Explaining preferred mental models in Allen inferences with a metrical model of imagery

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Abstract

We present a simple metrical representation and algorithm to explain putative imagery processes underlying the empirical mental model preferences found by Knauff, Rauh and Schlieder (1995) for Allen inferences (Allen, 1983). The computational theory is compared with one based on ordinal information only (Schlieder, in preparation). Both provide good fits with the data. They differ psychologically in background theories, visualisation strategies motivated by these, and model construction processes generating models with the properties indicated as desirable by the strategies. They differ computationally in assumptions about knowledge strength (ordinal: weaker) and algorithmic simplicity (metrical: simpler). Our theory and its comparison with the ordinal theory provide the basis for a discussion of issues pertaining to imagery in general: Using the assumption of imagery inexactness, we develop a sketch theory of mental images and motivate a new visualisation strategy ('regularisation'). We demonstrate systematic methods of modelling imagery processes and of analysing such models. We also outline some criteria for comparison (and future integration?) of cognitive modelling approaches.

name	symbol	diagram	point ordering
equals	A = B		sA=sB<eB=eA
before	A < B		sA<eA<sB<eB
meets	A m B		sA<eA=sB<eB
overlaps	A o B		sA<sB<eA<eB
starts	A s B		sA=sB<eA<eB
finishes	A f B		sB<sA<eA=eB
during	A d B		sB<sA<eA<eB
during-inverse	A di B		sA<sB<eB<eA
starts-inverse	A si B		sA=sB<eB<eA
finishes-inverse	A fi B		sA<sB<eB=eA
overlaps-inverse	A oi B		sB<sA<eB<eA
meets-inverse	A mi B		sB<eB=sA<eA
after	A > B		sB<eB<sA<eA

Figure 1: The 13 interval relations, adapted from (Schlieder, in preparation). White interval = A, black interval = B; sA, sB, eA, eB = start- and endpoints of A and B.

Allen inferences, preferred mental models, and a computational theory based on ordinal information

Allen relations are the 13 'qualitative' relations which can hold between two intervals, corresponding to relations between the start- and endpoints of the two intervals, between which only the ordinal relations 'is before/smaller than', 'is equal to', and 'is after/larger than' are distinguished, see fig. 1. They have been discussed by Allen (1983) in a logic for reasoning about temporal events and have been used also as a basis for spatial reasoning calculi , e.g. in (Guesgen, 1989; Mukerjee & Joe, 1990). Qualitative relations are of interest to Cognitive Science because they might be employed in programs like Geographical Information Systems (GIS) to model human temporal/spatial reasoning more adequately than models based on numerical specifications. **Allen inferences** are compositions of Allen relations answering the following question: If the Allen relation between intervals A and B is R_1 and that between intervals B and C is R_2, then which Allen relation(s) R_3 can hold between A and C? In some cases, there is only one possible answer (relation R_3); in others, there are several. For example, if A finishes-inverse B and B before C, then A can only be before C. If A finishes-inverse B and B during C, it is possible that A starts C, overlaps C, or is during C. The second example is shown in fig. 3; the first can easily be reconstructed from this drawing. For full details, see (Allen, 1983).

This composition can be seen as a three-term series in the sense of Johnson-Laird (1972). But how do people reason with Allen relations? Knauff, Rauh and Schlieder (1995) trained subjects in the understanding of Allen relations and then asked them to provide *one* answer to each of 12 × 12 Allen inference questions (they excluded the trivial compositions with equals). In cases where more than one Allen relation is a correct answer ('non-unique cases'), a great majority of subjects chose the same solution. The authors interpret these results as evidence of **preferred mental models**: In the sequential process of constructing the different possible mental models (= the several possibilities of arranging A, B and C such that different Allen relations R_3 hold), a strategy is employed which first leads to the construction of one particular model, the preferred model. Empirically, preferred models (aggregated over subjects) were never non-models; i.e. always yielded correct solutions.

When we try to find a **computational theory** of a strategy like this one, an interesting question concerns the **nature of the knowledge** used (cf., for example, (Huttenlocher, 1968) vs. (Johnson-Laird & Byrne, 1991)). If one assumes imagery in mental models, one needs an 'image' as model, which can be modelled as containing metrical information. The input relations R_1, R_2 and the intervals A, B, C linked by them are given a metrical interpretation, i.e. some placement in the image. This metrical representation is then inspected to obtain the required answer, the Allen relation between A and C. This is based on ordinal relations between A's and C's start- and endpoints. Considering, however, that the whole concept of Allen relations is based on ordinal information, one may want to avoid giving a metrical interpretation to the ordinal Allen relations and try to formulate a theory without imagery, i.e. without metrical representations.

Schlieder (in preparation) develops **a computational theory which operates on ordinal information only**. Development of the theory starts out from the discussion of formal properties of Allen inferences discussed by Ligozat (1990), namely two kinds of symmetries of pairs of inferences. These formal properties were not satisfied empirically: The pairs of inferences identified by Ligozat had asymmetrical empirical model preferences. It is not important to go into the details of the formal argument here; it suffices to note that the observation of formal symmetry and empirical asymmetry implies that in order to explain the data, a model must generate **order effects**: a relation R is sometimes conceptualised differently depending on whether it holds between A and B or between B and C (or B and A or C and B), i.e. the two intervals are placed differently with respect to each other (for an example, see fig. 2; for full details of the formal argument, see (Ligozat, 1990; Schlieder, in preparation)). Schlieder generates order effects by assuming a **focus** in the mental model, a point which, in addition to the start- and endpoints of the intervals already considered, is kept in the representation. Schlieder can explain all but 6 of the 60 non-unique empirical preferences he considers (see fig. 5).[1] He omits compositions of inverses, which are compositions of relations like before and after, i.e. compositions whose results lie in cells along the secondary diagonal of the composition table in fig. 5. A much simpler strategy seems to be applied there: "don't think" – just assume inverses always lead back to the original, i.e. to the Allen relation equals.

However, this model has the **drawback** of being rather **complicated**: There are 6 to 14 (depending on how one counts) 'scanning rules' and rather involved 'insertion schemes' specifying different processes and orders of the insertion of start- and endpoints, and the focus position has to be remembered. These complications were our motivation to devise a **strategy** that requires **stronger information**, namely, metrical knowledge, but is much **simpler** in terms of scanning and insertion.

[1] By 'computational theory', we mean that for all compositions, models are generated, i.e. correct Allen inferences. By 'cannot explain', we mean that the computational theory does not generate the empirically preferred mental model for a given composition.

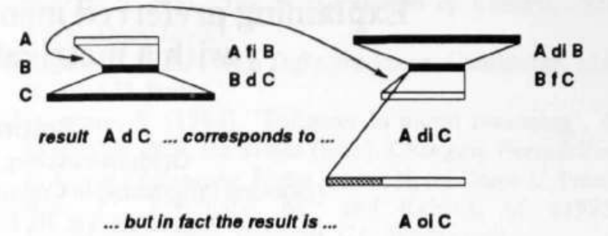

Figure 2: Order effects (example): If the Allen relations are always computed in the same way, and inverse relations are mirror images of each other, i.e. computed by inverting the operations generating the new interval in a composition, the preferred model of A finishes-inverse B and B during C ($AfiB$ and BdC) implies a preferred model for A during-inverse B and B finishes C ($AdiB$ and BfC). However, empirically the first inference has the preferred model A during C (AdC), while the second has A overlaps-inverse C ($AoiC$). An easy explanation is that the transformations linked by the curved line are different: f is *not* the mirror image of fi.

A computational theory based on metrical information

Our main **psychological background assumption** is that **mental images are inexact**. A simple example of empirical evidence for this claim is the finding that discoveries in mental images are common if the patterns to be rotated and the emerging patterns resulting from this transformation are sufficiently robust with respect to slight variations in shape and/or noise, e.g. (Finke & Slayton, 1988). If the given patterns are more complex and emergence is highly dependent on constructing and maintaining an exact representation of the given patterns' shape, however, mental discoveries are not the rule (Reisberg & Chambers, 1991); for discussion, see also (Logie, 1995). We interpret this to mean that imagery cannot represent and/or process fine details which are sensitive to small metrical changes. Image elements are represented metrically, relative to a reference frame specifying scale which is global to the whole image. However, within this frame of reference, they may only be represented inexactly, with noise. Inexactness may arise during construction, maintenance and/or inspection of the image.

For the present task, this **implies** one particular constraint on **what the images of the intervals should look like**: They should be **regular**. To understand this concept, consider the composition of A finishes-inverse B and B during C. One solution is A starts C. However, this result is extremely unstable: Any slight deviation in the lengths or placements of the intervals created by imagery inexactness would lead to A overlaps C or A during C (see fig. 3). As the reader may easily verify, the last two solutions are much more robust with respect to changes in metrical parameters. We call images involving solutions which are 'stable' in this sense (like overlaps and during) *regular*, as opposed to *singular* images involving 'unstable' solutions like starts. The data can be interpreted as showing a preference for regular images: Out of those compositions whose solution includes an unstable relation, this solution is the preferred model only in very few cases. We therefore need a **model construction process** which (a) generates correct Allen inferences, (b) generates order effects, and (c) generates regular images. The easiest metrical process contains

1. **distance parameters** specifying the length of the separation ($<, >$), the overlap (o, oi), or the offset (s, si, f, fi, d, di) of the relations. (m and mi must be 'separated' by a distance of 0.) We use 2 such parameters: Δ_n ("normal") and Δ_l ("large"). They are associated with relations depending on whether these are 'shifts' or 'deformations' and prescribe 'movements' to construct the new interval's start- and endpoints (see fig. 4).

A model containing only distance parameters generates correct Allen inferences, but fails because it cannot generate any order effects (Schlieder, in preparation). We therefore choose the second easiest, which also contains

2. a **correction parameter** ε. This is associated with relations depending on how many relations have been processed before. Using ε leads to slight, **progressive adjustments in the movements' and/or the intervals' lengths.**[2]

   These adjustments generate order effects, because Allen relations receive a different metrical interpretation depending on when they are processed, i.e. depending on whether they hold between A and B or between B and C. The adjustments also guarantee regularisation, because newly generated start- and endpoints cannot be equal to existing start- and endpoints 'by coincidence' (of course they must be equal when this is specified by relations s, si, f, fi, m, mi). As an example, consider fig. 3 again: If A finishes-inverse B ($AfiB$) and B during C (BdC), and we moved A's startpoint to the right to construct B and then moved B's startpoint to the left *by the same amount* when we construct C, we would obtain a singular image with A starts C. By moving left a bit more than we have moved right, we obtain the empirically correct A during C.

   By considering the changes made to the intervals in fig. 3 as 'shrinking or stretching intervals' instead of 'moving start- and endpoints', we can see that regularisation and (for this composition) the same order effect are generated when interval lengths instead of movement lengths are increased.

   It is not clear how to decide on psychological grounds whether movements and/or intervals should be shortened or lengthened. We shall therefore motivate the decision to lengthen them with a computational argument: lengthening leads to an algorithm satisfying the specification given by the data best. Computational reasoning also makes us prefer movement lengthening over interval lengthening (see below).

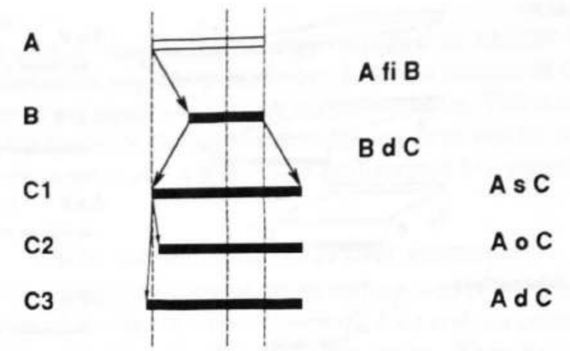

Figure 3: Example of an Allen inference with more than one solution. This shows the instability of singular images containing solutions like starts (s): Slight variations in the lengths or placements of the intervals change the obtained Allen relation.

Δ_n, Δ_l, and ε are defined relative to the standard interval length, i.e. we assume a scale-invariant imagery process.

The following algorithm and fig. 4 summarise how the three parameters control the construction process:[3]

X := first interval
insert X into the image at the standard first position
no-of-steps := 0
repeat
    no-of-steps := *no-of-steps* + 1
    R := next relation
    if R marks a shift ($\Leftrightarrow$ if $R \in \{<, m, o, oi, mi, >\}$) then
      $\Delta := \Delta_l$ else $\Delta := \Delta_n$
    adjust and place a copy of X according to R and Δ,
      using ε and *no-of-steps*
    insert the obtained interval into the image as Y
    $X := Y$
until no more relations are left
return the Allen relation obtained from reading
  the start- and endpoints of the first and the last interval

This theory, despite its great simplicity, fares extremely well when compared either to the data or to the fit of the ordinal theory. Only 9 empirical model preferences out of 60 are not explained when movements are lengthened (see fig. 5). When intervals are lengthened, another 4 preferences are not explained. As in the ordinal model, compositions of inverses were not considered.

We performed a **sensitivity analysis** of the results with respect to variations in the 3 parameters of the algorithm. We defined $errors(\Delta_n, \Delta_l, \varepsilon)$ to be the number of empirical

[2] When this affects interval length, it corresponds to scaled copying from the pattern activation subsystem into the visual buffer in (Kosslyn, 1994).

[3] All LISP program code used to compute the results reported in this paper can be obtained from the author on request.

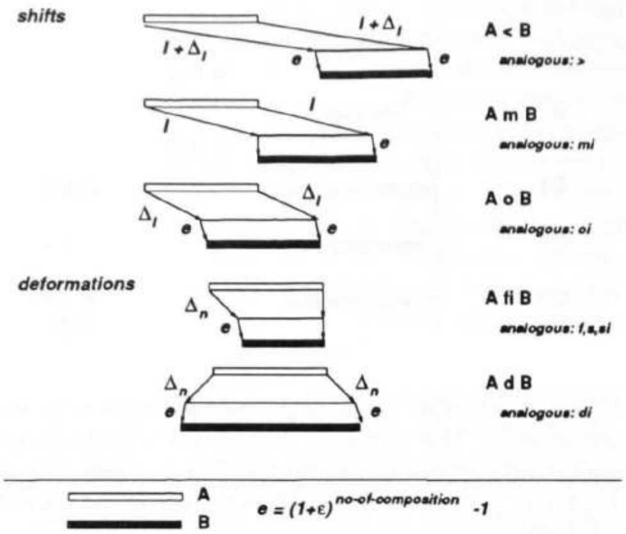

Figure 4: Constructing a new interval: computing start- and endpoints from the previous interval, the Allen relation, and the three parameters, lengthening movement. (Alternative: to lengthen intervals, shift the boundaries of the new interval obtained in the first step *outwards* by e in the second step.)

	<	m	o	fi	s	d	di	si	f	oi	mi	>
<						<,d			o [o, <]	o	o	
m						o			o	o		oi [>, oi]
o			<	< [<, o]		o	m [o, o]	o	o,d		oi	>
fi						d				oi	oi	>
s			o	o			fi [o, di]			oi		
d			o	o				oi		oi		
di	<	o	o		o				oi	oi	oi	>
si	<	o	o			d						
f			o				di,oi	oi		oi		
oi	<	o		oi	d [oi, oi]	oi	mi [oi, oi]	mi [oi, oi]		>		
mi	< [<, o]		oi		oi	oi						
>		oi	oi		oi [oi, >]	>						

Figure 5: Composition table for preferred models in Allen inferences: If AR_1B (R_1 at left end of row i) and BR_2C (R_2 at top of column j), then AR_3C (R_3 in cell ij). Table shows only compositions with non-unique solutions, and no compositions of inverses (see text). Cells containing 1 Allen relation: entry = empirical preference = preference generated by the ordinal theory = preference generated by the metrical theory. Cells containing 3 Allen relations: top entry = empirical preference; bottom left entry = preference generated by the ordinal theory; bottom right entry = preference generated by the metrical theory.

model preferences not explained by the algorithm for a given choice of parameters. *errors* yields "–" if for a given choice of parameters, non-models are produced, i.e. incorrect solutions. We computed *errors* for the relevant ranges, plotted the results, and described the constraints to ensure the best results obtainable geometrically.[4] These **geometrical constraints** turned out to be **qualitative constraints**:

$\varepsilon > 0$: Any value below 0 leads to a marked decrease in fit (up to 12 errors more for movement, 2 for intervals). (A value of exactly 0 leads to singular images, which increases the number of errors dramatically.) In other words, computational reasons suggest that the movements/intervals get progressively 'slightly lenghtened'; it is not just *any* deviation from singular results that happens.

For movement adjustments, there is one constraint to guarantee no non-models: $\Delta_l \leq [1-(3+10\varepsilon)\varepsilon] - \Delta_n$. This could be interpreted to mean that $\Delta_l + \Delta_n$ must not be more than 1 (remember that ε is supposed to be *small*). There are 3 constraints to guarantee not more than 9 errors. $\Delta_n \leq 0.33$ (plus some seemingly unsystematic variation): This could be interpreted as an upper bound of $\frac{1}{3}$ for Δ_n. $\Delta_l \geq 0.51 - 2\varepsilon$: this could be interpreted as a lower bound of $\frac{1}{2}$ for Δ_l. And $\Delta_l \geq -0.05 + 2.5\varepsilon + \Delta_n$: Δ_l should really be a 'larger movement' than Δ_n.

Interval length adjustments produce similar results, but are slightly less robust and produce a worse fit with the data (at least 13 errors). Here too, lengthening is superior to shortening. For reasons of space, we do not give details about constraints and errors here.

The case for preferring movement adjustment (lengthening) over interval adjustment is quite clear on computational grounds: The former produces a better fit.

What can this case study tell us about imagery in general?

As we mentioned above, our psychological background theory is that mental images are inexact. It is likely that people who reason with mental images know about this inexactness. We would therefore want to propose a new approach to inspection processes, which we call **the sketch theory of mental images**: Even though everything in the picture is determined, the picture as a whole is *treated as a sketch*, 'not to be taken literally'.[5] In other words, the meta-knowledge

[4]It is straightforward to determine absolute upper and lower bounds for the Δs to ensure that adding/subtracting them from start- and endpoints generates the prescribed Allen relations. ε was only examined systematically in the range (-0.1,0.1) because first, it is supposed to mark a 'slight adjustment' and second, because larger values showed no interesting change in behaviour.

The constraints should be regarded as approximations, since we only inspected the values at a certain resolution (down to 0.001 for the Δs and ε) and did not perform regression analysis etc. However, the demarcations between levels of goodness-of-fit were quite regular, and only the qualitative relations were of interest.

[5]This idea originally occurred in the context of an exploration of the importance of sketches in interactions between architects and their customers: A recurring problem of preliminary designs pre-

'this is a sketch' induces a different inspection process, which might be: '**only notice** which entities are in the picture and take this as an indication of which entities are in the represented scene', 'only notice how entities are ordered in the picture and take this as an indication of how these entities are ordered in the represented scene', or 'don't take metrical relations to be exact'. In terms of spatial knowledge content, this corresponds to: 'extract only containment relations', 'extract only order relations', or 'extract metrical relations, but at a relatively coarse level of granularity'.[6] The third kind of sketch interpretation is important whenever metrical distinctions count. It is important in our example, and constructing regular images aids in this process. For it is essential that the mental images can also depict intended equalities (if A finishes-inverse B, their endpoints *are* the same). So a visualisation strategy must enable the inspection process to distinguish between entities intended to be equal and entities not intended to be equal. There are two principal ways of solving this problem: (1) A model construction strategy generating regular images safeguards against inexactness both during model construction and during model inspection: Entities not intended to be equal are moved far enough apart by choosing a large enough ε. So even if, in construction, placement is inexact (i.e. may deviate from the value computed with the help of ε), the relative placement of entities not intended to be equal will still differ enough from the relative placement of entities intended to be equal to distinguish non-intended from intended equality during inspection: In the image, intended equalities are characterised by no gaps (if construction is exact) or by small gaps (if construction is inexact), other relations (not intended to be equalities) are characterised by large gaps. This can be distinguished both by exact inspection processes (the value read is the value depicted) and by inexact inspection processes (the value read may deviate from the value depicted). (2) The alternative would be to annotate intended equalities. In the image, intended equalities are then characterised by annotations, other relations by no annotations, and gaps can be zero, small or large in both cases. Obviously, (1) is more parsimonious and easily explainable by model construction processes, as we can show in the present case study. We would therefore assume that **regularity is generally a desired property of mental images. The image construction process** takes the **easiest** route towards achieving this goal. There is a **preference for copying** metrical prototypes from long-term memory into the image or within the image. If this is recognised as leading to undesired image properties, **movements are lengthened during copying and/or scaled copying is performed**; with the adjustment in length/size of the movements and/or image elements being as small as possible.

A second visualisation strategy employed in addition to regularisation might be *partitioning*: Relevant portions of the image are of equal size or small integer multiples. This is not directly linked to the argument emanating from mental image inexactness and will therefore be discussed in a separate paper.

Open questions and further research

A very straightforward application and test will be the comparison of the theory's results with the data and the ordinal theory predictions of **Allen four-term series**. These experiments are currently carried out by the authors of the three-term series paper, as are the generations of algorithmic predictions from both theories.

What could be a psychological explanation for construction involving movement or interval lengthening? This distinction reflects a difference either in the way in which temporal aspects of mental model construction are regarded, or the question what can be inspected in an image: image element properties or image element relations. Both construction processes change properties of image elements. Adjusting interval lengths is 'static' in the sense that the information needed to construct the next image element (= the previous interval's length) is 'in the picture', is a property of an image element. Adjusting movements is 'dynamic' in the sense that the information needed to construct the next image element (= the previous movement's length) is 'between the pictures', is a property of a relation between image elements. Only if image element relations can be inspected just like image element properties can this be considered 'static' in the same sense as interval length adjustment. (Of course, both methods also need access to the parameters $\Delta_n, \Delta_l, \varepsilon$, which are not usually 'in the picture'. We assume this does not create a problem.) It would be interesting to investigate further how possible dynamic effects in imagery could be formalised. In our assumption that dynamic effects *exist*, we follow the argument of Logie (1995), who explicitly introduces a spatial component into the visuo-spatial scratch pad of Baddeley and Lieberman (1980). Logie regards movement as the central feature and mechanism of spatial working memory/imagery. It seems plausible to assume that Allen inferences are as good an example of typical *spatial* tasks as is Logie's main example, the Brooks matrix task (Brooks, 1967). However, these thoughts do not answer the question what the **psychological reasons** could be **for preferring** movement or interval **lengthening over shortening.**

We should also address the question of **what criteria could be used to compare cognitive modelling approaches.** We show how our psychological background assumption, that mental images are inexact, motivates our visualisation strategy 'regularise', which in turn motivates our model construction process of movement or interval lengthening. Schlieder (in preparation) shows how his psychological background assumption, that search processes in mental models should be as simple as possible, motivates his visualisation strategy 'lin-

sented as CAD graphics is that they convey the impression of being finished, worked-out designs. Hand-drawn sketches, on the other hand, convey the intended impression of being unfinished, of not having the fixed details that, if the sketch were looked at as a picture, are of course fixed (Strothotte et al., 1994).

[6](Hobbs, 1985); see (Habel, 1991) for an example of a computational treatment of granularity in knowledge about space.

earise and center', which in turn motivates his model construction process of scanning and insertion in a memory structure describing the ordinal positions of interval start- and endpoints and a focus. It seems necessary to **relate the ordinal and the metrical theories**, which raises **psychological** as well as **computational** issues. How do the psychological arguments and their stages relate? In particular: (How) can the theories explain each other's visualisation strategy?[7] By what criteria should the computational tradeoff 'knowledge strength vs. computational simplicity' be judged in the context of cognitive modelling? These issues need further discussion.

Can we hope to be able to generalise these methods and results to other domains of imagery? The advantage of the data underlying the algorithm presented here are that they provide a yardstick that is very easy to employ: The task (Allen inferences) is a sufficiently simple visuo-spatial task, since the entities are abstract entities. Their abstractness also allows us to assume that the entities are conceptualised in 'screen-space'[8] and only there. Direct extension of the results and methods could be possible in the analysis of similar tasks. This could, for example, involve tasks with information learned from books, from programs (screen layout), maybe the layout of machines, control panels and other instruments (although a large tactile component may be involved here) and possibly also some of the tasks designed to test spatial abilities.

Extending the results and methods to an analysis of less abstract and larger spaces requires some more abstraction. This is because the representations are probably 'contaminated' by a lot of other information associated with the entities reasoned about (e.g. aesthetic and functional properties of landmarks along a route), because it is a generally open question what representational and processing assumptions may be transferred between 'spaces' (e.g. from 'screen-space' to 'large-scale space'), and because in large-scale space at least, long-term memory is involved. Nevertheless, some principles appear transferable: We are currently investigating the question of how the methods presented in this paper and, more generally, the sketch theory of mental images can be used in the analysis of distance cognition in large-scale space (Berendt, in preparation). The results of this work will be important for Allen inferences too, if these inferences are to be regarded not as abstract screen tasks, as they are here, but as foundations for entities in the world to be reasoned about, e.g. geographic entities in GIS.

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[7] Thanks to Christoph Schlieder for suggesting this way of parallelising the two theories to me.

[8] analogous to the often-made distinction between such spaces as 'table-top space', 'large-scale space' etc.

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