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X-ray Bi-prism Interferometry – A Design Study of Proposed Novel Hardware

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Running Title: X-ray Bi-prism Interferometry

11 **Abstract**

12 **Purpose:** Advances in X-ray phase-contrast imaging can obtain excellent soft-tissue contrast of phase-shift,
13 attenuation, and small angle scatter. Here we present fringe patterns for different design parameters of X-
14 ray bi-prism interferometry imaging systems. Our aim is to develop bi-prism interferometry imaging
15 systems with excellent polychromatic performance that produce high-contrast fringes with spatially
16 incoherent X-ray illumination. We also introduce a novel X-ray tube design that uses temporal multiplexing
17 to provide simultaneous virtual “electronic phase stepping” that replace “mechanical phase stepping”
18 popular with grating-based interferometry setups.

19 **Methods:** In our investigation we develop expressions for the irradiance distribution pattern of a bi-prism
20 interferometer comprised of multiple point sources and multiple bi-prisms. These expressions are used to
21 plot fringe patterns for X-ray design parameters including size of point source, number of point sources,
22 and point source separation; and bi-prism design parameters including material, angle, number of bi-
23 prisms, period, and bi-prism array to X-ray source and to detector distances.

24 **Results:** Results show that the fringe patterns for a bi-prism interferometry system are not longitudinally
25 periodic as with grating interferometers that produce a Talbot-Lau carpet. It is also shown that at 59 keV X-
26 rays the bi-prism material should be something comparable to nickel to obtain reasonable fringe visibility.

27 **Conclusion:** The irradiance distribution pattern demonstrates that bi-prism interferometry may provide
28 comparable or improved fringe visibility to that of gratings. Special care is given to present our findings
29 within the context of previous advancements. A single-shot image acquisition approach using a temporal
30 multiplexed, high-power X-ray source provides virtual electronic phase stepping without focal spot
31 sweeping. This provides multiple images, each at the same exposure and the same projection view, from
32 different fringe locations that allow one to derive the attenuation, phase, and dark-field images of the
33 sample without mechanical phase stepping of a grating.

34 1. INTRODUCTION

35 To take advantage of the full potential of X-ray CT, there is a need for imaging systems that measure the
36 full extent of the physics (attenuation, differential phase-shift, small angle scatter) involved in the
37 interaction of X-rays with soft tissues. In a recent review it was demonstrated that interferometry-based X-
38 ray imaging can provide excellent soft-tissue contrast of phase and directional small angle scatter
39 information in addition, attenuation properties of tissues common in present day X-ray CT.¹ We propose to
40 design such imaging systems using refractive bi-prisms^{2,3} with optimum material and geometric parameters
41 to provide fine spatial fringe modulation with high intensity, and to develop novel X-ray tubes that
42 eliminate the need for mechanical phase stepping.

43

44 1.A. Difference between grating and bi-prism interferometry

45 There are a variety of X-ray phase contrast techniques all of which are based on the observation of
46 interference patterns between deviated and un-deviated rays. Some approaches generate a Talbot-Lau
47 interference pattern shown in Fig. 1 using a grating. This produces a replication of the grating via
48 overlapping interference fringe patterns at regular distances away from the source. By placing the detector
49 at the particular distance where the signal of the intensity is amplified and using methods to measure phase
50 shifts in the interference fringes one is able to separate X-ray attenuation, phase, and scatter of the sample
51 being imaged by comparing the observed pattern with and without the sample. In Fig. 1 for a plane wave
52 passing through a grating, we see different fringe patterns mimicking the source images at regular distances
53 from the source. We will show for the bi-prism interferometer that the amplified fringe patterns are not
54 found at periodic longitudinal distances. Moreover, we will show that the visibility of many, but not all, of
55 the fringe patterns is diminished by the increased numbers of bi-prisms.

Fig. 1. (a) Schematic diagram of an X-ray grating interferometry system. (b) Talbot-Lau carpet. Illuminating plane wave passes through a grating producing a fringe pattern with replicating amplified fringe patterns at regular distances from the sources produced by the grating. At $Z_T/2$ there is a secondary Talbot image and at Z_T a replication of the original Talbot image that emerged from the grating. At $Z_T/4$ there is a double frequency fractional image and increased frequency of images at less fractional distances. (Modified from Wikipedia: https://en.wikipedia.org/wiki/Talbot_effect.)

In our lab, we are investigating a novel X-ray interferometer (Fig. 2) wherein an array of Fresnel bi-prisms produces interference fringes with X-rays from a source grating (G_0).⁴ The system does not have a phase stepping grating (G_2) common in most X-ray phase contrast imaging systems, but our intent is to use hardware, such as a detector/scintillator with small hexagonal elements,^{5,6} which acts as a G_2 analyser grating producing a Moiré pattern⁷ on the detector. The source grating G_0 forms multiple mutually-incoherent sources of X-ray illumination from a single X-ray source. The multiple mutually-incoherent X-rays sources refract through each element of a bi-prism, thus overlapping as if proceeding from two slightly separated virtual sources of coherent rays (Fig. 3). By selecting bi-prism design parameters of material, angle, number of bi-prisms, period, and bi-prism to X-ray source and to detector distances; the bi-prism array interferometer provides spatially modulated intensity across a wide field as each center fringe thus produced falls in a resonant position at the detector. An X-ray bi-prism material has widely varied refracting power relative to wavelength and thus the separation of the virtual sources also varies with X-ray

76 wavelength. The setup allows for rotating the sample itself to capture different angular views needed for
 77 analysing the X-ray scattering resulting from the orientation of a sample's internal microstructure.

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Fig. 2. Schematic diagram of an X-ray bi-prism interferometry system. (a) Operation of a Fresnel X-ray bi-prism, (b) experimental setup with multiple X-ray origins each producing fringes in resonant position, and (c) detail of production of Moiré fringes via a tilted grating G_2 at the detector. Our intention is to eliminate the G_2 grating and instead use only a detector/scintillator with small hexagonal elements to produce the Moiré fringe pattern. (Modified from our earlier publication.⁴)

Fig. 3. X-ray beam deflection by a bi-prism illustrating the distance d between the two virtual coherent point sources produced by a bi-prism. (Copied from The Fresnel Biprism, David-Alexander Robinson and Jack Denning; Daniel Tanner 08332461; 10th December 2009.)

1.B. Mathematical manipulation of a single exposure of an X-ray phase contrast interferometer

To reduce the acquisition time and dose, mathematical data processing methods have been developed to extract the data from a single exposure. One method⁸ “reverse projection” processing is based on manipulation of the phase relation between different X-ray projections and uses tomography to reconstruct the absorption coefficient and refractive index by filtered backprojection of sampled projections. Another method using a single lead grid,^{9,10} relies on Fourier analysis of spatial harmonics to obtain absorption, differential phase contrast, and diffraction images, all with a single exposure. The method assumes that refraction and diffraction in the imaged object is manifested as position shifts and amplitude attenuation of the fringes, respectively. In an additional method,¹¹ the scan of the phase-stepping grating is combined with the rotation of the sample so that a single detector frame per projection angle is obtained, yielding images corresponding to all three contrast modalities. Most single shot methods produce Moiré fringe patterns and depends on Fourier analysis to extract the attenuation, phase, and small angle scatter.¹²⁻¹⁷ In particular one paper¹³ used continuous wavelet transforms to extract the phase information from Moiré interferograms. In another paper,¹⁶ Moiré fringe patterns are produced by positioning a two-dimensional checkerboard grating at the first Talbot position beyond the object being imaged. Differential phase-contrast and absorption

107 images are obtained by Fourier analysis of the Moiré fringe patterns. In two other papers,^{15,17} in addition to
108 the source grating (G_0), the phase grating (G_1) and the analyzer grating (G_2) are designed and manipulated
109 to produce a Moiré pattern. In the future, our goal is to develop an imaging system that produces a Moiré
110 pattern without a G_2 grating.

111

112 **1. D. Applications of bi-prism interferometry**

113 Our bi-prism setup is the first to explore X-ray phase contrast bi-prism interferometry in a laboratory
114 setup. The bi-prism is a different approach from gratings to produce refracted waves for extracting the
115 properties of X-rays. Its fringe pattern is divergent in most setups. Bi-prisms have had many applications in
116 the optical regime.² Most of the investigations in X-ray bi-prism applications has been using hard X-rays
117 generated in synchrotrons.^{3,18-21} However, it has also found application to illustrate the wave-particle
118 behaviour in the single-photon regime.^{22,23} Bi-prisms have also been used in interference electron
119 microscopy.²⁴

120 **Outline of paper.** In the following we develop analytical expressions for the irradiance distribution of an
121 array of bi-prisms. Using the expressions, we demonstrate the non-periodic pattern characteristic of a bi-
122 prism interferometry system. We also evaluate results of various design parameters for a bi-prisms
123 interferometry system. We provide results for 17.5 keV X-rays with silicon and a plastic/epoxy photoresist
124 material (SU-8) demonstrating irradiance distributions for different number of point sources, point source
125 separations, and bi-prism angles. We then investigate the possibility of a nickel bi-prism array for 59 keV
126 that one might consider in the design of a high energy X-ray CT scanner. The results are followed by a
127 discussion of the potential merits of bi-prism interferometry.

128 **2. MATERIALS AND METHODS**

129 **2.A. X-ray bi-prism interferometry system**

130 In this paper, we propose a modified X-ray interferometer (Fig. 2) wherein the phase grating (G_1) is
131 replaced with an array of Fresnel micro-bi-prisms. A novel design for the source grating (G_0) is also
132 provided and the analyzer grating (G_2) might be eliminated, though in our present analysis we use a rotated
133 grating (G_2) to produce a Moiré pattern. The Fresnel bi-prism produces high-contrast fringes with spatially
134 incoherent X-ray illumination. The bi-prism array is composed of 7,000 sets of two counter-positioned
135 refractive prisms illuminated with multiple sources. X-rays refracting through each prism element overlap
136 and form divergent (magnified) interference fringes on a distant detector. A further advantage when using
137 polychromatic radiation is that the central fringe produced by a bi-prism is always a “white light” fringe.²⁵

138
139 Fig. 4. Lab setup of the bi-prism interferometry system at UCSF. The bi-prism interferometry system
140 includes a molybdenum-target X-ray tube operating at 35 kVp with predominate energy of the 17.5 keV
141 characteristic line (designed energy/wavelength). Shown also is a gold source grating with 0.8 μm apertures
142 separated by 4.8 μm , a bi-prism array made of plastic/epoxy photoresist material (SU-8), and a Stanford
143 Photonics iCCD camera.

Our laboratory bi-prism interferometry system (Fig.4) includes a Rigaku X-ray tube (Rigaku Corp., Akishima-shi, Tokyo, Japan) operating at 17.5 keV with a point-size of $100\ \mu\text{m}$ and a 15 mm wide gold source grating with $0.8\ \mu\text{m}$ apertures separated by $4.8\ \mu\text{m}$. The bi-prism array consisting of multiple $10\ \mu\text{m}$ bi-prisms with centers separated by $10\ \mu\text{m}$ is curved with radius of 210 mm (We assumed flat in our numerical calculations.), is located 0.53 m from the source grating, and is a plastic/epoxy photoresist material (SU-8) with refraction index decrement of $\delta=8.7\times 10^{-7}$. A gold source grating with $0.8\ \mu\text{m}$ apertures separated by $4.8\ \mu\text{m}$ provides multiple point sources to the bi-prisms of coherent sources of X-rays, which are each mutually incoherent. The X-ray tube source size determines the number of point source apertures that illuminate a single bi-prism. We assume that each point source aperture will radiate 17 separate bi-prisms. The detector is a Stanford Photonics CCD camera and is located 1.73 m from the source grating.

Our work is aimed at imaging small objects at energies of 17.5 keV; however, our X-ray source to detector distance is appropriate for a human size X-ray CT scanner. The equations developed in the following section provide parameters that one can change to design a bi-prism interferometry system at higher X-ray energies.

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2.B. Analytical expression for the irradiance distribution pattern

In the following section, we develop analytical equations for the irradiance distribution pattern for our bi-prism interferometry system. We use these equations to plot the distribution of fringes for different parameters of a bi-prism interferometry system.

Assuming an X-ray beam proceeding from a spatially incoherent planar source with wavelength λ illuminates a thin bi-prism (Fig. 3), we write the irradiance distribution I of X-ray sources on an arbitrary plane placed at a distance z beyond the bi-prism as²

$$I(\vec{x}, z; \eta) = \frac{\eta^2}{z^2} I_s\left(\frac{-\eta\vec{x}}{z}\right) \otimes_2 I_0(\vec{x}, z; \eta), \quad (1)$$

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where η is the distance between the source and the bi-prism, z is the distance between the bi-prism and the imaging plane, $\otimes_2$ is the 2D convolution performed over the transverse coordinates $\vec{x}=(x, y)$, I_s is the irradiance distribution of the planar source, I_0 is the irradiance distribution of the Fresnel bi-prism, and I is the irradiance distribution at the observation plane.

For a spherical wavefront proceeding from a point source illuminating a Fresnel bi-prism, the exiting wavefront produces an interference pattern I_0 beyond the Fresnel bi-prism whose irradiance distribution is

$$I_0(\vec{x}, z; \eta) = 1 + \cos\left(\frac{2\pi x}{p}\right),$$

where $p = \lambda(\eta + z)/(2\eta \tan \alpha)$ is the period of the interference pattern, $\alpha = \delta \tan \chi$ is the angle of the beam deflection,³ δ is the refraction index decrement, and χ is the angle of the bi-prism shown in Fig. 5. Here the expression for the period of the interference pattern for X-rays differ from the expression for light:²

$p = \lambda(\eta + z)/(2(n-1)\eta\theta)$, where n is the index of refraction and θ is the refringence angle of the bi-prism.

Note that light rays refract toward the normal to the material interface and X-rays refract away from the normal (Fig. 5).

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Fig. 5. X-ray beam deflection by bi-prism. Here is given the expression for the angle α of the beam deflection and the distance d between the two virtual coherent point sources produced by a bi-prism. δ is the refraction index decrement. Notice the difference in refraction between X-rays and light. (Modified from Chegg Study, www.x-ray-optics.de, and researchgate.net)

Of special interest is a source composed by an array of N_s mutually incoherent point sources with the same irradiance, I_p , and arranged equidistant perpendicular to the bi-prism edge and distributed symmetrically to the optical axis. In a realistic experimental situation, the width Δ of the sources along the x - and y -direction would not be infinitesimal. In that case, the irradiance distribution of each source can be written as the convolution between a delta function and a rectangular function of width Δ :

$$h(x, y) = I_p \delta(x, y) * \Pi\left(\frac{x}{\Delta}, \frac{y}{\Delta}\right).$$

(Note: Here $*$ refers to the 2D convolution.) If the point sources are located with x -coordinates

$$x_i = \left(\frac{N_s + 1}{2} - i\right) x_0, i = 1, \dots, N_s,$$

where x_0 is the separation between neighbor point sources, then from Eq. (1) the irradiance distributions of each point source is

$$g(x, y, z; \eta, x_i) = h\left(\frac{-\eta x}{z} - x_i, \frac{-\eta y}{z}\right) * I_0(x, y, z; \eta) = h\left(\frac{-\eta x}{z} - x_i, \frac{-\eta y}{z}\right) * \left[1 + \cos\left(\frac{2\pi x}{p}\right)\right],$$

where $-z/\eta$ is the magnification factor between the source and observation plane. The irradiance distribution of all the sources is therefore the summation of the distribution of source irradiances

$$I(\vec{x}, z; \eta) = \frac{\eta^2}{z^2} \sum_{i=1}^{N_s} g(x, y, z; \eta, x_i).$$

Using Mathematica (Wolfram Research, Champaign, Illinois), we come up with the following analytical expression for the irradiance distribution pattern for $N_b = 1$ (one bi-prism):⁴

$$I_{N_b=1}(x, z; \eta) = \begin{cases} I_p \left(\Delta^2 + \frac{p \eta \Delta}{\pi z} \cos\left[\frac{2\pi x}{p}\right] \sin\left[\frac{\pi z \Delta}{p \eta}\right] \right) & \text{if } N_s = 1 \\ \textcolor{red}{I}_p \left(N_s \Delta^2 + \frac{p \eta \Delta}{\pi z} \cos\left[\frac{2\pi x}{p}\right] \sin\left[\frac{\pi z \Delta}{p \eta}\right] \csc\left[\frac{\pi x_0 z}{p \eta}\right] \sin\left[\frac{\pi N_s x_0 z}{p \eta}\right] \right) & \text{if } N_s > 1 \end{cases}, \quad (2)$$

where

$$p = \lambda(\eta + z) / (2\eta \tan \alpha) = \lambda(\eta + z) / [2\eta \tan(\delta \tan \chi)]$$

is the period of the interference pattern. Notice that the expression is given only as a function of the x -coordinate, assuming the irradiation distribution in y is minimal.

Equation (2) gives an expression for one bi-prism with one and multiple point sources. To obtain an expression for multiple bi-prisms, we used Mathematica to sum the irradiance distribution of N_b shifted bi-prisms separated by x_b using the expression

$$I(x, z; \eta) = \sum_{j=1}^{N_b} I_{N_b=1}[(N_b + 1/2 - j)x_b] \quad ,$$

to obtain the following expression for multiple bi-prisms and multiple sources:

$$I(x, z; \eta) = I_p \left[N_b N_s \Delta^2 + \frac{p \eta \Delta}{\pi z} \cos\left(\frac{2\pi x}{p}\right) \csc\left(\frac{\pi x_0 z}{p\eta}\right) \csc\left(\frac{\pi x_b}{p}\right) \sin\left(\frac{N_s \pi x_0 z}{p\eta}\right) \sin\left(\frac{N_b \pi x_b}{p}\right) \sin\left(\frac{\pi z \Delta}{p\eta}\right) \right]. \quad (3)$$

215 3. RESULTS

216 Mathematica was used to numerically evaluate Eq. (2) to obtain fringe patterns behind a bi-prism for
 217 incoherent X-ray point sources located at $\eta=0.4$ m in front of the bi-prism (see Fig. 6). For the calculation
 218 we set $\lambda=7.1\times 10^{-11}$ m, $I_p=1/\Delta^2$, $\Delta=7.00\times 10^{-7}$, and $\alpha=\delta \tan(\chi)$, where $\delta=1.57\times 10^{-6}$. Using these
 219 parameters, calculations were made for 25-point sources with a bi-prism angle of $\chi=82^\circ$. The scale
 220 increases in proportion to the number of point sources.⁴ Non-periodic visibility is seen in Fig. 6. This is
 221 characteristic of a fringe pattern for a bi-prism and differs from the periodic pattern of the well-known
 222 Talbot–Lau interferometer with gratings in Fig. 1.

223 Fig. 6. Density plot of a fringe pattern for 25-point sources with 1 bi-prism. The amplification of the
 224 interference pattern is repeated at non-periodic distances away from the plane of the bi-prism. For the
 225 calculation we set $\lambda=7.1\times 10^{-11}$ m (17.5 keV), $I_p=1/\Delta^2$, $\Delta=7.00\times 10^{-7}$ m, $\alpha=\delta \tan(\chi)$, $\delta=1.58\times 10^{-6}$
 226 (silicon), $\chi=82^\circ$, $\eta=0.4$ m, and $x_0=36.7\ \mu\text{m}$. This figure is modified with new parameters from that of our
 227 previous publication.⁴
 228
 229

230 Notice that in our calculations using Eq. (2) we set the irradiance I_p of each point source to $1/\Delta^2$. One
231 might want to consider this to be a flux of 1 photon per Δ^2 . Then the resultant intensity in Eq. 2 is a total
232 number of photons. However, since we do not know the photon flux of the source, we interpret the result of
233 Eq. (2) as a relative measure or factor of increase in intensity. Keep in mind that our equations represent
234 refraction and do not include attenuation.

235 Mathematica was also used to numerically evaluate Eq. (3) to obtain a density plot in Fig. 7 of a fringe
236 pattern behind one bi-prism with 17-point sources for set of parameters used in our laboratory setup that
237 differed from those in Fig. 6.

Fig. 7. Density plot of a fringe pattern for 17-point sources with 1 bi-prism for a different set of parameters than those in Fig. 6. For the calculation we set $\lambda=7.1 \times 10^{-11} \text{m}$ (17.5 keV), $I_p=1/\Delta^2$, $\Delta=8.00 \times 10^{-7} \text{ m}$, $\alpha=\delta \tan(\chi)$, $\delta=8.7 \times 10^{-7}$ [plastic/epoxy photoresist material (SU-8)], $\chi=85.3^\circ$, $\eta=0.53 \text{ m}$, and $x_0=4.8 \mu\text{m}$, $x_b=10 \mu\text{m}$. There is a fringe separation of $1.09 \times 10^{-5} \text{ m}$ at $z=1.2 \text{ m}$.

If we change the source separation in Fig. 7 from $x_0=4.8 \mu\text{m}$ to $x_0=36.7 \mu\text{m}$, we see a very different fringe pattern in Fig. 8. This demonstrates that the fringe pattern is sensitive to the distance x_0 between the point sources.

248

249 Fig. 8. Density plot of a fringe pattern for 17-point sources with 1 bi-prism for the same set of parameters
 250 as those in Fig. 7, except x_0 was changed from $x_0=4.8\text{ }\mu\text{m}$ to $x_0=36.7\text{ }\mu\text{m}$. For the calculation:
 251 $\lambda=7.1\times 10^{-11}\text{ m}$ (17.5 keV), $I_p=1/\Delta^2$, $\Delta=8.00\times 10^{-7}\text{ m}$, $\alpha=\delta\tan(\chi)$, $\delta=8.7\times 10^{-7}$ [plastic/epoxy
 252 photoresist material (SU-8)], $\chi=85.3^\circ$, $\eta=0.53\text{ m}$, and $x_0=36.7\text{ }\mu\text{m}$, $x_b=10\text{ }\mu\text{m}$.
 253

254 The fringe pattern is sensitive to the bi-prism material, the bi-prism angle χ , and the distance x_0 between
 255 the point sources. In Fig. 6 the fringe pattern corresponds to silicon ($\delta=1.58\times 10^{-6}$) with parameters
 256 $\chi=82^\circ$, $\eta=0.4\text{ m}$, $x_0=36.7\text{ }\mu\text{m}$, and $\Delta=7.00\times 10^{-7}\text{ m}$; whereas, in Fig. 7 the fringe pattern corresponds to
 257 SU-8 material ($\delta=8.7\times 10^{-7}$) used in our laboratory setup with parameters $\chi=85.3^\circ$, $\eta=0.53\text{ m}$, $x_0=4.8\text{ }\mu\text{m}$
 258 m, and $\Delta=8.00\times 10^{-7}$. Since the refractive index decrement of SU-8 is less than silicon, thus decreasing
 259 the angle of deflection ($\alpha=\delta\tan\chi$); in our design we increased the angle of each bi-prism to $\chi=85.3^\circ$.
 260 Probably the significant increase in the number of non-periodic fringes in Fig. 8 is due to changing the
 261 source separation from $x_0=4.8\text{ }\mu\text{m}$ to $x_0=36.7\text{ }\mu\text{m}$.

262 The visibility, $(I_{\max}-I_{\min})/I(I_{\max}+I_{\min})$, was calculated for a few examples by finding the minimum and
 263 maximum intensity for $\pm 5\text{ }\mu\text{m}$ in x about the maximum central fringe. If we use 17-point sources with one

264 bi-prism, as in Fig. 8, we obtain a visibility of 0.93. If the point source separation is change from 36.7 μm
 265 to 4.8 μm the visibility is improved to 0.95. If for the same point source separation of 36.7 μm , the point
 266 source size Δ is changed from 8×10^{-7} to 4×10^{-7} , the visibility is improved even more to 0.98. These trends
 267 in the visibility calculations are as one would expect; however, are higher than one would expect in a
 268 laboratory X-ray phase contrast interferometer. The work of Isakovic et al.¹⁹ provided bi-prism
 269 measurements performed at the Brookhaven National Laboratory synchrotron where a single bi-prism was
 270 evaluated in an X-ray beam of 9.5 keV. Their data showed a difference in fringe visibility in the vertical
 271 and the horizontal axis, with a maximum fringe visibility in the vertical above 0.9 and approximately 0.6 in
 272 the horizontal axis. From this, they calculated an effective asymmetric source of their beamline as a vertical
 273 source of $10 \pm 5 \mu\text{m}$ and a horizontal source of $130 \pm 20 \mu\text{m}$. From generally known parameters, the
 274 transverse coherence length of that beamline would be $\sim 160 \mu\text{m}$ in the vertical and $\sim 12.3 \mu\text{m}$ in the
 275 horizontal. In the simulation of our laboratory setup, we use an array of 0.8 - μm sources at 0.53-meters in
 276 front of an array of bi-prisms. At 17.5 KeV, the transverse coherence length of the operable axis is 11.8 -
 277 μm and one would expect to produce one central fringe per each source/bi-prism pair. Of course, our
 278 proposed setup is an array of bi-prisms with a source grating as an array of small sources; thus, the detector
 279 records many such fringes each falling in resonant positions.

280 Again, using Mathematica to numerically evaluate Eq. (3), we obtained a fringe pattern behind 101 bi-
 281 prisms for 17-point sources. Notice in Fig. 7 that for a single bi-prism the maximum intensity is at
 282 approximately $z = 1.2 \text{ m}$, whereas, in Fig. 9 for 101 bi-prisms, the maximum intensity appears to be at $z = 1.05 \text{ m}$. Increasing the number of bi-prisms increases the fringe intensity and shifts the maximum intensity
 283 toward the sources while bringing the maximum fringe intensity more in focus and greatly reducing fringe
 284 intensity in other regions.

286

287

288 Fig. 9. Density plot of a fringe pattern for 17-point sources with 101 bi-prism for the same set of
 289 parameters as in Fig. 7. For the calculation we set $\lambda = 7.1 \times 10^{-11} \text{ m}$ (17.5 keV), $I_p = 1/\Delta^2$, $\Delta = 8.00 \times 10^{-7} \text{ m}$,
 290 $\alpha = \delta \tan(\chi)$, $\delta = 8.7 \times 10^{-7}$ [plastic/epoxy photoresist material (SU-8)], $\chi = 85.3^\circ$, $\eta = 0.53 \text{ m}$, and $x_0 = 4.8 \mu \text{ m}$,
 291 $x_b = 10 \mu \text{ m}$. There is a fringe separation of $1.00 \times 10^{-5} \text{ m}$ at $z = 1.05 \text{ m}$.
 292
 293

294 The equations developed in this section provide plots of the non-periodic fringe pattern, the intensity of
 295 the maximum fringe intensity, and the fringe separation at the maximum fringe intensity for our laboratory
 296 bi-prism design. Equation (3) was used to compare different possible designs of a bi-prism interferometry

system for parameters given in Table 1. Designs were compared between a bi-prism material of our present lab setup of plastic/epoxy photoresist material (SU-8) and silicon at 17.5 keV. The refractive index decrement for SU-8 was not listed in the CXRO online calculator,²⁷ but is a similar plastic to PMMA. The paper of Mamyrbayev et al.²⁶ states on page 7 that the refractive index of SU-8 is 9.23×10^{-7} at 17 keV, which is the value CXRO calculates for PMMA at 17 keV. The evaluations in our paper were performed for 17.5 keV, thus we used the value of $\delta = 8.7 \times 10^{-7}$ for SU-8 at 17.5 keV which is the value for PMMA at 17.5 keV in the CXRO online calculator.²⁷

By selecting different bi-prism materials, one can design a system that would be appropriate for an X-ray CT system with higher X-ray source energy than the 17.5 keV used here in our validations. In our demonstrations here, we have attempted to keep the distance between the X-ray source and detector with reasonable real world X-ray imaging applications. By modifying the bi-prism material and angle of the bi-prism one maintains reasonable X-ray source to detector distances. Silicon and our laboratory SU-8 material are not ideal for a higher energy of 59 keV. A better material would be nickel. A value of $\delta = 5.06 \times 10^{-7}$ was obtained for nickel at 59 keV from the X-Ray Optics Calculator.²⁸ As one sees from Table 1, the fringe separation decreased by a factor of approximately 3. The angle of the bi-prism is also a function of the material with silicon requiring a smaller angle to bring the detector distance close to that of our SU-8 material, whereas in Table 1 we see that an increased angle of 87° is required for nickel.

Table 1. Equation (3) was used to calculate the fringe separation and the fringe intensity for our laboratory bi-prism SU-8 material and compared to silicon at 17.5 keV. The fringe separation was nearly $10 \mu m$ for our laboratory setup and would be the same for the designated parameters for a bi-prism material of silicon.

323 The same equation was used to design a bi-prism nickel interferometry system at 59 keV. The distances
 324 between the bi-prisms and the selected detector positions were determined by observing the density plots
 325 like the one in Fig. 9.

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	Design 1 (Fig. 6)	Design 2	Design 3	Design 4 (Fig. 9)	Design 5
Bi-prism material	Silicon	plastic/epoxy photoresist (SU-8)	Silicon	plastic/epoxy photoresist (SU-8)	Nickel
Energy	17.5 keV	17.5 keV	17.5 keV	17.5 keV	59 keV
Wave length λ	7.1×10^{-11} m	7.1×10^{-11} m	7.1×10^{-11} m	7.1×10^{-11} m	2.14×10^{-11} m
Refractive index decrement δ	1.58×10^{-6} *	8.7×10^{-7} **	1.58×10^{-6} *	8.7×10^{-7} *	5.06×10^{-7} +
Bi-prism angle χ	82°	85°	82°	85.3°	87°
Angle of deflection α	11.2×10^{-6}	9.9×10^{-6}	11.2×10^{-6}	10.63×10^{-6}	9.66×10^{-6}
Bi-prism separation x_b	---	---	$10 \mu\text{m}$	$10 \mu\text{m}$	$3.65 \mu\text{m}$
Number of bi-prisms N_b	1	1	101	101	101
Number of X-ray sources N_s	25	25	17	17	17
Source size Δ	7.00×10^{-7} m	7.00×10^{-7} m	8.00×10^{-7} m	8.00×10^{-7} m	8.00×10^{-7} m
Pt. source separation x_0	$36.7 \mu\text{m}$	$36.7 \mu\text{m}$	$4.8 \mu\text{m}$	$4.8 \mu\text{m}$	$6 \mu\text{m}$
Distance between source and bi-prism η	0.4 m	0.4 m	0.53 m	0.53 m	0.51 m
Distance between bi-prism and detector z	1.38 m	1.4 m	1.14 m	1.05 m	1.19 m
Fringe separation	14.1×10^{-6} m	16.1×10^{-6} m	10.0×10^{-6} m	10.0×10^{-6} m	3.69×10^{-6} m
Factor of increase in fringe intensity	46.2	48.6	2176	2052	1733

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In a previous publication⁴ we showed that the distance to the maximum fringe pattern is sensitive to

336 small changes in the angle of the bi-prism. To study this, one can look at a simple case of one point source

337 and one bi-prism. We first write the intensity in Eq. (1) as a function of the angle of the bi-prism χ :

338 $Intensity[\chi, x, z] =$

$$339 \quad I_p \left(\Delta^2 + \frac{\Delta}{\pi z} ((\eta + z) * \lambda) / (2 \tan[\delta \tan(\chi)]) \cos \left[\frac{4 \pi x \eta \tan[\delta \tan(\chi)]}{(\eta + z) * \lambda} \right] \sin \left[\frac{2 \pi z \Delta \eta \tan[\delta \tan(\chi)]}{(\eta + z) * \lambda \eta} \right] \right), \quad (4)$$

340 where z is the distance between the bi-prism and the fringe pattern and x is the lateral distance on the
 341 detector. One can then take the partial derivative of the intensity with respect to the angle to come up with
 342 the expression:

$$343 \quad \frac{\partial Intensity(\chi, x, z)}{\partial \chi} = \frac{-1}{2 \pi z} I_p \delta \Delta \csc[\delta \tan(\chi)] \sec(\chi)^2$$

344 Using this expression, we obtain plots of the sensitivity of the Intensity as a function of the angle of the
 345 bi-prism (Fig. 10). One sees that the derivative of the Intensity is fairly flat near the central fringe $x=0$ for
 346 $z=1$ m until the angle nears $\pi/2=1.5708$; then it changes rapidly for an energy of 17.5 keV [Fig.10(a)]. For
 347 59 keV, the change is more dramatic near $\pi/2$ [Fig.10(b)].

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349 Fig. 10. Sensitivity as a function of bi-prism angle χ for X-ray source energies of (a) $\lambda = 7.1 \times 10^{-11}$ m
 350 (17.5 keV) and (b) $\lambda = 2.14 \times 10^{-11}$ m (59 keV). For the calculation we set $N_s=1$, $N_b=1$, $I_p=1/\Delta^2$,
 351 $\Delta=8.00 \times 10^{-7}$ m, $\delta=8.7 \times 10^{-7}$ (SU-8), $\eta=0.53$ m, $x=0.000001$ m, and $z=1$ m.

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354 Using Eq. (4) one can study the Intensity variation due to the size of the point source aperture Δ and the
 355 point source separation x_0 . For the parameters in Fig. 9, increasing the point source aperture Δ will as
 356 shown in Fig. 11 decrease the maximum fringe intensity until the aperture Δ approaches that of a point
 357 source aperture separation of $4.8 \mu\text{m}$ with a minimum around $7 \mu\text{m}$. The Intensity then increases as the

apertures continue to overlap the separation between point sources. In Fig. 12 we see that the Intensity will be maximum for point source separations x_0 at factors of $x_b/2=5\mu\text{m}$. This is helpful because we can see that if we change the value of x_b to 5 in Fig. 9, we would have come up with a better factor of increase in intensity of 3238 with a fringe separation of $10.0 \times 10^{-6} \text{ m}$ at $z=1.06 \text{ m}$.

Fig. 11. Intensity as a function of point source size Δ :

$Intensity = \frac{13736 \Delta + \sin(626988 \Delta)}{8 \Delta}$. For the calculation we set $N_s=17$, $N_b=101$, $I_p=1/\Delta^2$, $\delta=8.7 \times 10^{-7}$ (SU-8), $\chi=85.32^\circ$, $\eta=0.53 \text{ m}$, $x_0=4.8 \mu\text{m}$, $x_b=10 \mu\text{m}$, $x=0 \text{ m}$, and $z=1.06 \text{ m}$.

Fig. 12. Intensity as a function of separation between point sources x_0 . For the calculation we set $N_s=17$, $N_b=101$, $I_p=1/\Delta^2$, $\Delta=8.00 \times 10^{-7} \text{ m}$, $\delta=8.7 \times 10^{-7}$ (SU-8), $\chi=85.32^\circ$, $\eta=0.53 \text{ m}$, $x_b=10 \mu\text{m}$, $x=0 \text{ m}$, and $z=1.06 \text{ m}$.

369 **4. DISCUSSION**

370 This study provides expressions for fringe patterns of bi-prism interferometry and demonstrates
371 maximum fringe visibility regions that are not periodic as is common with Talbot–Lau interferometry
372 systems using diffraction gratings. Using the same analytical expressions, we calculate maximum fringe
373 contrast and fringe periods for different X-ray source sizes and separations; and bi-prism design parameters
374 of material, angle, size, number, period, and distances between the source and detector. The expressions
375 are used to design and calibrate our laboratory system so that experiments can be performed that would be
376 appropriate in the design of X-ray CT systems with higher energies and with high-power X-tubes capable
377 of providing phase images almost instantaneously.

378

379 **4.A. Advantages of a single exposure bi-prism interferometry system**

380 The well-established grating-based method of phase contrast X-ray imaging may yet prove capable of
381 clinical applications. Still, alternative approaches such as the bi-prism method herein discussed holds
382 several possible advantages. For good flux throughput, we have constructed a bi-prism array from the
383 polymer SU8, a popular photoresist useful with LIGA processes. SU8 is a photoactive compound with some
384 high Z components in low percentages. For 17 keV, the data yields 1/e absorption lengths for the polymer of
385 1825 μm , or a transparency of 97% for our optic design.

386 A bi-prism works well with polychromatic X-rays, as the fringe period is, in part, determined by the
387 separation of the two virtual sources. This spacing is not fixed, as with a grating or Young’s double slit, but
388 rather varies dependent on the refractive index of the prism material relative to X-ray wavelength. Whereas,
389 under polychromatic illumination the design of a grating system should be optimized for a particular
390 frequency of X-rays.²⁹

Moreover, the fringe period generated from a laboratory bi-prism setup has a relatively large magnification factor of approximately 2.5X. Some publications^{30,31} have shown that modifying the standard Talbot-Lau interferometer where by the G_1 grating is placed close to the source can also increase magnification factors similar to those in our bi-prism system. A larger fringe period at the detector could allow using a structured scintillator to record Moiré patterns. A hexagonal structured scintillator (Fig. 13) might aid the fast acquisition of phase contrast data of X-ray attenuation, phase, and small angle scatter. However, there are some challenges with using bi-prisms as well, mostly related to poorer spatial resolution. Even though, bi-prism-based interferometry, or other refractive-optical approaches, hold the potential to revolutionize phase-contrast and dark-field X-ray CT.

Fig. 13. Proposed cone-beam tensor tomography system. The X-ray source and optics rotate around an axis aligned to the center of an area detector with hexagonally segmented crosstalk-free scintillation elements. The components are mounted on a revolving gantry (for clarity, not shown here). X-ray scatter data is captured for three specific orientations 120° apart for every angular view.

In our previous paper³² we performed simulations using wave optics to evaluate X-ray phase contrast imaging with grating interferometry using phase stepping and with bi-prism interferometry using single

411 exposure Moiré fringe analysis. Results of reconstructed vector coefficients of small angle scatter showed
412 that bi-prism interferometry with single-exposure Moiré fringe analysis has an increase in noise compared
413 to grating interferometry using phase stepping because of 1/8th the number of projection samples but
414 provides better CNR due to better photon flux with less dose and faster acquisition times.

415

416 ***4.A.1. Design of structured scintillators***

417 The implementation of a structured scintillator^{5,6} fits well into our design of an X-ray bi-prism
418 interferometry system with single-exposure Moiré fringe analysis because it reduces crosstalk between
419 pixels of our high-resolution large fringe separation. Though the authors of this referenced work aimed
420 their development for grating systems, a structured scintillator is ideal for our bi-prism interferometry
421 system. The structure scintillator consists of filling an array of silicon pores with a thallium-doped cesium
422 iodide [CsI(Tl)] scintillator. The pores act as a mold which are filled with melted CsI(Tl) powder. Before
423 filling, the walls are coated thermally with an oxide layer to provide a wave guide to direct the light and
424 reduce noise between pixels. The design of a structured scintillator prevents crosstalk and allows
425 opportunity for Moiré fringe development when illuminated with spatially modulated X-ray illumination.
426 Moiré patterns occur because of the interaction between the pixelated image sensor and the periodic pore
427 structure of the scintillator. The magnitude of this occurrence depends on the ratio of the respective pitches,
428 the geometries involved and the amount of angular misalignment. The Moiré pattern is ideal for the
429 extraction of dark field scatter by taking the first harmonic of the Fourier transform. The work of
430 Rutishauser et al.⁶ showed an increase in fringe visibility at X-ray energies around 60 keV (greater than our
431 simulations of 17.5 keV), leading to an improvement in the quality of the differential phase and dark field
432 data compared to a conventional grating interferometry system.

433

434 ***4.A.2. Parameter range for a bi-prism interferometry system***

435 In our laboratory setup, we have been using a Rigaku X-ray tube with molybdenum anode operating at
436 35kVp, where 17.5 keV is the predominate energy with a point-size of 100 μm and a 15 mm wide gold
437 source grating with 0.8 μm apertures separated by 4.8 μm . We assume the multiple point source apertures
438 are the “open windows” of the source grating. These are much smaller than the “closed bars” of a typical
439 source grating and provide to the bi-prism array multiple coherent sources of X-rays, which are each
440 mutually incoherent. There is some reduction in image resolution by using laboratory X-ray tube sources.
441 The actual laboratory X-ray source size is that of a typical X-ray tube and determines the number of “open
442 window” point sources that illuminate a single bi-prism. We assume that each point source aperture will
443 radiate 17 separate bi-prisms. The intensity of the fringe visibility increases with the number of sources and
444 the number of sources seen by each bi-prism. The intensity is maximum if the source separation is a factor
445 of one-half the width of the bi-prism. The intensity also increases with the increase in the angle of the bi-
446 prism. The rate of change of this increase is fairly flat until the angle of the bi-prism nears 90°.

447 Our bi-prism system design, like that proposed by Yaroshenko *et al.*³³ for a grating interferometry
448 system, offers the potential for future X-ray CT application by providing high fringe visibility at shorter
449 propagation distances. They demonstrated that using phase gratings with triangular-shaped structures in an
450 X-ray interferometer can yield high visibilities for significantly shorter propagation distances than
451 conventional gratings with binary structures. This is especially important in the design of human systems
452 where bore sizes are approximately 75-85 cm.

453 Although similar in construction as Yaroshenko *et al.*³³ our interferometry methods follow Doblus *et al.*² -
454 albeit with X-rays rather than visible light. Where Doblus *et al.* use multiple coherent sources and a single
455 Fresnel bi-prism, we use an array of Fresnel bi-prisms. The full setup yields multiple fringe patterns in
456 resonant superposition with a large field of illumination. Fundamentally, this is a different way to make
457 interference fringes. A single coherent X-ray source can illuminate a single Fresnel bi-prism and produces
458 interference fringes downfield. The fringe patterns produced by our array of bi-prisms are only indirectly

related to the array period, but are the direct result of the periodicity of the virtual sources created by the individual bi-prisms in the array. Interference fringes produced by a grating are directly related to the period of the grating, and this includes a triangular-profile grating such as used by Yaroshenko *et al.*³³ Moreover, their work highlights the benefit of closer spacing of 10 cm between the phase grating and analyzer grating. This is the opposite effect we are pursuing in our approach with a bi-prism array to analyzer/structured detector of 1.06 m.

Periodic replication of a grating, which is the Talbot effect, results from the overlapping zero-order, 1st order and -1st order diffracted beams, when those beams encounter a screen (detector). Yaroshenko *et al.*³³ rightly calls for future work on sinusoidal amplitude gratings as those gratings produce only those diffraction orders. Their triangular-shaped grating system clearly demonstrates periodic longitudinal spacing of fringe visibility regions.

Our goal has been to keep the dimensions of our laboratory system within something that would fit a present-day large bore X-ray CT scanner. Though our initial work is aimed at imaging small objects at energies of 17.5 keV, we have performed calculations using Eq. (3) to see how the parameters of the bi-prism system would change at higher X-ray energies. For example, if we were to image at 59 keV, because of the change in refraction index decrement at higher energies, we would have to modify the material of the bi-prism to something like nickel in Table 1 to obtain reasonable fringe visibility at 1.19 m from the bi-prism array with a fringe separation of $3.69 \mu m$.

Recent advances in grating fabrication techniques^{34,35} alleviate some concerns regarding high-energy X-ray phase contrast imaging. The pattern height of our laboratory G_0 grating is 24 microns, however 200 microns are now available, which could be available for higher energies. Much of the X-ray flux is absorbed by G_0 . Thus, with higher energies the thicker G_0 grating will have its effect on flux. However, current CT tube technology, can operate at >100 kW for the scan duration necessary to acquire clinical tensor tomographic data including attenuation, phase, and small angle scatter information.

483

484 **4.A.3. Advantages of single shot approaches**

485 There are single shot approaches,⁷⁻¹¹ which render phase, scattering, and attenuation data from single
486 projection measurements without mechanical phase stepping measurements at each projections angle. The
487 challenges of mechanical phase stepping are the need to obtain precise grating posing at multiple
488 measurements per projection angle, whereas single shot approaches of data acquisition have shorter
489 acquisition times and have better mechanical stability. However, the paper of von Teuffenbach *et al.*³⁶ does
490 point out some negative aspects of previous single shot methods. For example, the reverse projection
491 method⁸ combines two interferograms taken exactly 180 degrees apart, which requires stability of the
492 grating alignment for each view and gives only access to attenuation and differential phase-contrast
493 projection images. The implementation of helical and cone beam tomography would be difficult. The Moiré
494 analysis method⁷ using a slight detuning of the interferometer suffers from loss in resolution in one
495 dimension of the detector. The sliding window method¹¹ requires only a single interferogram per projection
496 angle, but changes the grating position at each angular position. Missing interferograms are simply
497 interpolated from adjacent projection angles. This works well for scans with high angular sampling.

498 The paper of von Teuffenbach *et al.*³⁶ compares three single shot approaches and demonstrate that it is
499 possible to successfully reconstruct the linear attenuation coefficient, the refractive index and the linear
500 diffusion coefficient (measure of small angle scatter attenuation) using a single measurement per projection
501 angle without grating movements. In their paper using the interferogram-based statistical iterative
502 reconstruction (IBSIR) method of data reconstruction, they compare: 1. an equidistant phase stepping (PS)
503 acquisition where the analyzing grating was shifted by 1/3rd the grating period to obtain 3 measurements at
504 101 equal angular positions over 360° (identified as PS-IBSIR); 2. a sliding window phase-stepping (SW)
505 acquisition that records only one measurement at each of 303 equal angular positions over 360° with the
506 analyzing grating shifted by 1/3rd its period before the next acquisition (SW-IBSIR); and 3. a single shot

electromagnetic phase-stepping acquisition (SSEPS) recording a single image at each of 303 equal angular positions over 360° with the position of the X-ray focal spot shifted from the previous acquisition before each measurement (SSEPS-IBSIR). These three methods provided the same number of measurements. The PS-IBSIR showed artifacts at sharp boundaries, probably due to the finer angular sampling even though the same number of measurements. The SW-IBSIR and SSEPS-IBSIR methods showed similar X-ray attenuation reconstruction with the SW-IBSIR method showing a slightly better contrast but showing more artifacts in the phase reconstruction and somewhat more noise in the reconstruction of the scatter diffusion coefficient. As can be seen from the author's very nice display of the Radon space sampling, even though both methods provide the same number of angular sampling, the regular sampling of SW-IBSIR method probably has more aliasing than the irregular sampling of the SSEP-SIBSIR method with the motion of the X-ray focus spot. This Radon space sampling proposes interesting future investigations similar to those that have been pursued using data compression with irregular Fourier space sampling in MRI.^{37,38} The proposed electronic phase stepping acquisition protocol incorporated into an interferometry system would be perfectly suited for application with a clinical CT system because the required focal spot sweeping is already available in most modern clinical CT systems and the technique is well suited for a continuously rotating gantry; however, it supposes one could design an X-tube with electronic phase stepping without focal spot sweeping.

4.B. X-ray tube design for virtual electronic phase stepping

Electromagnetic phase-stepping (EPS)³⁹ moves an electrical beam to move projections of the object across the detector whereby the shifted images are combined to form pseudo phase-stepping images. Here we propose a single-shot image acquisition approach using a novel X-ray source that provides for "virtual" electronic phase stepping without focal spot sweeping. This approach involves using an X-ray interferometer imaging system where the X-ray tube has multiple X-ray origins/spots. The tube is operated

531 in such a way that each spot is temporally modulated at differing frequencies, and spatially modulated to
532 form illumination beams with differing fringe locations as observed in the interferometer. Composite
533 images of an object acquired with the interferometer are subsequently separated according to the flashing
534 source that provided the illumination. This is accomplished via de-multiplexing transform processes⁴⁰ based
535 on the known temporal multiplexing transform of the source. After an alignment step, the isolated images
536 from a single composite image/exposure have the same view of the object but with different fringe
537 locations relative to a feature in the object. An isolated image group of each single view are then analyzed
538 to yield images representing the differential phase shift, attenuation, and X-ray scattering features of the
539 object. All of the X-ray optical components in the interferometer, either a Talbot-Lau or a biprism setup,
540 remain fixed and motionless.

541 A high-power X-ray tube design that does this full work of virtual electronic phase stepping during a
542 single exposure is illustrated in Fig. 14. Here, a binary multiplexing transform of the X-ray output is
543 conducted by normal tube operation. Multiple X-ray spots are generated on the side of a rotating and
544 linearly-translated cylinder. An X-ray window allows that G_0 is illuminated by each spot, but at slightly
545 different angles. The oscillating linear translation, about 2 Hz, carries the anode past separate, adjacent,
546 focused, and identical high-current electron guns. As the anode cylinder walls are patterned, either with
547 metal/insulator or with ridges/grooves, in varying stepped spatial periods, each X-ray spot turns on/off at
548 different frequencies.

549 Other multiplexing transforms can result from different patterning, which might work better with slower
550 detectors. Lower power tubes can control the electron guns independently to accomplish a similar
551 multiplexed X-ray tube output.

552

553

554 Fig. 14. Illustration of multiple-spot cylindrical rotating anode X-ray tube with oscillating translation for
 555 phase/dark-field single-shot data acquisition. (a.) Anode at minimum extension, (b.) Anode at maximum
 556 extension, **detail**: showing multiple spot tracks/flash rates 1.) Low frequency spot, 2.) Lower mid-
 557 frequency spot, 3.) Higher mid-frequency spot, 4.) High-frequency spot, 5.) Very high-frequency spot.
 558 (Image courtesy Rigaku Corp., Akishima-shi, Tokyo, Japan.)

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563 **5. CONCLUSION**

564 Phase-contrast and dark-field X-ray CT for clinical human use will require advancements in X-ray optics,
565 X-ray sources, and X-ray detectors. As an alternative X-ray optical component, a Fresnel bi-prism array
566 adds to the set of tools available for the development of phase contrast imaging systems. Here, we present
567 one possible approach that is somewhat like an “inverse geometry Talbot-Lau” setup, but with improved
568 fringe visibility. Other bi-prism-based interferometry approaches are possible, including combinations with
569 phase gratings. We also present an advanced X-ray tube design that offers single-shot “virtual electronic
570 phase stepping.” These two technologies, when combined with previously demonstrated structured-
571 scintillator detectors, can provide separate phase, dark-field, and attenuation signals without the need for
572 mechanical phase stepping. A system employing these components could be made to capture X-ray dark
573 field projections for direct tomographic tensor reconstruction of small angle scatter. The review of the
574 literature presents evidence of advancement in hardware (without grating phase stepping) and of new
575 algorithm developments that could provide fast single shot acquisition with mechanical simplicity
576 delivering less radiation exposure and faster acquisition times.

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580

581 **CONFLICT OF INTEREST**

582 The authors have no conflict of interest.

583

584 **DATA AVAILABILITY**

585 Data distribution is available through an appropriate data sharing agreement between institutions.

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FIGURE CAPTIONS

680

681 Fig. 1. (a) Schematic diagram of an X-ray grating interferometry system. (b) Talbot-Lau carpet.
682 Illuminating plane wave passes through a grating producing a fringe pattern with replicating amplified
683 fringe patterns at regular distances from the sources produced by the grating. At $Z_T/2$ there is a secondary
684 Talbot image and at Z_T a replication of the original Talbot image that emerged from the grating. At $Z_T/4$
685 there is a double frequency fractional image and increased frequency of images at less fractional distances.
686 (Modified from Wikipedia: https://en.wikipedia.org/wiki/Talbot_effect.)

687

688 Fig. 2. Schematic diagram of an X-ray bi-prism interferometry system. (a) Operation of a Fresnel X-ray
689 bi-prism, (b) experimental setup with multiple X-ray origins each producing fringes in resonant position,
690 and (c) detail of production of Moiré fringes via a tilted grating G_2 at the detector. Our intention is to
691 eliminate the G_2 grating and instead use only a detector/scintillator with small hexagonal elements to
692 produce the Moiré fringe pattern. (Modified from our earlier publication.⁴)

693

694 Fig. 3. X-ray beam deflection by a bi-prism illustrating the distance d between the two virtual coherent
695 point sources produced by a bi-prism. (Copied from The Fresnel Biprism, David-Alexander Robinson and
696 Jack Denning; Daniel Tanner 08332461; 10th December 2009.)

697

698 Fig. 4. Lab setup of the bi-prism interferometry system at UCSF. The bi-prism interferometry system
699 includes a molybdenum-target X-ray tube operating at 35 kVp with predominate energy of the 17.5 keV
700 characteristic line (designed energy/wavelength). Shown also is a gold source grating with $0.8 \mu m$ apertures
701 separated by $4.8 \mu m$, a bi-prism array made of plastic/epoxy photoresist material (SU-8), and a Stanford
702 Photonics iCCD camera.

703

Fig. 5. X-ray beam deflection by bi-prism. Here is given the expression for the angle α of the beam deflection and the distance d between the two virtual coherent point sources produced by a bi-prism. δ is the refraction index decrement. Notice the difference in refraction between X-rays and light. (Modified from Chegg Study, www.x-ray-optics.de, and researchgate.net)

708

Fig. 6. Density plot of a fringe pattern for 25-point sources with 1 bi-prism. The amplification of the interference pattern is repeated at non-periodic distances away from the plane of the bi-prism. For the calculation we set $\lambda=7.1 \times 10^{-11}$ m (17.5 keV), $I_p=1/\Delta^2$, $\Delta=7.00 \times 10^{-7}$ m, $\alpha=\delta \tan(\chi)$, $\delta=1.58 \times 10^{-6}$ (silicon), $\chi=82^\circ$, $\eta=0.4$ m, and $x_0=36.7 \mu\text{m}$. This figure is modified with new parameters from that of our previous publication.⁴

714

Fig. 7. Density plot of a fringe pattern for 17-point sources with 1 bi-prism for a different set of parameters than those in Fig. 6. For the calculation we set $\lambda=7.1 \times 10^{-11}$ m (17.5 keV), $I_p=1/\Delta^2$, $\Delta=8.00 \times 10^{-7}$ m, $\alpha=\delta \tan(\chi)$, $\delta=8.7 \times 10^{-7}$ [plastic/epoxy photoresist material (SU-8)], $\chi=85.3^\circ$, $\eta=0.53$ m, and $x_0=4.8 \mu\text{m}$, $x_b=10 \mu\text{m}$. There is a fringe separation of 1.09×10^{-5} m at $z=1.2$ m.

719

Fig. 8. Density plot of a fringe pattern for 17-point sources with 1 bi-prism for the same set of parameters as those in Fig. 7, except x_0 was changed from $x_0=4.8 \mu\text{m}$ to $x_0=36.7 \mu\text{m}$. For the calculation: $\lambda=7.1 \times 10^{-11}$ m (17.5 keV), $I_p=1/\Delta^2$, $\Delta=8.00 \times 10^{-7}$ m, $\alpha=\delta \tan(\chi)$, $\delta=8.7 \times 10^{-7}$ [plastic/epoxy photoresist material (SU-8)], $\chi=85.3^\circ$, $\eta=0.53$ m, and $x_0=36.7 \mu\text{m}$, $x_b=10 \mu\text{m}$.

724

Fig. 9. Density plot of a fringe pattern for 17-point sources with 101 bi-prism for the same set of parameters as in Fig. 7. For the calculation we set $\lambda=7.1 \times 10^{-11}$ m (17.5 keV), $I_p=1/\Delta^2$, $\Delta=8.00 \times 10^{-7}$ m,

727 $\alpha = \delta \tan(\chi)$, $\delta = 8.7 \times 10^{-7}$ [plastic/epoxy photoresist material (SU-8)], $\chi = 85.32^\circ$, $\eta = 0.53$ m, and
 728 $x_0 = 4.8 \mu\text{m}$, $x_b = 10 \mu\text{m}$. There is a fringe separation of 1.00×10^{-5} m at $z = 1.05$ m.

729

730 Fig. 10. Sensitivity as a function of bi-prism angle χ for X-ray source energies of (a) $\lambda = 7.1 \times 10^{-11}$ m
 731 (17.5 keV) and (b) $\lambda = 2.14 \times 10^{-11}$ m (59 keV). For the calculation we set $N_s = 1$, $N_b = 1$, $I_p = 1/\Delta^2$,
 732 $\Delta = 8.00 \times 10^{-7}$ m, $\delta = 8.7 \times 10^{-7}$ (SU-8), $\eta = 0.53$ m, $x = 0.000001$ m, and $z = 1$ m.

733

734 Fig. 11. Intensity as a function of point source size Δ : $Intensity = \frac{13736 \Delta + \sin(626988 \Delta)}{8 \Delta}$. For the
 735 calculation we set $N_s = 17$, $N_b = 101$, $I_p = 1/\Delta^2$, $\delta = 8.7 \times 10^{-7}$ (SU-8), $\chi = 85.32^\circ$, $\eta = 0.53$ m, $x_0 = 4.8 \mu\text{m}$,
 736 $x_b = 10 \mu\text{m}$, $x = 0$ m, and $z = 1.06$ m.

737

738 Fig. 12. Intensity as a function of separation between point sources x_0 . For the calculation we set $N_s = 17$,
 739 $N_b = 101$, $I_p = 1/\Delta^2$, $\Delta = 8.00 \times 10^{-7}$ m, $\delta = 8.7 \times 10^{-7}$ (SU-8), $\chi = 85.32^\circ$, $\eta = 0.53$ m, $x_b = 10 \mu\text{m}$, $x = 0$ m,
 740 and $z = 1.06$ m.

741

742 Fig. 13. Proposed cone-beam tensor tomography system. The X-ray source and optics rotate around an
 743 axis aligned to the center of an area detector with hexagonally segmented crosstalk-free scintillation
 744 elements. The components are mounted on a revolving gantry (for clarity, not shown here). X-ray scatter
 745 data is captured for three specific orientations 120° apart for every angular view.

746

747 Fig. 14. Illustration of multiple-spot cylindrical rotating anode X-ray tube with oscillating translation for
 748 phase/dark-field single-shot data acquisition. (a.) Anode at minimum extension, (b.) Anode at maximum
 749 extension, **detail**: showing multiple spot tracks/flash rates 1.) Low frequency spot, 2.) Lower mid-

750 frequency spot, 3.) Higher mid-frequency spot, 4.) High-frequency spot, 5.) Very high-frequency spot.

751 (Image courtesy of Rigaku Corp., Akishima-shi, Tokyo, Japan.)