STRINGS WITH DIMENSION-DEPENDENT INTERCEPT

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January 28, 1976

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ABSTRACT

By changing the boundary conditions of the relativistic string in extra dimensions, the intercept $\alpha(0)$ is lowered to $1 - \dfrac{26 - D_0}{16}$ in the modified orbital model, where D_0 is the dimensionality of the Poincaré-invariant subspace of space-time. In the modified model of the spinning string, the boson intercept becomes $\dfrac{1}{2} - \dfrac{10 - D_0}{8}$, while the fermion intercept stays at zero. The projective invariance of the ground state is broken, giving the "photon" mass by a Higgs-like mechanism. Unfortunately, the strings have a negative "G-parity", so the usual, unshifted strings appear as intermediate states in the scattering amplitudes. Also, some of the amplitudes are not dual.

I.   INTRODUCTION[1]

The two major problems of dual models are the restrictions of the intercept[2] and dimensionality to unphysical values. In this paper the "extra" dimensions are treated as internal degrees of freedom in such a way as to lower the intercept: One end of the string is fixed in the extra dimensions, while both ends are left free, as usual, in physical space-time. Quantizing in the light-cone gauge, Lorentz invariance in the D_0 "ordinary" dimensions requires $\alpha(0) = 1 - \dfrac{26 - D_0}{16}$

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and $D_0 + D_E = 26$ in the modified orbital model; here D_E is the number of "extraordinary" dimensions in which one end of the string is fixed. By modifying the spinning string in the same way, one finds $\alpha(0) = \dfrac{1}{2} - \dfrac{10 - D_0}{8}$ for the bosons and $\alpha(0) = 0$ for the fermions, with $D_0 + D_E = 10$. In these new models, the "photon" has been given mass by a Higgs-like mechanism: Its longitudinal mode is created by the (D_0-dimensional Lorentz group) scalar operators of the extra dimensions.

In the simplest interacting-string picture, continuity of the boundary conditions requires the number of intercept-shifted strings at a vertex to be even ("G-parity" conservation). Therefore, the use of a three-string vertex requires that one string be unshifted. Though unsatisfactory as it stands, the model is at least not equivalent to any old model. Calculation of the four-point functions with this vertex gives the amplitude for scattering of a shifted and an unshifted ground-state string as Veneziano's Beta-function amplitude, with $\alpha_s(s) = s + 1 - \dfrac{26 - D_0}{16}$ and $\alpha_t(t) = t + 1$. However, the amplitude for shifted-shifted scattering turns out to be non-dual.

II.   FREE STRINGS WITH SHIFTED INTERCEPT

The results of this section (concerning the modified orbital model) were previously derived by M. B. Halpern[3], and are given here for completeness.

A.   The classical string.

The simplest way to modify the dynamics of the free string is to change the boundary conditions without changing the equations of motion. With δx not necessarily arbitrary, the variational principle for the relativistic string is

$$0 = \delta \int d^2\sigma \frac{1}{2\pi\alpha'} \sqrt{-g} = \oint d\sigma^\alpha \epsilon_{\alpha\beta} P_\mu^{\ \beta} \delta x^\mu - \int d^2\sigma \partial_\alpha P_\mu^{\ \alpha} \delta x^\mu \ ,$$

$$P_\mu^{\ \alpha} \equiv \frac{1}{2\pi\alpha'} (\sqrt{-g}\ g^{\alpha\beta}) \partial_\beta x_\mu = \frac{1}{2\pi\alpha'} \eta^{\alpha\beta} \partial_\beta x_\mu$$

$$(\alpha,\beta = 0,1;\ \mu = 0,\ldots,D-1;\ \epsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ \eta^{\alpha\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}).$$

(2.1)

($d\sigma^\alpha \epsilon_{\alpha\beta} = d\ell n_\beta$, where n is the unit vector normal to the boundary.)
Allowing δx to be arbitrary inside the string, the usual field equations follow from setting the second term equal to zero. The first term can be made zero by a combination of two types of (local) boundary conditions: At either end, we can either allow δx to be arbitrary (free end), which requires $x' = 0$, or we can use the constraint $x =$ constant (fixed end). In order to have Poincaré invariance in the first D_0 of the D dimensions, we choose both ends to be free in these dimensions. In the remaining D_E dimensions, we have four choices: The first choice is to have both ends free (the standard string model). Secondly, we may choose to fix both ends at the same point, again giving essentially the standard string model (the zero mode is eliminated, but we will choose $p^\mu \equiv 0$ for $\mu \gtrless D_0$ in any case). If they are fixed at different points, we have the massive string of Chodos and Thorn[4]. The last choice is to fix one end, leaving the other free:

$$x_\mu'(\sigma^0, 0) = 0;\ x_\mu'(\sigma^0,\pi) = 0 \ \text{for}\ \mu < D_0,\ x_\mu(\sigma^0,\pi) = 0 \ \text{for}\ \mu \gtrless D_0.$$

(2.2)

This choice differs from the others in that it gives the string half-integral modes[5] for $\mu \gtrless D_0$. Explicitly:

$$\mu < D_0 : x^\mu = q^\mu + 2\alpha'p^\mu\sigma^0 + \sqrt{2\alpha'} \sum_{n=1}^\infty \frac{i}{n} \cos n\sigma^1 (a_n^\mu e^{-in\sigma^0} - c.c.)$$

$$\mu \gtrless D_0 : x^\mu = \sqrt{2\alpha'} \sum_{n=\frac{1}{2}}^\infty \frac{i}{n} \cos n\sigma^1 (a_n^\mu e^{-in\sigma^0} - c.c.) \ .$$

(2.3)

B. Canonical quantization.

In the rest of this section we discuss only the string with one end fixed in the extra dimensions. The commutation relations of p^μ, q^μ, a_n^μ, and $a_n^{\dagger\mu}(\mu < D_0$ for p and $q)$ have the usual form in both the covariant and light-cone gauges $(x^+ \equiv x^0 + x^{D_0-1})$. The easiest way to determine the intercept and dimension of the string is to prove the $(D_0$-dimensional) covariance of the light-cone gauge. This involves as the only non-trivial calculation the commutators of the Lorentz generators $M^{i-}(i < D_0)$. M^{i-} has the usual form in terms of q^i, q^-, p^+, a_n^i, $\alpha(0)$, and L_n, but now

$$L_n = \frac{1}{2} \sum_{\substack{m=-\infty \\ m\epsilon\mathbb{Z}}}^\infty \sum_{i=1}^{D_0-2} : a_m^i a_{n-m}^i : + \frac{1}{2} \sum_{\substack{m=-\infty \\ m+\frac{1}{2}\epsilon\mathbb{Z}}}^\infty \sum_{i=D_0}^{D-1} : a_m^i a_{n-m}^i : . \quad (2.4)$$

The commutator is calculated to be[6]

$$\left[M^{i-}, M^{j-}\right] = \frac{1}{\alpha'p^{+2}} \sum_{n=1}^\infty \frac{1}{n} (a_{-n}^i a_n^j - a_{-n}^j a_n^i).$$

$$\cdot \left\{ n^2 \left(\frac{D-2}{24} - 1\right) + \left[\alpha(0) - \left(\frac{D-2}{24} - \frac{D-D_0}{16}\right)\right] \right\} . \quad (2.5)$$

Since (D_0-dimensional) Lorentz covariance requires this commutator equal zero, we have

$$D = 26, \quad \alpha(0) = 1 - \frac{26 - D_0}{16} . \tag{2.6}$$

Therefore, depending on the choice of D_0, the intercept can vary by steps of $1/16$ from the usual 1 down to $-3/8$ ($D_0 \geq 4$ to keep physical space-time Poincaré covariant).

We now study the way in which the "photon" becomes massive ($D_0 < 26$) by examining the operators which create its states. To find the states we calculate ($\mu, \nu, \sigma, \tau = 0, \ldots, D_0 - 1$; choose $p^i = 0$)

$$W^2 = (\tfrac{1}{2} \epsilon_{\mu\nu\sigma\tau} M^{\nu\sigma} p^\tau)^2 =$$

$$\frac{1}{2\alpha'} (L_0 - \alpha(0)) \sum_{i,j=1}^{D_0-2} \left[\sum_{n=1}^{\infty} \frac{1}{n} (a_{-n}^i a_n^j - a_{-n}^j a_n^i) \right]^2 + \cdot$$

$$\frac{1}{2\alpha'} \sum_{i=1}^{D_0-2} \left[\sum_{n=1}^{\infty} \frac{1}{n} (L_{-n} a_n^i - a_{-n}^i L_n) \right]^2 = -M^2 \vec{S}^2 , \tag{2.7}$$

where $M^2 = \frac{1}{\alpha'} (L_0 - \alpha(0))$ is the (mass)^2 and $\vec{S}$ the spin. From this we find the "photon's" states are the usual $a_{-1}^i |0>(i = 1, \ldots, D_0 - 2)$ and the new state $\sum_{i=D_0}^{25} (a_{-\frac{1}{2}}^i)^2 |0> \sim L_{-1}|0> \sim a_{-1}^-|0>$. The latter, longitudinal state could also be found by noting that it is the only other state of that mass which is a singlet under the internal-symmetry group of the string: the $O(D_E)$ of the D_E dimensions where one end of the string is fixed. This is analogous to the Higgs mechanism in that the vector boson has gained mass by acquiring a longitudinal mode

from scalar operators ($a_{-\frac{1}{2}}^i$ for $i \geq D_0$); projective invariance of the ground state is broken because $L_{-1}|0> \neq 0$.

C. The modified spinning string.

The same choices of boundary conditions for x are possible for the spinning string; we again study the case where one end is fixed in the extra dimensions. The requirement that the model have the same number of subsidiary conditions as when $D_E = 0$ (so that the system is not overdetermined) gives the following boundary conditions for the spin density at the fixed end: If $S_1^\mu(\sigma^0, \pi) = \pm S_2^\mu(\sigma^0, \pi)$ for $\mu < D_0$ (for fermions or bosons, respectively, as in the conventional model), then $S_1^\mu(\sigma^0, \pi) = \mp S_2^\mu(\sigma^0, \pi)$ for $\mu \geq D_0$. This means that the operators G_n have either all indices integral (fermions) or all indices half-integral (bosons). Covariance in the light-cone gauge requires

$$D = 10; \quad \alpha(0) = 0 \text{ for fermions}, \quad \frac{1}{2} - \frac{10 - D_0}{8} \quad \text{for bosons.} \tag{2.8}$$

Here $\alpha(0)$ for bosons is the intercept of the pion trajectory; the leading trajectory is the rho with intercept $1 - \frac{10 - D_0}{8}$. The rho intercept can thus take values from 1 to $1/4$. For the rest of the article, we consider only strings without spin.

III. INTERACTING STRINGS

A. Path-integral quantization[7].

For studying the interactions of strings, path-integral quantization is more convenient than canonical quantization. In this formalism the choice of boundary conditions follows from the functional integration, which leaves the amplitude in terms of only the volume element and the Green's function. The definition of the Green's function which allows this reduction is equivalent to the field equations

of x derived in the canonical quantization. This is accomplished by the standard path-integral change of variables (we now work in Euclidean σ^α-space by Wick rotating σ^0; $\alpha' = 1$; the index on σ^α is dropped):

$$x^\mu(\sigma) \to x^\mu(\sigma) - i \int d^2\sigma' G^\mu{}_\nu(\sigma,\sigma') J^\nu(\sigma');$$

$$\left. \begin{array}{l} J^\mu(\sigma) \equiv \sum_r P_r{}^\mu(\sigma') \delta(\sigma^0 - \sigma^0{}_{1r}), G_{\mu\nu} = \delta_{\mu\nu} G_\mu \text{ (not summed)}, \\[2mm] \partial_\sigma{}^2 G_{\mu\nu}(\sigma,\sigma') = 2\pi \delta_{\mu\nu} \delta^{(2)}(\sigma - \sigma'). \end{array} \right\} \quad (3.1)$$

P_r is the momentum density of the rth string ($p_r{}^+ < 0$ for outgoing strings), and $\sigma^0{}_{1r} = \sigma^0{}_i$ or $\sigma^0{}_f$ when the rth string is incoming or outgoing, respectively. This changes the exponent of the functional integral[8]:

$$\left. \begin{array}{l} \int d^2\sigma \mathcal{L} + i \sum_r \int d\sigma' P_r{}^i(\sigma') x^i(\sigma_{1r}{}^0, \sigma') = \int d^2\sigma \left[-\frac{1}{4\pi}(\partial_\alpha x^i)^2 + i J^i x^i \right] \\[3mm] \to \int d^2\sigma \left[-\frac{1}{4\pi}(\partial_\alpha x^i)^2 \right] + \frac{1}{2} \int d^2\sigma d^2\sigma' J^i(\sigma) G^i(\sigma,\sigma') J^i(\sigma') + \\[3mm] \frac{i}{2\pi} \oint d\sigma_\alpha \epsilon^{\alpha\beta} \left[x^i \partial_\beta \int d^2\sigma' G^i(\sigma,\sigma') J^i(\sigma') + \frac{1}{4} \partial_\beta \left(\int d^2\sigma' G^i(\sigma,\sigma') J^i(\sigma') \right)^2 \right]. \end{array} \right\} (3.2)$$

(Sum over $i = 1, \ldots, D_0 - 2, D_0, \ldots, 25$.) The first and second terms give the volume element and Green's-function dependence, as usual. The last term (surface integral) can be made zero again by two kinds of choices of boundary conditions: $\frac{\partial}{\partial n_\sigma} G^i(\sigma,\sigma') = 0$; or $G^i(\sigma,\sigma') = 0$, with the constraint $x^i(\sigma) = $ constant. These are the two standard choices for boundary conditions in solving Laplace's equation (the equation of motion of the string for Wick-rotated σ^0): The former choice is the Neumann boundary condition used in the ordinary string

model, the latter is Dirichlet. The boundary conditions on x are the same as those on G due to

$$x^i(\sigma) = \frac{1}{2\pi} \oint d\sigma'_\alpha \epsilon^{\alpha\beta} x^i(\sigma') \overset{\leftrightarrow}{\partial_\beta} G^i(\sigma',\sigma) + i \int d^2\sigma' G^i(\sigma',\sigma) J^i(\sigma'), \quad (3.3)$$

where the surface term must be made zero. For scattering amplitudes we can use the more general condition $\frac{\partial}{\partial n_\sigma} G^i(\sigma,\sigma') = f^i(\sigma)$ (for arbitrary $f^i(\sigma)$), due to momentum conservation $\int d^2\sigma \, J^\mu = \sum_r p_r{}^\mu = 0$ (for $\mu < D_0$). Corresponding to (2.2), we choose the Neumann condition for G^i when $i < D_0$ and the Dirichlet condition when $i \geq D_0$.

B. Evaluating the Green's-function[7,9].

In general $G^i(\sigma,\sigma')$ for $i \geq D_0$ is difficult to evaluate: One needs to solve Laplace's equation in two dimensions with a combination of Neumann and Dirichlet boundary conditions. When calculating the N-string scattering amplitude, G can be found for $N < 8$ by the usual Schwartz-Chrisotffel transformation of the string to the upper-half z-plane, followed by another transformation to the interior of a regular N- or (N - 1)-gon for N even or odd, respectively. (We define a 2-gon as a quadrant.) The sides of the polygon alternate between Neumann and Dirichlet boundary conditions. The map to the quadrant and the Green's-function in it can be expressed in terms of elementary functions, the square needs elliptic functions, and the hexagon requires functions which have no closed form (calculation of the Green's-function in the last case is achieved by dividing up the plane into hexagons and using the method of images). We will avoid these computational problems by calculating only the three-string Green's-function: This gives the three-string vertex, from which all amplitudes can be found by use of the operator formalism. (The question of ghosts in changing from

the light-cone gauge of the interacting-string picture to the covariant gauge of the operator formalism is no problem, since the ghosts are due to operators in the Poincaré-invariant subspace, and so can be treated in the standard way.)

The map from the three-string diagram in the $\rho\,(=\sigma^0 + i\sigma')$-plane to the upper-half z-plane is the usual $\rho = \alpha_1 \ln(z - 1) + \alpha_2 \ln z$, and the map to the upper-right quadrant of the x-plane is simply $x = z^{\frac{1}{2}}$ (strings 2 and 3 have shifted intercepts, so continuity of boundary conditions requires string 1 to be an ordinary unshifted string: see Figure 1). The Green's function ($i \geqq D_0$ suppressed) is then given by:

$$G(x,x') = G_0(x,x') - G_0(x, -x'), \quad G_0(x,x') \equiv \ln|x - x'| + \ln|x - x'^*|. \tag{3.4}$$

The Green's function G is found by the method of images from the Green's function G_0 of the upper-half plane used in the standard string model.

To find the Fourier coefficients, we first find the coefficients of $M \equiv (\partial/\partial\sigma^0 + \partial/\partial\sigma^{0\prime})G$. The partial derivatives are expressed in terms of x by

$$\frac{\partial}{\partial\rho} = \frac{1}{2}\frac{x(1 - x^2)}{\alpha_2 + \alpha_3 x^2}\frac{\partial}{\partial x} \ , \qquad \frac{\partial}{\partial\sigma^0} = \frac{\partial}{\partial\rho} + \frac{\partial}{\partial\rho^*} \ , \tag{3.5}$$

which, after some algebra, gives

$$M = 2\alpha_1\left[\frac{\partial}{\partial\sigma^0}\left(\ln|x + 1| - \ln|x - 1|\right)\right]\left[\frac{\partial}{\partial\sigma^{0\prime}}\left(\ln|x' + 1| - \ln|x' - 1|\right)\right]. \tag{3.6}$$

Therefore, we only need to find the Fourier coefficients of

$$A(x) = \ln|x + 1| - \ln|x - 1| = -\delta_{r1}\xi_1 + \sum_{n>0} A_{rn}\begin{Bmatrix}\cos\\\sin\end{Bmatrix}n\eta_r\,e^{n\xi_r} + \text{constant}, \tag{3.7}$$

$$\frac{\alpha_1}{\alpha_r\alpha_s}(-\delta_{m0} + mA_{rm})(-\delta_{n0} + nA_{sn}) \equiv \delta_{rs}\delta_{mn}\delta_{n0}\frac{1}{\alpha_r} + \left(\frac{m}{\alpha_r} + \frac{n}{\alpha_s}\right)G_{rsmn}. \tag{3.8}$$

(Because of the boundary conditions, in (3.7) the cosine is used for $r = 1, 2$, the sine for $r = 3$; n is integral for $r = 1$, half-integral for $r = 2,3$.) The coefficients

$$A_{rn} = \frac{2}{\pi n}\int_0^\pi d\begin{Bmatrix}\sin\\-\cos\end{Bmatrix}n\eta_r\,e^{-n\xi_r}A(x) \tag{3.9}$$

can be evaluated[10] by use of the identities

$$\sin n\eta_1 = \frac{i}{2}e^{n\xi_1}\left\{\left[(x^2 - 1)x^{2\alpha_2/\alpha_1}\right]^{-n} - \text{c.c.}\right\} ,$$

$$\sin \frac{n}{2}\eta_2 = \frac{i}{2}e^{n\xi_2/2}\left\{\left[x(1 - x^2)^{\alpha_1/2\alpha_2}\right]^{-n} - \text{c.c.}\right\} , \tag{3.10}$$

$$\sin \frac{n}{2}\eta_3 = \frac{i^{-n}}{2}e^{n\xi_3/2}\left\{\left[y(1 - y^2)^{\alpha_1/2\alpha_3}\right]^{-n} - \text{c.c.}\right\} \ (y = \frac{1}{x}) .$$

These follow from the definitions of η_r and ξ_r and the expression for ρ in terms of x. Using the (anti) symmetry of $A(x)$ under $x \leftrightarrow x^*$ and $x \leftrightarrow -x$, (3.9) becomes an integral over a closed contour in the x-plane. Integration by parts then eliminates the $\ln(x \pm 1)$ factors. The $\ln(x^* \pm 1)$ terms vanish by identities such as

$$\left[(x^2 - 1)x^{2\alpha_2/\alpha_1}\right]^{-1} = e^{-2\xi_1}(x^{*2} - 1)x^{*\,2\alpha_2/\alpha_1} .$$

We are then left with integrals like

$$\oint_0 dx\, x^{-n}\left[(1-x)^{-n\alpha_1/2\alpha_2}(1+x)^{-n\alpha_1/2\alpha_2-1}\right] = \frac{2\pi i}{(n-1)!}\, \frac{d^{n-1}}{dx^{n-1}} \cdot$$

$$\cdot\left[(1-x)^{-n\alpha_1/2\alpha_2}(1+x)^{-n\alpha_1/2\alpha_2-1}\right]\Bigg|_{x=0}.$$

These can be expressed as Jacobi polynomials by Rodrigues' formula[11]

$$\frac{1}{n!}\,\frac{d^n}{dx^n}\left[(1-x)^a(1+x)^b\right] = (-2)^n(1-x)^{a-n}(1+x)^{b-n}P_n^{(a-n,b-n)}(x).$$

(3.11)

The final result is

$$\left.\begin{array}{l}
A_{1n} = -\dfrac{2^{-2n}}{n}\, P_n^{(-n[2\alpha_2/\alpha_1+1],-2n-1)}(3),\\[2mm]
A_{2n} = \dfrac{2^{2n-1}}{n}\, P_{2n-1}^{(-2n[\alpha_1/2\alpha_2+1]+1,-2n[\alpha_1/2\alpha_2+1])}(0),\\[2mm]
A_{3n} = (-1)^{(2n-1)/2}\cdot(A_{2n}\ \text{with}\ \alpha_2\to\alpha_3),
\end{array}\right\}$$

(3.12)

with the Green's-function Fourier coefficients G_{rsmn} given by (3.8).

For calculating the four-point functions, we only need the special case of the above Green's function for $\alpha_2 = 0$, since it only contributes to the volume element and not to the momentum dependence ($p^\mu \equiv 0$ for $\mu \geq D_0$). We also only need string 2 in the ground state. In this special case we have

$$A_{1n} = -\frac{1}{n}(-1)^n\binom{n-\frac{1}{2}}{n},\ A_{3n} = \frac{1}{n}\binom{n-1}{n-\frac{1}{2}};\ G_{rsmn} = \frac{m+n}{mn}\,A_{rm}A_{sn}\ (m,n\neq 0).$$

(3.13)

These are also the Green's-function coefficients for the two-string mixing vertex (Figure 2), which will be discussed in the conclusions.

C. The four-point functions.

The four-point amplitude (Figure 3) can be written as

$$T = \int_0^1 \frac{dz}{z}\left[z^{-2p_1\cdot p_2}(1-z)^{-2p_2\cdot p_3}\right]\langle 0|e^{\frac{1}{2}aGa}a_z^\dagger a\, e^{\frac{1}{2}a^\dagger Ga^\dagger}|0\rangle.$$

$$\cdot z^{\alpha_1(0)+\alpha_2(0)-\alpha_{INT}(0)}$$

(3.14)

for s-channel exchange, plus the corresponding integral for the t-channel exchange. The factor in brackets is the usual momentum-dependent contribution for $\mu < D_0$. The vacuum expectation value is the contribution for $\mu \geq D_0$ (with all indices suppressed). The final factor comes from the propagator

$$(L_0 - \alpha_{INT}(0))^{-1} = \int_0^1 \frac{dz}{z}\, z^{L_0-\alpha_{INT}(0)} =$$

$$\int_0^1 \frac{dz}{z}\, z^{-a_\mu^\dagger a^\mu - 2p_1\cdot p_2 + \alpha_1(0)+\alpha_2(0)-\alpha_{INT}(0)}.$$

(3.15)

Evaluating the vacuum expectation value, (3.14) becomes:

$$T = \int_0^1 dz\, z^{-(s+\alpha_{INT}(0))-1}(1-z)^{-t-\alpha_2(0)-\alpha_3(0)}\left[\det(1-\tilde{G}_{OUT}\tilde{G}_{IN})\right]^{-D_E/2},$$

$$\tilde{G}_{mn} \equiv \sqrt{mn}\, G_{mn}\, z^{(m+n)/2}.$$

(3.16)

G_{mn} are the Fourier coefficients of the Green's function between the internal string and string 1 or 4 for G_{IN} or G_{OUT}, respectively. (Strings 2 and 3 have infinitesimal width.) The coefficients are given by (3.13) between a shifted and an unshifted string, and are zero between two strings of the same type (as in the standard orbital model). The only two non-trivial determinants to be evaluated are[12,13]

$$\det(1 - \tilde{G}_{11}{}^2) = (1 - z)^{1/4} \frac{2}{\pi} K(\sqrt{z}), \quad \det(1 - \tilde{G}_{33}{}^2) = (1 - z)^{1/4}, \quad (3.17)$$

with G_{11mn} and G_{33mn} as in (3.13). The latter determinant gives the shifted-unshifted elastic scattering amplitude as

$$T_{SU} = \int_0^1 dz \, z^{-(s + \alpha(0)) - 1}(1 - z)^{-(t + 1) - 1} = B(-\alpha_S(s), -\alpha_U(t))$$

$$(\alpha(0) = 1 - \frac{26 - D_0}{16}). \qquad (3.18)$$

The contribution from t-channel exchange equals that from s-channel exchange, as in the usual orbital model. However, the amplitude for shifted-shifted scattering is

$$T_{SS} = \frac{1}{2} \left(\int_0^1 dz \, z^{-(s + 1) - 1}(1 - z)^{-(t + 1) - 1} \left[\frac{2}{\pi} K(\sqrt{z}) \right]^{-D_E/2} + s \leftrightarrow t \right). \qquad (3.19)$$

Not only are the s-channel and the t-channel contributions different, but the s-channel-exchange resonances do not generate t-channel poles (except, of course, for $D_E = 0$), due to the $\ln(1 - z)$ behavior of the complete elliptic integral of the first kind $K(\sqrt{z})$. The s-channel poles generate branch-point singularities in the t-channel for $D_E = 1$ or 2, and no singularities for $D_E > 2$. Consequently the amplitude is not Regge-behaved.

IV. CONCLUSIONS

We have shown how to construct a simple generalization of the free string which lowers the boson trajectories. It thus gives mass to the "photon" by breaking the projective invariance of the ground state. This indicates that the method of boundary constraints may be

related to the problem of spontaneous symmetry breakdown in dual models.

In the interacting-string picture, ordinary unshifted strings are required. It may be possible to shift the unshifted strings by introducing a shifted-unshifted two-string interaction. It would be interesting to see what effect this dynamical symmetry breaking has on the model (e.g., are trajectories still linear?).

The non-dual behavior of one of the four-point functions is similar to the behavior of the fermion-fermion scattering amplitude in a dual model of Schwarz[14]. An operator similar to the three-string vertex also occurs in a recent model for off-mass-shell dual amplitudes by Schwarz[12] and by Corrigan and Fairlie[5]. Whether there is some deeper significance to these relationships is unknown.

V. ACKNOWLEDGMENTS

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(14) J. H. Schwarz, Nucl. Phys. B68 (1974) 221. See also Reference 12.
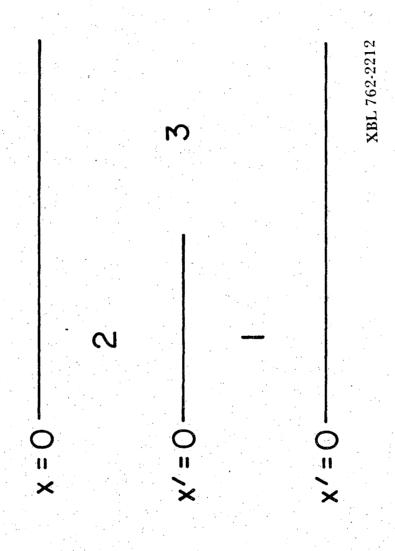
FIGURE CAPTIONS

Figure 1: Three-string vertex.

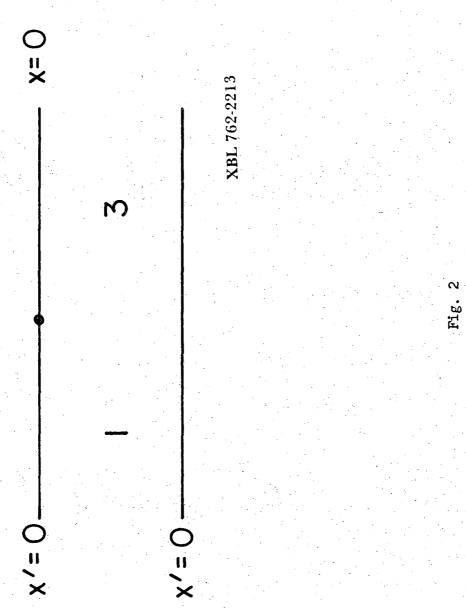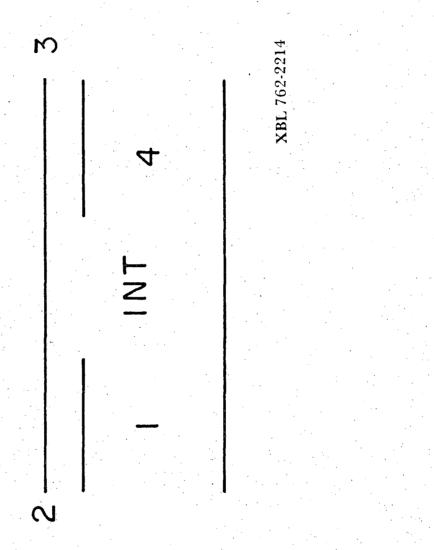Figure 2: Two-string vertex.

Figure 3: Four-point function.

Fig. 1

XBL 762-2212

Fig. 2

XBL 762-2213

Fig. 3

XBL 762-2214