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Localization length at the conductivity minima of the quantum Hall effect

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Abstract

The quantum localization is known to be responsible for the deep conductivity minima of the quantum Hall effect. In this paper we calculate the localization length ξ as a function of magnetic field B at such minima for several models of disorder (“white-noise”, short-range, and long-range random potentials). We find that $\xi \propto B^{-\alpha}$ with the exponent α between one and $\frac{10}{3}$, depending on the model. In particular, for the “white-noise” random potential ξ roughly coincides with the *classical* cyclotron radius. Our results are in agreement with available experimental data. © 1997 Elsevier Science B.V. All rights reserved.

Keywords: Quantum Hall effect; Localization; Weak magnetic field

The appearance of narrow conductivity peaks separated by deep minima is a defining feature of the quantum Hall effect (QHE). The explanation of such a dependence of σ_{xx} on the magnetic field B is based on the idea of localization. The localization length ξ is believed to diverge at certain discrete values B_N of the magnetic field, near which its calculation is notoriously difficult. In this paper we calculate ξ at the midpoints of the divergencies, $B \simeq (B_N + B_{N+1})/2$. They correspond to the minima of σ_{xx} in the transport measurements. The localization length ξ is defined as follows:

$$\frac{1}{\xi} = - \lim_{r \rightarrow \infty} \frac{1}{r} \langle \ln |G(0, r)| \rangle, \quad (1)$$

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where $G(0, r)$ is the one-particle Green’s function. In our model the disorder is described by a Gaussian random potential with correlation length d and rms amplitude $W \ll E_F$, where E_F is the Fermi energy. The electron–electron interaction is ignored.

It is convenient to present the results using the phase diagram shown in Fig. 1. The vertical axis stands for the dimensionless parameter $k_F d$, where k_F is the Fermi wave vector of the two-dimensional electron gas. The horizontal axis is the Landau level (LL) index $N \simeq (k_F l)^2/2$, where $l = \sqrt{\hbar c/eB}$ is the magnetic length. The Fermi level is assumed to be at the midpoint between the centers of N th and $(N+1)$ st LLs. The axes are in the logarithmic scale. The ratio W/E_F is assumed to be fixed. Several lines drawn in Fig. 1 divide the phase plane into the regions with different dependence of ξ on B . The physical meaning

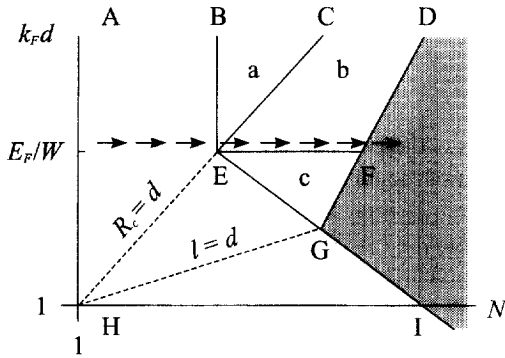


Fig. 1. The parameter space for a random potential with a single characteristic length d . Different regions correspond to different dependence of ξ on N and $k_F d$ (see text). The arrows show the ‘trajectory’, traced by a ‘standard’ sample as the magnetic field decreases.

of these lines is as follows. The line BEGI separates the regimes of overlapping LLs (to its right) from the regime of discrete LLs (to its left). The equation of the line EGI is $k_F d \sim (E_F/W)^2/N$. The line GFD separates the regions of different dynamic properties. To the left of this line the guiding centers of the cyclotron orbits would perform the regular drift along certain closed contours. This phenomenon has been termed the ‘classical localization’ in Ref. [1]. To the right of the line GFD (shaded sector in Fig. 1) the motion of the guiding center is diffusive (on not too large length scales).

The equation for the line GFD is [1] $k_F d \sim N(W/E_F)^{2/3}$. In the diffusive region ξ can be expressed in terms of the dimensionless ‘classical’ conductivity σ_{xx} by means of the ansatz [2]

$$\xi \propto e^{\pi^2 \sigma_{xx}^2}, \quad \sigma_{xx} \gg 1. \quad (2)$$

The ‘classical’ σ_{xx} is to be calculated as the product of the density of states, the classical diffusion coefficient, and the Planck’s constant $\hbar$. The physical mechanism of the localization in this region is the destructive interference of the classical diffusion paths. In fact, the classical motion is diffusive in the entire shaded sector to the right of DFGI, and the corresponding ‘classical’ σ_{xx} is given by the usual Drude–Lorentz formula

$$\sigma_{xx} = \frac{\sigma_0}{1 + (\omega_c \tau)^2}. \quad (3)$$

Eqs. (2) and (3) enable one to calculate ξ up to a pre-exponential factor. In the sector DFGI ξ is exponentially large. As N decreases and the boundary of the diffusive region is crossed, the ‘classical’ σ_{xx} rapidly falls off. Above the point G this is brought about by the aforementioned ‘classical localization’; below the point G it is caused by the rapid decrease of the density of states at the Fermi level. Already slightly to the left of the line DFGI the ‘classical’ σ_{xx} becomes much less than one and the ansatz, Eq. (2), loses its domain of applicability. The nature of the particle propagation also changes: the diffusion is replaced by the quantum tunneling.

The derivation of ξ in the region above the line BEGFD (overlapping LLs) goes on along the lines of Ref. [3] with some modifications appropriate for the case of high LLs. For example, along the segment EF ξ is given by

$$\xi \sim R_c \frac{N^{3/2}}{(k_F d)^{5/2}}, \quad (4)$$

where $R_c = \sqrt{2N+1} l$ is the cyclotron radius. In fact, there are three separate regimes labeled by a, b, and c in Fig. 1 with different power-law dependencies of ξ on N and $k_F d$. The boundary EC, which separates the regions a and b, is given by $k_F d \sim N$.

In contrast, the dependence of ξ on N in the region of discrete LLs (AHIGEB) is given by a single formula

$$\xi \sim \frac{R_c}{k_F d} \quad (5)$$

whose derivation is based on the locator expansion. The propagation of the particle over a distance r requires about rd/l^2 scattering acts. Each scattering amplitude is of the order of $\hbar\omega_c/W$. Thus, the total propagation amplitude is of the order of $(\hbar\omega_c/W)^{rd/l^2}$. On the other hand, this amplitude should be of the order of $e^{-r/\xi}$, which leads to $\xi \sim l^2/d = R_c/(k_F d)$. A similar formula for limited range of tunneling distances (i.e., with the ‘lim’ in Eq. (1) omitted) and for the lowest LL has previously appeared in the work of Raikh and Shahbazyan [4].

Finally, below the line HI, i.e., at $k_F d \leq 1$, ξ is given by

$$\xi \sim R_c, \quad (6)$$

some logarithmic factors being neglected. Previously, similar formulas have been obtained in Refs. [5, 6] for the lowest LL where $R_c = l$. It is quite amazing that the quantum localization length ξ is determined by a purely classical quantity, the cyclotron radius!

To make the connection with the experimental practice we will consider the model where the random potential is created by randomly positioned ionized donors with two-dimensional density n_i set back from the electron gas by an undoped layer of width d . The parameter $k_F d$ is almost always larger than one in the experiment; therefore, in strong fields (boot-shaped region AHIGEB of Fig. 1) ξ is expected to be given by Eq. (5). The corresponding range of N can be expressed in terms of the dimensionless parameters $k_F d$ and n_i/n . We will concentrate on the case $n_i \sim n$, which we call ‘standard’. In this case Eq. (5) is valid for $N \lesssim 2k_F d$. At larger N , the localization length is given by Eq. (4) so that the dependence of ξ on N is super-linear. Note that the ‘standard’ case corresponds to the straight-line ‘trajectory’ passing through the points E and F and shown by the arrows in Fig. 1. Finally, at $N \gtrsim C(k_F d)^{5/3}$ where C is some undetermined numerical factor, ξ becomes exponentially large.

To verify our predictions we used the magnetoresistance data of Ebert et al. [7, 8]. In most of their samples d was equal to 6 nm, which corresponds to $k_F d = 0.9$. The data of Ebert et al. are in agreement with the following dependence of σ_{xx} on the temperature T , $\sigma_{xx} \propto \exp[-(T_0/T)^{1/2}]$, which can be interpreted [9] in terms of the variable-range hopping in the presence of the Coulomb gap. In this theory T_0 is

directly related to ξ defined by Eq. (1),

$$k_B T_0 \simeq \frac{6e^2}{\kappa \xi}, \quad (7)$$

where e is the electron charge and κ is the dielectric constant of the medium. With the help of Eq. (7), we converted T_0 reported by Ebert et al. [7, 8] for even filling factors $\nu = 2N = 2, 4, \dots, 12$ into ξ . We found good agreement without theoretical predictions, Eq. (5) for $N \leq 2$ and Eq. (4) for $3 \leq N \leq 6$.

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