,4]; Δ_1 and Δ_2 signify the difference of hypergeometric functions ${}_1F_1 [-1/2; 1; -(\mu^2 + x^2 + 2x\mu\cos\psi)/2] - {}_1F_1 [-1/2; 1; (\mu^2 + x^2 - 2x\mu\cos\psi)/2]$ and ${}_1F_1 [-1/2; 1; -(\mu + x)^2/2] - {}_1F_1 [-1/2; 1; -(\mu - x)^2/2]$, respectively. Essential nonlinearity results from the inherent behaviour of the detector used in the system when signal is being detected in the presence of noise. Based on previous work [2] and comments made by A. R. Johnson [5], careful investigations show that the necessary and sufficient condition for nonlinearity minimum N_{BMIN} is given by a transcendental generalized criterion in the form

$$w[f(x_B)] \left\{ [x_B^2/2 + (\mu x_B \cos \psi)/2] \cdot s[v(x_B)] - [x_B^2/2 - (\mu x_B \cos \psi)/2] m[t(x)] - \Delta_{1B} \right\} + \Delta_{1B} (x_B/2)^2 K[f(x_B)] = 0, \quad (1)$$

where functions $w[f(x_B)]$, $s[v(x_B)]$, $m[t(x_B)]$, and $K[f(x_B)]$ are given by

$$w[f(x_B)] = {}_1F_1 \left[1/2; 2; -(\mu^2 + x_B^2)/2 \right], \quad (2)$$

$$s[v(x_B)] = {}_1F_1 \left[1/2; 2; -(\mu^2 + x_B^2 + 2\mu x_B \cos \psi)/2 \right], \quad (3)$$

$$m[t(x_B)] = {}_1F_1 \left[1/2; 2; -(\mu^2 + x_B^2 - 2\mu x_B \cos \psi)/2 \right], \quad (4)$$

$$K[f(x_B)] = {}_1F_1 \left[3/2; 3; -(\mu^2 + x_B^2)/2 \right]. \quad (5)$$

Terms Δ_{1B} and Δ_{2B} signify the previously defined difference of hypergeometric functions, only x should be changed into x_B .

* Work done under auspices of the U.S. Atomic Energy Commission.

By means of computer-aided analysis, using numerical solutions of Eq. (1) and high-density discrete-value calculations, the minimum nonlinearity expressed $N_{BMIN} = f^{**}(x_B)_{\mu, \psi}$ is calculated and plotted in Fig. 2. From curves it can be seen that N_{BMIN} is a monotonically decreasing function of x_B having a fast rate of decrease of almost a half order of magnitude for $x_B \leq 10$. N_{BMIN} varies less than 16% for $x_B \geq 10$ and $\psi \leq \pi/6$. For $x_B \geq 10$ and $\psi \geq \pi/6$, N_{BMIN} has approximately a constant value with variation of x_B . Furthermore, there are N_{BMIN} accumulation points at $x_B = 2.37295$ for $\mu \leq 0.1$ and for any value of ψ . The N_{BMIN} accumulation points are maximum values of N_{BMIN} for a given value of ψ .

Similarly, a generalized criterion for the maximum nonlinearity N_{CMAX} is given by

$$\begin{aligned} & \{\phi[g(x_C)] - z[h(x_C)]\} \{[x_C/2 - (\mu \cos \psi)/2] m[t(x_C)] \\ & - [x_C/2 + (\mu \cos \psi)/2] s[v(x_C)]\} \\ & + \Delta_{1C} \{[(x_C + \mu)/2] \rho[g(x_C)] - [(x_C - \mu)/2] \ell[h(x_C)]\} = 0, \end{aligned} \quad (6)$$

where functions $m[(x_C)]$ and $s[v(x_C)]$ are given by relations (2) and (3), respectively. Other functions are defined by

$$\phi[g(x_C)] = {}_1F_1[-1/2; 1; -(\mu + x_C)^2/2], \quad (7)$$

$$z[h(x_C)] = {}_1F_1[-1/2; 1; -(\mu - x_C)^2/2], \quad (8)$$

$$\rho[g(x_C)] = {}_1F_1[1/2; 2; -(\mu + x_C)^2/2], \quad (9)$$

$$\ell[h(x_C)] = {}_1F_1[1/2; 2; -(\mu - x_C)^2/2]. \quad (10)$$

Terms Δ_{1C} signifies the previously defined difference of hypergeometric functions, only x should be changed into x_C . The maximum nonlinearity expressed as $N_{CMAX} = \phi^{**}(x_C)_{\mu, \psi}$ is calculated by use of a high-density discrete-value approach and plotted in Fig. 3. From the curves in Fig. 3 we see that N_{CMAX} is a monotonically increasing function of x_C , having a fast rate of increase depending upon ψ . N_{CMAX} accumulation points are again at $x_C = 2.37295$ for $\mu \leq 0.1$ and for any value of ψ . Generally N_{CMAX} accumulation points are minimum values of N_{CMAX} for a given value of ψ .

Furthermore, applying the same method as in previous considerations, it is also of interest to calculate over a wide dynamic range of operating conditions the normalized form of characteristics of the phase-sensitive system as a function of ψ , for various values of μ and calculated values of x_B and x_C , considering above criteria. According to [2], normalized forms of the detector characteristics as a function of ψ , with μ , x_B , and x_C as parameters, are given by

$$(V_o/\eta_d V_\sigma)_B = (\pi/2)^{1/2} \Delta_{1B} \quad (11)$$

and

$$(V_o/\eta_d V_\sigma)_C = (\pi/2)^{1/2} \Delta_{1C} \quad (12)$$

where V_o and η_d are the system output signal and detector efficiency.

Calculations show that the numerical values of x_B and x_C are very close over a wide range of μ and ψ , although x_B gives the condition for minimum nonlinearity, and x_C for maximum nonlinearity. Consequently, both functions (11) and (12) can be represented by one curve for a set of values of μ , ψ , and x_B or x_C . Calculations show that the normalized output signal is almost independent of the phase angle for a ratio $\psi \leq 0.2$. For a $\psi \geq 0.2$ ratio, the normalized output signal considerably decreases its value, achieving $V_o/\eta_d V_\sigma = 0$ for $\psi = \pi/2$.

FOOTNOTE AND REFERENCES

The author wishes to express appreciation to E. Schroeder for writing the computer program.

1. B. Leskovar, Phase-sensitive detector nonlinearity at the signal detection in the presence of noise, IEEE Transactions on Instrumentation and Measurements, Vol. IM-16, No. 4, pp. 285-294, 1967.
2. B. Leskovar, Essential nonlinearity of phase-sensitive detector characteristics, IEEE Transactions on Instrumentation and Measurements, Vol. IM-18, No. 2, pp. 81-87, 1969. Also in Proceedings of the 6th Allerton Conference on Circuit and System Theory, pp. 122-130, Urbana, Illinois, 1969.
3. D. Middleton, An Introduction to Statistical Communication Theory (McGraw-Hill Book Company, Inc., New York, 1960), pp. 1075-1076.
4. A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, Higher Transcendental Functions, Vol. 1 (McGraw-Hill Book Company, Inc., New York, 1953), pp. 277-282.
5. A. R. Johnson, private communication, 1969.

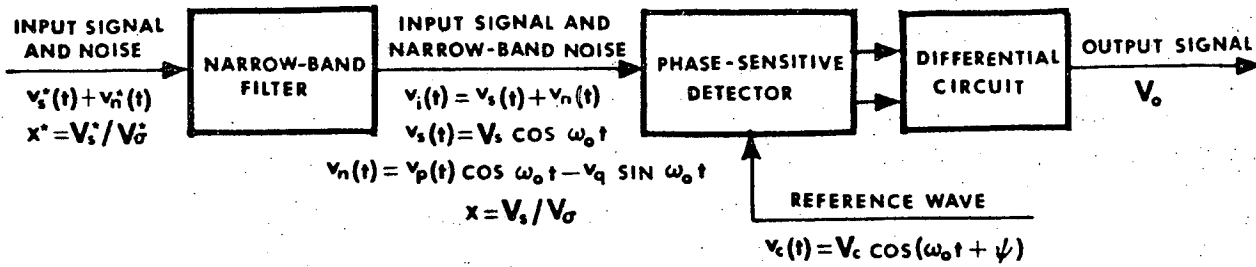


Fig. 1. Idealized phase-sensitive detection system.

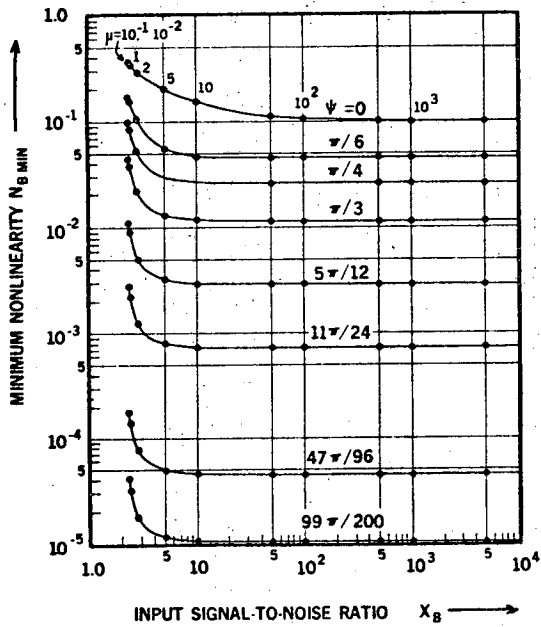


Fig. 2. Minimum nonlinearity $N_{B \text{ MIN}}$ as a function of the optimum value of the input signal-to-noise ratio x_B , with the phase angle ψ and the reference wave-to-input noise ratio as parameters.

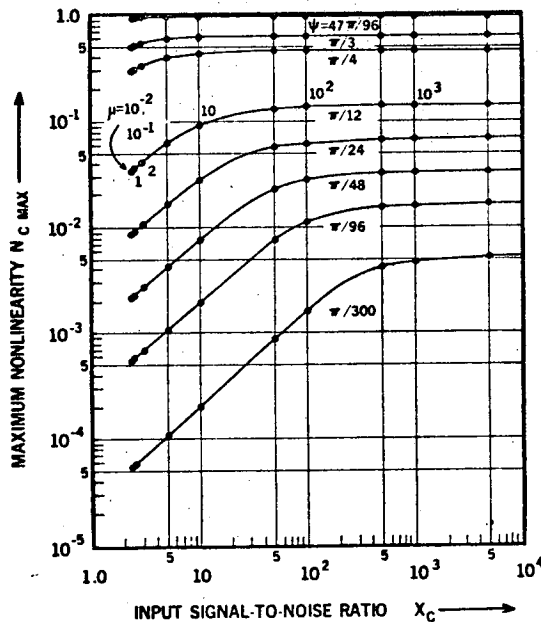


Fig. 3. Maximum nonlinearity $N_{C \text{ MAX}}$ as a function of the nonoptimum value of the input signal-to-noise ratio x_C , with the phase angle ψ and the reference wave-to-input noise ratio as parameters.

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or*
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.*

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

TECHNICAL INFORMATION DIVISION
LAWRENCE RADIATION LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720