Title
The Speed of Gravity in the Lights of LIGO and Mercury

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Most of the fun in these things lies in doing the arithmetic for yourself and deciding whether the answer feels likely. The *LIGO*–gamma-ray event of 2017 August 17 is the best opportunity since we all pulled out our envelope backs to see what we could estimate from the arrival times of the neutrinos from SN 1987A. Oh? You were in kindergarten in 1987? Never mind. The Appendix lets you pretend you were there then.

The LIGO result

Let's start with *LIGO*, both because it's easier than Mercury and because the official numbers were just published in 2017[1,2], making them easy to find. If a source emits both photons and gravitational waves, which travel at speeds c and v_g for a distance d, the difference in arrival times will be t, given by

$$t = d/v_g - d/c. \tag{1}$$

The *Fermi* satellite photons lagged the *LIGO* event by 1·7 seconds, and the best estimate of the distance was 40 Mpc. Equation (1) then gives you … . Go on, do it! We all found v_g smaller than c by 4·25 parts in 10^{-16}, if the photons and the waves left the source at the same time.

The *LIGO* folks[1,2] allowed for the source possibly being as close as 26 Mpc and the electromagnetic and gravitational waves leaving the source at slightly different times, to conclude that

$$c - v_g = (-5 \times 10^{-15} \text{ to } +7 \times 10^{-16})\ c. \tag{2}$$

There is a corresponding limit on the mass of the graviton that comes from a relativistic particle having a total energy, E, given by

$$E = mc^2 / (1 - v^2/c^2)^{1/2}. \tag{3}$$

Now, you already have the limit on v, so all you need is the energy, E. Well, gravitons are also waves, for which

$$E = hf \tag{4}$$

and the measured frequencies, f, were in the range 50–400 Hz. Carry on to get something like

$$m \leq 1 \cdot 2 \times 10^{-22}\ \text{eV}/c^2 \approx 2 \times 10^{-54}\ \text{grams}. \tag{5}$$

Is this the smallest mass we have ever seen? It is not. Fritz Zwicky[3] claimed that the range of the gravitational force is only about 10 Mpc, because there is no clustering of clusters of galaxies. I heard him say it in a seminar talk at Caltech before I had ever heard of the Yukawa argument for the mass of a particle carrying the nuclear force (a prediction of mesons of some sort as then understood)*.

Now use a form of the Heisenberg uncertainty principle

$$\Delta E\,\Delta t \geq h/2\pi \qquad (6)$$

and require that Δt be long enough for the particle to travel a distance of 10 Mpc to discover that if $E = mc^2$ then

$$m \leq 10^{-63} \text{ grams.} \qquad (7)$$

This was an actual value for Zwicky[3,4], but a limit if you think there is a second-order clustering of galaxies. Others claimed to find super-clustering in the Zwicky catalogues, including Igor Karachentsev[5] and Emil Herzog. The latter said so at a Caltech colloquium in about 1966 with Zwicky in the audience. Herzog was offered 'political asylum' at UCLA by George Abell, whose catalogues[6] definitely show super-clustering.

I think the *LIGO* graviton mass has to be very much an upper limit, or the range of the gravitational force would be something like 4000 AU, and the Oort Cloud would be well on its way out of the Milky Way, which would be well on its way to … oh, never mind.

Now about Mercury

Why Mercury? Because a number of commentators have commented that the *LIGO* result for the pair of merging neutron stars[1] was the first quantitative determination of the speed of gravity[7]. But my husband told me in the mid-1970s that we know that speed to be the same as the speed of light to 5% or so because of the Mercuric … Mercurial … Mercurian perihelion advance†.

LIGO and, I think, most of the arguments in ref. 7 concern Special Relativistic contexts. But back in 1905, in the lead-up to what eventually became General Relativity, Henri Poincaré wrote[8], " … *la propagation de la gravitation n'est pas instante, mait se fait avec la vitesse de la lumière*."

If all one has met is the Schwarzschild solution, one might reasonably suppose that deviations from Newtonian results would be of order $2GM/Rc^2$. Start by evaluating this at the distance from the Sun to Mercury (0·387 AU semi-major axis). Convert from radians to arcsec by multiplying by 206 265 (a number

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* Cecil Powell won the 1950 Nobel Prize in Physics for finding the right sort of meson. Even Powell felt badly enough about the omission of Giuseppe Occhialini from that prize to nominate him two years later. Two women named Marietta, Blau and Kurz, are also part of that story.

† Is this the sort of thing on which to take one's husband's word? Well, mine was Joseph Weber and typically reliable in such matters. Truthfully, I only 'did the numbers' for myself a few weeks ago as part of trying to find a few calculations that could be done approximately by undergraduate non-physics students in a seminar on 'The Impact of World War I on the Sciences'. If you have never taught such a course, you may not share my worry "Are we who perhaps had our last history courses in secondary school as far away from 21st-Century historical research as these students are from 20th- or 21st-Century physics, given that they had their last, and probably only, physics course in secondary school?"

written upon my heart, like Calais on Mary Tudor's) per 0·24-year orbit period (about $0''\!\cdot\!01$ at this point). There are some 420 periods in a century, leading to about $4''\!\cdot\!3$ per century. Are we off by a power of 10? Well, only sort of.

The time has come to abandon simple approximations and let Jim Hartle[9] do the integration along the path of Mercury correctly. There are several integrals. One looks like a mess but always comes out 2π. Another really is a mess, but the outcome is

$$\Delta\varphi = 6\pi GM/c^2 a(1-e^2) \text{ per orbit.} \tag{8}$$

Keep in mind that GM is much better known than either separately and that the path is an ellipse of eccentricity e or there wouldn't be a perihelion to advance. We get $42''\!\cdot\!98$ per century as the relativistic prediction.

What do the observations say? The perihelion advance from a perfectly closed orbit is some 5599″ per century. Most of this is our own fault for living on a non-inertial coordinate system called Earth, whose own rotation axis and therefore its celestial coordinates rotate or precess with a period of about 26000 years. Get rid of that (about 5025″ per century), and there are still some 574″ per century not accounted for. At this point, Hartle hands us over to Irwin Shapiro, a connoisseur of Solar System tests of GR and owner of his very own Shapiro delay*. The potential felt by Mercury is not just the nearly-spherical one of the Sun. Venus and Earth count most, but also Jupiter. They introduce another 565″ per century[10], leaving a modern residual of $42''\!\cdot\!98$ per century[10], so close to the predicted number that you cannot insert even an anecdote between them.

The historical situation was not quite so tidy. Urbain J. J. Le Verrier was the first to collect enough good observations of the position of Mercury to attempt the full Newtonian fit to the four inner planets. He used both daytime meridian-crossing times and some solar transits of Mercury, announcing in 1859 a 38″ per century residual[11,12].

Our own Learned Astronomer, Simon Newcomb, who devoted a large part of his career to determining the most accurate possible parameters for the motions of the planets and satellites of the Solar System, confirmed the effect and gradually improved the value to about 43″ per century, the target Einstein had to shoot for. Thus he worried that his original *Entwurf*[13] theory of gravity accounted for only 17″ per century. It was also, he felt, inadequately Machian for rotating frames and did not provide unique field equations from a variational principle[14].

Gerald M. Clemence (1908–1974), Newcomb's eventual successor as director of the US Nautical Almanac Office, thought accurate ephemerides were important not only for navigation and prediction of eclipses, occultations, and such, but also to allow comparison of observations with theory, to improve astronomical constants and the actual theories of motion. Much of his work, like Newcomb's, is buried fairly deeply in USNO papers, but a version for the unwashed† appeared in 1947[15]. His numbers are $43''\!\cdot\!03$ per century for the relativistic prediction (remember this includes having to know the length of the

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*Insert your own optional witticism about the relative probabilities of you and Shapiro being delayed in arrival for some event, given that you probably flew and he declines to.

†Other interesting tidbits in the paper include use of analysis by Jan Oort[16] and thanks to Jan Schilt and Martin Schwarzschild of Columbia, who were elsewhere described as "director and directee", being the only astronomers there at the time.

AU and the speed of light, less precise then than now), or rather $43''{\cdot}03\pm0''{\cdot}03$, and for the observed precession $42''{\cdot}56\pm0''{\cdot}94$ per century. That $1''$ out of $43''$ in $(vc)^2$ is Joe's 5% in v_g compared to c. Shapiro's number allows you maybe 0·05% in c minus v_g.

Is this the final word (or number) on what we can expect from General Relativity? Not quite (and the issue was a live one around 1972). The Sun might, of his own accord, have a non-spherical potential, if, for instance, the inside were spinning much faster than the outside. Carl Brans and Robert Dicke[17] thought so at one time, as part of an effort to make gravitation still more Machian than the Einstein version. Expecting such oblateness, Dicke found it[18]. Others did not[19]. Dicke stuck to his oblate guns[20], but was eventually overruled[21]. The actual solar oblateness shows up as p-mode splitting in helioseismology studies at a level too small to drag Mercury away from the $42''{\cdot}98$.

So, in summary, the neutron-star-merger event has told us that gravitational waves travel at the speed of light to within a part in 10^{15} or so, Poincaré and Einstein knew the speeds had to be equal, and in between Mercury was telling us equal to within astronomical accuracy.

Yours faithfully,
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Appendix: The envelope back of 1987 February

"The first mark of supernovae might be high neutrino flux while the surface of the star has yet no sign of this sudden internal collapse." So wrote Hong-Yee Chiu (b. 1932, Shanghai, China) & Philip Morrison (1915–2005) in 1959[23].* They were proven correct on 1987 February 24 (in at least some time zones). It was SN 1987A not because of its enormous importance but because it was the first one spotted that year, and was indeed a naked-eye object for southern -hemisphere observers.

The neutrino flux was not as high as Chiu & Morrison had thought of — their reference SN was 30 parsecs away and would have showered us with 10^{13} neutrinos $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ for a few seconds, thus drowning out a solar flux of less than 10^{10} neutrinos $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$.

On that day in 1987 February, five devices existed capable of detecting MeV neutrinos that found their way to Earth. The most famous, Ray Davis' tank of C_2Cl_4, probably caught one, lost in the noise of a month's accumulation of solar-induced transformations of Cl^{37} to A^{37}. The other four all reported events. See ref. 22 for the details and a majority decision to consider only the Japanese (*Kamiokande*) and American (Irvine–Michigan–Brookhaven) counts.

The first person to do a careful calculation published it, but most of us got out our pencils and papers when all we had heard was that about 20 antineutrinos had been scattered in the IMB and Kamioka detectors (originally built to look for proton decay) over an interval of about 10 seconds, and that the energies were sort of 10 to 20 MeV.

The simplest calculation is the difference in arrival time between a photon and a neutrino of mass m over the distance to the LMC (55 kpc according to that expert Trimble[26], but 50 kpc was used by the neutrino folks). The photon and neutrino speeds are allowed to differ by at most a couple of parts in 10^{11}. (Check it!) And, following the same procedure,

$$mc^2 = E(1 - v^2/c^2)^{1/2}, \tag{9}$$

the neutrino mass ends up as, at most, some tens of eV, less if you consider a time window around 2 seconds as having included half the neutrino arrival times.

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*The paper was submitted on 1960 November 28 and appeared in the December 15 issue, suggesting that the editor thought it important. It was cited a good deal in the 1960s and 1970s, tapering off to the present. The two supernova-neutrino papers that remained longer in their colleagues' memories were those of George Gamow & M. Schoenberg[24] with "a neutrino theory of stellar collapse" meant seriously despite the April 1 date of publication, and of Colgate & White[25], whose 1961 detailed calculation of neutrino transport through dense gas was state of the art until neutrino physics changed.