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Sediment Discharge Scaling in Large Rivers of the World

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ABSTRACT

The variation of suspended sediment discharge with respect to changes in drainage area follows a series of scaling laws that express sediment discharge as a power function of drainage area. Four such scaling laws for sediment discharge were discovered for large rivers, with average annual runoff exceeding 10 km^3. These scaling laws require that rivers be categorized in non-overlapping ranges of specific sediment yield. An analysis of the variation of sediment discharge with respect to runoff revealed four other scaling laws for sediment discharge as function of annual runoff categorized by sediment concentration. The applicability of scaling laws is highlighted.

INTRODUCTION

Loáiciga (1997) examined the scaling properties of runoff in large rivers, i.e., those with a mean annual runoff of 10 km^3 or more. There are 47 such rivers, which are listed in Table 1 where they ranked according to decreasing specific runoff. The regions or countries in which these rivers lie are listed at the bottom of Table 1. If mean runoff and drainage area are denoted by Q and A, respectively, the specific runoff (q) is defined as q = Q/A, in meters. It represents the volume of water produced per unit of land surface in a drainage area. Specific runoff is listed in column 5 of Table 1, while drainage area and runoff appear in columns (3) and (4) respectively. Table 1, in addition, shows (suspended) sediment discharge (y), specific sediment yield (s), and sediment concentration (C). Drainage area, mean runoff, and sediment discharge data in Table 1 were first published by Milliman and Meade (1983), who discussed the sources and uncertainty of those data. Specific runoff, specific sediment yield, and sediment concentration in Table 1 are derived variables calculated by this author.

Several authors have researched empirical relationships that relate sediment discharge to drainage area and/or runoff in watersheds. Vanoni (1975, p. 481-484), for example, provided a summary of empirical sediment discharge equations for small watersheds. Strand and Pemberton (1982) related annual suspended sediment yield (in m^3/km^2) to drainage area (in km^2) for rivers in the United States. Many studies dealing with the estimation of sediment production in river basins have been concerned with the assessment of sedimentation of reservoirs and ports, also (see Krynine and Judd, 1972). Studies focused on sediment yield in large rivers (Holeman, 1968; Milliman and Meade, 1983, for example) did not address the issue of sediment discharge scaling examined in this article. The scaling properties of natural phenomena are of scientific and practical interest (see, for example, Kalma and Sivapalan, 1995). The scaling of runoff or sediment discharge, is commonly determined with respect to the river basin's drainage area, a fixed quantity. Other empirical equations have used additional predictor variables (Langbein and Schumm, 1958; Waananen and Crippen, 1977). Loáiciga (1997) showed that the variation of mean runoff with respect to changes in drainage area in large rivers is governed by a power law. The scaling law so discovered for runoff, however, required the classification of rivers according to specific runoff. Specifically, Loáiciga (1997) showed that rivers with specific runoff equal to or larger than 1.0 m/yr (those ranked 1 through 7 in Table 1) follow a specific scaling law, while those with specific runoff between 0.15 and 1.0 m/yr (ranked 8 through 39 in Table 1) scale according to a different power law. Those rivers with specific runoff less than 0.15 m/yr (ranked 40-47 in Table 1) cannot be described by any scaling law.

Figure 1 summarizes the findings of Loáiciga (1997), where it is shown that in rivers with specific runoff q ≥ 1.0 m/yr, the scaling law for mean runoff is:

$$Q_1 = 17.535 A_1^{0.8866} \qquad q \geq 1.0 \text{ m/yr} \qquad (1)$$

in which mean runoff Q_1 is given in km^3/yr and drainage area (A_1) in multiples of 10^4 km^2. The regression coefficient of equation (1) is $R^2 = 0.97$. Equation (1) is drawn as a dashed regression line in Figure 1. The rivers fit by the scaling law in (1) lie within equatorial latitudes or in subtropical latitudes affected by monsoonal activity. Those rivers include, among others, the Amazon (rank 5 in Table 1) -located within equatorial latitudes- and the subtropical Hungho (Vietnam).

The regression equation for rivers with intermediate specific runoff was found to be (Loáiciga (1997):

$$Q_2 = 7.8971 A_2^{0.7708} \qquad 0.15 \leq q < 1.0 \text{ m/yr} \qquad (2)$$

where mean runoff Q_2 is expressed in km^3/yr and drainage area (A_2) in multiples of 10^4 km^2. The regression coefficient of equation (2) is $R^2 = 0.88$. The regression line for equation (2) is shown as a solid line in Figure 1. The rivers fit by the scaling law in (2) are located in a wide range of geographical-climatic zones. The excellent fit exhibited by the power laws (1) and (2) is remarkable because they span changes in mean runoff and drainage area ranging over at least three orders of magnitude.

The scaling of certain natural phenomena according to power laws of the type exemplified by equations (1) and (2) has been the focus of scientific inquiry covering a broad spectrum of problems (e.g., Feder, 1988). There are, moreover, practical reasons for the searching of scaling laws in hydrologic analysis. In the case of runoff and sediment discharge, for example, scaling laws provide in some cases accurate and simple ways for their estimation in ungaged or poorly-sampled river basins. In other instances, predictions of hydrologic variables such as sediment discharge derived from models calibrated to a specific river basin can be "scaled-up or -down" to bigger or smaller river basins, respectively (e.g., Loáiciga et al., 1996). It has been shown

(Loáiciga, 1997), however, that scaling laws of hydrologic phenomena do not reveal themselves in a simplistic fashion. Rather, the discovery of such scaling laws may require a categorization of the scaling variable by means of indexes such as specific runoff and specific sediment yield. Following this line of reasoning, this paper examines the scaling of sediment yield in large rivers of the world, and categorizes the scaling laws using suitable variables.

SCALING LAWS FOR SEDIMENT DISCHARGE

Figure 2 is a scattergram of the sediment discharge (y) vs. drainage area (A) data shown in Table 1. The numbers shown adjacent to points in the scattergram of Figure 2 are the ranks assigned to rivers in Table 1. Those ranks correspond to the specific runoff of rivers. A simple regression line was fitted to the entire data set in Figure 2. It is evident that there is a weak statistical association between sediment discharge and drainage area when the data is analyzed as a whole ($R^2 = 0.037$ in Figure 2). This section postulates the hypothesis that there might be a power law of the type:

$$y = m\,A^{n} \qquad (3)$$

which relates sediment discharge and drainage area, where m and n are constants. It is known from first principles that the sediment discharge (y) is related to the sediment concentration (C) and runoff (Q) as follows (where dimensions must be chosen properly):

$$y = C \cdot Q \qquad (4)$$

It was seen in Figure 1 that, after appropriately accounting for specific runoff, runoff scales according to power laws that can be generally written as follows (see equations (1) and (2)):

$$Q = a\,A^{b} \qquad (5)$$

where a and b are constants. If equations (4) and (5) are combined, the sediment discharge can then be re-written in the following manner:

$$y = C\,a\,A^{b} \qquad (6)$$

Were the sediment concentration constant for all rivers, then equations (3) and (6) indicate that $m = C\cdot a$ and $n = b$ and the sediment discharge would scale in exactly the same way as does runoff times a factor C. In a log-log plot the sediment discharge power laws would be parallel to the runoff scaling laws but shifted an amount log C.

Figure 3 shows a plot of sediment concentration (C) vs. drainage area. It is evident that sediment concentration is hardly a constant with respect to drainage area. Instead, the scattergram of Figure 3 implies a weak statistical association between sediment concentration and drainage area. Specifically, rivers ranked 1 through 7 (those with specific runoff equal to or larger than 1.0 m/yr) and those ranked 40 – 47 (those with specific runoff less than 0.15 m/yr) in Table 1 are well scattered and mixed with rivers of intermediate specific runoff (between 0.15 and 1.0 m/yr, not labeled in Figure 3). This suggests that one must look for a categorizing variable other than specific runoff in the search for scaling laws of sediment discharge. A review of the sediment discharge vs. drainage area data depicted on Figure 2 and of the specific sediment yield associated with them (given in Table 1) suggested the following (non-overlapping) categories of specific sediment yields to classify scaling laws for sediment discharge: (1) $s > 500$ t/ km^{2}yr; (2) $140 \leq s \leq 500$ t/ km^{2}yr; (3) $50 \leq s < 140$ t/ km^{2}yr; (4) $10 \leq s < 50$ t/ km^{2}yr; (5) $s < 10$ t/ km^{2}yr.

Rivers were organized into the latter five categories according to their specific sediment yield. Regression analyses were run for each subset of sediment discharge vs. drainage area data. Figure (4) shows the results obtained. Excellent fits were provided by power laws to each subset of sediment discharge vs. drainage area data, except for the case $s < 10$ t/km^2 yr, i.e., for rivers with the lowest specific sediment yields. Notice that in Figure 4 points are labeled from 1 to 47 and that these numbers correspond to the ranks assigned to rivers in Table 1. This is intended to facilitate comparison of runoff scaling laws depicted in Figure 1 with the scaling laws for sediment discharge shown in Figure 4, while keeping track of the rivers included in each scaling law. Table 1 provides the key to the river name and other statistics for each river.

The identified sediment discharge scaling laws are (see Figure 4):

For rivers with specific sediment yield $s > 500$ t/ km^{2}yr (ranks 1, 4, 7, 11, 12, 18, 42):

$$y_1 = 17.206\, A_1^{0.8689} \qquad R^2 = 0.93, \tag{7}$$

For rivers with specific sediment yield $140 \leq s \leq 500$ t/km^2 yr (ranks 2, 3, 5, 8, 9, 10, 14, 25):

$$y_2 = 5.150\, A_2^{0.8054} \qquad R^2 = 0.95, \tag{8}$$

For rivers with specific sediment yield $50 \leq s < 140$ t/km^2 yr (ranks 13, 15, 20, 24, 26, 27, 28, 32, 34, 43):

$$y_3 = 0.9212\, A_3^{0.9514} \qquad R^2 = 0.95 \tag{9}$$

For rivers with specific sediment yield ranks $10 \leq s < 50$ t/km^2 yr (ranks 16, 21, 22, 23, 31, 33, 35, 36, 38, 40, 45, 46):

$$y_4 = 0.1321\, A_4^{1.0914} \qquad R^2 = 0.87 \tag{10}$$

Rivers that have specific sediment yield less than 10 t/km^2 yr do not exhibit any scaling law with respect to drainage area. Those rivers have ranks 20, 29, 30, 37, 39, 41, 44, 47.

SCALING LAWS, SPECIFIC RUNOFF, AND SEDIMENT YIELD

The first striking feature of the sediment discharge scaling laws (equations (7)-(10)) is that they mix rivers located in widely different climatic regimes. For example, the scaling law expressed by equation (7) –and for which the specific sediment yield is larger than 500 t/km^2 yr- includes the river with the largest specific runoff (the Purari river, rank 1, located in a wet equatorial latitude, in New Guinea) and a river with one of the lowest specific runoffs (the Huangho river, rank 42, located in an semi-arid mid-latitudinal region of China). Likewise, equation (10) applies to rivers of diverse specific runoff, such as the Mehandi (rank 16, specific runoff = 0.515 m/yr, in India) and the Orange (rank 46, specific runoff = 0.0107 m/yr, in South Africa). The mixing of rivers located in widely varying climates in each of the scaling laws (7)-(10) demonstrates that sediment scaling is not well characterized by the amount of runoff produced per unit area (i.e., by specific runoff), but, rather, by the amount of sediment generated per unit area (= specific sediment yield). A corollary of the former conclusion is that specific runoff is not always a good indicator of specific sediment yield, a fact that can be proven by statistical analysis of the specific runoff and specific sediment yield data in Table 1. The nature of equations (7)-(10) indicates that the specific sediment yield integrates very well the features that categorize rivers into suitable scaling laws.

The stronger role that specific sediment yield has on the characterization of sediment discharge scaling laws, relative to that of specific runoff, is not entirely surprising. Sediment production depends on several factors. One of them is the erosivity of water. Runoff, therefore, plays an undeniable role on sediment production. But there are other factors that influence sediment production as well. The erodibility of soils and rocks is a case in point. Erodibility, in turn, may depend on human impacts such as, for example, the large-scale conversion of forests to cultivated land. Therefore, it is not unreasonable to hypothesize, for example, that the presence of the Huangho river (rank 42) and of the Orange river (rank 46) in the scaling laws (7) and (10), respectively, may be caused to a considerable degree by long term land-use changes in their watersheds. The latter hypothesis is substantiated by the (relatively) large sediment concentrations in the Huangho river (22,041 g/m^3, the largest of all the rivers in Table 1) and in the Orange river (1,545 g/m^3).

Another important characteristic of the scaling laws for sediment discharge in equations (7)-(9) is that their exponents are less than 1. In the case of equations (7)-(9) this means that the specific sediment yield ($s = y/A$) decreases with increasing drainage area. The scaling law given by equation (10), however, has an exponent slightly larger than 1. In contrast to all other scaling laws reported in this article, the rivers covered by this law are such that their specific sediment yields increase as their drainage areas increase. The latter pattern is atypical, given that natural phenomena tend to be diluted in strength, measured on a per unit area, when that the total area that they encompass increases. This suggests that some unusual factor may be at play. For example, the presence of the Murray (rank 45, in Australia), and Orange (rank 46) rivers along scaling law (10) in Figure 4 points to the possibility that land-use impacts in their arid watersheds have greatly increased their rates of sediment production. This possibility is supported by the high sediment concentrations in the Murray and Orange rivers, which equal 1,364 g/m^3 and 1,545 g/m^3, respectively.

SEDIMENT DISCHARGE AND RUNOFF SCALING LAWS

Given that the variations of sediment discharge and runoff with respect to drainage area follow a series of scaling laws, shown in the previous sections, the question arises as to whether they scale with respect to each other. In other words, does sediment discharge scale with respect to runoff? It might seem intuitive that sediment discharge should increase when runoff increases. An analysis of this question yielded several regression equations, which are shown in Figure 5. It is seen in Figure 5 that four distinct scaling laws govern the variation of sediment discharge with respect to annual runoff. Those scaling laws apply over non-overlapping ranges of sediment concentration (C) as follows:

For $C > 900$ g/m^3 (rivers with ranks 1, 4, 7, 11, 12, 18, 25, 43, 45, 46):

$y_1 = 1.5895 Q_1^{0.9226}$ $R^2 = 0.78$ (11)

For $200 \leq C \leq 900$ g/m^3 (rivers with rank 2, 6, 8, 9, 10, 14, 15, 19, 24, 26, 27, 28, 32, 34, 36):

$y_2 = 0.2801 Q_2^{1.043}$ $R^2 = 0.92$ (12)

For $100 \leq C < 200$ g/m^3 (rivers with ranks 3, 5, 17, 33, 35, 40):

$y_3 = 0.1159 Q_3^{1.0524}$ $R^2 = 0.99$ (13)

For $10 \leq C < 100$ g/m^3 (rivers with rank 13, 16, 21, 22, 23, 29, 30, 31, 37, 39, 41):

$y_4 = 0.1522 Q_4^{0.76}$ $R^2 = 0.86$ (14)

Rivers with sediment concentrations below 10 g/m^3 (rivers with rank 20, 44, 47) do not follow any particular scaling law involving sediment discharge and runoff. Two of these rivers, the Colorado (USA, rank 44) and the Nile (Africa, rank 47) owe their low sediment discharge to sediment deposition behind large dams. The Saint Lawrence (Canada, rank 20) has most of its sediment trapped in the Great Lakes. The Huangho river (China, rank 42) with a sediment concentration of slightly over 22,000 g/m^3 is not covered by any of the above scaling laws, either. The Huangho river's high sediment concentration is caused by land-use practices that induce large rates of sediment generation within its drainage area. Human impacts are, again, seen to play a role on the behavior of sediment discharge scaling laws.

CONCLUSIONS

A data set of suspended sediment discharge for large rivers of the world (mean runoff equal to or larger than 10 km^3/yr) was analyzed in search for scaling laws of sediment discharge. Following prior work by the author, power functions expressing sediment discharge as a function of drainage area were entertained as plausible scaling laws. Results show that:

- Specific sediment yield (s) is a suitable index to categorize sediment discharge (y) scaling laws. Four, non-overlapping, ranges of specific sediment yield were used to identify scaling laws for sediment discharge. The generic statement of the scaling laws is a power function of the type $y = m A^n$, in which m and n are coefficients that vary with the range of specific sediment yield and A is drainage area. The exponent n was found to be less than unity in three of the scaling laws, implying a decrease of specific sediment yield with increasing area. A fourth scaling law exhibited an exponent larger than unity, for which the specific sediment yield then increases with increases in drainage area. Rivers with specific sediment yield below 10 t/km^2 yr do not scale with respect to drainage area in any systematic fashion.
- The categorization of large rivers by specific sediment yield indicates that their sediment discharges scale homogeneously with respect to drainage area even

though the rivers might belong to very different climates and runoff regimes (the latter measured by their specific runoff). Apparently, all that matters for the existence of a scaling law of sediment discharge is uniformity in specific sediment yield. Human activities that induce large-scale erosion appear to play a role on the observed pattern of sediment discharge scaling.

- An analysis of the variation of sediment discharge with respect to changes in mean runoff (Q) showed that sediment discharge scales as a power law in this instance also. Four scaling laws were identified in this case, with a generic statement of the form $y = r Q^s$, where r and s are constants that depend on the range of sediment concentration (C). Rivers obstructed by large-scale impoundments, such as the Colorado, Nile, and Saint Lawrence, do not scale with respect to mean runoff. Neither does the Huangho river, in China, which has abnormally large losses of loess sediments via runoff.

The existence of hydrologic scaling laws of the type discovered in this work holds promise for the characterization of ungaged or poorly sampled river basins other than those studied herein, especially in rivers of moderate and small size that are important sources of water supply. Scaling of hydrologic model predictions to non-calibrated river basins is another area of potential applicability of hydrologic scaling laws.

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TABLE 1. Runoff and sediment data for large rivers of the world.

Rank (1)	River (2)	Drainage area (10^4 km^2) (3)	Mean runoff (km^3/yr) (4)	Specific runoff (m/yr) (5)	Sediment discharge (10^6 t /yr) (6)	Specific sed. yield (t/ km^{2}yr) (7)	Sed. conc. (g/m^3) (8)
1	Purari	3.1	77	2.48	80	2581	1039
2	Fly	6.1	77	1.26	30	492	390
3	Orinoco	99	1100	1.11	210	212	191
4	Hungho	12	123	1.03	130	1083	1057
5	Amazon	615	6300	1.02	900	146	143
6	Irrawaddy	43	428	1.00	265	616	619
7	Magdalena	24	237	0.988	220	917	928
8	Susitna	5	40	0.800	25	500	625
9	Zhu Jiang	44	302	0.686	69	157	228
10	Po	7	46	0.657	15	214	326
11	Ganges/ Brahmaputra	148	971	0.656	1670	1128	1720
12	Copper	6	39	0.650	70	1167	1795
13	Hudson	2	12	0.600	1	50	83.3
14	Mekong	79	470	0.595	160	203	340
15	Rhone	9	49	0.544	10	111	204
16	Mehandi	13	67	0.515	2	15.4	29.9
17	Fraser	22	112	0.509	20	90.9	179
18	Damodar	2	10	0.500	28	1400	2800
19	Yangtze	194	900	0.464	478	246	531
20	St. Lawrence	103	447	0.434	4	3.88	8.95
21	Columbia	67	251	0.375	8	11.9	31.9
22	Zaire	382	1250	0.327	43	11.3	34.4
23	Severnay Dvina	35	106	0.303	4.5	12.9	42.5
24	Negro	10	30	0.300	13	130	433
25	Godavari	31	84	0.271	96	310	1142
26	Danube	81	206	0.254	67	82.7	325
27	Indus	97	238	0.245	100	103	420
28	Yukon	84	195	0.232	60	71.4	308
29	Yenisei	258	560	0.217	13	5.04	23.2
30	Lena	250	514	0.206	12	4.8	23.3
31	Zambesi	120	223	0.186	20	16.7	89.7
32	Mississippi	327	580	0.177	210	64.2	362
33	Amur	185	325	0.176	52	28.1	160
34	MacKenzie	181	306	0.169	100	55.2	327
35	La Plata	283	470	0.166	92	32.5	196
36	Niger	121	192	0.159	40	33.1	208
37	Ob	250	385	0.154	16	6.4	41.6

38	Indigirka	36	55	0.152	14	38.9	255
39	Sao Francisco	64	97	0.151	6	9.38	61.9
40	Yana	22	29	0.132	3	13.6	103
41	Kolyma	64	71	0.111	6	9.38	84.5
42	Huangho	77	49	0.063	1080	1403	22041
43	Tigris/Euphrates	105	46	0.043	53	50.5	1152
44	Colorado	64	20	0.031	0.1	0.156	5
45	Murray	106	22	0.0208	30	28.3	1364
46	Orange	102	11	0.0107	17	16.7	1545
47	Nile	296	30	0.0101	0	0	0

Notes:

1. Columns 3, 4, and 6 from Milliman and Meade (1983).
2. Country/regions were rivers lie are: 1. New Guinea; 2. New Guinea; 3. Venezuela; 4. Vietnam; 5. Brazil; 6. Burma; 7. Colombia; 8. USA; 9. China; 10. Italy; 11. Bangladesh; 12. USA; 13. USA; 14. Vietnam; 15. France; 16. India; 17. Canada; 18. India; 19. China; 20. Canada; 21. USA; 22. Zaire; 23. Russia; 24. Argentina; 25. India; 26. Romania; 27. Pakistan; 28. USA; 29. Russia; 30. Russia; 31. Mozambique; 32. USA; 33. Russia; 34. Canada; 35. Argentina; 36. Nigeria; 37. Russia; 38. Russia; 39. Brazil; 40. Russia; 41. Russia; 42. China; 43. Iraq; 44. USA; 45. Australia; 46. South Africa; 47. Egypt.

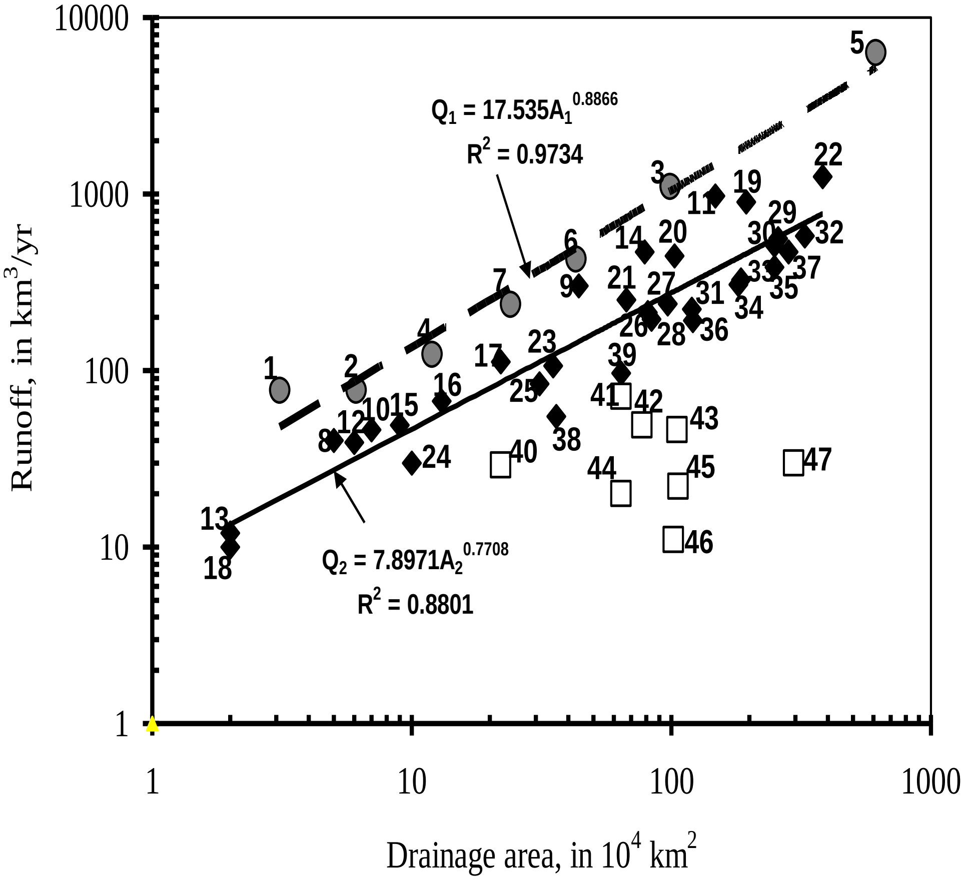

Figure 1. Runoff scaling laws (see equations (1) and (2) in the text for explanation of equations, and Table 1 for interpretation of ranks 1-47 shown in the Figure). Adapted from Loáiciga (1997).

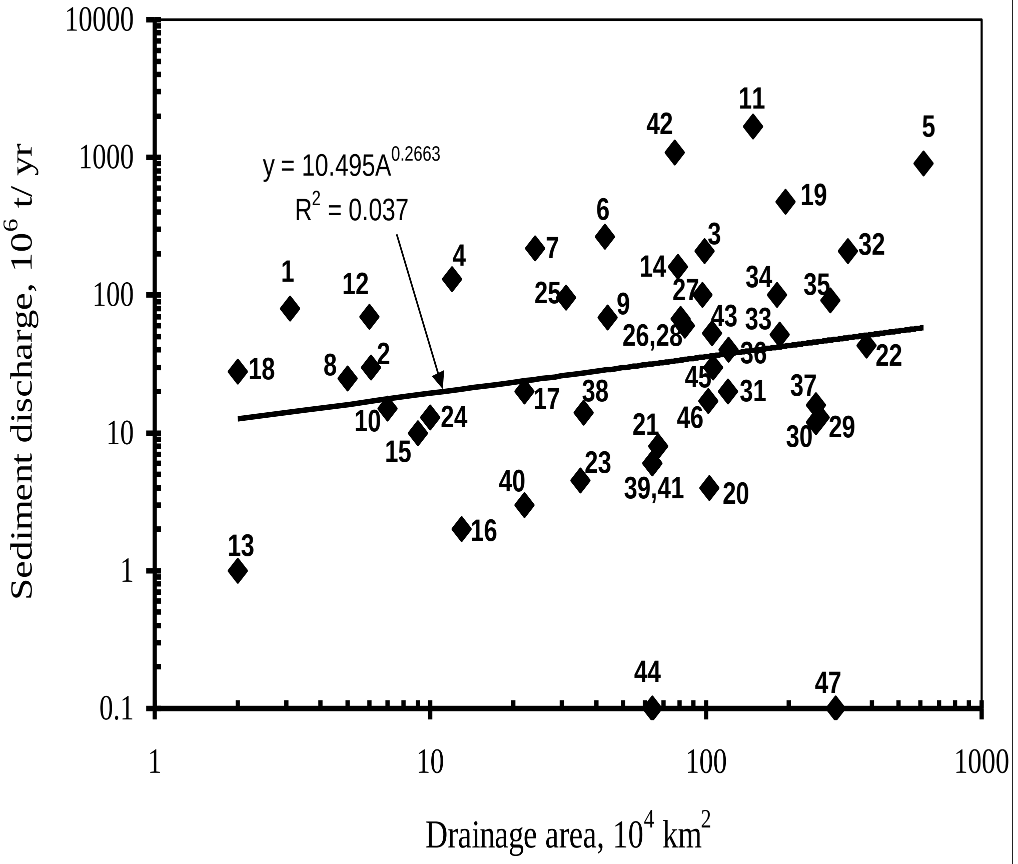

Figure 2. Scattergram of sediment discharge vs. drainage area (see Table 1 for an interpretation of ranks 1-47 shown in the Figure).

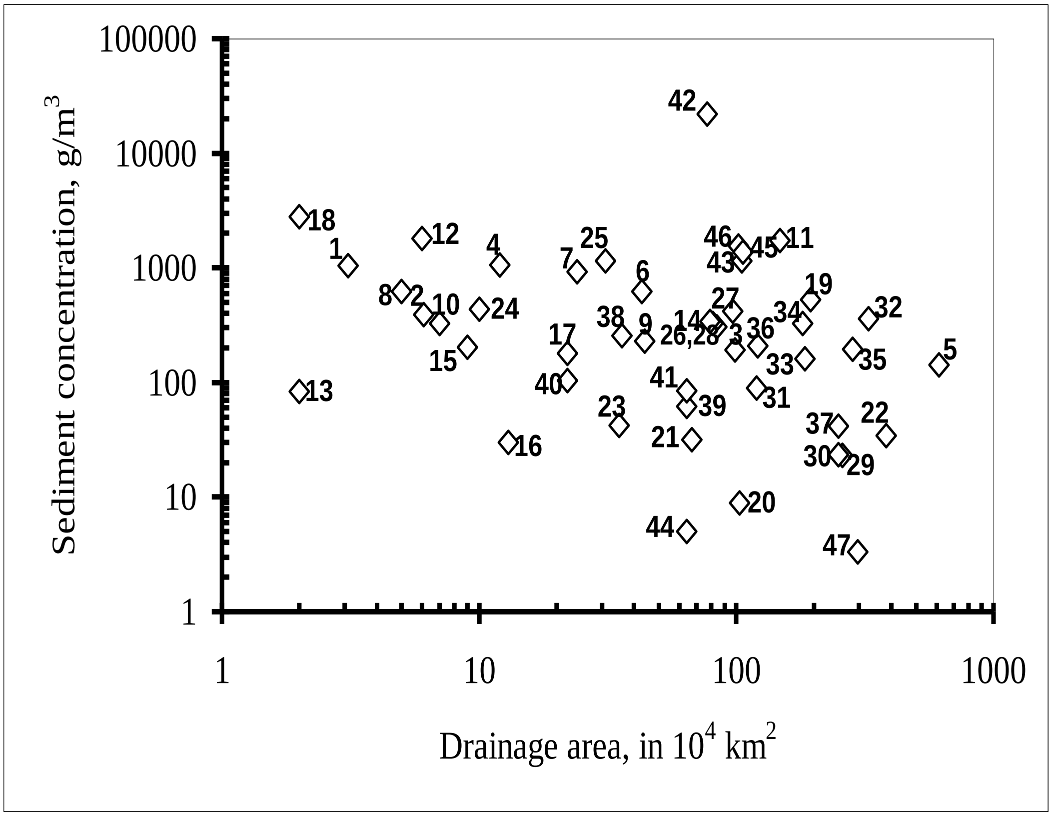

Figure 3. Scattergram of sediment concentration vs. drainage area (see Table 1 for an interpretation of ranks 1-47 shown in the Figure).

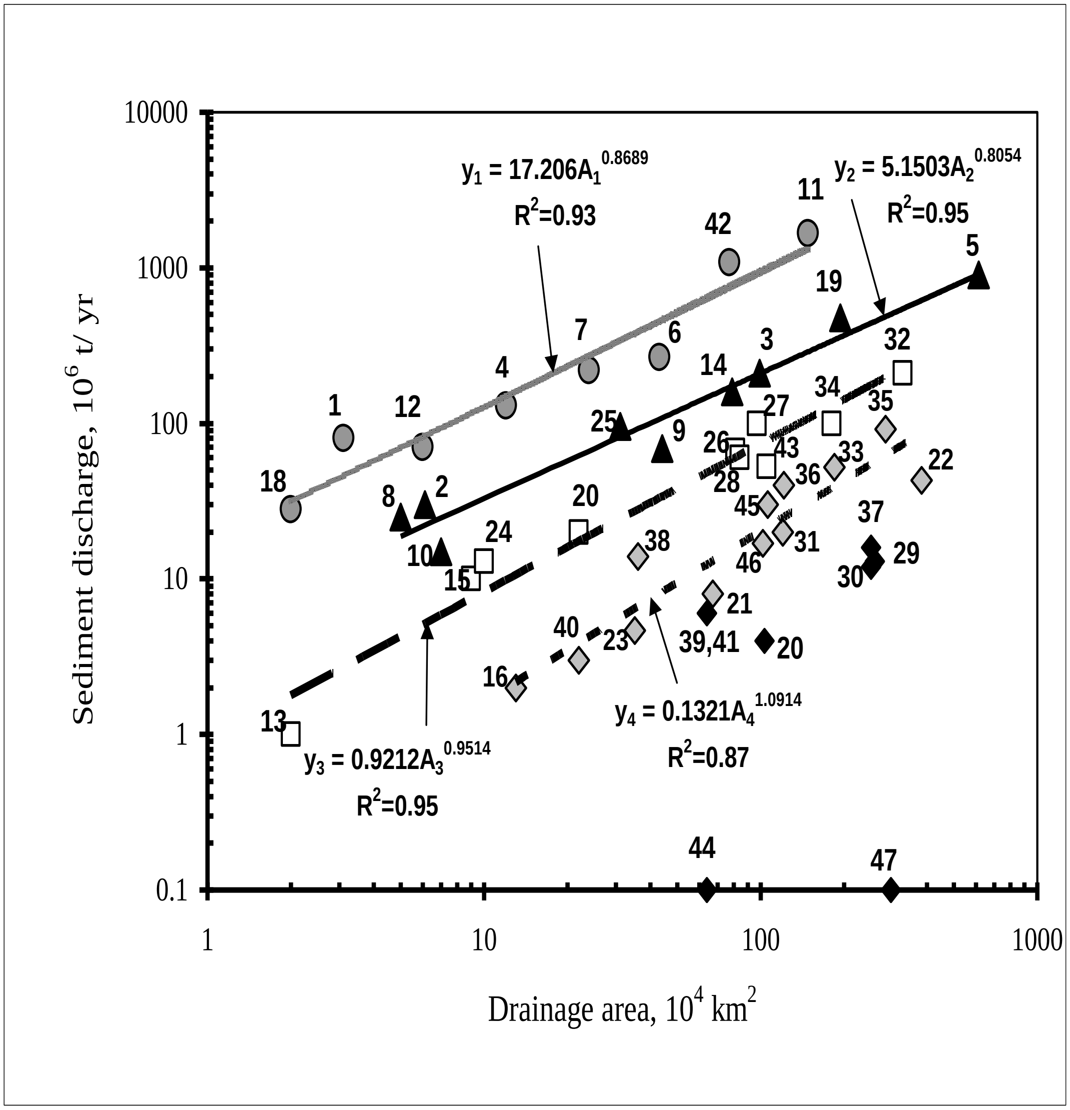

Figure 4. Scaling laws for sediment discharge (see equations (7)-(10) in the text for explanation of equations, and Table 1 for interpretation of ranks 1-47 shown in the Figure).

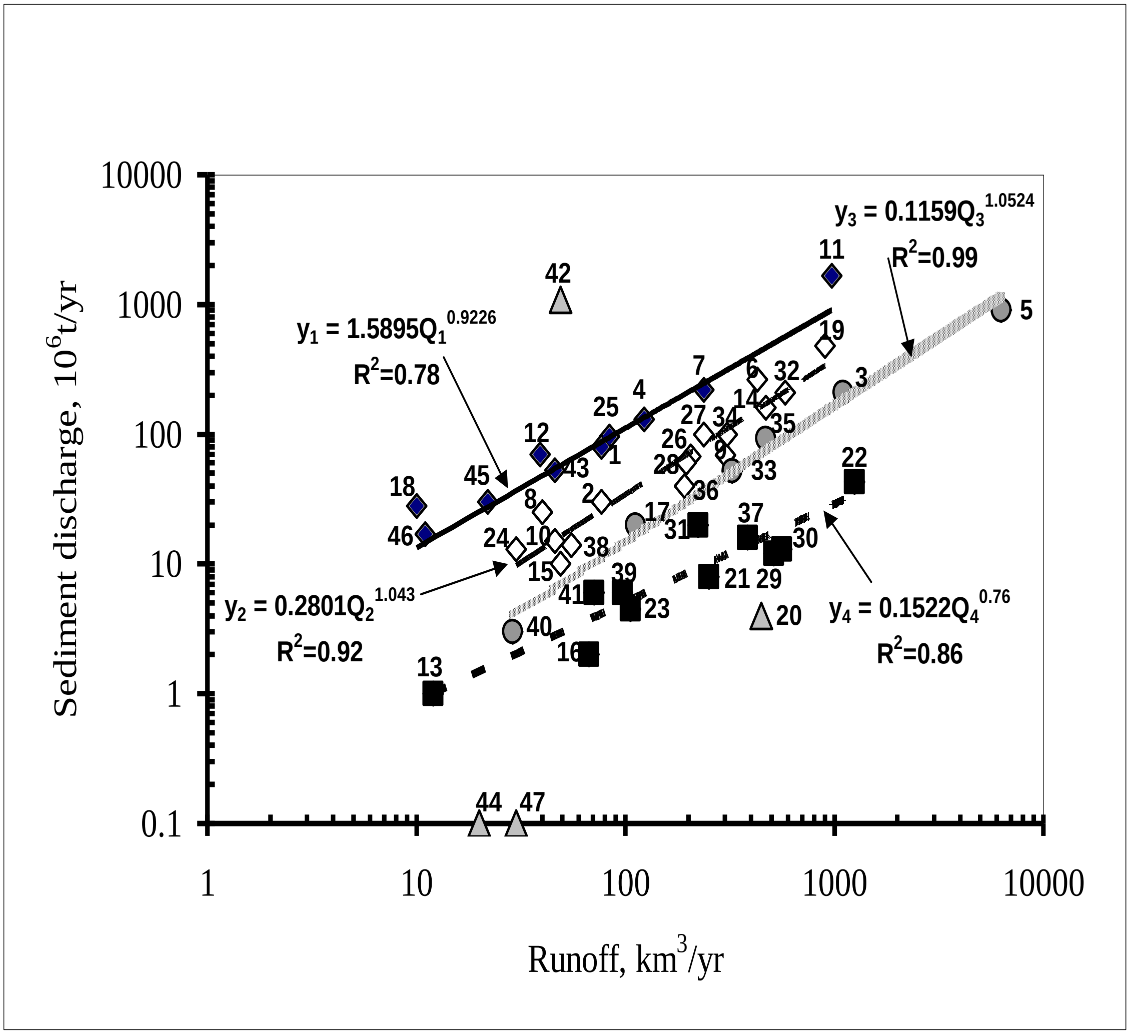

Figure 5. Scaling laws involving sediment discharge and runoff (see equations (11)-(14) in the text for explanation of equations, and Table 1 for interpretation of ranks 1-47 shown in the Figure).