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The bremsstrahlung emitted by an electron scattered in a Coulomb field was first calculated by Bethe and Heitler.[1] The total cross section for production of photons with wave number between k and k + dk by a nonrelativistic electron of kinetic energy ϵ is

$$\frac{d\sigma}{dk}\,dk = \frac{16}{3}\,Z^2 r_0^{\,2}\left(\frac{e^2}{\hbar c}\right)\left(\frac{mc^2}{\epsilon}\right)\log\left\{\sqrt{\frac{\epsilon}{\hbar ck}}+\sqrt{\frac{\epsilon}{\hbar ck}-1}\right\}\frac{dk}{k}\,, \qquad (1)$$

where Ze is the charge of the (heavy) ion, and r_0 is the classical electron radius.

Bremsstrahlung in a plasma has been computed by a number of authors in the approximation of replacing the Coulomb field by a cut-off Coulomb or static Debye potential.[2] It is the purpose of this communication to call attention to another important effect of the medium upon the rate of emission of bremsstrahlung. This may be described as a modification of the relation of the photon's energy to its wave number, due to the index of refraction of the medium. Equivalently, we note that one must include in the calculation of bremsstrahlung in a medium the photon-medium interactions which result in the "clothing" of a "bare" photon. The replacement of a particle by a quasi-particle has long been known to be important in the description of strongly interacting systems of massive particles such as liquid helium;[3] the effect can be particularly dramatic for a photon because the medium gives a nonzero effective mass to the quasi-photon.[4]

The interaction between the electromagnetic field and the plasma leads to the "dressing" of photons as well as their emission and absorption. When the quasi-photon lifetime is finite, no general theory exists for systematically treating both of these effects. However, one can take the first step toward such a theory by using the results of the simple pair or random-phase approximation in which the lifetime is still infinite. In this approximation, the index of refraction (in the long wavelength limit) is given by $n = 1 - \omega_p^2/\omega^2$, which implies the dispersion relation

$$\hbar\omega = \left[(\hbar\omega_p)^2 + (\hbar ck)^2\right]^{1/2} , \tag{2}$$

where ω_p is the plasma (angular) frequency. Thus, in first approximation, a quasi-photon has an effective mass $m^* = \hbar\omega_p/c^2$. The radiation of quasi-photons by electrons in collision with ions is readily calculated by using perturbation theory in the manner of Bethe and Heitler, due account being taken of the change in normalization of the electric field and of the change in the density of final states.[5] If one approximates the screened potential by its static approximation then the total cross section for production of photons with wave number between k and $k + dk$ by an electron of kinetic energy ϵ is

$$\frac{d\sigma}{dk}\,dk = \frac{4}{3} Z^2 r_0^{\,2} \left(\frac{e^2}{\hbar c}\right) \frac{mc^2}{\epsilon} \left\{ \frac{(|\underset{\sim}{p}_1|+|\underset{\sim}{p}_2|)^2}{(|\underset{\sim}{p}_1|+|\underset{\sim}{p}_2|)^2+(\hbar k_D)^2} - \frac{(|\underset{\sim}{p}_1|-|\underset{\sim}{p}_2|)^2}{(|\underset{\sim}{p}_1|-|\underset{\sim}{p}_2|)^2+(\hbar k_D)^2} \right. \tag{3}$$

$$\left. + \ln \left[\frac{(|\underset{\sim}{p}_1|+|\underset{\sim}{p}_2|)^2+(\hbar k_D)^2}{(|\underset{\sim}{p}_1|-|\underset{\sim}{p}_2|)^2+(\hbar k_D)^2}\right] \right\} \frac{c^3k^2dk}{\omega^3} ,$$

where $\underset{\sim}{p}_1$ and $\underset{\sim}{p}_2$ are the initial and final momenta of the electron, k_D is the Debye wave number, and the energy loss of the electron is

$$h\omega = \frac{|\underset{\sim}{p}_1|^2 - |\underset{\sim}{p}_2|^2}{2m} = \left[(\hbar\omega_p)^2 + (\hbar ck)^2\right]^{1/2} . \quad (4)$$

Screening may be neglected by setting k_D to zero, in which case Eq. (3) differs from Eq. (1) by the replacement of dk/k by c^3k^2dk/ω^3. This modification removes the infrared divergence of Eq. (1), because ω approaches ω_p as k approaches zero. Physically, the nonzero mass of the quasi-photon means that an electron of given energy can radiate at most a finite number of quasi-photons. The resulting difference in the energy loss of an electron is significant only for a very slow electron or in a very dense plasma. A second qualitative difference between Eq. (3) and Eq. (1) is the absence of any radiation for ω less than ω_p. In particular, if the correct frequency-dependent shielded potential is employed in the derivation, the classical limit taken, and the result then averaged over a thermal distribution, one easily obtains Dawson and Oberman's[6] result for the plasma emission coefficient.

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Footnotes and References

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