Title
MEASUREMENT AND ANALYSIS OF THE REACTION n+p → noA++

Permalink
https://escholarship.org/uc/item/5h21r9rr

Author
Grether, D.

Publication Date
1973
MEASUREMENT AND ANALYSIS OF THE REACTION

\[ \pi^+ p \rightarrow \eta^0 \Delta^{++} \]

D. Grether, G. Gidal, and G. Borreani

January 1973

Prepared for the U. S. Atomic Energy Commission under Contract W-7405-ENG-48

For Reference
Not to be taken from this room
This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
MEASUREMENT AND ANALYSIS OF THE REACTION $\pi^+ p \rightarrow n^0 \Delta^{++}$

D. Grether, G. Gidal, and G. Borreani

E. O. Lawrence Berkeley Laboratory
University of California
Berkeley, California

ABSTRACT

New data for the reaction $\pi^+ p \rightarrow n^0 \Delta^{++}$ are presented at eleven momenta between 1.28 and 2.67 GeV/c. Existing data at higher momenta are included in an analysis of the reaction in terms of $A_2$ exchange. An effective trajectory parameterization of the data above 2 GeV/c is shown to adequately describe that data, although it yields an effective trajectory steeper than expected from $\rho, A_2$ exchange degeneracy. An existing Regge Pole model is refitted to the data above 2 GeV/c with generally satisfactory results. Both the effective trajectory parameterization and the Regge model are extrapolated to the lower momenta data and shown to give remarkably good agreement with the data. Evidence is presented against a dominant contribution to the lower momenta data from s-channel resonances.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

†Present address: Istituto di Fisica dell'Università, Torino, Italy.
I. INTRODUCTION

We present new measurements for and an analysis of the reaction

\[ \pi^+ p \to n^0 \Delta^+ \]  

(1)

The new measurements consist of cross sections and angular distributions at 1.28, 1.35, 1.39, 1.45, 1.55, 1.62, 1.67, 1.75, 1.86, 2.30 and 2.67 GeV/c incident beam momentum. Selected aspects of this data have been previously published.\(^1\) We include in the analysis published data between 3 and 4 GeV/c\(^2\) (henceforth referred to as 3.5 GeV/c), 3.7 GeV/c,\(^3\) 5 GeV/c,\(^4\) 8 GeV/c\(^5\) and 13 GeV/c.\(^6\)

Reaction (1) and the closely related reaction

\[ \pi N \to n N \]  

(2)

are unique in that of the generally accepted Regge trajectories only the \(A_2\) may be exchanged. In Ref. 1 we showed that the combined data at 2.3, 2.67, 3.5 and 3.7 GeV/c gives a picture of \(A_2\) exchange that generally agrees with that expected of a nonsense wrong-signature zero (NWSZ) Regge model. In particular, we observed a dip in the differential cross section near \(t = -1.4\) (GeV/c)\(^2\) that would correspond to the \(A_2\) trajectory passing through -1. The calculated effective trajectory was steeper than, but in fair agreement with, an \(A_2\) trajectory that would be exchange degenerate with that of the \(\rho\). Recent work on reaction (2) has yielded similar results. Harvey et. al.\(^7\) observe a dip in the differential cross section for

\[ \pi^- p \to n^0 n \]

that is consistent in position and width with that in Ref. 1. Spiro and
Derem have recalculated the effective $A_2$ trajectory for reaction (2) and for sufficiently high energies find a trajectory in closer agreement with that of the $\rho$, than that from previous analyses.

In the present paper we enlarge upon the study of the relationship of reaction (1) to $A_2$ exchange. The various momenta are considered individually rather than combined (as in Ref. 1), and the analysis is extended to the momenta below 2 GeV/c. In Section II the acquisition of the data is discussed. The total and differential cross sections and density matrix elements are presented in Section III. In Section IV the method of determining the effective trajectory and reduced residue is examined in some detail. In Section V the Regge pole model of Krammer and Maor is refit to reaction (1) alone and the results are compared to various distributions of the data. Finally, in Section VI, the lower momentum data are examined for evidence of resonance formation in the s channel.

II. DATA ACQUISITION

The data of laboratory momenta 1.28, 1.39, 1.55, 1.62, 1.75, 1.85, and 2.67 GeV/c were obtained in the LBL 25-inch hydrogen bubble chamber; the data at 1.35, 1.45, and 1.67 GeV/c were obtained in the 72-inch chamber. Both chambers were exposed to a separated $\pi^+$ beam with proton contamination less than 0.5%. The film from the 25-inch chamber for momenta less than 2 GeV/c was scanned for four prong events and measured on conventional measuring machines (COBWEB). The remainder of the film was scanned and roads made for the Flying Spot Digitizer (FSD).
The events were processed through the FOG-CLOUDY-FAIR series of spatial reconstruction, kinematic fitting, and hypothesis selection programs.

Some 15% of the film at each momenta was scanned a second time to establish scanning efficiencies, which were typically 97%. Events failing to reconstruct in FOG, or which were judged to be inadequately measured, were measured a second time. Upon completion of the second measurement, a typical momentum would have 93-95% of the scanned events successfully reconstructed and judged to be adequately measured. A beam track count in every 50th frame was used for normalization purposes in the cross section calculations.

The events were kinematically constrained to the hypotheses involving non-strange mesons and baryons. There are three four-constraint (4C) hypothesis to the reaction

\[ \pi^+ p + \pi^+ p \pi^+ \pi^- \]  (3)

corresponding the possible assignment of a proton to each of the three positive tracks. There are three 1C hypothesis to

\[ \pi^+ p \rightarrow \pi^+ p \pi^+ \pi^- \]  (4)

and a single 1C hypothesis to

\[ \pi^+ p \rightarrow \pi^+ \pi^+ \pi^- n \]  (5)

Events satisfying any of the 4C hypotheses were assigned to reaction (3) and eliminated from further consideration. Further selection was somewhat different for events measured on COBWEB than on the FSD. For COBWEB, events satisfying the kinematic constraints for only one of the 1C hypotheses were assigned to that hypothesis. Ambiguities between
hypotheses were resolved by examining the ionization of the tracks on the scan table. At these low momenta, essentially all of the ambiguities could be resolved. For the FSD events, the automatic ionization measurements were used to construct an ionization $\chi^2$ analogous to the usual kinematic $\chi^2$. This ionization $\chi^2$ was used in two ways: first, to distinguish between kinematically ambiguous hypotheses; second, to avoid mis-assignment of an event with multiple neutral particles (and hence not constrainable) to one of the reactions (4,5).

Events from the 25-inch chamber that were assigned to reaction (4) were also fitted to the 2C hypothesis

$$\pi^+ p \rightarrow \pi^+ p\pi^0$$

$$\rightarrow \pi^+ \pi^- \pi^0$$

(6)

The use of this 2C fit will be described in Section III.

The data at 3.5 GeV/c and 3.7 GeV/c were available in the form of Data Summary Tapes, and these tapes were used in the analysis rather than the distributions that appear in Refs. 2 and 3. For the data at 5 GeV/c, 8 GeV/c, and 13 GeV/c we used the published cross sections.

III. CROSS SECTIONS AND DENSITY MATRIX ELEMENTS

A. Total Cross Sections

The total cross section for reaction (1) at a given momentum may be written as

$$\sigma_{tot} = X (\text{mb/event}) \cdot N_{\text{mb}} / \text{BR}$$

(7)

The factor $X$ was calculated in the usual way taking into account the beam track count, fiducial volume considerations, scanning and measuring efficiencies, $\chi^2$ cutoffs, etc. $X$ for the various momenta are tabulated
in Table I. BR is the branching ratio of \( \eta^0 \rightarrow \pi^+ \pi^- \eta^0 \) and is taken to be 0.23. \( N_{\eta\Delta} \) is the true number of \( \eta \Delta \) events within the sample of events assigned to reaction (4). We describe the method of estimating \( N_{\eta\Delta} \) using the data at 2.67 GeV/c as an example.

Figure 1 shows the mass of \( p\pi^+ \) combinations plotted vs. the mass of the \( \pi^- \), the \( \pi^0 \), and the other \( \pi^+ \). A clear \( \eta\Delta \) signal is apparent, with a relatively small background. We observe empirically that the \( \eta\Delta \) events are essentially confined to the area within the intervals \( M(p\pi^+) \leq 1.35 \text{ GeV} \) and \( 0.52 \leq M(\pi^+\pi^-\pi^0) \leq 0.58 \text{ GeV} \); henceforth referred to as the "\( \Delta \) region" and "\( \eta \) region" respectively. Fig. 2a shows the distribution of \( \pi^+\pi^-\pi^0 \) mass combinations for the other \( \pi^+ \) in the \( \Delta \) region. The solid portion of the histogram is for combinations in the \( \eta \) region; the dashed portion for combinations in an "\( \eta \) background region," \( 0.49 \leq M(\pi^+\pi^-\pi^0) \leq 0.52 \text{ GeV/c} \) and \( 0.58 < M(\pi^+\pi^-\pi^0) < 0.61 \text{ GeV/c} \). This background region is chosen so that the mass interval on either side of the \( \eta \) region is one-half of the \( \eta \) region mass interval. Thus, on the assumption of a linear background (solid curve in Fig. 2a) the number of combinations in the background region corresponds to our estimate of the amount of non-\( \eta \) background in the \( \eta \) region. Fig. 2b shows the distribution of \( p\pi^+ \) mass combinations for the other \( \pi^+ \) in the \( \eta \) region (solid histogram), and in the background region (dashed histogram). For each histogram we estimate the true number of \( \Delta \) events in the \( \Delta \) region by counting the number of combinations above a hand drawn background (solid and dashed curves). The results are designated \( N_{\Delta}^\eta \), the true number of \( \Delta \) events for the other \( \pi^+ \) in the \( \eta \) region, and \( N_{\Delta}^b \), the true number of \( \Delta \) events for the other \( \pi^+ \) in the background region. Referring again to our assumption that the background region represents the amount of non-\( \eta \) background in the
In the $n$ region, we have:

$$N_{n\Delta} = N_{\Delta}^n - N_{\Delta}^b$$

(8)

The statistical error is just

$$\delta N_{n\Delta}^S = (N_{\Delta}^n + N_{\Delta}^b)^{1/2}$$

while the error assigned to $N_{n\Delta}$ was taken to be

$$\delta N_{n\Delta} = N_{n\Delta} \cdot ((\delta N_{n\Delta}^S/N_{n\Delta})^2 + Q^2)^{1/2}.$$ 

$Q$ is our estimate of the uncertainty of the method of obtaining $N_{n\Delta}$. For the momenta above 2 GeV/c, $Q = 0.10$; below 2 GeV/c, $Q = 0.15$. For the momenta at 1.67 GeV/c and below, phase space essentially limits $M(p\pi^+)$ to the $\Delta$ region and we cannot separate the $\Delta$ signal from background. For these momenta we take the number of events in the $\Delta$ region as the "true" number of $\Delta$ events and the resulting cross sections are thus upper limits. The values of $X$, $N_{n\Delta}$ and the total cross section are given in Table I. The cross section values are plotted as a function of lab momenta in Fig. 3 together with the values of 3.7, 5, 8 and 13 GeV/c.

The lowest momentum in this experiment, 1.28 GeV/c, corresponds to an energy in the overall center-of-mass of 1.82 GeV, which is only slightly above threshold for reaction (1) (the threshold would be 1.78 GeV for an $n$ of mass .55 GeV and a $\Delta$ of mass 1.23 GeV). Figure 3 then shows the cross section for reaction (1) from essentially threshold to the Regge asymptotic region. The cross section is seen to rise from threshold, peak at roughly 1.5 GeV/c and then decrease smoothly, going like $p^{-1.5}$ (solid curve) above 2 GeV/c. The dotted curve in Fig. 3 represents the total cross section calculated from a particular Regge pole model fitted
to the data for lab momenta \( > 2.3 \) GeV/c and discussed in Section IV. From phase space limitations alone, the model reproduces to some extent the peaking of the data at low momenta. Possible interpretation of the peak as evidence for s-channel resonance production will be discussed in Section VI.

B. Differential Cross Section

The method of Section IIIA established the number of \( n\Delta \) events at each momentum. However the mass intervals of the "\( n \) region" and "\( \Delta \) region" were deliberately chosen to be sufficiently wide to include all \( n\Delta \) events and hence include the resonance tails where the background is as large or larger than the resonance signal. For further analysis we select an enriched sample of events as follows.

For the data taken in the 25-inch chamber, the 2C fit to hypothesis (6) provides an effective means of selecting \( n \) events. An event was selected if

\[
\chi^2(2C) - \chi^2(1C) \leq 3
\]

For the so-called "double \( n \)" events (both combinations of a \( \pi^+ \) together with the \( \pi^- \) and \( \pi^0 \) satisfy the criterion), the combination with the smaller \( \chi^2(2C) \) was chosen. The number of double \( n \) events varied from around 10% of the \( n \) events at the lower momenta to 1.5% at 2.67 GeV/c. Since, at worst, the choice of the lower \( \chi^2(2C) \) would be wrong half the time, the contamination from wrong choices would be no more than 5% even at the lower momenta.

For the data in the 72-inch chamber, including that at 3.5 GeV/c and 3.7 GeV/c, an event was selected as an \( n \) event if
for the double $n$ events, one combination was chosen at random. The number of double $n$ events at the lower momenta was again about 10% and was negligible at 3.5 and 3.7 GeV/c.

The events from both chambers were further selected as $\Delta$ events if

$$1.15 \leq M(p^+) \leq 1.30 \text{ GeV}$$

for the $\pi^+$ not in the $n$. Let the total number of selected $n\Delta$ events be $N_{Total}^{n\Delta}$ and the number in a given $t$ interval, $\Delta t$, be $N_{\Delta}(t)$. Then we take the differential cross section to be

$$d\sigma/dt = \left( N_{\Delta}(t)/\Delta t \right) \cdot \left( \sigma_{Total}/N_{Total}^{n\Delta} \right)$$  \hspace{1cm} (9)$$

(Equation (9) guarantees that the integral of $d\sigma/dt$ over all $t$ will yield the correct total cross section.) Implicit in (9) is the almost certainly incorrect assumption that whatever background events are included in $N_{\Delta}(t)$ have the same $t$ dependence as the true $n\Delta$ events.

The limited statistics at the individual momenta do not allow a correction for the background as was done for the combined data in Ref. 1. However, we expect minimal distortion of $d\sigma/dt$ for two reasons. First, the background in the vicinity of the $n\Delta$ signal is small (e.g. for 2.67 GeV/c in Fig. 1). Second, the correction calculated in Ref. 1 had little effect on the shape of the $t$ distribution in the $t$ region considered in this paper.

The differential cross sections are then displayed in Fig. 4 and given in Table II. To increase statistics some nearby momenta were
combined: 1.28 and 1.35 GeV/c (mean value of 1.32 GeV/c), 1.39 and 1.45 GeV/c (1.42 GeV/c), 1.62 and 1.67 GeV/c (1.64 GeV/c), 1.75 and 1.85 GeV/c (1.80 GeV/c), 3.5 and 3.7 GeV/c (3.64 GeV/c). For comparison we display in Fig. 5 the published cross sections at 5 GeV/c\(^4\) and 8 GeV/c\(^5\) plotted as a function of t.\(^{14}\) The various curves will be discussed in subsequent sections. We observe that the distributions are similar in appearance from 1.32 to 8 GeV/c; d\(\sigma/dt\) rises gently from the kinematic threshold to a broad maximum and falls off at higher \(|t|\). The rate of fall-off increases with increasing momentum (i.e., the distributions are "shrinking"), which is characteristic of reactions that proceed by meson exchange. These general observations will be made quantitative in Sections IV and V.

C. Density Matrix Elements

We parameterize the decay of \(\Delta^{++}\) in terms of the usual density matrix elements in the t-channel helicity (Jackson) frame, calculated using a moments analysis. The results are displayed in Fig. 6 for the various momenta. The distributions are generally consistent with each other. Deviations at a given t bin and momentum are unmatched at adjacent momenta. The dotted lines are the predictions of the M1 Dominance model\(^{15}\) and agree rather well with the data. (The dashed curves will be discussed in Section V.) Although the M1 dominance model was based on a \(\rho\)-photon analogy, strong exchange degeneracy between the \(\rho\) and \(A_2\) would require that the respective helicity amplitudes for \(\rho\) exchange and \(A_2\) exchange be the same to within an overall coupling constant. Hence the M1 predictions would apply to reaction (1).\(^{16}\)

Given the large errors and general isotropic behavior of the distributions, we have not tabulated the density matrix elements directly.
Rather, we have fitted each distribution at each momentum to a straight line of zero slope, a procedure essentially equivalent to taking a weighted average over all t. The results are presented in Table III together with the $\chi^2$, the degrees of freedom, and the M1 predictions. The $\chi^2$'s do not show any particular pattern of deviation of the density matrix elements from isotropy. The fitted values of the elements show a slight energy dependence, tending to approach the M1 predictions with increasing energy.

IV. THE EFFECTIVE TRAJECTORY

From exchange degeneracy one expects an $A_2$ trajectory similar to that of the $\rho$. A recent analysis of reaction (2) yielded a trajectory, $\alpha = (0.43 \pm 0.03) + (0.64 \pm 1.10)t$, that is closer to the $\rho$ trajectory than that from previous analyses. Further, the dip in the differential cross section near $t = -1.4$ GeV/c$^2$ observed in both reaction (1) and reaction (2) is consistent (in the context of NWSZ models) with a $\rho$-like trajectory. In Ref. 1 we presented an effective trajectory for reaction (1) from a fit of the data at 2.3, 2.67, 3.64, 5 and 8 GeV/c to the formula

$$\frac{d\sigma}{dt} = \left[\frac{G(t)}{P_{\text{lab}}^2}\right] \cdot \left[\frac{(s-u)}{2}\right]^2 \alpha(t)$$

(10)

where $\alpha$ is the effective trajectory and $G$ the effective residue. The resulting trajectory was in only fair agreement with a nominal $\rho$ trajectory, having a larger slope and intercept. One potential explanation for the difference is that the data cannot be adequately parameterized by (10) and the calculated effective trajectory is therefore misleading. Such a possibility is suggested by the work of Spiro and Derem on reaction (2). They show that for the existing data, the effective trajectory is a strong function of $s$, approaching the $\rho$ trajectory only above 4 GeV/c laboratory momentum. Of course, in equation (10), $\alpha$ is a function of $t$ only. We now
examine this potential explanation for reaction (1) in some detail.

Figure 7 displays log-log plots of $P_{lab}^2 \cdot \frac{d\sigma}{dt}$ vs (s-u)/2 for various t values for all data for reaction (1). The solid curves are straight-line fits to the five momenta at 2.3 GeV/c and above, except at $t = -1.05$ GeV/c where data is lacking for 5 and 8 GeV/c. At a given t value, the slope of the straight line is $2a(t)$ and the intercept (at $(s-u)/2 = 1$) is $G(t)$. These straight lines are clearly adequate representations of the data at 2.3 GeV/c and above. In addition, there is general agreement between the lower momentum data and the extrapolation of the straight lines below 2.3 GeV/c. Exceptions are 1.32 GeV/c systematically at all kinematically accessible t values (-.25 to -.75 GeV/c²); and 1.42 GeV/c at its kinematic minimum and maximum values of t (-.15 and -1.05 GeV/c²). Certainly from 1.55 to 8 GeV/c the differential cross section is adequately parameterized by $G$ and $a$ in Eq. (10) functions of t only.

The values for $a(t)$ and $G(t)$ obtained from the straight line fits in Fig. 7 are plotted versus t in Fig. 8. The effective trajectory is parameterized in the usual way,

$$a(t) = a_0 + a' t$$  \hspace{1cm} (11)

This form was fitted to the five solid data points (-.15 to -.75 GeV/c²) in Fig. 8a. The result (solid curve in Fig. 8a) is the same effective trajectory as given in Ref. 1, but with reevaluated errors,$^{17}$

$$a_0 = .87 \quad a' = 1.75 \quad \delta a_0 = .09 \quad \delta a' = .23 \quad \delta a_0 \delta a' = .019$$

The points at $t = -.075$ and -1.05 GeV/c² (dotted in Fig. 8) were excluded from the fit$^{18}$ but are consistent with it.
Although Fig. 7 compares Eq. 10 to the data in a particularly useful way, it is nevertheless interesting to compare it to the more familiar differential cross sections. To accomplish this we must parameterize G(t). Lacking a simple apriori form, we choose

$$G(t) = -t [A(1-e^{Bt}) + C]$$  \hspace{1cm} (12)

A fit to the solid data points in Fig. 8b yields

$$A = 4.42 \quad B = 2.99 \quad C = .573$$

The result of the fit is shown in Fig. 8b (solid curve). Given (11) and (12) we then use (10) to calculate $d\sigma/dt$ at the various momenta, with the results (solid curves) shown in Figs. 4 and 5. The agreement is quite good from 1.55 to 8 GeV/c. Only the momenta nearest threshold for reaction (1), 1.32 and 1.42 GeV/c, show significant deviations.

We conclude that an effective trajectory and residue approach provides a quite adequate parameterization of reaction (1) with no evidence for an s-dependence of the effective trajectory. A numerical comparison\textsuperscript{17} of this trajectory to a nominal \(\rho\) trajectory of \(\alpha_\rho = .57 + .91t\) yields \(\chi^2 = 13\) for two degrees of freedom; roughly a three standard deviation effect. At this statistical level we cannot determine whether the difference between our \(A_2\) effective trajectory and the \(\rho\) trajectory is a statistical fluctuation or a real physical effect of some as yet unknown origin. The \(A_2\) trajectory determined here is in considerable disagreement with those given in Refs. 8 and 10 for reaction (2), with \(\chi^2\)'s of 23 and 35 respectively.

V. A REGGE POLE MODEL

Krammer and Maor\textsuperscript{11} have fitted a \(\rho\) and \(A_2\) NWSZ Regge Pole model simultaneously to reaction (1) and the reactions
The contribution to the fit from reaction (1) was the least significant of the three reactions, including only the differential cross sections at 3.5 and 8 GeV/c. Nevertheless, as shown for the combined data in Ref. 1, the model manifests a number of features of reaction (1) such as the general shape of the t distribution, the dip in the forward direction and at \( t = -1.4 \) GeV/c, and the general trend of the density matrix elements. However, the model does not predict particularly well the magnitude of the differential cross sections at the individual momenta. In order to obtain a better representation of reaction (1) by a NWSZ model, we have refit the model of Krammer and Maor to that reaction alone. A preferable procedure would be to extract the helicity amplitudes directly from the data in a model independent way. However, we measure insufficient parameters to allow such an extraction and thus rely on the model to provide the necessary constraints.

We have refit to the formula used by Krammer and Maor to obtain their solution 1, and to a variation. To explain this variation we rewrite Eq. (5) of Ref. 11 as

\[
f_{ij} = H_{ij}(t) \left( \sin \theta_t \right)^{|i-j|} \left( \frac{4q' \cos \theta_t}{S_0} \right)^\alpha(t) / S_{ij} \]

where \( f_{ij} \) is the t-channel helicity amplitude for an incident proton of helicity \( i = \pm 1/2 \), and final \( \Delta^{++} \) of helicity \( j = \pm 1/2, 3/2 \). \( H_{ij}(t) \) encompasses all factors depending only on \( t \); \( q,q' \) and \( \theta_t \) are the incoming momenta, outgoing momenta, and scattering angle in the t-channel center of mass; \( S_0 \) a scaling constant; and \( \alpha \) the \( A_2 \) trajectory.
Assuming the existence of daughter trajectories as in Ref. 11 we may write

$$\cos \theta_t = \frac{(s-u)}{(4qq')} \quad (14)$$

Using (14) we write (13) as

$$f_{ij} = H_{ij}(t) \left[ \frac{S_0}{4qq'} \right]^{i-j} \left[ \tan \theta_t \right]^{i-j} \left[ \frac{(s-u)}{S_0} \right]^{\alpha(t)} \quad (15)$$

Now q and q' depend only on t, but \( \tan \theta_t \) depends also on s and so the various helicity amplitudes have different s dependencies according to the value of |i-j|. In contrast, the derivation of the formula for the effective trajectory and residue (Eq. (10)) assumes the same s dependency for all amplitudes.\(^\text{10}\) This assumption is valid in the asymptotic region since

$$\lim_\limits{s \to \infty} \tan \theta_t = i$$

and in this limit

$$f_{ij} = H_{ij}(t) \left[ \frac{\alpha s}{4qq'} \right]^{i-j} \left[ \frac{(s-u)}{S_0} \right]^{\alpha(t)} \quad (16)$$

Cross sections derived from (16) will then have the same form for the s dependence as Eq. (10). The success of this latter equation in describing the data (Section IV) suggests that the data be fit not only to Eq. (15) but (16) as well. We label the corresponding fits Fit A and Fit B respectively. The input data were the differential cross sections (\(|t| < 1.2\)) at 2.3, 2.67, 3.64, 5 and 8 GeV/c and the density matrix elements at 2.3, 2.67, and 3.64 GeV/c. We varied \( \alpha' \) and \( \alpha_0 \), the slope and intercept of the trajectory; \( S_0 \); and the four parameters X1-X4 (see Ref. 11) that determine the relative amounts of the various helicity amplitudes.
Table IV gives the parameters from Krammer and Maor's Sol. 1, and our fits A and B. Both fits show an improvement over Sol. 1 in $\chi^2$ probability but the probability is not particularly good. Although all of the parameters have changed somewhat from those of Sol. 1, none has changed drastically. The resulting $A_2$ trajectories are quite similar to a nominal $\rho$ trajectory; they are compared to the effective trajectory in Fig. 8a. The difference between the trajectories for Fits A and B and the effective trajectory is somewhat surprising since essentially the same data was used. The explanation is that in the fits to Eq. (15) or (16) the $H_{ij}(t)$ are of fixed functional form, while in the fit to (10) the corresponding term, $G(t)$, is not constrained at all. The difference can be seen in Fig. 8b where an effective residue has been derived from Fit B. (For Fit A, the non-asymptotic $s$ dependence prevents an unambiguous extraction of an effective residue.) As for the trajectory, this effective residue is only in fair agreement with the data.

The differential cross sections from Fits A and B are compared to the data in Figs. 4, 5, and 7. When the fits give visually indistinguishable results, only Fit B is shown. We observe that in the energy region included in the fits (lab momentum $\geq 2.3$ GeV/c) they give nearly identical and generally adequate descriptions of the data. The non-asymptotic $s$-dependence of Fit A is most evident in Fig. 7 for lab momentum $< 2.3$ GeV/c (and especially for the higher $|t|$ values) where the predictions (dot-dashed lines) show considerable curvature. The general trend of the data favors the asymptotic $s$-dependence of Fit B and suggests that in Regge Pole model fits to reaction (1) and similar reactions, the asymptotic form of the $s$-dependence should be used.

The density matrix elements for Fits A and B are compared to the
data in Fig. 6. For $\rho_{33}$ and $\text{Re}(\rho_{31})$ the two fits give visually indistinguishable results and only Fit B is shown. For $\text{Re}(\rho_{3-1})$ both fits give results that cannot be distinguished from those of the M1 dominance model and are not shown. $\rho_{33}$ and $\text{Re}(\rho_{31})$ are only slightly different from the M1 predictions and all the elements share with the M1 dominance model the general agreement with the data.

VI. S-CHANNEL RESONANCES

The lower momenta data (1.28 to 1.85 GeV/c) correspond to a region of energy in the overall center-of-mass of 1.83 to 2.10 GeV. This energy region includes the central mass values of four resonances with isospin 3/2 identified from pion-nucleon elastic scattering phase shift analyses;\textsuperscript{13,19} the well-known $F7(1950)$, the relatively well established $F5(1890)$ and $P1(1910)$, and the more speculative $D5(1950)$. For the $F7$ the branching ratio into the elastic channel is \textasciitilde .45; the $F5$, $P1$ and $D5$ are rather more inelastic with branching ratios of roughly .17, .25 and .14 respectively into the elastic channel.\textsuperscript{13} It is of interest to examine potential decay channels such as $n\Delta^{++}$ for evidence of the inelastic decay modes of these resonances. Particularly relevant to the examination of reaction (1) is the work of Mehtani et al.\textsuperscript{20,21} on the closely related (e.g. by $SU_3$) reaction

$$\pi^+ p + \pi^0 \Delta^{++} \quad (17)$$

These authors have undertaken a study and partial wave analysis of reaction (17) based on data from the same film (and hence energy range) as was used in the experiment reported here. The reaction is found to be dominated by formation of the $F7$, with evidence for the $F5$, an indication of the $D5$, but no indication of the $P1$.\textsuperscript{20} We lack sufficient statistics to undertake
a partial wave analysis\textsuperscript{22} and thus are limited to some largely qualitative observations concerning dominance of reaction (1) by s-channel resonances.

Perhaps the most striking indication of a s-channel resonance is a peak in the cross section above some slowly varying background. The cross section for reaction (1) does peak in the low energy region (Fig. 3).

Recall, however, the solid line in Fig. 3 which represents the behavior of the cross section above 2.3 GeV/c. If this line is taken as "background" and extrapolated to the lower energy region the peaking is roughly a 15\% effect (\(~0.1\) mb) above the extrapolation. This small excess is in marked contrast to a similar extrapolation that may be performed for reaction (17) using the \(p_{1\text{lab}}^{-1.6}\) behavior of the cross section at higher energies.\textsuperscript{23} The data\textsuperscript{21} for reaction (17) lie some 20\% below the extrapolation around 1.8 GeV/c but rise above it with decreasing momenta until at 1.42 GeV/c (slightly below the central mass value of the F7) the data is \(~3\) mb above the extrapolation, out of a total cross section of \(~5\) mb. A similar contrast may be made with respect to Regge Pole models. As shown by the dashed line in Fig. 3, Fit B of the Regge Pole model discussed in Section V (when extrapolated to the lower momenta) reproduces, albeit crudely, the peaking at low energy. For the model the peaking is just due to phase space limitations. For \(\pi^0A^{++}\), the data agrees with an extrapolation to low energies of a similar model (Solution 1 of Ref. 11) around 1.8 GeV/c, then rises with decreasing momenta \(~4\) mb above it. For reaction (17) the dominant resonance, the FF7, accounts for some 85\% of the cross section,\textsuperscript{20} or about \(~4\) mb. This is roughly the same amount that the data exceed the extrapolations just discussed, suggesting that such extrapolations be interpreted as non-resonant background. If this interpretation
is applied to reaction (1), then we conclude that only some ~.1 mb (out of a total cross section of ~.7 mb) could be resonant.

We now inquire whether the above conclusion is consistent with SU_3, in particular with respect to the F7. We write

\[ \sigma_{\eta \Delta} = C \cdot R \cdot \sigma_{\pi^0 \Delta} \]  

(18)

where \( \sigma_{\eta \Delta} \) is the cross section for a particular resonance decaying into \( \eta \Delta^{++} \) (and similarly for \( \sigma_{\pi \Delta^0} \)), C is the SU_3 Clebsch-Gordon coefficient relating the two decays, and R is a correction factor for phase space and centrifugal barrier effects. For the F7 in a SU_3 decouplet, C = 1/5.24

For R we use the suggestion of Trilling,25

\[ R = \left( \frac{p_{\eta}}{p_{\pi^0}} \right)^{2\lambda+1} \]

where \( p_{\eta} \) and \( p_{\pi^0} \) are the magnitude of momenta of the \( \eta \) and the \( \pi^0 \) in the resonance rest frame, and \( \lambda \) is the orbital angular momentum of the decay.

For a FF7 of nominal mass 1.95 GeV and a \( \Delta^{++} \) of nominal mass 1.22 GeV, R = .06. With \( \sigma_{\pi \Delta^0} = 4 \text{ mb} \), \( \sigma_{\eta \Delta} \) from Eq. (18) would be about .8 mb from the SU_3 coefficient but is reduced to \( \approx .05 \text{ mb} \) by the phase space-centricrifugal barrier factor. This value is quite compatible with the .1 mb or so suggested above as a limit on a resonance contribution to reaction (1).

The other resonance in reaction (17) for which there is some evidence, the F5, appears as a FF5.20 Thus it has about the same R factor and (assuming that it is in a SU_3 decuplet) the same C factor as for the FF7.

Since the contribution to reaction (17) from the FF5 is considerably smaller than from the FF7, the expected contribution of the FF5 to reaction (1) is even less than the .05 mb for the FF7.
We have shown that if an extrapolation of the high energy cross section or of Regge models to the low energy region is interpreted as non-resonant background, then only a small fraction of the low energy cross section for reaction (1) could be considered as resonant. In contrast, the concept of duality suggests that the extrapolation of, in particular, the Regge Pole model should be interpreted as describing the s-channel resonances in some average way. We thus continue our analysis by examining the angular distributions for evidence of such resonances. We choose to discuss the angular distributions in terms of the $A$ coefficients in the expansion

$$\frac{d\sigma}{dz} = \sum A_L P_L(z)$$

(19)

where the $P_L$ are the Legendre Polynomials and the $z$ the cosine of the scattering angle in the overall center of mass. $A_0$ is just the total cross section. Higher coefficients were obtained by a moments analysis. $A_1$-$A_6$ are displayed in Fig. 9 ($A_7$ and above are consistent with zero) for the lower momenta data with 2.3 GeV/c and 2.67 GeV/c included for reference. The dot-dashed and dashed curves are the A coefficients obtained for fits A and B respectively of the Regge pole model discussed in Section V. The similarity between the model and the data in the general trend of the coefficients is striking. However, this similarity is not unexpected since the model predicts fairly well the shape of the differential cross sections (Fig. 4). All of the coefficients are consistent with zero at the lowest momenta (essentially threshold) and undergo a variety of changes with increasing momenta. The most prominent coefficients are $A_1$, which rises almost linearly to a large, positive value; and $A_2$, which dips to a large negative value around 1.6 GeV/c, then rises through
zero to positive values. The dip is significantly greater for the data than the model and suggestive of a resonance-like effect. $A_3$ and $A_4$ are significantly negative in the region below 2 GeV/c. Within the rather large errors, $A_5$ and $A_6$ tend to be negative and of smaller value than the lower order coefficients.

We now examine the A coefficients with respect to the partial waves possible for the four above-mentioned resonances. Each resonance in the elastic channel corresponds to a particular partial wave amplitude. However, for reactions (1) and (7) the final orbital angular momentum may differ by two units from the initial value for the same parity and total spin, and each resonance may contribute to two partial waves. Thus the $F_7$ may appear as a $F F_7$ or $F H_7$. Similarly, for the $F_5$ ($F P_5$, $F F_5$) and the $D_5$ ($D D_5$, $D G_5$); the $P_1$ may only appear as a $P P_1$. The relationship between the Legendre polynomial expansion coefficients and the partial waves for reactions of the type (1) and (17) is given by Roberts. Some general properties are of particular note. The odd coefficients can result only from interference between waves of opposite parity (e.g. $F F_5$ and $D D_5$). The even coefficients can result either from a pure wave, or from interference between two waves of the same parity (e.g. $F F_5$ and $F F_7$). In contrast to elastic scattering, some pure waves yield negative even coefficients (e.g. $F F_7$ yields a negative $A_4$ and $A_6$). We remember that in the presumed resonance region $A_1$ is large and positive and $A_3$ significantly negative. Thus no single partial wave may account for the data. Furthermore, from Ref. 26 none of the seven partial waves possible for the four resonances under consideration give a negative $A_2$. For completeness we mention that for all waves up through $G G_7$, only $P P_3$ can, by itself, yield a negative $A_2$. However, there is no evidence for a $P 3$
resonance in the elastic channel in this energy region. Neglecting PP3, no two of the four resonances may interfere in such a way as to account both for $A_1$ and $A_2$ since the former requires two waves of opposite parity, and the latter requires two of the same parity. Thus the prominent features of the $A$ coefficients are not readily explained by one or two dominant resonances. Again, this situation is in contrast to that for reaction (17), in which the dominance of the F7 manifests itself in a large, negative $A_6$ with the other coefficients interpreted as interferences between the F7 and other, smaller waves.\footnote{21}

Since the partial wave amplitudes form a complete set, the angular distributions for reaction (1) correspond to some combination of partial waves. These waves may in general be either "background" or resonant and we cannot exclude the possibility that some are resonant in the energy range considered. We may conclude, however, that no one or two resonant partial waves dominate the reaction. This conclusion is quite consistent with the earlier comparison to reaction (17) in which it was shown that SU$_3$ predicted essentially negligible contributions to reaction (1) from the resonances observed in (17).
VII. SUMMARY

New data have been presented for the reaction

$$\pi^+ p \rightarrow \eta^0 \Delta^{++}$$

that, together with existing data at higher energies, allow a study of the reaction from near threshold to the Regge asymptotic region.

The higher energy data (laboratory momentum $\geq 2.3$ GeV/c) were analyzed in terms of $A_2$ exchange. An effective trajectory formula that was fitted to the differential cross-sections yielded a quite adequate parameterization of the data. The effective trajectory obtained was only in fair agreement with that expected from $\rho$-$A_2$ exchange degeneracy, having a larger slope and intercept. The differential cross sections and density matrix elements were fitted to two versions of a NWSZ Regge pole exchange model. The resulting fits gave a generally satisfactory representation of the data. These fits, under the constraint of a residue of fixed form, yielded a $A_2$ trajectory closer to that of the $\rho$ than for the effective trajectory formula, but were correspondingly poorer fits.

The lower energy data (laboratory momentum $< 2$ GeV/c) were shown to be similar in character to the high energy. The differential cross sections were of similar shape and, except at the lowest momentum, agreed rather well with extrapolations of the effective trajectory formula and a version of the Regge Pole model. The density matrix elements were of about the same value as at the higher energies and as extrapolations of the Regge models. The similarity to the high energy data was explored relative to $s$-channel resonance production, since the energy region of the low energy data is dominated by resonances in the elastic channel and in the SU$_3$ related reaction $\pi^+ p \rightarrow \pi^0 \Delta^{++}$. The interpretation
that the low energy data for $n\Delta^{++}$ is essentially non-resonant was shown to be consistent, with the dominance of resonances in $\pi^0\Delta^{++}$, based on a calculation incorporating $SU_3$, phase space and centrifugal barrier effects. The scattering angular distributions were studied in terms of the coefficients of the Legendre Polynomials. This study could not rule out the possibility of resonance production but did show that no one or two resonances dominated the data.

We conclude that the reaction $\pi^+p \rightarrow n\Delta^{++}$ may be adequately described from near threshold to the Regge asymptotic region by $A_2$ Regge Pole exchange, and that the lower energy data is consistent with the interpretation that there is no significant contribution from s-channel resonances.

ACKNOWLEDGEMENTS

We thank the Bevatron and Bubble Chamber staff and the Powell-Birge group data reduction personnel for their efforts in obtaining and processing the data.
References

3. G. S. Abrams et al., Lawrence Radiation Laboratory Report No. UCRL-20067, 1970 (unpublished). We thank the Trilling-Goldhaber group for supplying us with a data summary tape.
12. The value of X for 3.5 GeV/c was taken from Ref. 2. Ref. 2 does not quote a total cross-section and we calculate it in the same manner as for the data of this experiment.
14. The original distributions were plotted vs t' = t-t_{min}. In Fig. 5 the distributions are shifted by a t_{min} calculated for a nominal n and \Delta^{++} of masses .55 and 1.23 GeV respectively.

16. Other approaches have yielded this same result. See, e.g., R. Dashen and S. Frautschi, Phys. Rev. 152, 1450 (1966).

17. We quote the joint error between $\alpha_0$ and $\alpha'$, $\delta\alpha_0\delta\alpha'$, since the two quantities are highly (- 90\%) correlated. Proper numerical comparisons of our result to other trajectories must include this joint error.

18. The point at $t = -0.075 \text{ (GeV/c)}^2$ is not used since it is near kinematic threshold at the lower momenta. The point at $t = -1.05 \text{ (GeV/c)}^2$ is not used because there is no data at this $t$ value at 5 and 8 GeV/c.

19. We adopt a standard notation, $LL'2J$ where $L$ refers to the angular momentum of the initial state, $L'$ to the final state, and $J$ is the total spin. Since all resonances discussed here have Isospin = 3/2, we delete the usual reference to that quantity. When referring to the elastic channel, we delete $L'$.


22. At a typical momentum in this low energy region, the number of $\eta\Delta$ events (with the decay $\eta \to \pi^+\pi^-\pi^0$) is only - 1/15th of the number of $\pi^0\Delta$ events.

23. G. Gidal, G. Borreani, D. Grether, F. Lott, R. W. Birge, S. Y. Fung, W. Jackson, and R. Poe, Phys. Rev. Lett. 23, 994 (1969). From Fig. 1 of this reference we take the normalization of the $P_{\text{lab}}^{-1.6}$ behavior as .6 mb at 3 GeV/c.


Figure Captions

Fig. 1. Scatter plot at 2.67 GeV/c of the $p\pi^+$ mass plotted against the mass of the $\pi^-,\pi^0$ and the other $\pi^+$. The $M(\pi^+\pi^-\pi^0)$ distribution is not shown above 0.66 GeV in order to avoid the much larger $\omega^0$ signal.

Fig. 2. Invariant mass distributions at 2.67 GeV/c.

(a) Mass of $\pi^+\pi^-\pi^0$ combinations for the other $\pi^+$ in the "$\Delta$ region," $M(p\pi^+) < 1.35$ GeV. The solid portion of the histogram is for events in the "$\eta$ region," $0.52 < M(\pi^+\pi^-\pi^0) < 0.58$ GeV; the dotted portion is for the "background region," $0.49 < M(\pi^+\pi^-\pi^0) < 0.52$ GeV and $0.58 < M(\pi^+\pi^-\pi^0) < 0.61$ GeV. The solid curve is our estimate of the background in the $\eta$ region.

(b) Mass of $p\pi^+$ combinations for the other $\pi^+$ in the $\eta$ region (solid histogram) and in the background region (dashed histogram). The solid and dashed curves are the respective estimates of the non-$\Delta^{++}(1238)$ background.

Fig. 3. Total cross section vs laboratory momentum for the reaction $\pi^+p \rightarrow n\Delta^{++}$. The solid curve is $p_{lab}^{-1.5}$ from a fit to the data above 2 GeV/c. The cross section values for $p_{lab} < 1.7$ GeV/c are upper limits. The dashed curve is from the Regge model fit B, as discussed in the text.

Fig. 4. Differential cross sections vs $t$ for the reaction $\pi^+p \rightarrow n\Delta^{++}$ for eight momenta between 1.32 and 3.64 GeV/c. The solid curve is from an effective trajectory parameterization; the dot-dashed and dashed curves are from the Regge model Fits A and B respectively. When the two fits are visually indistinguishable, only Fit B is shown.
Fig. 5. Differential cross section for the reaction $\pi^+ p + n\Delta^{++}$ at 5 GeV/c (Ref. 4) and 8 GeV/c (Ref. 5). The curves are as described for Fig. 4.

Fig. 6. Density matrix elements in the t-channel helicity frame for the reaction $\pi^+ p + n\Delta^{++}$ at eight momenta between 1.32 and 3.64 GeV/c. The dotted curves are the $M_1$ dominance model predictions. The Regge model Fit B results (dashed curves) are shown when visually distinct from the $M_1$ predictions. The Fit A results are essentially identical to those for B and are not shown.

Fig. 7. Plot of $P_{\text{lab}}^2 \cdot d\sigma/dt$ vs. $(s-u)/2$ at seven $t$ values between -0.075 (GeV/c)$^2$ and -1.05 (GeV/c)$^2$. The correspondence between lab momenta and $(s-u)/2$ is indicated in the upper right hand corner of the figure. The curves are as described for Fig. 4.

Fig. 8. Results of the effective trajectory parameterization of the reaction $\pi^+ p + n\Delta$ for the data above 2 GeV/c.

a) $\alpha_{\text{eff}}$, the effective trajectory, vs. $t$. The solid curve is a linear fit to the solid data points, yielding $\alpha_{\text{eff}} = (0.87 \pm 0.03) + (1.75 \pm 1.3)t$. The dot-dashed and dashed curves are from the Regge pole model fits A and B with respective results $\alpha_A = 0.58 + 1.0t$, $\alpha_B = 0.62 + 0.98t$.

b) $G_{\text{eff}}$, the effective residue, vs. $t$. The solid curve is a fit to the data points using a particular parameterization as discussed in the text. The dot-dashed curve is for the Regge model Fit B.

Fig. 9. Coefficients $A_1 - A_6$ from the expansion $d\sigma/dt = \sum_{L=0} A_L P_L(z)$. Higher coefficients are consistent with zero. The dot-dashed and dashed curves are from the Regge model Fits A and B respectively.
Table I. Total Cross Sections

<table>
<thead>
<tr>
<th>$p_{\text{lab}}$ (GeV/c)</th>
<th>$x$ ($\mu$b/event)</th>
<th>$N_{\eta \Delta}$ (events)</th>
<th>$\sigma$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.28</td>
<td>.71 ± .05</td>
<td>88</td>
<td>.27 ± .06</td>
</tr>
<tr>
<td>1.35</td>
<td>.71 ± .07</td>
<td>119</td>
<td>.36 ± .08</td>
</tr>
<tr>
<td>1.39</td>
<td>.66 ± .04</td>
<td>208</td>
<td>.59 ± .11</td>
</tr>
<tr>
<td>1.45</td>
<td>.93 ± .09</td>
<td>130</td>
<td>.52 ± .11</td>
</tr>
<tr>
<td>1.55</td>
<td>.91 ± .05</td>
<td>191</td>
<td>.75 ± .14</td>
</tr>
<tr>
<td>1.62</td>
<td>.52 ± .03</td>
<td>298</td>
<td>.67 ± .09</td>
</tr>
<tr>
<td>1.67</td>
<td>.83 ± .08</td>
<td>197</td>
<td>.71 ± .11</td>
</tr>
<tr>
<td>1.75</td>
<td>.80 ± .04</td>
<td>189</td>
<td>.66 ± .10</td>
</tr>
<tr>
<td>1.85</td>
<td>1.15 ± .07</td>
<td>103</td>
<td>.51 ± .09</td>
</tr>
<tr>
<td>2.30</td>
<td>.541 ± .025</td>
<td>141</td>
<td>.33 ± .05</td>
</tr>
<tr>
<td>2.67</td>
<td>.26 ± .01</td>
<td>226</td>
<td>.253 ± .035</td>
</tr>
<tr>
<td>3.50</td>
<td>.384 ± .015</td>
<td>104</td>
<td>.174 ± .033</td>
</tr>
</tbody>
</table>

a Corrected for branching ratio of $\eta^0 + \pi^+ \pi^- \pi^0$ of .23.
Table II. Differential Cross Sections

<table>
<thead>
<tr>
<th>$P_{lab}$ (GeV/c)</th>
<th>1.32</th>
<th>1.42</th>
<th>1.55</th>
<th>1.64</th>
<th>1.80</th>
<th>2.30</th>
<th>2.67</th>
<th>3.64</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-t$ intervals $\left[(GeV/c)^2\right]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.05 - .10</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>.10 - .20</td>
<td>$0.03 \pm 0.02$</td>
<td>$0.14 \pm 0.05$</td>
<td>$0.51 \pm 0.17$</td>
<td>$0.44 \pm 0.09$</td>
<td>$0.40 \pm 0.10$</td>
<td>$0.23 \pm 0.08$</td>
<td>$0.30 \pm 0.07$</td>
<td>$0.28 \pm 0.06$</td>
</tr>
<tr>
<td>.20 - .30</td>
<td>$0.48 \pm 0.11$</td>
<td>$0.76 \pm 0.15$</td>
<td>$0.70 \pm 0.21$</td>
<td>$0.84 \pm 0.14$</td>
<td>$0.62 \pm 0.12$</td>
<td>$0.40 \pm 0.11$</td>
<td>$0.39 \pm 0.08$</td>
<td>$0.20 \pm 0.05$</td>
</tr>
<tr>
<td>.30 - .40</td>
<td>$0.60 \pm 0.13$</td>
<td>$0.87 \pm 0.17$</td>
<td>$0.89 \pm 0.25$</td>
<td>$0.75 \pm 0.13$</td>
<td>$0.84 \pm 0.16$</td>
<td>$0.45 \pm 0.12$</td>
<td>$0.35 \pm 0.08$</td>
<td>$0.27 \pm 0.06$</td>
</tr>
<tr>
<td>.40 - .60</td>
<td>$0.58 \pm 0.11$</td>
<td>$0.83 \pm 0.14$</td>
<td>$0.95 \pm 0.22$</td>
<td>$0.74 \pm 0.11$</td>
<td>$0.58 \pm 0.10$</td>
<td>$0.42 \pm 0.09$</td>
<td>$0.24 \pm 0.05$</td>
<td>$0.18 \pm 0.04$</td>
</tr>
<tr>
<td>.60 - .90</td>
<td>$0.26 \pm 0.05$</td>
<td>$0.57 \pm 0.10$</td>
<td>$0.73 \pm 0.17$</td>
<td>$0.58 \pm 0.08$</td>
<td>$0.29 \pm 0.05$</td>
<td>$0.25 \pm 0.06$</td>
<td>$0.12 \pm 0.03$</td>
<td>$0.06 \pm 0.02$</td>
</tr>
<tr>
<td>.90 - 1.2</td>
<td>----</td>
<td>$0.14 \pm 0.03$</td>
<td>$0.38 \pm 0.10$</td>
<td>$0.38 \pm 0.06$</td>
<td>$0.29 \pm 0.05$</td>
<td>$0.06 \pm 0.02$</td>
<td>$0.06 \pm 0.02$</td>
<td>$0.006 \pm 0.004$</td>
</tr>
</tbody>
</table>
Table III. Fitted Density Matrix Elements

<table>
<thead>
<tr>
<th>$P_{\text{lab}}$</th>
<th>$\rho_{33}$</th>
<th>$\chi^2$</th>
<th>ND</th>
<th>$\Re \rho_{31}$</th>
<th>$\chi^2$</th>
<th>ND</th>
<th>$\Re \rho_{31}$</th>
<th>$\chi^2$</th>
<th>ND</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.32</td>
<td>.278 ± .026</td>
<td>3.5</td>
<td>3</td>
<td>.151 .025</td>
<td>11.3</td>
<td>3</td>
<td>-.107 .023</td>
<td>.9</td>
<td>3</td>
</tr>
<tr>
<td>1.42</td>
<td>.316 ± .019</td>
<td>8.3</td>
<td>5</td>
<td>.159 .020</td>
<td>5.4</td>
<td>5</td>
<td>-.030 .018</td>
<td>15.5</td>
<td>5</td>
</tr>
<tr>
<td>1.55</td>
<td>.296 ± .027</td>
<td>1.2</td>
<td>5</td>
<td>.149 .027</td>
<td>1.3</td>
<td>5</td>
<td>-.071 .025</td>
<td>5.8</td>
<td>5</td>
</tr>
<tr>
<td>1.64</td>
<td>.283 ± .019</td>
<td>3.2</td>
<td>5</td>
<td>.183 .017</td>
<td>14.1</td>
<td>5</td>
<td>.051 .017</td>
<td>6.6</td>
<td>5</td>
</tr>
<tr>
<td>1.80</td>
<td>.325 ± .022</td>
<td>5.6</td>
<td>5</td>
<td>.180 .024</td>
<td>8.1</td>
<td>5</td>
<td>.031 .020</td>
<td>7.1</td>
<td>5</td>
</tr>
<tr>
<td>2.30</td>
<td>.280 ± .034</td>
<td>2.6</td>
<td>4</td>
<td>.181 .037</td>
<td>6.5</td>
<td>4</td>
<td>-.019 .029</td>
<td>6.6</td>
<td>4</td>
</tr>
<tr>
<td>2.67</td>
<td>.426 ± .023</td>
<td>3.7</td>
<td>5</td>
<td>.297 .027</td>
<td>9.0</td>
<td>5</td>
<td>.027 .021</td>
<td>3.8</td>
<td>5</td>
</tr>
<tr>
<td>3.64</td>
<td>.306 ± .025</td>
<td>2.7</td>
<td>5</td>
<td>.199 .026</td>
<td>4.9</td>
<td>5</td>
<td>.002 .023</td>
<td>5.2</td>
<td>5</td>
</tr>
<tr>
<td>M1</td>
<td>.373</td>
<td></td>
<td></td>
<td></td>
<td>.216</td>
<td></td>
<td></td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

*aND* - degrees of freedom
Table IV. Parameters for Regge Pole Fits

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sol. 1</th>
<th>Fit A</th>
<th>Fit B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>1.63</td>
<td>1.97</td>
<td>1.73</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>.4</td>
<td>.58</td>
<td>.62</td>
</tr>
<tr>
<td>$\alpha'$</td>
<td>.9</td>
<td>1.01</td>
<td>.98</td>
</tr>
<tr>
<td>$x_1$</td>
<td>-.268</td>
<td>-.231</td>
<td>-.275</td>
</tr>
<tr>
<td>$x_2$</td>
<td>.965</td>
<td>.783</td>
<td>.741</td>
</tr>
<tr>
<td>$x_3$</td>
<td>1.129</td>
<td>.925</td>
<td>.768</td>
</tr>
<tr>
<td>$x_4$</td>
<td>-.042</td>
<td>-.038</td>
<td>-.041</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>244</td>
<td>123</td>
<td>135</td>
</tr>
</tbody>
</table>

$\chi^2$ - degrees of freedom
Fig. 1
Fig. 3

\[ \sigma_{\text{total}} \text{ (mb)} \]

\[ \pi^+ p \rightarrow \eta \Delta^{++} \]

\[ P_{\text{lab}} \text{ (GeV/c)} \]

- THIS EXPERIMENT
- ABRAMS ET AL
- SCHOTANUS ET AL
- ADERHOLZ ET AL
- SCH'GUIVEL ET AL

XBL721-2210
Fig. 5
Fig. 7
Fig. 8
Fig. 9
LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.