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Publication Date
2013-01-01

Peer reviewed|Thesis/dissertation
UNIVERSITY OF CALIFORNIA, SAN DIEGO

Channel aware scheduling and resource allocation with cross layer optimization in wireless networks

A dissertation submitted in partial satisfaction of the requirements for the degree
Doctor of Philosophy

in

Electrical Engineering (Communication Theory and Systems)

by

Sheu Sheu Tan

Committee in charge:

James Zeidler, Chair
Bhaskar Rao, Co-Chair
Robert Bitmead
Pamela Cosman
William Hodgkiss
Larry Milstein

2013
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Co-Chair

__________________________________

Chair

University of California, San Diego

2013
DEDICATION

To my family.
EPIGRAPH

All the art of living lies in a fine mingling of letting go and holding on.
—Havelock Ellis
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ACKNOWLEDGEMENTS

The journey of my PhD has been remarkable. I would like to express my heartfelt gratitude to my advisors, colleagues, friends and family for their support and contribution in making this dissertation possible.

First and foremost, my sincere thanks go to both of my advisors, Prof. Zeidler and Prof. Rao for the support, guidance and insights for my research and beyond. I am thankful to Prof. Zeidler for introducing me to the rich area of cross-layer optimizations for cognitive radio, ad-hoc networks and MIMO systems and for always encouraging me and appreciating my work. I am grateful to Prof. Rao for taking deep interest in my research and for his meticulous, critical inputs for the subtle technical aspects of my research work.

I thank Prof. Milstein and Prof. Cosman for their initial guidance, introducing me to cross-layer design for image transmission, and always taking active interest in my research. I am thankful to Prof. Milstein for being a tremendous mentor and his passion for digital communications has always inspired to me. I am also thankful to Prof. Cosman for her encouragement and helpful advices for research, professional growth and presentation skills. I thank Prof. Hodgkiss and Prof. Bitmead, for being part of my thesis committee and providing me invaluable feedback and suggestions.

I am indebted to my other professors who have helped and inspired me at various points of my education. To my master thesis's advisor, Prof. Nallanathan, thank you for giving me a lot of inspiration on how to become a better scholar and researcher. I specially thank Prof. Orlitsky for the personal attention and care over the years and for providing me teaching assistantship opportunities.

I am thankful to my collaborators, Prof. Junshan Zhang, Dong Zheng and Adam Anderson. I am grateful to Prof. Zhang for inviting me as a visiting student at ASU and for his guidance and support. My special thanks go to Dong Zheng for introducing me to the framework of distributed opportunistic scheduling which appears as the main focus of this dissertation.

I also thank all the graduate students at the Digital Signal Processing lab as well as the graduate students at ECE department, especially Hirakendu Das,
Abhijeet Bhorkar, Matt Pugh, Sagnik Gogh, Seok-Ho Chang, Seong-Ho Hur, Steve Cho, Natan Jacobson and many others. I fondly remember and appreciate the technical discussions we had over the past several years. My sincere thanks go to my friends Hao Zheng, Omer Lang, Gina Tuazon, Kathy Nyugen, Jianjian Gao, Ellen Yueh and Ivy Tan for being the wonderful friends to me who supported me through the ups and downs of my PhD. Special thanks to Chee-Wooi Ten for encouraging me to pursue my PhD. There is a long list of people which can not fit into this page have been helpful to me during my whole journey of PhD and prior to that. I am thankful to have come across every one of them.

I would like to thank my parents Cheng-Bak Tan and Siew-Lan Heng for their unconditional love and support. Throughout my whole growing life, I am grateful that they have always believed in me and allowed me to realize my own potential. I am also thankful to my sisters Sheu-Maen Tan and Sheu-Wei Tan for their pleasant companies and encouragements. Despite being away from me, they always made me feel close to them and they are always close to my heart.


Chapter 6 contains materials from the paper “Adaptive Modulation for OFDM-based Multiple Description Progressive Image Transmission” presented at *IEEE Global Telecommunications Conference (Globecom)* in December of 2008 and “Variance-Aware Adaptive Modulation for OFDM-based Multiple Description Progressive Image Transmission” presented at *IEEE International Conference on Communications (ICC)* in May of 2010. Both these papers were coauthored with Minjoung Rim, Pamela Cosman, and Lawrence Milstein.

The dissertation author is the primary researcher and author of all the works presented here. The co-authors advised and contributed to the work.
VITA

2001 B. S. in Computer and Communications Engineering, National University of Malaysia, Malaysia
2004 M. S in Electrical Engineering, National University of Singapore, Singapore
2013 Ph. D. in Electrical Engineering (Communications Theory and Systems), University of California, San Diego

PUBLICATIONS


ABSTRACT OF THE DISSERTATION

Channel aware scheduling and resource allocation with cross layer optimization in wireless networks

by

Sheu Sheu Tan

Doctor of Philosophy in Electrical Engineering (Communication Theory and Systems)

University of California, San Diego, 2013

James Zeidler, Chair
Bhaskar Rao, Co-Chair

We develop channel aware scheduling and resource allocation schemes with cross layer optimization for several problems in multiuser wireless networks. We consider problems of distributed opportunistic scheduling, where multiple users contend to access the same set of channels. Instead of scheduling users to the earliest available idle channels, we also take the instantaneous channel quality into consideration and schedule the users only when the channel quality is sufficiently high. This can lead to significant gains in throughput compared to system where PHY and MAC layers are designed separately and the wireless fading channels are
abstracted as time-invariant, fixed rate channels for scheduling purposes.

We first consider opportunistic spectrum access in a cognitive radio network, where a secondary user (SU) shares the spectrum opportunistically with incumbent primary users (PUs). Similar to earlier works on distributed opportunistic scheduling (DOS), we maximize the throughput of SU by formulating the channel access problem as a maximum rate-of-return problem in the optimal stopping theory framework. We show that the optimal channel access strategy is a pure threshold policy, namely the SU decides to use or skip transmission opportunities by comparing the channel qualities to a fixed threshold. We further increase the spectrum utilization by interleaving SU’s packets with periodic sensing to detect PU’s return. We jointly optimize the rate threshold and the packet transmission time to maximize the average throughput of SU, while limiting interference to PU.

Next, we develop channel-aware opportunistic spectrum access strategies in a more general cognitive radio network with multiple SUs. Here, we additionally take into account the collisions and complex interaction between SUs and sharing of resources between them. We derive strategies for both cooperative settings where SUs maximize their sum total of throughputs, as well as non-cooperative game theoretic settings, where each SU tries to maximize its own throughput. We show that the optimal schemes for both scenarios are pure threshold policies. In the non-cooperative case, we establish the existence of Nash equilibrium and develop best response strategies that can converge to equilibria, with SUs relying only on their local observations. We study the trade-off between maximal throughput in the cooperative setting and fairness in the non-cooperative setting, and schemes based on utility functions and pricing that mitigate this tradeoff.

In addition to maximizing throughput and fair sharing of resources, it is important to consider network/scheduling delays for QoS performance of delay-sensitive applications. We study DOS under both network-wide and user-specific average delay constraints. We take a stochastic Lagrangian approach and characterize the corresponding optimal scheduling policies accordingly, and show that they have a pure threshold structure.

Next, we consider the use of different types of channel quality information,
i.e., channel state information (CSI) and channel distribution information (CDI) in the opportunistic scheduling design for MIMO ad-hoc networks. CSI is highly dynamic in nature and provides time diversity in the wireless channel, but is difficult to track. CDI offers temporal stability, but is incapable of capturing the instantaneous channel conditions. We design a new class of cross-layer opportunistic channel access scheduling framework for MIMO networks where CDI is used in the network context to group the simultaneous transmission links for spatial channel access and CSI is used in the link context to decide when and which link group should transmit based on a pre-designed threshold. We thereby reap the benefits of both the temporal stability of CDI and the time diversity of CSI.

Finally, we consider a novel application of cross layer optimization for communication of progressive coded images over OFDM wireless fading channels. We first consider adaptive modulation based on the instantaneous channel state information. An algorithm is proposed to allocate power and constellation size at each subchannel by maximizing the throughput. We next consider both the variance and the average of the throughput when deciding the constellation size for adaptive modulation. Simulation results confirm that cross-layer optimization with adaptive modulation enhances system performance.
Chapter 1

Introduction

Wireless communications networks are under more strain than ever before due to the rapid growth of smartphones, tablets and other mobile devices, together with their data hungry and always-connected multimedia and social applications. At the same time, wireless communications have improved remarkably due to many technological breakthroughs in various layers and subsystems, e.g., multiple-input-multiple-output (MIMO) technologies, massive MIMO, adaptive modulation and coding, and advanced routing and scheduling designs, to name a few. Cross-layer optimization provides another avenue for increasing the performance of the wireless systems by exploiting the interplay and dependencies across the different OSI layers [1]. This dissertation is concerned with several problems of cross-layer optimization involving the physical, MAC and application layers for cognitive radio, MIMO and OFDM systems. The properties of these layers and the connections we exploit are briefly stated below and illustrated in Fig. 1.1.

Physical layer (PHY): We exploit the time variant nature of wireless fading channels in the design of upper layers, instead of treating them as fixed rate channels.

Medium access layer (MAC): In addition to considering the idle-busy states of channels, we include various other factors like instantaneous channel quality, fairness and delay constraints.

Application layer: We jointly optimize the progressive image transmission in application layer with the instantaneous channel quality information from
Throughout this dissertation, we study joint optimization and design of these layers by taking the physical layer properties into account for upper layer protocol design. In chapters 2 and 3, we develop channel aware spectrum access for the MAC layer in cognitive radio network by incorporating channel quality of the physical layer into the scheduling design. In chapters 4 and 5, we consider an ad-hoc network. We address the issue of the delay constraint for the delay sensitive applications in Chapter 4. We design a new scheduler based on different types of information in a MIMO networks in Chapter 5. Finally, we investigate how to jointly optimize the adaptive modulation in the physical layer for progressive image transmission at the application layer in Chapter 6. The following is a brief description of the studies in the various chapters.

1.1 Opportunistic Channel-Aware Spectrum Access for Cognitive Radio Networks

Cognitive radio appears as one very viable technology that can optimize the use of available radio frequency spectrum [2]. In the current spectrum allocation framework, most of the frequency bands are exclusively assigned to specific licensed
services. However, a lot of licensed bands are experiencing low utilization, such as those for TV broadcasting, resulting in inefficient usage. In view of this, cognitive radio technology has been proposed to optimize the use of available radio frequency spectrum and meet the increasing demands [2–4]. Cognitive radio allows unlicensed users, i.e., secondary users (SUs), to reuse spectral white spaces of licensed users, i.e. primary users (PUs), in an opportunistic manner without causing harmful interference to PUs [5].

The conventional approaches for SU to access channels mainly focus on sensing the channels and transmitting on the ones that are deemed idle regardless of channel quality [6]. Recent results in [7, 8], show that by taking the channel conditions into account, in addition to the idle/busy status, the network throughput can be improved. Here, when a SU senses a channel as idle, it estimates the channel quality and then decides either to proceed with data transmission or give up the opportunity and continue to explore for a potentially better channel. Further exploration increases the likelihood for finding a better channel, but at the cost of additional time for sensing and probing the channels. This tradeoff is characterized in the optimal stopping theory framework [9] to determine the best spectrum access strategy.

In general, in such channel-aware scheduling schemes, the SU transmits for a fixed time after finding an idle channel of a sufficiently high quality. If the SU’s transmission time is small, the spectrum is underutilized, but if this time is too large, the return of PU would cause the transmission of SU to fail. Moreover, while SU is transmitting, it has no knowledge of the return of PU. In Chapter 2, we therefore propose a spectrum access strategy where in addition to taking channel quality into consideration, in order to fully utilize the transmission opportunities and increase the spectrum utilization, we interleave SU’s transmission with periodic sensing to track the return of PU and minimize collision. The benefit of periodic sensing is that when PU returns, only the data transmitted since the last successful sensing may be lost – prior transmitted packets are not affected. Furthermore, we jointly optimize the rate threshold and the packet transmission time to maximize the average throughput of SU, while limiting interference to PU. For simplicity, in
Chapter 2, we primarily consider a system where there is only one SU, although it can be readily extended to the case when there are multiple SUs.

In Chapter 3, we derive channel-aware opportunistic spectrum access strategies for general cognitive radio networks consisting of multiple secondary users. The design of a channel access strategy in such a setup is challenging due to possible collisions between the SUs and PUs, and also among the SUs. This is in contrast to prior works that consider ad-hoc networks with no incumbent users and a single channel [7], or those that consider cognitive radio networks, but do not account for collisions between SUs [8], for designing the channel access strategies. In addition to maximizing the network throughput, it is also of great interest to optimize the sharing of resources. Borrowing heavily from the work by Zheng et al. [7], we derive strategies for both cooperative settings where SUs maximize their sum total of throughputs, as well as non-cooperative, game theoretic settings where each SU tries to maximize its own throughput. In the non-cooperative case, we establish the existence of Nash equilibrium and develop best response strategies that can converge to equilibria, with SUs relying only on their local observations. We study the trade-off between maximal throughput in cooperative setting and fairness in the non-cooperative setting, and schemes based on utility functions and pricing that mitigate this tradeoff.

1.2 Distributed Opportunistic Scheduling for Ad-hoc Networks Under Delay Constraints

While the first two works are focused on throughput and fairness, another important factor to consider in the context of multimedia communications is that of delay. With the convergence of multimedia applications and wireless communications, there is an urgent need for developing new scheduling algorithms to support real-time traffic with stringent delay requirements. However, distributed scheduling under delay constraints is not well understood and remains an underexplored area. In chapter 4, we take some steps in this direction and explore the distributed opportunistic scheduling (DOS) with delay constraints. Consider
an ad-hoc network with $M$ links which contend for the channel using random access. Distributed scheduling in such a network requires joint channel probing and distributed scheduling. Using optimal stopping theory, we explore DOS for throughput maximization, under two different types of average delay constraints: 1) a network-wide constraint where the average delay should be no greater than $\alpha$; or 2) individual user constraints where the average delay per user should be no greater than $\alpha_m$, $m = 1, \ldots, M$. Since the standard techniques for constrained optimal stopping problems are based on sample-path arguments and are not applicable here, we take a stochastic Lagrangian approach instead. We characterize the corresponding optimal scheduling policies accordingly, and show that they have a pure threshold structure, i.e., data transmission is scheduled if and only if the rate is above a threshold. Specifically, in the case with a network-wide delay constraint, somewhat surprisingly, there exists a sharp transition associated with a critical time constant, denoted by $\alpha^*$. If $\alpha$ is less than $\alpha^*$, the optimal rate threshold depends on $\alpha$; otherwise it does not depends on $\alpha$ at all, and the optimal policy is the same as that in the unconstrained case. In the case with individual user delay constraints, we cast the threshold selection problem across links as a non-cooperative game, and establish the existence of Nash equilibria. Again we observe a sharp transition associated with critical time constants $\{\alpha_m^*\}$, in the sense that when $\alpha_m \geq \alpha_m^*$ for all users, the Nash equilibrium becomes the same one as if there were no delay constraints.

1.3 Opportunistic Scheduling for MIMO Ad-hoc Networks Using Channel Distribution Information

The next problem of cross-layer optimization we consider is that of designing opportunistic channel access scheduling frameworks in MU-MIMO networks using both Channel state information (CSI) and channel distribution information (CDI). CSI provides instantaneous information about the channel and scheduling
protocols that employ multiple antennas have often been based on perfect knowledge of CSI to achieve maximum network throughput [10–14]. While the use of CSI in the physical layer generally involves only the transmitting and receiving nodes of a single link, the use of CSI in the MAC layer involves a large number of nodes, i.e., the entire network. Maintaining accurate CSI in the MAC layer is therefore impractical, especially in ad-hoc networks with high mobility and interference where the channel conditions are time-varying [15,16].

Channel distribution information (CDI) provides long-term statistical information about the channel. Researchers have explored the usage of CDI at the physical layer for Multiple-Input-Multiple-Output (MIMO) systems [15–21]. CDI is effective for longer times and allows for reduced frequency of feedback in time-varying channels. It also allows the MAC and the physical layers to share common information about link characteristics, thereby enabling truly cross-layer optimization of the link schedule. However, CDI does not accurately capture the instantaneous channel conditions. The MAC layer therefore has the option of a throughput-stability tradeoff depending on the type of information used for scheduling [22,23].

We design a new class of cross-layer opportunistic channel access scheduling framework for multiple-input multiple-output (MIMO) networks by exploiting the respective strengths of both CDI and CSI. The proposed scheduling protocol implements a threshold policy derived from the optimal stopping theory and consists of two phases. In the first phase, CDI is used in the network context to group the simultaneous transmission links for spatial channel access. In the second phase, CSI is used in the link context to decide when and which link group should transmit based on a pre-designed threshold.

Given that CSI is very effective in cancelling the interference with beamforming on PHY layer links, results show that deploying CDI at the MAC layer is desirable as it would benefit the network with multi-user diversity, stability and less frequency of feedback. By utilizing CDI for the network decisions and using CSI/CDI in the individual links, it is shown that one can reap the benefits of both the temporal stability of CDI and the time diversity of CSI.
1.4 Adaptive Modulation Schemes for Progressive Image Transmission

In Chapter 6, we consider a novel illustration of cross layer optimization across the physical and application layers. We use adaptive modulation in progressive image transmission with multiple description coding in conjunction with an Orthogonal Frequency Division Multiplexing (OFDM) system. We build on the schemes described in [24], where each of the descriptions of the image is mapped into one of the subchannels of the OFDM waveforms. Most previous works on progressive image coding [25–27] employ temporal coding. We use a cyclic redundancy check (CRC) to check the validity of each description, and erase all descriptions that do not pass the CRC. Then, Reed Solomon (RS) erasure decoding is used across the descriptions.

In much of the literature, the same constellation size is used for all the subchannels when applying adaptive modulation for image transmission [25, 26]. However, we propose adopting different constellations for different subchannels to avoid the problem of overwhelming some of the subchannels by imposing a higher order modulation size sustainable by the quality of the channels. To achieve minimal image distortion, we need to optimize the constellation size and code rates jointly. However, due to the complexity of jointly optimizing adaptive modulation and channel coding, the problem is decomposed into two sub-problems. First, we decide the constellation sizes to maximize the system throughput prior to RS decoding, then we decide the code rates to minimize distortion.

For determining the constellation sizes, we consider two schemes of M-QAM adaptive modulation. The first is a variable rate, fixed power scheme; for each subchannel, a constellation size is assigned which maximizes the system throughput prior to RS decoding, with equal power allocation for all subchannels. The second is a variable rate, variable power scheme, which maximizes the system throughout prior to RS decoding by changing the constellation size and the allocated power at each subchannel.

While average throughput maximization has commonly been used as an al-
ternative to image or video average distortion minimization, since the throughput-distortion curve is non-linear, throughput maximization can be very different from distortion minimization if the variance of the throughput is large. Under high channel quality variability, the variance of the throughput will be high. Multiple description coding with unequal error protection is a promising technique for the transmission of progressive images when there exists a high variability of the channel conditions. We consider both the variance and the average of the throughput when deciding the constellation size for adaptive modulation in an OFDM system used for transmitting progressively-coded images with multiple description coding.

Simulation results show that our proposed adaptive modulation technique for transmitting progressive images with multiple description coding gives significant performance gains especially at high SNR region.
Chapter 2

Opportunistic Channel-Aware Spectrum Access for Cognitive Radio Networks with Interleaved Transmission and Sensing

2.1 Introduction

2.1.1 Motivation

The ever-increasing demand for higher spectrum efficiency in wireless communications due to limited or under-utilized spectral resources has infused a great interest in finding techniques for improving the spectrum usage. Cognitive radio appears as one very viable technology that can optimize the use of available radio frequency spectrum [2]. The concept of cognitive radio allows secondary users (SUs) to reuse spectral white spaces of primary users (PUs) in an opportunistic manner, without causing harmful interference to PUs [5].

It is essential for SU to make good sensing decisions in real-time to explore and utilize such opportunities for data transmission [6]. The conventional approaches for SU to access channels mainly focus on sensing the channels and transmitting on the ones that are deemed idle regardless of channel quality [6].
Recent results in [8, 28] show that by taking the channel conditions into account, in addition to the idle/busy status, the network throughput can be improved.

2.1.2 Main Contributions

In this chapter, we propose an optimal spectrum access strategy involving transmission interleaved with periodic sensing that leverages sensing, channel-aware scheduling and optimization of transmission time in a joint manner to maximize SU’s throughput. One of the key observations on cognitive radio is that the successful transmission of SU depends on PUs’ activities. The return of PU would cause the transmission of SU to fail. However, while SU is transmitting, it has no knowledge of the return of PU. We therefore propose periodic sensing while transmission to track PU. In channel-aware scheduling and transmission with periodic sensing, there are two stages [29]. First, channel sensing is carried out to explore a spectrum hole for SU’s transmission. Second, while a channel is used by SU, periodic sensing is deployed to detect the return of PU. The benefit of periodic sensing is that when PU returns, only the data transmitted since the last successful sensing may be lost – prior transmitted packets are not affected.

In transmission with periodic sensing, there exists a tradeoff between data lost due to PU’s return using long packets, and the time cost of frequent sensing using short packets. If the transmission time is long, i.e., the frequency of periodic sensing is low, the time cost of tracking the return of PU is small but the amount of lost data when PU returns is large. On the contrary, if the transmission time is small and the frequency of periodic sensing is high, the amount of lost data when PU returns is small but at the expense of high cost of tracking PU. Motivated by this, we optimize the transmission time of SU between consecutive sensing phases to maximize the network throughput, which is equivalent to optimizing the frequency of periodic sensing.

We consider a system consisting of multiple channels. For channel searching, we adopt sequential channel scanning without recall [30] since SU may not be able to sense many channels at once due to the limitation on hardware and/or sensing capability. We characterize the joint sensing, probing and channel access
with optimal transmission duration in a stochastic decision-making framework and formulate the decision problem as an optimal stopping problem [7]. When the sensing indicates that a channel is idle, probing is carried out to estimate the channel quality and the highest data rate it can support. Based on this estimate, one can decide either to proceed with transmission on this channel or to give up the opportunity and continue sensing for a potentially better channel. Clearly, further sensing/probing increases the likelihood of finding an idle channel with better rate, but at the cost of additional time. We show that the optimal channel access strategy exhibits a threshold structure, i.e., the channel access decision can be made by comparing the rate to a threshold. Furthermore, we jointly optimize the threshold and the transmission time between consecutive sensing phases to maximize the average throughput. This is done by alternately optimizing the threshold while keeping transmission time fixed using fixed-point iterations similar to [7], and followed by optimizing the transmission time keeping the threshold fixed using Newton’s method.

In a practical cognitive radio network, spectrum sensing is not always accurate due to feedback delays, estimation errors and quantization errors. We say that a misdetection occurs if a channel is being used by PU but is incorrectly determined to be idle by SU. On the other hand, a false alarm happens if SU incorrectly determines that a channel is busy when in fact it is idle. Both situations are caused by sensing errors, which leads to the degradation of spectrum efficiency. In this chapter, we take the sensing errors into account and determine their impact on our proposed channel access scheme. Given a certain probability of misdetection and false alarm, we determine the optimal transmission time and threshold under the sensing errors.

2.1.3 Related Works

The emergence of cognitive radio technology has stimulated a flurry of research activities in the area of dynamic spectrum access. We highlight some of the related channel access schemes.

Motivated by the rich channel diversity inherent in wireless communica-
tions, channel knowledge can be used as one criterion for channel selection to improve spectrum efficiency in wireless networks [7, 8, 28, 31]. Zheng et al. [7] use optimal stopping theory to develop distributed opportunistic scheduling (DOS) for exploiting multiuser diversity and time diversity in a single channel model for wireless ad hoc networks. Chang et al. [31] address the optimal channel selection problem in a multichannel system by considering the channel conditions. In our work, besides gaining the benefits of channel knowledge, we consider cognitive radio networks with incumbent PUs and also optimize the transmission time of SU to maximize throughput.

Shu et al. [8] show that joint channel sensing/probing scheme for cognitive radio can achieve significant throughput gains over conventional mechanisms that use sensing alone. They consider multiple channels and the throughput maximizing decision strategy is formulated as an optimal stopping theory problem. Our channel access scheme is an extension of optimal stopping results in [7], [8] and is more complex due to the variable transmission times, probing of the channels only when they are sensed to be idle and consideration of sensing errors. Additionally, we consider periodic sensing while transmission to track the return of PU and minimize collision. We further optimize the transmission time, i.e. the frequency of sensing. This helps us to efficiently utilize the idle state of channels that are explored.

There are few works in the literature that explicitly optimize the transmission time or perform periodic sensing while transmission [32], [33], [28]. Pei et al. [32] optimize the frame duration to maximize the throughput of the cognitive radio network subject to a fixed sensing time. They address the tradeoff that larger frame sizes allow for higher fraction of transmission time, but at a higher risk of collision and frame loss when the PU returns. They consider a slotted single channel – the SU does not transmit in a frame when an active PU is detected and waits until the next frame. In contrast, our work considers multichannel unslotted system and also takes channel quality into consideration before accessing an idle channel. Huang et al. [33] consider a model consisting of a single channel with a PU and SU and develop a scheme where the SU decides to transmit a packet
or sense the channel based on its instantaneous estimate of PU's idle probability under a POMDP framework. The SU may therefore transmit multiple consecutive packets after sensing the channel each time and can be considered as optimizing the transmission time between the sensing phases. They do not utilize channel quality information in their scheme. Li et al. [28] consider a scheme that is closely related to ours in a multichannel ad hoc network. They consider a model where the channel quality gradually changes with time and therefore monitor the channel quality periodically while transmission, stopping when the quality falls below a threshold. But the packet transmission time is fixed and not optimized.

Apart from the optimal stopping theory approach to the channel access problem, another popular approach in the literature is based on the POMDP framework. Zhao et al. [6] and Chen et al. [34, 35] study such spectrum access schemes for slotted multichannel cognitive radio networks. POMDP-based schemes attempt to dynamically track the idle state of various channels and maximize throughput by exploiting the temporal spectrum opportunities. Like most schemes based on optimal stopping theory, our scheme explores channels uniformly at random and doesn’t dynamically track the idle channels, but it does fully utilize the idle state of the channels it accesses. As we show in the numerical results, our scheme that is unslotted, takes channel quality into consideration, performs periodic sensing while transmission and jointly optimizes the packet duration and the channel quality threshold, outperforms the POMDP scheme in [6] for slotted systems. Zhao et al. [36] study the dynamic access using a periodic sensing strategy under a constrained Markov decision process framework. However, they consider the packet transmission time to be given and fixed, and do not optimize it.

Most recent works relate to a wide variety of other important concerns like energy-efficient transmission for cognitive radio sensor networks, e.g., [37], [38], game theoretic and security considerations in non-cooperative multiuser setups, e.g., [39], [40], and machine learning approaches when the various channel parameters and related probability distributions are not known, e.g., [41], [42] and the references therein. While a treatment of periodic sensing and joint optimization packet duration and channel quality threshold in such contexts is beyond the
scope of this chapter, we believe that these concepts can be used in conjunction with existing schemes and provide significant performance benefits.

The remainder of the chapter is organized as follows. We present the channel and system model in Section 5.2. In Section 2.3, we present the throughput-optimal channel access strategy and in Section 2.4, we provide the average throughput analysis. We then present the joint optimization of threshold and transmission duration in Section 2.5. In Section 2.6, we consider interference of SU to PUs and in Section 2.7, we consider extensions of our scheme to more general scenarios. We present the numerical results in Section 5.5. Finally, we conclude the chapter with Section 6.6.

2.2 Channel and System Model

2.2.1 Channel Model

We consider a frequency-selective multi-channel system such as orthogonal frequency-division multiple access (OFDMA) that is commonly used for cognitive radios, e.g., the IEEE 802.22 [43] wireless standard. The entire frequency spectrum is assumed to be divided into $L$ independent and identically distributed (i.i.d.) channels. We assume that the coherence bandwidth is bigger than the signal bandwidth of the individual channels or the subcarriers in it, and thus each channel experiences flat fading. Furthermore, we assume that each channel experiences slow fading, i.e., its condition varies slowly over time.

We further assume that all channels have the same statistics, and are subject to Rayleigh fading. While the homogeneous setup is assumed for simplicity and may correspond to the case when channels belong to the same network, we consider extensions to the heterogeneous case in subsection 2.7.1. The distribution of rate $R$ is continuous and is given by the Shannon channel capacity $R = \log(1 + \rho|h|^2)$ nats/s/Hz, where $\rho$ is the normalized average SNR, and $h$ is the random channel coefficient with a complex Gaussian distribution $CN(0, 1)$. 
Accordingly, the distribution of the rate is given by

\[ F_R(r) = 1 - \exp \left( -\frac{\exp(r) - 1}{\rho} \right) \]  

for \( r \geq 0 \), and \( F_R(r) = 0 \) otherwise.

### 2.2.2 System Model

#### PU and SU Model

We assume that each channel has only one designated PU. The \( L \) channels are opportunistically available to SU. Although we consider a system where there is only one SU, it can be readily extended to the case when there are multiple SUs, as discussed later in Subsection 2.7.2.

Each channel’s status is modeled as a continuous-time random process that alternates between busy and idle states depending on whether PU is using the channel. Specifically, we consider a system in which the idle/busy states of different PU channels are homogeneous, independent and identically distributed. This is motivated by the common scenario that all the \( L \) channels belong to the same primary licensed network [8] and may therefore have similar usage statistics. We assume that for all PUs, the time durations of the idle and busy states are exponentially distributed with parameters \( a \) and \( b \) [44]. In other words, for any PU, the duration \( T_I \) of any idle state has distribution \( f_{T_I}(t) = ae^{-at} \) and the duration \( T_B \) of any busy state has distribution \( f_{T_B}(t) = be^{-bt} \). The expected durations of each of the idle and busy states are \( \frac{1}{a} \) and \( \frac{1}{b} \) respectively. The fraction of time for which PU is idle in the long term is the idle probability \( P_I = \frac{1/a}{1/a + 1/b} = \frac{b}{a+b} \).

#### Channel Sensing, Probing and Data Transmission

For selecting a channel, SU uses the scheme of sequential sensing and probing without recall [30]. Here, SU senses/probes the channels sequentially and does not have the memory of the previously sensed/probed channels and their outcomes. Therefore, SU cannot recall or select a previously sensed/probed channel once it forgoes the opportunity to transmit on that channel, unless the sensing/probing is repeated on that channel.
To obtain a better sense of the dynamics of channel access, a sample realization of the sensing/probing for channel selection followed by data transmission on that channel with periodic sensing is depicted in Fig. 2.1.

![Diagram of channel access](image)

**Figure 2.1:** A sample realization of channel sensing, probing and data transmission with PU returns

When SU intends to transmit, it searches for an available channel by randomly choosing channels one at a time and sensing/probing them. The total time spent for channel searching depends on the activities of PUs and the channel conditions. Specifically, if the outcome of the sensing stage is busy, the probing stage is skipped and SU randomly selects another channel for sensing. In this case, the time spent for sensing/probing a busy channel is $\tau_s$. However, if the sensed channel is idle, SU proceeds with probing to determine the channel quality for deciding whether to transmit on the channel. During the probing stage, a channel probing packet (CPP) and a probing feedback packet (PFP) are exchanged between the transmitter and receiver [8]. The time spent on a CPP/PFP exchange is denoted by the channel probing time $\tau_p$ and in this case, the time cost for sensing/probing an idle channel is $\tau_s + \tau_p$. With the feedback information on the channel quality, the transmitter compares the maximum achievable data rate to an optimal threshold ($\lambda^*$) pre-designed using the optimal stopping theory. If the data rate is less than the threshold due to poor channel condition, then SU forgoes its transmission opportunity and continues with sensing/probing another randomly selected channel.
However, if the data rate is high and exceeds the threshold, then SU proceeds with the data transmission.

During SU’s data transmission, it has no knowledge of the return of PU. If PU returns during the transmission of a SU’s packet, then that entire packet is lost. Hence, to maximize the chances of successful transmission of SU’s packets and to reduce the interference of SU to PU, it is necessary for SU to track the activity of PU. We therefore propose periodic sensing during the transmission. Specifically, SU will periodically sense the channel after transmitting for time $T_s$. Note that $T_s$ is the duration of SU’s transmission between two consecutive sensing phases and is also equivalent to the length of a sub-packet of SU. The transmission of SU stops once it senses the return of a PU during a sensing phase. If PU returns during SU’s transmission, then interference occurs and the current sub-packet being transmitted is destroyed, but the previously transmitted sub-packets are still valid. During SU’s transmission, only sensing is performed periodically, but not the probing. This is because the channel condition is assumed to be constant over a long period of time. The transmitter and the receiver are assumed to be synchronized. Under the same spectrum access strategy, the transmitter and the receiver will always sense, probe and access the same channel.

**Spectrum Sensing Model**

Spectrum sensing can be modeled as hypothesis testing. It is equivalent to distinguishing between the two hypotheses:

$$\begin{cases}
H_0 : y(t) = n(t), & \text{PU is inactive} \\
H_1 : y(t) = x(t) + n(t), & \text{PU is active}
\end{cases}$$

(2.2)

where $x(t)$ denotes PU’s transmitted signal, $n(t)$ is additive white Gaussian noise and $y(t)$ denotes sample collected by SU. The notation $H_0$ represents the hypothesis that PU is inactive (idle channel) whereas $H_1$ indicates that PU is active (busy channel).

In a practical system, there may be sensing errors and accordingly, we define
the probability of false alarm $P_{fa}$ and miss detection $P_{md}$ as

$$P_{fa} = \Pr(I = 0|H_0) \quad \text{and} \quad P_{md} = \Pr(I = 1|H_1),$$

where $I = 0$ indicates that SU decides the channel is busy and $I = 1$ indicates idle.

### 2.3 Derivation of Throughput-Optimal Channel Access Strategy

We consider the problem of finding an optimal strategy for SU to decide whether or not to transmit on an idle channel based on its quality, so as to maximize the long-term average throughput. We show that for any given packet length $T_s$, an optimal strategy for the SU is to select the first idle channel whose rate exceeds a fixed threshold $\lambda^* \triangleq \lambda^*(T_s)$. For this, we consider a maximum rate-of-return problem in the optimal stopping theory framework [9, 45]. An optimal stopping rule is a strategy to decide as to when one should take a given action based on the past events in order to maximize the average return. The return is defined as the net gain between the reward achieved and the cost spent. In our problem, the reward is the rate of the channel probed and the cost is the total time taken to explore the channels so far.

As illustrated in Fig. 2.1, after finding an idle channel, a stopping rule $N$ decides whether SU should carry out the data transmission, or skip this transmission opportunity, based on the channel quality. As such, $N$ is the number of idle channels considered by SU before deciding to transmit on the last idle channel based on the channel qualities and the time spent so far. One can see that further sensing/probing would certainly increase the probability of getting an available channel with a better channel quality, but at the expense of spending additional time in searching. Using the optimal stopping theory, this tradeoff can be characterized in a stochastic decision making framework.

Suppose that the process of successful sensing/probing followed by transmission is carried out for $U$ rounds. Let $\{N_1, \ldots, N_U\}$ be the corresponding number
of idle channels considered in these rounds, and are independent realizations of \( N \).
Let \( T_{Nu} \) denote the total duration of round \( u \) which includes sensing/probing with transmission and periodic sensing. And let \( R_{Nu} \) be the data rate of the channel used in round \( u \). Based on the Renewal Theorem [7], the average throughput after \( U \) rounds is given by

\[
x_U \triangleq \frac{\sum_{u=1}^{U} R_{Nu} T_{u}'}{\sum_{u=1}^{U} T_{Nu}} \xrightarrow{U \to \infty} \frac{E[R_N T']}{E[T_N]} \text{ a.s. (2.4)}
\]

Here, \( x \triangleq \frac{E[R_N T']}{E[T_N]} \) is the long-term average rate-of-return for SU, \( T_N \) is the total duration of a round (i.e., time spent for channel searching and transmission), \( R_N \) is the transmission rate in a round and \( T' \) is the effective data transmission time in a round. Clearly, the distributions of \( R_N \) and \( T_N \) depend on the stopping rule \( N \). The total time \( T_N \) of a round consists of the time \( T'_N \) spent in sensing and probing to acquire a good channel and the time \( T_{tr} \) for transmitting SU’s packets over this channel. The time \( T_{tr} \) includes both the successfully and unsuccessfully transmitted packets (due to PU’s return and sensing errors) until SU senses PU’s return, and the time spent due to periodic sensing between the packets. We have \( E[T_N] = E[T_N'] + E[T_{tr}] \).

It follows that the problem of maximizing the long-term average throughput can be formulated as a maximal-rate-of-return problem [7]. Our goal is to find an optimal stopping rule \( N^* \) that maximizes the average rate-of-return \( x \), and the corresponding maximal throughput \( x^* \):

\[
N^* \triangleq \arg \max_{N \in Q} \frac{E[R_N T']}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_N T']}{E[T_N]}, \quad (2.5)
\]

where \( Q \triangleq \{N : N \geq 1, E[T_N] < \infty \} \) is the set of all possible stopping rules. We exploit optimal stopping theory to solve (5.18).

**Proposition 2.1.** There exists an optimal stopping rule \( N^* \) for the opportunistic spectrum access and is a pure threshold policy given by

\[
N^* = \min \{n \geq 1 : R_n \geq \lambda^* \}, \quad (2.6)
\]

where the optimal threshold \( \lambda^* \) is the unique solution for \( \lambda \) in

\[
E[(R - \lambda)^+] = \lambda (E[K_s] \tau_s + E[K_p] \tau_p) \quad \frac{1}{E[T_{tr}]}. \quad (2.7)
\]
Here, $R$ is a r.v. which refers to the rate whose CDF is $F_R(r)$ shown in (4.50), and $K_s$ and $K_p$ are the number of channels sensed and probed respectively to find a channel in which PU is idle for the time $(\tau_s + \tau_p)$. (Thus, $E[K_s]\tau_s + E[K_p]\tau_p$ is the expected time spent until SU finds an idle channel to probe completely.) Furthermore, the maximum throughput is given by $x^* = \frac{x^* E[R]}{E[T]}$.

**Proof:** The proof of Proposition 2.1 uses methods from optimal stopping theory [9] and closely follows a similar result in [7]. In order to maximize the average throughput $\frac{E[R_nT]}{E[T_N]} = \frac{E[R_n]E[T']}{E[T_N] + E[T_n]}$, a standard technique [9, Ch. 6] is to consider for all $x \in (0, \infty)$, the reward function $Z_n(x) \Delta = R_nE[T'] - x(T_n + E[T_n])$ and an optimal stopping rule $N(x)$ that maximizes the expected reward $E[R_NT' - xT_N]$. Let the corresponding maximum reward be

$$V(x) \Delta = \sup_{N \in Q} E[Z_N(x)] = \sup_{N \in Q} E[R_NT' - xT_N] = E[R_N(x)T' - xT_N(x)].$$

The motivation behind considering the reward function $Z_n(x)$ is [9, Ch. 6, Th. 1], which states that if the maximum rate, i.e., throughput is $x^* \triangleq \sup_{N \in Q} \frac{E[R_NT]}{E[T_N]}$, then $V(x^*) = 0$, and furthermore, $N(x^*)$ is the stopping rule that maximizes throughput.

Using [9, Ch. 3, Th. 1], the existence of $N(x)$ is guaranteed if

$$E[\sup_n Z_n(x)] < \infty \quad \text{and} \quad \limsup_{n \to \infty} Z_n(x) = -\infty \text{ a.s.}$$

We show that both these conditions are satisfied in our setup. We express the time spent in a round for successfully accessing a channel as $T'_n = \sum_{i=1}^n (K_i\tau_s + K'_i\tau_p)$, where $K_i$ and $K'_i$ are the number of channels sensed and probed respectively to find the $i$-th idle channel, for $i = 1, 2, \ldots, n$. Note that the $K_1, K_2, \ldots, K_n$ are i.i.d. and have the same distribution as $K_s$, and likewise $K'_1, K'_2, \ldots, K'_n$ are i.i.d. copies of $K_p$. It is hence easy to see that $\limsup_{n \to \infty} E[Z_n(x)] = -\infty$ almost surely. This is because both $R_n$, the channel rate under Rayleigh fading, and $T'$, which is related to a geometrically distributed r.v., have finite mean and variance. Furthermore, $K_s \geq 1$, $K_p \geq 1$ and $x > 0$. We show that $E[\sup_n Z_n(x)] < \infty$ by a
similar reasoning. Observe that

\[
E[\sup_n Z_n(x)] \leq E[\sup_n R_n T' - n x \epsilon (E[K_s] \tau_s + E[K_p] \tau_p)] \\
+ E[\sup_n x \sum_{i=1}^{n} \epsilon (E[K_s] \tau_s + E[K_p] \tau_p) - (K_i \tau_s + K'_i \tau_p)],
\]

for any \( \epsilon \in (0,1) \). The contribution due to \( T_{tr} \) is negative and safely ignored. Again using the fact \( R_n \) and \( T' \) are positive random variables with finite mean and variance, we use [9, Ch. 4, Th. 1 and Th. 2] to conclude that both the terms on the right hand side of above inequality are finite. Thus, the existence of \( N(x) \) is guaranteed for all \( x \in (0, \infty) \).

We proceed to find \( N(x) \) and \( x^* \). Using the principle of optimality [9, Ch. 3, Th. 3], an optimal stopping rule is

\[
N(x) = \min \{ n \geq 1 : R_n E[T'] - x(E[T_{tr} + T'_n]) \geq V(x) - x T'_n \} \\
= \min \{ n \geq 1 : R_n E[T'] \geq V(x) + x E[T_{tr}] \},
\]

and the optimality equation [9, Ch. 3, Th. 2] gives

\[
V(x) = E[\max \{ R_1 E[T'] - xE[T_{tr}], V(x) \} - x(K_1 \tau_s + K'_1 \tau_p)].
\]

Using \( V(x^*) = 0 \) and the above expressions for \( N(x) \) and \( V(x) \), we conclude that the stopping rule that maximizes the throughput is

\[
N(x^*) = \min \{ n \geq 1 : R_n \geq x^* \frac{E[T_{tr}]}{E[T']}, \}
\]

and the maximal throughput \( x^* \) is a solution for \( x \) in

\[
E\left( (R_n - x \frac{E[T_{tr}]}{E[T']})^+ \right) = \frac{x(E[K_s] \tau_s + E[K_p] \tau_p)}{E[T']}. 
\]

Lastly, we show that the above equation for \( x^* \) has a unique solution. We perform a change of variable \( \lambda \triangleq x \frac{E[T_{tr}]}{E[T']} \) and equivalently show that there is a unique solution for \( \lambda \) in

\[
E[(R_n - \lambda)^+] = \lambda \frac{(E[K_s] \tau_s + E[K_p] \tau_p)}{E[T_{tr}]}.
\]
The left hand side of equation (3.13) can be written as

\[ g(\lambda) \triangleq E[(R_n - \lambda)^+] = \int_{\lambda}^{\infty} (r - \lambda) f_R(r) dr. \]

Clearly, \( g(\lambda) \) is continuous and decreases from \( E[R_n] \) to 0, since \( f_R(r) \) is positive, continuous and differentiable and hence for \( \lambda_1 < \lambda_2 \), we have

\[
g(\lambda_2) - g(\lambda_1) = \int_{\lambda_2}^{\infty} (r - \lambda_2) f_R(r) dr - \int_{\lambda_1}^{\infty} (r - \lambda_1) f_R(r) dr
\]
\[
= \int_{\lambda_2}^{\lambda_1} (\lambda_1 - \lambda_2) f_R(r) dr - \int_{\lambda_1}^{\lambda_2} (r - \lambda_1) f_R(r) dr
\]
\[
\leq 0.
\]

The right hand side of equation (3.13), \( \lambda \frac{E[K_s] \tau_s + E[K_p] \tau_p}{E[T_{tr}]} \) is continuous and increasing from 0 to \( \infty \). Hence, the equation (3.13) has a unique solution in \( \lambda \). Note that the solution \( \lambda = \lambda^* \) is the threshold in the optimal stopping rule, i.e., the throughput maximizing stopping rule is \( \{ n \geq 1 : R_n \geq \lambda^* \} \) and the maximum throughput is \( x^* = \lambda^* \frac{E[T']}{E[T_{tr}]} \).

Proposition 2.1 suggests that an optimal scheduling strategy has the following form: the successfully contended link will start the data transmission if the transmission rate from the probing is bigger than or equal to the \( \lambda^* \). Else, the link will forgo the transmission opportunity.

### 2.4 Throughput Analysis

To analyze the maximal throughput \( x^* \) and the optimal stopping rule \( N^* \), we consider the calculation of \( x^* \) and \( \lambda^* \) in terms of the various channel and system model parameters. We first calculate the various expectations that were encountered in Proposition 2.1.

**Proposition 2.2.** For any stopping rule that is a pure threshold policy \( N = \min\{n : R_n \geq \lambda\} \), the expected times of effective transmission \( E[T'] \), channel access \( E[T_{N}] \), transmission with periodic sensing \( E[T_{tr}] \), and the rate of trans-
mission \( \mathbb{E}[R_N] \) are given by

\[
\begin{align*}
\mathbb{E}[T'_N] &= \frac{T_s \cdot e^{-aT_s}}{1 - e^{-a(T_s + \tau_s)(1 - P_{fa})}}, \\
\mathbb{E}[T_N] &= \frac{\tau_s + Q'_1 \tau_p}{(\frac{b}{a+b})e^{-a(\tau_s + \tau_p)}(1 - P_{fa})(1 - F_R(\lambda))}, \\
\mathbb{E}[T_{tr}] &= \frac{1 - P_{md}e^{-a(T_s + \tau_s)}}{1 - P_{md}} \frac{(T_s + \tau_s)}{1 - e^{-a(T_s + \tau_s)(1 - P_{fa})}}, \\
\mathbb{E}[R_N] &= \frac{\int_{\lambda}^{\infty} r \, dF_R(r)}{1 - F_R(\lambda)}. 
\end{align*}
\]

Here, \( Q'_1 = \left( \frac{a}{a+b} \right) P_{md} + \left( \frac{b}{a+b} \right) \left( (1 - e^{-a\tau_s})P_{md} + e^{-a\tau_s}(1 - P_{fa}) \right) \) is the probability of finding a channel in which PU is sensed to be idle. The expected number of channels sensed \( \mathbb{E}[K_s] \) and probed \( \mathbb{E}[K_p] \) for finding a channel in which PU is idle for the time \( (\tau_s + \tau_p) \) are given by

\[
\begin{align*}
\mathbb{E}[K_s] &= \frac{1}{(\frac{b}{a+b})e^{-a(\tau_s + \tau_p)}(1 - P_{fa})} \quad \text{and} \quad 
\mathbb{E}[K_p] &= \frac{Q'_1}{(\frac{b}{a+b})e^{-a(\tau_s + \tau_p)}(1 - P_{fa})}.
\end{align*}
\]

The expressions for \( \mathbb{E}[T'_N], \mathbb{E}[T_{tr}], \mathbb{E}[K_s], \mathbb{E}[K_p] \) hold irrespective of the stopping rule being used.

**Proof:** The proof uses properties of exponential and geometric distributions, especially the memoryless property of exponential distributions.

**Expected Effective Transmission Time (\( \mathbb{E}[T'_N] \))**

If \( K \) is the number of packets transmitted successfully by SU in a round, then

\[
\Pr(K = k) = e^{-a(kT_s + (k-1)\tau_s)} \cdot (1 - P_{fa})^{k-1} \cdot \left( (1 - e^{-a(T_s + \tau_s)}) + e^{-a(T_s + \tau_s)} P_{fa} \right)
= e^{-aT_s} \left( e^{-a(T_s + \tau_s)}(1 - P_{fa}) \right)^{k-1} \cdot (1 - e^{-a(T_s + \tau_s)}(1 - P_{fa})).
\]

This is because PU should be idle during the transmission of first \( k \) packets, i.e., for a time of \( kT_s + (k-1)\tau_s \) from the start of transmission. Also, there should be no false alarm in the first \( k - 1 \) sensing phases. Lastly, either PU should return during the following sensing phase or packet transmission (i.e., the following duration of \( \tau_s + T_s \)), or the transmission should be terminated due to false alarm if PU does not return. The above distribution of \( K \) closely resembles a geometric distribution.
with parameter $e^{-a(\tau_s + T_s)}(1 - P_{fa})$ and $E[K] = \sum_k k \Pr(K = k) = \frac{e^{-aT_s}}{1 - e^{-a(\tau_s + \tau_p)(1 - P_{fa})}}$.

Since the packet duration is $T_s$, we have $E[T'] = \frac{T_s e^{-aT_s}}{1 - e^{-a(\tau_s + \tau_p)(1 - P_{fa})}}$.

**Derivation of Expected Time for successfully accessing the channel (E[T''])**

The number of different channels $K$ that are explored, i.e., sensed and possibly probed, for finding good channel in a round is distributed geometrically as $\Pr(K = k) = (1 - z)^{k-1}z$ for $k \in \{1, 2, \ldots\}$, where $z = \frac{b}{a+b} e^{-a(\tau_s + \tau_p)}(1 - P_{fa})(1 - F_R(\lambda))$. This is because a good channel must satisfy the following conditions:

1. PU should be idle at the start of the sensing phase, the probability of which is $\frac{b}{a+b}$.

2. PU should continue to be idle during the duration $\tau_s + \tau_p$ of sensing and probing, which happens with probability $e^{-a(\tau_s + \tau_p)}$. (This is the conditional probability, given that the channel was idle to begin with.)

3. There should be no false alarm, which happens with probability $(1 - P_{fa})$.

4. The rate of the channel should be higher than threshold and $\Pr(R > \lambda) = 1 - F_R(\lambda)$.

If any of these conditions are not satisfied, SU proceeds to explore another channel. Hence,

$$E[K] = \frac{1}{z} = \frac{b}{a+b} e^{-a(\tau_s + \tau_p)}(1 - P_{fa})(1 - F_R(\lambda)).$$

(2.9)

Of these $K$ channels, the first $K - 1$ are bad. Probability that a channel is bad and probed, is

$$p_{bad, probe} = \left( \frac{a}{a + b} \right) P_{md}$$

$$+ \left( \frac{b}{a+b} \right) \left( e^{-a\tau_s}(1 - P_{fa})((1 - e^{-a\tau_p}) + e^{-a\tau_p}F_R(\lambda)) + (1 - e^{-a\tau_s})P_{md} \right).$$

Here, the first summand is the probability of the case that PU is busy to begin with, but is misdetected as idle. The second summand corresponds to the case when PU is idle to begin with. There are two subcases here when the channel is probed. In the first subcase, PU is idle during sensing duration of $\tau_s$ and there is
no false alarm. But the channel is bad either because PU returns during probing
duration of \( \tau_p \) or the rate is low. The second subcase is that PU returns during
the sensing phase but is misdetected as idle. All the \( K \) explored channels are
sensed for a duration of \( \tau_s \). The \( K - 1 \) bad channels are probed with probability
\( \frac{p_{\text{bad,probe}}}{1 - z} \) (which is the conditional probability that a channel is probed given that it
is bad). If \( K' \) of the \( K - 1 \) bad channels are probed, then
\( E[K'] = E[K - 1] \frac{p_{\text{bad,probe}}}{1 - z} \).
The \( K \)-th channel which is good, is also probed for a duration of \( \tau_p \). By putting
together these observations, we have

\[
E[T'_N] = E[K \tau_s + K' \tau_p + \tau_p] = E[K] \tau_s + \left( E[K - 1] \frac{p_{\text{bad,probe}}}{1 - z} + 1 \right) \tau_p
\]

\[
= \frac{1}{z} \tau_s + \left( \frac{1}{z} - 1 \right) \frac{p_{\text{bad,probe}}}{1 - z} \frac{1}{z} \tau_p = \frac{\tau_s + (p_{\text{bad,probe}} + z \tau_p)}{z} \tau_p
\]

\[
= \frac{\tau_s + \left( \left( \frac{a}{a+b} \right) P_{\text{md}} + \left( \frac{b}{a+b} \right) \left( (1 - e^{-a \tau_s}) P_{\text{md}} + e^{-a \tau_s} (1 - P_{\text{fa}}) \right) \right) \tau_p}{\frac{b}{a+b} e^{-a(\tau_s + \tau_p)} (1 - P_{\text{fa}}) (1 - F_R(\lambda))}
\]

\[
Q_1 \tau_p
\]

The number of channels explored, \( K_s \), for finding a channel that can be
probed completely is distributed geometrically with parameter \( \frac{b}{a+b} e^{-a(\tau_s + \tau_p)} (1 - P_{\text{fa}}) \) and hence

\[
E[K_s] = \frac{1}{\frac{b}{a+b} e^{-a(\tau_s + \tau_p)} (1 - P_{\text{fa}})}.
\]

If \( K_p \) out of these \( K_s \) channels are considered for probing, then

\[
E[K_p] = \frac{\left( \frac{a}{a+b} \right) P_{\text{md}} + \left( \frac{b}{a+b} \right) \left( (1 - e^{-a \tau_s}) P_{\text{md}} + e^{-a \tau_s} (1 - P_{\text{fa}}) \right)}{\frac{b}{a+b} e^{-a(\tau_s + \tau_p)} (1 - P_{\text{fa}})}
\]

\[
= \frac{Q_1}{\frac{b}{a+b} e^{-a(\tau_s + \tau_p)} (1 - P_{\text{fa}})}
\]

using arguments similar to that for calculating \( E[T'_N] \) earlier.

**Expected Time for Transmission with Periodic Sensing** (\( E[T_{tr}] \))

The time \( T_{tr} \) is spent by SU in each round for transmitting its packets
along with periodic sensing, until it detects the return of PU, either correctly or
due to false alarm. Observe that \( T_{tr} \) is a multiple of \( (T_s + \tau_s) \) since SU alternately
transmits a packet followed by sensing for PU’s return. In the case when there is
no misdetection, i.e., \( P_{\text{md}} = 0 \), the probability that the transmission lasts for \( k \geq 1 \)
periods is \((1 - z)^{k-1}z\) where \(z = (1 - e^{-a(T_s + \tau_s)} + e^{-a(T_s + \tau_s)}P_{fa} = 1 - e^{-a(T_s + \tau_s)}(1 - P_{fa})\). Hence, the expected number of such periods is \(E[T_{tr}] = \frac{T_s + \tau_s}{1 - e^{-a(T_s + \tau_s)}(1 - P_{fa})} + \frac{(1 - e^{-a(T_s + \tau_s)})}{1 - e^{-a(T_s + \tau_s)}(1 - P_{fa})} \frac{P_{md}}{1 - P_{md}}(T_s + \tau_s)\). Thus,

\[
E[T_{tr}] = \frac{T_s + \tau_s}{1 - e^{-a(T_s + \tau_s)}(1 - P_{fa})} + \frac{(1 - e^{-a(T_s + \tau_s)})}{1 - e^{-a(T_s + \tau_s)}(1 - P_{fa})} \frac{P_{md}}{1 - P_{md}}(T_s + \tau_s)
\]

\[
\frac{1 - P_{md}}{1 - P_{md}}(T_s + \tau_s)
\]

**Expected Transmission Rate (E[R_N])**

Under a stopping rule \(N = \min\{n : R_n \geq \lambda\}\) that is a pure threshold policy,

\[
E[R_N] = E[R|R > \lambda] = \frac{\int_{\lambda}^{\infty} r dF_R(r)}{1 - F_R(\lambda)}. \quad (2.11)
\]

Using Proposition 2.2, it follows that for a threshold rule \(N = \min\{n \geq 1 : R_n \geq \lambda\}\) with threshold \(\lambda\), the rate of return in (2.4) is given by

\[
x = \frac{E[R_N T'']}{E[T_N]} = \frac{E[R_N] E[T']}{{E[T_N']} + E[T_{tr}]} = \frac{\int_{\lambda}^{\infty} r dF_R(r) \cdot T_s e^{-aT_s}}{(a+b) \cdot e^{a(T_s + \tau_p)(\tau_s + Q')}(1 - (1 - P_{fa}) e^{-a(T_s + \tau_s)})} \cdot \frac{P_{md}}{1 - P_{md}}(T_s + \tau_s)(1 - P_{fa})(1 - P_{md}) = \phi(\lambda, T_s). \quad (2.12)
\]

Using Proposition 2.1, since \(\lambda^*\) and \(x^*\) satisfy \(x^* = \frac{\lambda^*}{E[T']}{E[T]}\), we have \(\lambda^* = \frac{E[T_N]}{E[T_{tr}]} x^* = \frac{E[T_N]}{E[T']} \phi(\lambda^*, T_s)\), i.e., \(\lambda^*\) is a solution to the fixed-point equation in \(\lambda\), given by

\[
\lambda = \frac{E[T_N]}{E[T']} \phi(\lambda, T_s) = \frac{E[R_N]}{E[T_N]} \frac{E[T_N]}{E[T']} + 1 = \frac{\int_{\lambda}^{\infty} r dF_R(r)}{c_0 - F_R(\lambda)} \triangleq \psi(\lambda). \quad (2.13)
\]

Here,

\[
c_0 = 1 + \frac{E[T_N]}{E[T_{tr}]}(1 - F_R(\lambda))
\]

\[
= 1 + \frac{(a+b) \cdot e^{a(T_s + \tau_p)(\tau_s + Q')}(1 - (1 - P_{fa}) e^{-a(T_s + \tau_s)})}{1 - P_{fa}} \frac{1 - P_{md}}{1 - P_{md} e^{-a(T_s + \tau_s)}}(T_s + \tau_s)(1 - P_{md})
\]
is a constant that does not depend on $\lambda$ using Proposition 2.2.

Similar to [7, Prop. 3.4], we have the following result for finding $\lambda^*$ when $T_s$ is given.

**Proposition 2.3.** For a given $T_s$, the fixed-point iteration

$$\lambda_{k+1} = \psi(\lambda_k),$$

(2.14)

for $k \in \{0, 1, 2, \ldots\}$ and for any nonnegative $\lambda_0$ converges to the optimum threshold $\lambda^*$.

**Proof:** The proof is along the same lines as that of [7, Prop. 3.4]. Using (5.18), (2.12), (2.13), and Proposition 2.1, it follows that

$$\lambda^* = \max_{\lambda} \psi(\lambda) = \psi(\lambda^*).$$

(2.15)

Proposition 2.1 and (2.15) imply that the functions $y = \lambda$ and $y = \psi(\lambda)$ for $\lambda > 0$ intersect only at $\lambda = \lambda^*$. Together with $\psi(0) > 0$, we have

$$\psi(\lambda) > \lambda \text{ for } \lambda < \lambda^*, \quad \text{and } \psi(\lambda) < \lambda \text{ for } \lambda > \lambda^*.$$  

(2.16)

If $\lambda_0 > \lambda^*$, then $\lambda_1 = \psi(\lambda_0) \leq \psi(\lambda^*) = \lambda^*$, i.e., equivalent to starting with $\lambda_1 \leq \lambda^*$. Hence, we assume that $\lambda_0 \leq \lambda^*$. Then, by induction for $k \in \{0, 1, 2, \ldots\}$, we have $\lambda_{k+1} = \psi(\lambda_k) \geq \lambda_k$. Furthermore, $\lambda_{k+1} = \psi(\lambda_k) \leq \psi(\lambda^*) = \lambda^*$ for all $k$. Thus, $\{\lambda_k\}_{k=0}^\infty$ is a monotonically increasing sequence upper bounded by $\lambda^*$, and therefore converges to a limit, say $\lambda_\infty$.

We finally show that $\lambda_\infty = \lambda^*$. We have $\psi(\lambda_k) - \lambda_k = \lambda_{k+1} - \lambda_k$. By taking the limit $k \to \infty$ on both sides, we have $\psi(\lambda_\infty) - \lambda_\infty = 0$. Since $\psi(\lambda) - \lambda = 0$ has a unique solution $\lambda^*$, we conclude $\lambda_\infty = \lambda^*$. \qed

### 2.5 Joint Optimization of Threshold and Transmission Duration

We jointly optimize the transmission time $T_s$ and the threshold $\lambda$ to maximize the throughput $\phi(\lambda, T_s)$ in (2.12). An illustration of the function $\phi(\lambda, T_s)$ is given in Fig. 2.2.
Figure 2.2: The average throughput $\phi(\lambda, T_s)$ as a function of threshold $\lambda$ and transmission time $T_s$

We show that for a given threshold rule $N = \min\{n \geq 1 : R_n \geq \lambda\}$, i.e., for a fixed threshold $\lambda$, the optimum transmission time $T_s$ that maximizes throughput can be obtained by taking the derivative with respect to $T_s$ and equating to zero, i.e., solving for $T_s$ in $\frac{\partial}{\partial T_s} \phi(\lambda, T_s) = 0$. To simplify this process, we express $\phi(\lambda, T_s)$ in (2.12) as

$$\phi(\lambda, T_s) = \frac{c_1 T_s e^{-aT_s}}{c_2 (1 - c_3 e^{-aT_s}) + c_4 (T_s + \tau_s) (1 - c_5 e^{-aT_s})} = \frac{c_1 T_s}{c_4 T_s e^{aT_s} + c_6 e^{aT_s} - c_7 T_s - c_8},$$

(2.17)

where

$$c_1 = \int_{\lambda}^{\infty} r \, dF_R(r), \quad c_2 = \frac{(\frac{a+b}{b}) e^{a(\tau_s + \tau_p)} (\tau_s + Q'_p)}{1 - P_{fa}}, \quad c_3 = (1 - P_{fa}) e^{-a\tau_s},$$
$$c_4 = \frac{(1 - F_R(\lambda))}{1 - P_{md}}, \quad c_5 = P_{md} e^{-a\tau_s}, \quad c_6 = c_2 + c_4 \tau_s, \quad c_7 = c_4 c_5, \quad c_8 = c_2 c_3 + c_4 c_5 \tau_s$$

(2.18)

do not depend on $T_s$. We therefore have

$$\frac{\partial}{\partial T_s} \phi(\lambda, T_s) = \frac{c_6 e^{aT_s} - c_8 - ac_4 T_s^2 e^{aT_s} - ac_6 T_s e^{aT_s}}{(c_4 T_s e^{aT_s} + c_6 e^{aT_s} - c_7 T_s - c_8)^2}.$$  

(2.19)
We solve for \( T_s \) in \( \frac{\partial}{\partial T_s} \phi(\lambda, T_s) = 0 \), i.e.,
\[
c_6 e^{a T_s} - c_8 - a c_4 T_s^2 e^{a T_s} - a c_6 T_s e^{a T_s} = 0,
\]
i.e.,
\[
\zeta(T_s) \triangleq c_6 - c_8 e^{-a T_s} - a c_6 T_s - a c_4 T_s^2 = 0. \tag{2.20}
\]

The next proposition shows that the above equation has a unique solution \( T_s^* \) for \( T_s > 0 \) and \( T_s^* < \frac{1}{a} \). Hence, \( T_s^* \) can be obtained by solving (2.20) using Newton’s method with initial value \( \frac{1}{a} \) and update equation or by bisection on the range \((0, \frac{1}{a})\).

**Proposition 2.4.** For a threshold rule with given threshold \( \lambda \), the optimal transmission time \( T_s^* \) that maximizes throughput \( \phi(\lambda, T_s) \) is the unique solution to Equation (2.20). And \( T_s^* \leq \frac{1}{a} \).

**Proof:** From (2.19), we observe that for any given \( \lambda \), \( \phi'_T(\lambda, T_s) \triangleq \frac{\partial}{\partial T_s} \phi(\lambda, T_s) \) is continuous in \( T_s \). We have \( \phi'_T(\lambda, T_s)|_{T=0} = \frac{c_6-c_8}{(c_6-c_8)^2} > 0 \) since \( c_6 = c_2 + c_4 T_s > c_2 c_3 + c_4 c_5 T_s = c_8 \). It is easy to see that \( \phi'_T(\lambda, T_s)|_{T=\infty} = -\infty \). If \( \phi'_T(\lambda, T_s) = 0 \) for some value of \( T_s \), then it satisfies (2.20). We show that (2.20) has a unique solution for \( T_s > 0 \), i.e., the function \( \zeta(T_s) \triangleq c_6 - c_8 e^{-a T_s} - a c_6 T_s - a c_4 T_s^2 \) has only one positive root. To see this, we observe that \( \zeta(T_s) \) is concave since \( \frac{\partial^2}{\partial T_s^2} \zeta(T_s) = -a^2 c_6 e^{-a T_s} - 2 a c_4 < 0 \). Hence, it can have at most two roots. Furthermore, \( \zeta(0) = c_6 - c_8 > 0 \). Hence, it has exactly one positive and negative root. Let this positive root be \( T_s^* \). Combining these arguments, it follows that for \( 0 < T_s < T_s^* \), \( \phi'_T(\lambda, T_s) > 0 \), i.e., \( \phi(\lambda, T_s) \) is increasing. And for \( T_s > T_s^* \), \( \phi(\lambda, T_s) \) is decreasing. Thus, \( \phi(\lambda, T_s) \) is maximized at \( T_s = T_s^* \). \( \square \)

Based on propositions 2.3 and 2.4, we propose Algorithm 1 for finding \( \lambda^* \) and \( T_s^* \) that jointly maximize the throughput \( \phi(\lambda, T_s) \).

By propositions 2.3 and 2.4, the inner loops in the above algorithm converge to the best \( \lambda \) and \( T_s \) for the current \( T_s \) and \( \lambda \) respectively, and each inner loop leads to an increase in the rate \( \phi(\lambda, T_s) \). Therefore, the algorithm converges to a local maximum of \( \phi(\lambda, T_s) \). While propositions 2.3 and 2.4 show that \( \phi(\lambda, T_s) \) has a unique maximum, i.e., is quasi-concave, in \( \lambda \) for a given \( T_s \) and has a unique maximum in \( T_s \) for a given \( \lambda \), it does not guarantee that \( \phi(\lambda, T_s) \) has a unique local maximum. For example, the function \( g(x, y) = -x^4 + 6x^3 - 11y^2 + 6y \) has a unique maximum in \( x \) (resp. \( y \)) for a given \( y \) (resp. \( x \)), but has two local maxima, as seen
Algorithm 1 Joint optimization of $\lambda$ and $T_s$ to maximize throughput $\phi(\lambda, T_s)$

1: Given: sufficiently small error bounds $\epsilon_{\lambda}, \epsilon_{T_s}$
2: Initialize $\lambda = 1, T_s = \frac{1}{a}$
3: repeat
4: $\lambda^{old} = \lambda, T_s^{old} = T_s$
5: repeat {Optimize $\lambda$ for current $T_s$ by fixed-point iterations}
6: $\lambda = \psi(\lambda)$
7: until $|\lambda - \psi(\lambda)| \leq \epsilon_{\lambda}/2$
8: repeat {Optimize $T_s$ for current $\lambda$ by Newton’s method}
9: $T_s = \frac{e^{c_s-c_{T_s}}(aT_s+1)-ac_1T_s^2-c_6}{a(e^{c_s-c_{T_s}}-2c_4T_s-c_6)} \ (= T_s - \frac{\zeta(T_s)}{\sigma_f(T_s)})$
10: until $|T_s - \frac{e^{c_s-c_{T_s}}(aT_s+1)-ac_1T_s^2-c_6}{a(e^{c_s-c_{T_s}}-2c_4T_s-c_6)}| \leq \epsilon_{T_s}/2$
11: until $|\lambda^{old} - \lambda| \leq \epsilon_{\lambda}$ and $|T_s^{old} - T_s| \leq \epsilon_{T_s}$
12: Return $\lambda$ and $T_s$ as approximations of $\lambda^*$ and $T_s^*$

from the fact that $g(x, x) = x(1 - x)(2 - x)(3 - x)$. Thus, the algorithm is not guaranteed to converge to the global maximum, except when $\phi(\lambda, T_s)$ has a unique local maximum. Based on our numerical results, e.g., see Fig. 2.2, we strongly suspect that this is indeed true for rate distributions under Rayleigh fading.

While we do not have theoretical guarantees on the speed of convergence of Algorithm 1, in our experiments that we describe in Section 5.5, the algorithm converges very fast, within 10 iterations, to within a small error of $\epsilon_{\lambda} = \epsilon_{T_s} = 10^{-5}$.

Before we proceed to show numerical results for our scheme, in the next two sections, we consider modifications of our scheme to take into account the interference caused by SU to PUs and extensions to more general scenarios.

### 2.6 Interference to Primary User

If PU returns during SU’s transmission, there may be a collision, leading to interference to PU. The collision will continue until SU detects PU’s presence in one of the following sensing phases. To minimize this interference, the transmission power of SU should be small or alternatively, the transmission time $T_s$ of SU
between two consecutive sensing phases should be small. One way of quantifying
this interference is in terms of the fraction of time for which each PU experi-
ences interference in the long term. We compute this fraction when the SU uses a
threshold policy for channel access and periodic sensing while transmission, with
corresponding threshold \( \lambda \) and packet length \( T_s \). Let \( T_c \) be the random time dura-
tion at the end of a round of transmission for which the SU experiences collision
due to the return of the PU. In order to calculate the expected collision time in a
round \( E[T_c] \), we split \( T_c \) in two components. The first component \( T_{c,1} \) is due to PU
returning between two sensing phases of SU, and is the time between the return
of PU to the next sensing phase. The second is the additional collision time \( T_{c,2} \)
if the SU continues to transmit due to misdetection in one or more of the sensing
phases. To calculate \( E[T_{c,1}] \), we observe that if the PU returns at time \( t \) from
the start of SU’s transmission, then the rest of SU’s transmission causes collision,
i.e., \( T_{c,1} = T_s - t \). This happens with probability density \( ae^{-at} \) and we thus have
\[
E[T_{c,1}] = \int_0^{T_s} (T_s - t)ae^{-at}dt = T_s - \frac{(1-e^{-aT_s})}{a}.
\]

To calculate \( E[T_{c,2}] \), we see that for every misdetection, there is an addi-
tional collision time of \( T_s \). The number of such misdetections is a geometrically dis-
tributed random variable with expected value of \( \frac{P_{md}}{1-P_{md}} \). Thus, \( E[T_{c,2}] = T_s \cdot \frac{P_{md}}{1-P_{md}} \),
and
\[
E[T_c] = E[T_{c,1}] + E[T_{c,2}] = T_s - \frac{(1-e^{-aT_s})}{a} + T_s \cdot \frac{P_{md}}{1-P_{md}}. \tag{2.21}
\]
Since the expected duration of a round is \( E[T_N] \), the fraction of time for which the
SU experiences collision during transmission in the long term is \( \frac{E[T_c]}{E[T_N]} \). Since the
SU is equally likely to transmit on each channel, the average fraction of time for
which each PU experiences collision is
\[
\eta_c = \frac{1}{L} \cdot \frac{E[T_c]}{E[T_N]}, \tag{2.22}
\]
where \( E[T_c] \) and \( E[T_N] = E[T_{N}'] + E[T_{tr}] \) are given by Equation (2.21) and Propo-
sition 2.2 respectively.

The collision \( \eta_c \) increases with \( P_{md} \) and high \( P_{md} \) may cause significant
interference to PU. Hence, in a practical system, the requirement for \( P_{md} \) needs to
be very small, i.e., \( P_{md} \ll 1 \) [8].
We also see that $\eta_c$ is an increasing function of $T_s$. Given a bound $\hat{\eta}_c$ on the interference, we consider the following modification of Algorithm 1 to find $\lambda$ and $T_s$ that maximize the throughput, while causing low interference. If the outputs $\lambda^*$ and $T_s^*$ from Algorithm 1 are such that the corresponding $\eta_c \leq \hat{\eta}_c$, then we use them as the threshold and packet time respectively. If not, starting with $\lambda^*$ and $T_s^*$, we use a modified Algorithm 1 where in the inner loop for optimizing $T_s$, each time the $T_s$ obtained at the end of the loop is such that corresponding $\eta_c > \hat{\eta}_c$, we lower $T_s$ to the solution of $\hat{\eta}_c = \frac{1}{L} \cdot \frac{\mathbb{E}[T_c]}{\mathbb{E}[T_N]}$, obtained by Newton’s method. Since there is a risk that at the end of the outer loop, the new $\lambda$ and $T_s$ are such that the throughput $\phi(\lambda, T_s)$ is lower, in such an event, we terminate the procedure and output the $\lambda$ and $T_s$ obtained at the end of previous iteration. (We perform at least one iteration.) If not, we iterate till convergence.

The procedure has similar convergence guarantees as Algorithm 1 when $\eta_c(\lambda^*, T_s^*) \leq \hat{\eta}_c$. In other cases, the throughput is lower and not guaranteed to be the best possible under the given interference constraints. A similar problem setup with similar solution structure and conclusions can be found in [46].

2.7 Extensions to More General Scenarios

2.7.1 General Channel Statistics

Throughout the chapter, we consider a homogeneous channel and system model where all channels have the same statistics, i.e., the rate distribution of SU is $F_R(r)$ for all channels and all the PUs have the same exponential distribution parameters $a$ and $b$ for the idle and busy times. The assumption can be justified due to consideration of the common scenario that the channels belong to the same licensed network [8], typically consisting of equal quality channels with equal usage constraints on the PUs.

To make our results more useful, we consider an extension to a heterogeneous scenario where the rate distribution of SU is not necessarily the same on the $L$ channels, and given by $F_1(r)$, $F_2(r), \ldots, F_L(r)$ respectively. In such a scenario, it is natural to consider strategies where instead of exploring channels
uniformly at random, we explore channels of higher quality more frequently. Accordingly, we consider schemes that explore the channels with unequal probabilities $p_1, p_2, \ldots, p_L$, where $\sum_{i=1}^{L} p_i = 1$. Using similar techniques as Section 2.3, it can be shown for each channel $l$, the optimal stopping rule for deciding whether to transmit on that channel after probing is a threshold policy. Let the thresholds corresponding to the different channels be $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_L)$ respectively. By noting that the probability of transmitting on channel $i$ is proportional to $p_i(1 - F_i(\lambda_i))$, the corresponding throughput can be seen to be

$$x(\lambda) = \frac{\sum_{i=1}^{L} p_i \int_{\lambda_i}^{\lambda} r d F_i(r) E[T']}{\sum_{i=1}^{L} p_i (1 - F_i(\lambda_i))} = \frac{\sum_{i=1}^{L} p_i \int_{\lambda_i}^{\lambda} r d F_i(r) E[T']}{\tau' + E[T_t]} \sum_{i=1}^{L} p_i (1 - F_i(\lambda_i)), \quad (2.23)$$

where $\tau' = \frac{\tau_s + Q_1}{1 + a b_A(1 - P_{fa})}$, and $E[T']$, $E[T_t]$ and $Q_1$ are given by Proposition 2.2. From the expression above, and as pointed out in [8], it is easy to see that for a given $\lambda$, selecting $p_l = 1$ for the channel $l$ that has the highest average throughput, and $p_i = 0$ for $i \neq l$ maximizes the throughput. However, for such a choice of $p_i$'s, Equation (2.23) for throughput no longer holds because we assume that a large number of channels are explored and hence the exploration outcomes are independent. As a counter example, consider a scenario consisting of a large number of channels $L$, where $L - 1$ of them have equal rate distributions and the last channel has a slightly better rate distribution. And let the expected busy time $\frac{1}{b}$ be really large. In such a scenario, there is clear benefit of exploring all the channels, as opposed to stuck with the single best channel for a long time when PU is busy. Accounting for the higher order terms in the calculation of channel access times due to SU exploring previous channels is difficult and is not considered in this chapter. Instead, we motivate and propose the following strategy for the heterogeneous case at hand. We first consider strategies that explores each channel $i$ with equal probability $p_i = \frac{1}{L}$. To find $\lambda$ that maximizes throughput $x(\lambda)$, we equate the derivative of $x(\lambda)$ with respect to each $\lambda_i$ to zero, and deduce that all $\lambda_i$ are equal. Letting the common threshold to be $\lambda$, Equation (2.23) for throughput simplifies to

$$x(\lambda) = \frac{\int_{\lambda}^{\lambda} r d F(r)}{E[T'](1 - F(\lambda))} \frac{E[T']}{\tau' + E[T_t]} \sum_{i=1}^{L} p_i (1 - F_i(\lambda_i)) \quad (2.24)$$
where \( F(r) \stackrel{\Delta}{=} \frac{1}{L} \sum_{i=1}^{L} F_i(r) \). This is therefore equivalent to a scenario that all channels have the same rate distribution equal to the average of the rate distributions of the original channels. And the best threshold \( \lambda^* \) can be found by optimization techniques considered earlier. While this strategy ensures that we are able to achieve the optimal performance corresponding to the average rate distribution, we may be able to get a better throughput using the following tweak. Suppose for this common choice of thresholds \( \lambda^* \), we want to minimize the channel access time or equivalently maximize the channel access probability, by suitably selecting the probabilities \( p_1, \ldots, p_L \). Note that this is not the same as maximizing the throughput, which would again lead to the degenerate solution \( p_l = 1 \) for the best channel \( l \). The channel access probability is proportional to \( \sum_{i=1}^{L} p_i(1 - F_i(\lambda^*)) \), which is maximized when \( p_l = \frac{1 - F_l(\lambda^*)}{\sum_{i=1}^{L}(1 - F_i(\lambda^*))} \), by Cauchy-Schwarz inequality or otherwise. This choice of \( p_i \)'s also favors better channels, albeit in a moderate way. To summarize, we propose using the best threshold \( \lambda^* \) and packet time \( T_s^* \) as obtained in Algorithm 1 corresponding to the average rate distribution \( F(r) \) and use channel exploration probabilities \( p_i \propto (1 - F_i(\lambda^*)) \).

We do not consider the heterogeneous case of different idle and busy parameters \( a \) and \( b \) across the channels, which is much more difficult because of the presence of exponential terms in throughput and it also affects the transmission times \( E[T'] \) and \( E[T_{tr}] \). A simple strategy in a heterogeneous case is to take \( \frac{1}{a} \) and \( \frac{1}{b} \) as the average idle and busy times across the channels. Lastly, while we only consider the rate distributions under Rayleigh fading, most of the results in this chapter, including Algorithm 1, apply to other continuous, well-behaved rate distributions.

### 2.7.2 Multiple Secondary Users

In this chapter, we only consider a setup with only one SU. In general, there can be multiple SUs sharing the channels with the PUs. In such a scenario, we not only have to consider collisions between each SU and the PUs, but also collisions among SUs. Let the number of SUs be \( M \). When \( M \ll L \), one way to extend the scheme and results in our work to this scenario is to divide the \( L \) channels
into $M$ disjoint groups, each having $L/M$ channels. And each SU exclusively explores channels in one of these groups. In such a case, each SU can use the scheme presented in this chapter and the throughput optimality results hold as is. However, if $M$ is larger than $L$, such a grouping is not possible. Moreover, even if $M$ is smaller, but comparable to $L$, the number of channels $L/M$ is no longer large, which is used as an assumption for the calculation of channel access times and throughput. The throughput is lower since the channel access time is higher due to the SU exploring a previous channel where PU may still be busy or rate may still be low.

An alternative is to consider schemes where each SU explores all the $L$ channels uniformly at random. Assuming $L$ is large, we can now neglect the higher order terms in the channel access time due to re-exploration of channels, which happens very infrequently. If $M \ll L$, we may neglect collisions with other SUs as well. If $M$ is comparable to $L$, while we can still neglect excess access time due to re-exploration, we do need to address the excess due to collisions with other SUs. Accounting for excess times is beyond the scope of this chapter and will be considered in a future work [47] along with other concerns like cooperation and fairness across the SUs.

Note that if $M$ is even moderately larger than $L$, we need to consider other standard techniques for multiple access and explore channels infrequently. For example, the $m$-th SU may explore channels with probability $p_m < 1$ and remain inactive at other times. If $p_m$ is small enough that collisions among SUs are infrequent, we can again directly use the results in this chapter with the channel access times now increased by a factor of $\frac{1}{p_m}$.

### 2.8 Numerical Results

We present numerical results to evaluate the performance of our proposed scheme. Unless otherwise stated, the values of the various parameters used are $\rho = 10$, $\tau_s = 20$ ms, $\tau_p = 30$ ms, $\frac{1}{a} = 500$ ms, $\frac{1}{b} = 666.67$ ms, $P_{fa} = 0.1$ and $P_{md} = 0.05$. For simplicity, we assume that the optimal transmission time meets
the interference requirements, i.e., the $T_s^*$ obtained in Algorithm 1 satisfies $\eta_c \leq \tilde{\eta}_c$. If not, the easy modification mentioned in Section 2.6 can be used.

We study the performance of the proposed channel access scheme as a function of the key operational parameters. Specifically, we examine the maximal throughput $x^*$, the optimal threshold $\lambda^*$ and the optimal transmission time $T_s^*$ as a function of $\frac{1}{a}$, $P_{fa}$ and $P_{md}$ when each one of these is varied while keeping others fixed.

The dynamic behavior of PU directly affects SU’s performance. The effect of PU’s average idle time \( \frac{1}{a} \) on SU’s throughput is shown in Fig. 2.3. When \( \frac{1}{a} \) increases, since SU has the opportunity to transmit for longer times, we observe that the optimal threshold $\lambda^*$ and transmission time $T_s^*$ increase, and consequently the throughput $x^*$ increases.

**Figure 2.3:** The effect of average idle time: (a) Maximal throughput $x^*$ versus average idle time, $\frac{1}{a}$ (b) Optimal $T_s^*$ versus average idle time, $\frac{1}{a}$ (c) Optimal threshold $\lambda^*$ versus average idle time, $1/a$

Sensing errors have a negative impact on the performance of the proposed
scheme. The impact of sensing errors in the form of various values of false alarm probabilities $P_{fa}$ is shown in Fig. 2.4 and for misdetection probabilities $P_{md}$ is shown in Fig. 2.5. Note that $P_{fa}$ and $P_{md}$ are decreasing functions of $\tau_s$. But they are also functions of other channel and system parameters, e.g., the channel bandwidth, the SNR of PU at SU’s receiver, and the detection threshold used in sensing systems based on energy detection [8, 48]. Even though the value of $\tau_s$ is fixed in our numerical results, we attribute the different values of $P_{fa}$ and $P_{md}$ implicitly to the remaining parameters.

![Graphs showing the effect of false alarm (Pfa varies but Pmd = 0.05).](image)

**Figure 2.4:** The effect of false alarm ($P_{fa}$ varies but $P_{md} = 0.05$): (a) Maximal throughput $x^*$ versus probability of false alarm, $P_{fa}$ (b) Optimal $T_s^*$ versus probability of false alarm, $P_{fa}$ (c) Optimal threshold $\lambda^*$ versus probability of false alarm, $P_{fa}$

In Fig. 2.4, when $P_{fa}$ increases, $x^*$ decreases as expected whereas $T_s^*$ increases. The reason for the increment in $T_s^*$ is that when $P_{fa}$ is high, i.e., when PU is detected as idle less often, SU increases its transmission time whenever it
gets the chance to transmit. Similarly, \( \lambda^* \) decreases when \( P_{fa} \) increases because the transmission opportunity is smaller and thus \( \lambda^* \) is small so that transmission can take place more readily.

In Fig. 2.5, when \( P_{md} \) increases, \( x^* \) decreases similar to the effect of \( P_{fa} \). Unlike the case of \( P_{fa} \), when \( P_{md} \) increases, \( T_s^* \) decreases to reduce the amount of collision and data loss when PU returns but is misdetected as idle. As in the case of \( P_{fa} \), for small values of \( P_{md} \), when \( P_{md} \) increases, \( \lambda^* \) decreases to facilitate channel access more readily. However, for high values of \( P_{md} \), there may be an increase in \( \lambda^* \) so that transmission is carried out at a higher rate, albeit less often, thereby also reducing frequent collision and data loss due to misdetection. This phenomenon is not observed in Fig. 2.5.

**Figure 2.5:** The effect of misdetection (\( P_{md} \) varies but \( P_{fa} = 0.1 \)): (a) Maximal throughput \( x^* \) versus probability of misdetection, \( P_{md} \) (b) Optimal \( T_s^* \) versus probability of misdetection, \( P_{md} \) (c) Optimal threshold \( \lambda^* \) versus probability of misdetection, \( P_{md} \)

It is of interest to compare our proposed scheme for opportunistic channel-
aware spectrum access with periodic sensing to other schemes. One such comparison is shown in Fig. 2.6, our scheme is compared with the one without periodic sensing and one without probing. To obtain a fair comparison, the transmission time $T_s'$ used for the scheme without periodic sensing is the same as expected time of transmission $E[T_{tr}]$ for the periodic sensing scheme. A threshold based channel access strategy is used for the scheme without periodic sensing, where the threshold that maximizes throughput is derived using optimal stopping theory, similar to the scheme with periodic sensing. The results show that significant throughput gains are achieved for the proposed scheme over the schemes without periodic sensing and without probing.

![Figure 2.6](image.png)

**Figure 2.6:** Comparison between our scheme and one without channel probing and one without periodic sensing: Maximal throughput $x^*$ versus average SNR $\rho$

Next, we evaluate the benefit of optimizing the transmission time $T_s$ – if it is too large, the return of PU will lead to loss of the entire packet, and if it too small, we spend too much time in periodic sensing. In Fig. 2.7, we compare our proposed scheme with a scheme without optimal transmission time, arbitrarily set to a large value of 500 ms and small value of 50 ms. We observe significant improvements by optimizing $T_s$ as shown in the figure.
Figure 2.7: Comparison between scheme with and without optimal transmission time: Maximal throughput $x^*$ versus average SNR $\rho$. Note that optimal transmission time $T_s^*$ varies with $\rho$.

To further demonstrate the benefit of our proposed scheme, we compare our scheme with a POMDP-based scheme in [6]. For fair comparison, we set the rates of all channels to $E[R]$, slot length to $\tau_s + T_s$ and the transition probabilities $p_{\text{busy}} \rightarrow \text{idle} = b(\tau_s + T_s)$ and $p_{\text{idle}} \rightarrow \text{busy} = a(\tau_s + T_s)$ so that average idle and busy times of PUs are $1/a$ and $1/b$. POMDP-based schemes are popular in the literature for solving the opportunistic spectrum access problem. Such schemes dynamically track the idle state of various channels in a slotted system and maximize throughput by exploiting the spectrum opportunities. Our scheme fully utilizes the idle state of the channels it accesses by periodic sensing, and together with exploitation of channel quality information and optimization of transmission time, it outperforms the POMDP-based scheme as seen in Fig. 2.8.

2.9 Conclusions

In this chapter, we proposed an opportunistic channel access framework where the transmissions are interleaved with periodic sensing. For the proposed
Figure 2.8: Comparison between our proposed channel-aware with periodic sensing scheme and POMDP-based scheme: Maximal throughput \( x^* \) versus average SNR \( \rho \)

scheme, we obtained the optimal threshold and the optimal transmission period that jointly maximize the average throughput. We consider the effect of sensing errors throughout the analysis. Numerical results show that our scheme can offer a much higher throughput than other well-known schemes. We also studied numerically the effect of some of the important channel and system parameters on our scheme as they vary over a range of values.

Chapter 3

Opportunistic Spectrum Access for Cognitive Radio Networks with Multiple Secondary Users

3.1 Introduction

3.1.1 Motivation

The rapid growth of wireless communications has increased the demand for higher spectrum efficiency. In the current spectrum allocation framework, most of the frequency bands are exclusively assigned to specific licensed services. However, a lot of licensed bands are experiencing low utilization, such as those for TV broadcasting, resulting in inefficient usage. In view of this, cognitive radio technology has been proposed to optimize the use of available radio frequency spectrum and meet the increasing demands [2–4]. Cognitive radio allows unlicensed users, i.e., secondary users (SUs), to reuse spectral white spaces of licensed users, i.e. primary users (PUs), in an opportunistic manner without causing harmful interference to PUs [5].

To efficiently explore and utilize the unused spectrum, it is essential for the SUs to use good channel access strategies. In a cognitive radio network with multiple channels and multiple SUs, the design of a channel access strategy is
challenging due to possible collisions between the SUs and PUs, and also among the SUs. In addition, one needs to consider the varying qualities of the channels. Under these constraints, it is of great interest to derive rules for managing the SUs so as to maximize the network throughput and optimize the sharing of resources.

3.1.2 Main Contributions

We derive channel-aware opportunistic spectrum access strategies for cognitive radio networks consisting of multiple channels and multiple SUs. Similar to several related works [7, 8, 49], we treat the problem of joint sensing, probing and channel access for SUs using optimal stopping theory [9]. Here, when a SU senses a channel as idle, it proceeds to probe and estimate the channel quality and obtains the data rate it can support. Based on the rate, the SU can decide either to proceed with data transmission or give up the opportunity and continue to explore for a potentially better channel. Further exploration will increase the likelihood for finding a better channel, but at the cost of additional time for sensing and probing the channels. We characterize this tradeoff in an optimal stopping theory framework and study opportunistic channel-aware spectrum access to exploit multiuser diversity, multi-channel diversity and time diversity.

We analyze the opportunistic spectrum access of SUs in two different sharing scenarios, cooperative and non-cooperative. In the cooperative, network centric setting, all the SUs cooperate to maximize the sum network throughput of the SUs. In contrast, in the user-centric setting, the problem of opportunistic spectrum access can be formulated as a non-cooperative game. Here, each SU seeks to maximize its individual throughput in a greedy manner. For both cooperative and non-cooperative scenarios, we show that the optimal schemes are pure threshold policies, i.e., SU proceeds to transmit on an idle channel if the supported data rate exceeds a particular threshold. For the non-cooperative scenario, we establish the existence of Nash equilibrium and show its uniqueness under certain conditions. We provide several best response strategies, i.e., time evolving strategies for SUs that achieve equilibrium – a two-stage best response strategy, a simplified one-stage pseudobest response strategy that is robust but slower, and two online
strategies. We provide convergence guarantees for these algorithms. The first two algorithms require that the SUs be aware of each others’ strategies, which may be impractical in non-cooperative scenarios. The online algorithm doesn’t have such a requirement and the SUs indirectly estimate each others’ strategies using the time taken for finding an idle channel and accordingly adjust their strategies dynamically. The drawback is that the algorithm has slower convergence guarantees. Hence, we propose a fourth online heuristic algorithm that aims to fasten the convergence and has lesser, more practical sensing requirements for tracking other SUs’ strategies.

For both the cooperative and non-cooperative settings, we address several side issues related to sharing of resources. In the cooperative approach, the overall network throughput maximization is rather disadvantageous to SUs with low SNR’s – not only are their average transmission rates lower, they are given lesser opportunity to transmit to make way for the SUs with better SNRs so as to maximize the total throughput of all SUs. We take the standard approach of considering concave utility functions of the throughput to address these fairness issues, albeit at the cost of slightly lower total throughput. At another extreme, the non-cooperative setup is very inefficient compared to the cooperative setup in terms of overall throughput due to the selfish decisions made by the SUs. To counter this, we use a standard pricing mechanism and consider a modified utility function that charges the SUs by the proportion of total transmission time of SUs they use and subtract it from their average transmission rates. As a result, SUs with lower SNRs reduce their transmissions, i.e., charges to keep their utility high. Thus, the overall effect is that the SUs are less greedy and the total throughput of the network is better and comparable to that in the cooperative setup. To address the spectrum sharing with the PUs, we analyze the interference of SUs to PUs. We derive the impact of SUs’ strategies and transmission times so that they can be adjusted appropriately to limit the interference.
3.1.3 Related Works

In a related previous paper [49], we consider only a single SU scenario and propose a similar channel aware optimal spectrum access strategy that additionally involves periodic sensing to optimize the transmission time and maximize the SU’s throughput as well as minimize the interference to PUs. In this chapter, we consider the problem in a much more general and complex cognitive radio network with multiple SUs. The primary focus in this chapter is to analyze and optimize the interactions between the SUs to maximize the throughput. Although the transmission times can be optimized by periodic sensing, for simplicity we assume that the transmission times are fixed and given, and already optimized.

Our work heavily borrows from the work by Zheng et al. [7]. They consider channel-aware distributed opportunistic scheduling in a simpler setup consisting of multiple users in an ad hoc network with only one channel. The derivation of our results is complicated due to the presence of PUs as well as consideration of multiple channels. While many of our results are extensions with mathematical similarity, bulk of our work involves modeling and approximating the complex interactions between the network users.

Our setup is essentially the same as that in the work of Shu et al. [8]. There, the authors mostly limit themselves to studying opportunistic access for a setup involving a single SU and analyze the performance of this scheme when used in a setup consisting of multiple SUs by considering a Markov-chain model. However, such an approach is not very insightful and not easily amenable to rigorous analysis along the lines of [7]. Indeed, in our work, we consider a close approximation of the throughput of SUs that is suitable for game theoretic analysis. In a later stage, we use the Markov chain model for accurate throughput calculations and for validating our approximations. We also use the Markov chain model to simplify the simulations in the numerical results section.

While schemes based on optimal stopping theory explore channels uniformly at random, another popular approach in the literature is based on the Partially Observable Markov Decision Process (POMDP) framework. Zhao et al. [6] and Chen et al. [34, 35] study such spectrum access schemes for slotted multichannel
cognitive radio networks. POMDP-based schemes try to accurately track the idle state of channels. Our scheme doesn’t track the idle channels like POMDP schemes, but it does consider the channel quality in channel selection and efficiently utilizes the idle state of the explored channel by optimal selection of the transmission time.

Game theory has been used as a mathematical tool to model and analyze the interaction process between users in a cognitive radio network, as well as to design efficient spectrum sharing schemes. Wang et al. [50] survey game theoretic applications for cognitive radios and classify them into several categories: non-cooperative spectrum sharing [51], spectrum trading and mechanism design [52], [39], cooperative spectrum sharing [53] and stochastic spectrum sharing game [54]. We consider both non-cooperative and cooperative channel access.

Most of the recent works on cognitive radios relate to a wide variety of other important topics like energy-efficient transmission schemes e.g., [38], machine learning approaches when the various channel parameters and related probability distributions are not known, e.g., [41], [42], and the references therein. While a treatment of channel-aware opportunistic channel access in such contexts is beyond the scope of this chapter, we believe that these concepts can be used in conjunction with our schemes and provide significant performance benefits.

The rest of the chapter is organized as follows. We describe the channel and system model in Section 3.2. We derive the opportunistic spectrum access in the network-centric setting in Section 3.3. We then analyze opportunistic spectrum access in the user-centric setting as a non-cooperative game in Section 3.4. Section 3.5 presents the interference of SU to PU and Section 3.6 provides the evaluation of throughputs using Markov chains. Numerical results are presented in Section 3.7. Finally, we conclude the chapter in Section 3.8.

3.2 Channel and system model

3.2.1 Channel model

The channel model is essentially the same as that in [49]. We consider a frequency-selective multi-channel system such as the orthogonal frequency-division
multiple access (OFDMA) system that is commonly used for cognitive radios, e.g., the IEEE 802.22 [43] wireless standard. The entire frequency spectrum is assumed to be divided into $L$ independent and identically distributed (i.i.d.) channels. We assume that the coherence bandwidth is larger than the signal bandwidth of the individual channels or the subcarriers in it, and thus each channel experiences flat fading. Furthermore, we assume that each channel experiences slow fading, i.e., its condition varies slowly over time.

We assume that all channels are homogeneous, i.e., have identical statistics, and are subject to Rayleigh fading. The instantaneous rate $R$ is given by the Shannon channel capacity $R = \log(1 + \rho|h|^2)$ nats/s/Hz, where $\rho$ is the normalized average SNR, and $h$ is the random channel coefficient with a complex Gaussian distribution $CN(0, 1)$. Accordingly, the distribution of $R$ is given by

$$F_R(r) = 1 - \exp\left(-\frac{\exp(r) - 1}{\rho}\right)$$

for $r \geq 0$, and $F_R(r) = 0$ otherwise.

### 3.2.2 System model

**PU and SU model**

In each of the $L$ licensed channels, there is only one designated PU and each PU is assigned exactly one channel. The $L$ channels are also opportunistically available to $M$ SUs and the SUs are not bound to any particular channels. The PUs are the incumbent users and have higher priority over SUs. The status of each channel is modeled as a continuous-time random process that alternates between idle and busy states. The busy and idle states respectively indicate whether the PU is transmitting over the channel or not. We assume that the idle/busy states of the different channels are driven by homogeneous and independent random processes, and the time durations of the idle and busy states are exponentially distributed with parameters $a$ and $b$. Thus, for any PU, the duration $T_I$ of any idle state has distribution $f_{T_I}(t) = ae^{-at}$ and the duration $T_B$ of any busy state has distribution $f_{T_B}(t) = be^{-bt}$. The expected durations of each of the idle and busy states are $\frac{1}{a}$ and $\frac{1}{b}$, respectively.
and \( \frac{1}{b} \) respectively. The fraction of time for which a PU is idle in the long term is the idle probability \( P_I = \frac{1/b}{1/a+1/b} = \frac{b}{a+b} \).

**Channel sensing, probing and data transmission**

In order to look for transmission opportunities, the SUs use a scheme of sequential sensing and probing without recall [30]. Each SU senses/probes the channels one at a time and cannot recall or select a previously sensed/probed channel once it gives up the opportunity to transmit on that channel, unless the sensing/probing is repeated on that channel.

The details of the sensing, probing and transmission scheme are as follows. Each SU \( m \in \{1, 2, \ldots, M\} \) selects one of the channels at random, with equal probability \( \frac{1}{L} \). The SU takes time \( \tau_s \) to sense a channel. During the sensing stage, if the channel is idle, i.e., the PU is not transmitting, and other SUs are not exploring or transmitting on that channel, the SU will proceed to probe the channel for time \( \tau_p \) to estimate the channel quality. Thus, the time taken to explore, i.e., sense and probe a channel is \( \tau = \tau_s + \tau_p \). If the sensed channel is busy, i.e., there is a collision with the PU or other SUs, it will proceed to explore another channel. In this case, although no probing is performed, the SU still waits for time \( \tau_p \) time to guarantee perfect transmitter-receiver synchronization. After exploring one or more channels, the SU eventually contends a channel successfully, i.e., finds a channel where the PU and other SUs are not present for the duration \( \tau \) of sensing/probing. Then, depending on the channel quality found by probing, the SU decides whether to transmit on that channel for a predetermined time \( T_s \) or continue exploration to find an idle channel of better quality. Clearly, for the objective of maximizing throughput, there is a tradeoff between the time spent on exploring the channels and the quality of the idle channel obtained. In the next two sections, we derive optimal channel access strategies in the cooperative and non-cooperative settings.

The dynamics of channel access is illustrated by a sample realization of sensing, probing and transmission process for one of the SUs in Fig. 3.1. In the first channel the SU explores, the PU is present, so it continues to explore another
channel. In the second and third channels, it detects the presence of other SUs who are transmitting and exploring respectively. The fourth channel is idle, but the quality is low, so the SU decides to continue exploring. The fifth channel is idle and the channel quality is determined to be sufficiently high, so the SU transmits data for time $T_s$. Thereafter, the SU repeats the channel exploration process, looking for transmission opportunity.

**Figure 3.1:** A sample realization of the channel sensing, probing and data transmission by a SU

### 3.3 Maximum throughput under cooperative channel access strategies

In the cooperative setup, SUs select a strategy so as to maximize the total throughput. In general, a SU may forgo its transmission opportunity to seek a better quality channel and increase its throughput. Here, a SU may also forgo to help other SUs with better channel conditions access the channel so as to maximize the total throughput, even if it is detrimental to its own throughput.

#### 3.3.1 Analysis of total throughput under general cooperative channel access strategies

A channel access strategy $N$ governs whether or not a SU transmits on an idle channel it finds, depending on the channel quality and the time spent so
far in exploring transmission opportunities. For calculating the total throughput \( x \) under a strategy \( N \), we consider the channel access and transmission process by all SUs from the perspective of a particular channel \( l \in \{1,2,\ldots,L\} \). The total throughput \( x \) is the sum of throughputs \( x_l \) on each of the channels \( l \), i.e., \( x = \sum_{l=1}^{L} x_l \). To calculate \( x_l \), we make the following simplifying assumptions. We refer to the time \( \tau \) taken for exploring a channel as a time slot. We assume that the various SUs are synchronized and explore the channels at multiples of time \( \tau \). For simplicity, we assume that \( T_s \) is an integer multiple of \( \tau \). We further assume that while an SU is transmitting, if some other SU explores the channel, the transmission is unaffected, but if the PU returns, the transmission fails. If the PU returns at any point during the exploration time \( \tau \), the sensing process fails.

We consider several rounds of transmission in the timeline of \( l \)-th channel. Each such round consists of one or more exploration slots in which different SUs try to access the channel and is followed by transmission for a period of \( T_s \) by one of the SUs. Each round can be further divided into one or more subrounds, where at the end of each subround, one of the SUs successfully contends the channel. Let \( N_u \) denote the number of subrounds in the \( u \)-th round. The number of subrounds \( N \) in a round is governed by the stopping rule – based on the channel quality, it is used to decide that the successfully contending SUs at the end of the first \( N - 1 \) subrounds forgo their transmission opportunities due to poor channel quality, and the SU at the end of \( N \)-th subround transmits. Accordingly, let \( T' \) be the effective transmission time of the SU at the end of a round and \( T_N \) denote the total duration of a round. Let \( R_N \) be the transmission rate of the SU that transmits at the end of the round. It is easy to see that the total throughput of the \( l \)-th channel is \( x_l = \frac{E[R_N]E[T']}{E[T_N]} \), since the long-term average throughput at the end of a large number of rounds \( U \) is \( x_{l,U} = \frac{\sum_{u=1}^{U} R_{N_u} T'_u}{\sum_{u=1}^{U} T_{N_u}} \xrightarrow{U \to \infty} \frac{E[R_N T']}{E[T_N]} \) a.s., where \( T'_u, T'_N, R_{N_u} \) are the \( T', T'_N, R_N \) corresponding to \( u \)-th round. Since the idle and busy statistics of the \( L \) channels are the same, we have by symmetry,

\[
x = \sum_{l=1}^{L} x_l = L \cdot \frac{E[R_N]E[T']}{E[T_N]}.
\] (3.2)

We briefly analyze the various expectations in the above equation:
1. We have $E[R_N] = E[R|N]$, where the rate $R$ is a compound random variable corresponding to the rate of the successfully contending SU at the end of a subround and is therefore distributed according to

$$F(r) = \frac{1}{M} \sum_{i=1}^{M} F_i(r). \quad (3.3)$$

This is because each SU $i \in \{1, 2, \ldots, M\}$ successfully contends a particular channel with the same probability $\left(\frac{b}{a+b}\right)e^{-a\tau}(1 - P_{fa}) \cdot \frac{1}{L} \left(1 - \frac{1}{L}\right)^{M-1}$, which is the probability that the PU is idle to begin with and remains idle for time $\tau$ (without false alarm) and only the $i$-th SU accesses the channel. Note that $P_{fa}$ is the probability of false alarm.

2. We have $T_N = T'_N + T_s$, where $T'_N$ is the time taken in the round for the SUs to contend for the channel before one of them transmits. The time spent in sensing and probing in each subround is $K\tau$, where the number of slots $K$ in a subround is distributed geometrically with parameter $M \cdot p_s$ such that

$$p_s \triangleq \left(\frac{b}{a+b}\right)e^{-a\tau}(1 - P_{fa}) \cdot \frac{1}{L} \left(1 - \frac{1}{L}\right)^{M-1}, \quad (3.4)$$

is the probability that a particular SU successfully contends a particular channel. Therefore,

$$E[K] = \frac{1}{M p_s} \quad (3.5)$$

and the time spent by the SUs in actively contending for the channel is $E[K] \tau = \frac{1}{M p_s} \tau$. However, this does not include the time the different SUs are stuck in transmitting in other channels and therefore do not participate in the contention process. To account for this additional time, we observe that between any two consecutive instances of transmission by a particular SU on the same channel, the expected time it spends transmitting on the other channels is $(L - 1)T_s$, since it gets equal opportunity to transmit on each channel and therefore transmits once on an average on each of the other channels. Since the channel contention and transmission processes of the SUs occur parallely, when calculating the throughput of all the SUs, it is sufficient to consider this additional time due to only one of them. Hence,
the average additional time per round over all SUs for a given channel is \((L-1)T_s/M\). Accounting for both these times,

\[
E[T'_N] = E[N]E[K]\tau + \frac{L-1}{M}T_s = \frac{\tau}{Mp_s}E[N] + \frac{L-1}{M}T_s
\]  

(3.6)

and

\[
E[T_N] = E[T'_N] + T_s = \frac{\tau}{Mp_s}E[N] + \frac{M + L - 1}{M}T_s.
\]  

(3.7)

3. We observe that \(T'_0 = T_s\) if the PU does not return during the transmission period, which happens with probability \(e^{-aT_s}\) and \(T'_0 = 0\) if the PU returns. Thus,

\[
E[T'_0] = T_se^{-aT_s}.
\]  

(3.8)

### 3.3.2 Derivation of a throughput-maximizing strategy using optimal stopping theory

The problem of finding the stopping rule \(N^*\) that maximizes total throughput can be formulated as a maximal-rate-of-return problem [7]. The following proposition exploits optimal stopping theory to to find an optimal stopping rule \(N^*\) and the corresponding maximal throughput \(x^*\):

\[
N^* = \arg\max_{N \in Q} L \cdot \frac{E[R_N]E[T'_N]}{E[T_N]}, \quad x^* = \max_{N \in Q} L \cdot \frac{E[R_N]E[T'_N]}{E[T_N]},
\]  

(3.9)

where \(Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}\) is the set of all possible stopping rules.

**Proposition 3.1.** There exists an optimal stopping rule \(N^*\) for the opportunistic spectrum access in the cooperative setting and is a pure threshold policy given by

\[
N^* = \min \{n \geq 1 : R_n \geq \lambda^*\},
\]  

(3.10)

where the optimal threshold \(\lambda^*\) is the unique solution for \(\lambda\) in

\[
E[(R - \lambda)^+] = \lambda \cdot \frac{\tau}{(M + L - 1)p_sT_s}.
\]  

(3.11)

Here, \(R\) is a r.v. that refers to the average rate of the SUs and its CDF is \(\frac{1}{M} \sum_{i=1}^{M} F_i(r)\). Furthermore, the corresponding maximum total throughput is given by

\[
x^* = \lambda^* \cdot \frac{ML}{M + L - 1} e^{-aT_s}.
\]  

(3.12)
Proof: The proof uses methods from optimal stopping theory [9] and is very similar to [7, Prop. 3.1] and [49, Prop. 3.1]. Using equations (3.9), (3.7), (3.6), the goal is to maximize the throughput \( \frac{LE[R_N T']}{E[T_N]} \). For this, a standard technique [9, Ch. 6] is to consider for all \( x \in (0, \infty) \), the reward function \( Z_n(x) = LR_n E[T'] - x T_n = R_n L T_s e^{-a T_s} - x (\tau M n + M + \frac{1}{M} - 1) T_s \) and an optimal stopping rule \( N(x) \) that maximizes the expected reward \( E[LR_n T' - x T_N] \). Note that throughout this proof, by an abuse of notation, we use \( T_n \) and \( T'_n \) to imply \( E[T_N | N = n] \) and \( E[T'_N | N = n] \) respectively. Let the corresponding maximum reward be

\[
V(x) = \sup_{N \in Q} E[Z_N(x)] = \sup_{N \in Q} E[LR_n T' - x T_n] = E[LR_n(x) T' - x T_N(x)].
\]

The reason for considering the reward function \( Z_n(x) \) is [9, Ch. 6, Th. 1], which states that if the maximum rate, i.e., throughput is \( x^* = \sup_{N \in Q} \frac{LE[R_n T']}{E[T_N]} \), then \( V(x^*) = 0 \), and furthermore, \( N(x^*) \) is the stopping rule that maximizes throughput.

Using [9, Ch. 3, Th. 1], the existence of \( N(x) \) is guaranteed if \( E[\sup_n Z_n(x)] < \infty \) and \( \lim \sup_{n \to \infty} Z_n(x) = -\infty \) almost surely. Both conditions hold true in our setup by arguments similar to the proof of [7, Prop. 3.1] and [49, Prop. 3.1] that rely on the fact that the various random variables involved, namely \( K_1, K_2, \ldots, K_n, R_n \) and \( T' \) have finite means and variances. We therefore proceed to find \( N(x) \) and \( x^* \). We have \( E[T'] = T_s e^{-a T_s} \) and \( T_n = \frac{M + \frac{1}{M} - 1}{M} T_s + \sum_{j=1}^{n} K_j \tau \). Using the principle of optimality [9, Ch. 3, Th. 3], an optimal stopping rule is

\[
N(x) = \min \left\{ n \geq 1 : LR_n T_s e^{-a T_s} - x \left( \frac{M + \frac{1}{M} - 1}{M} T_s + \sum_{j=1}^{n} K_j \tau \right) \geq V(x) - x \sum_{j=1}^{n} K_j \tau \right\},
\]

and the optimality equation [9, Ch. 3, Th. 2] gives

\[
V(x) = E \left[ \max \left\{ LR_1 T_s e^{-a T_s} - x \frac{M + \frac{1}{M} - 1}{M} T_s, V(x) \right\} - x K_1 \tau \right].
\]

Using \( V(x^*) = 0 \) and the above expressions for \( N(x) \) and \( V(x) \), we conclude that the stopping rule maximizing the throughput is

\[
N(x^*) = \min \left\{ n \geq 1 : R_n \geq x^* \frac{M + \frac{1}{M} - 1}{M} e^{a T_s} \right\},
\]

and the maximal throughput \( x^* \) is a solution for \( x \) in...
\[
\text{E}[\left( R_n - x^{\frac{M+L-1}{ML}}e^{aT_s} \right)^+] = \frac{x e^{aT_s} \text{E}[K] \tau}{L T_s} = \frac{x e^{aT_s} (M + L - 1)}{ML} e^{aT_s} \cdot \frac{\tau}{(M + L - 1)p_s T_s}.
\]

Finally, we show that the above equation has a unique solution in \( x \). By a change of variable \( \lambda \overset{\Delta}{=} x^{\frac{M+L-1}{ML}}e^{aT_s} \), it is equivalent to show that there is a unique solution for \( \lambda \) in
\[
\text{E}[\left( R_n - \lambda \right)^+] = \lambda \frac{\tau}{(M + L - 1)p_s T_s}.
\]
(3.13)
The left hand side can be written as
\[
g(\lambda) \overset{\Delta}{=} \text{E}[\left( R_n - \lambda \right)^+] = \int_{\lambda}^{\infty} (r - \lambda) f_R(r) dr.
\]
We observe that \( g(\lambda) \) is continuous and decreases from \( \text{E}[R_n] \) to 0. This is because \( f_R(r) \) is positive, continuous and differentiable, and hence, if \( \lambda_1 < \lambda_2 \), then
\[
g(\lambda_2) - g(\lambda_1) = \int_{\lambda_2}^{\infty} (\lambda_1 - \lambda_2) f_R(r) dr - \int_{\lambda_1}^{\lambda_2} (r - \lambda_1) f_R(r) dr \leq 0.
\]
The right hand side \( \lambda \frac{\tau}{(M + L - 1)p_s T_s} \) is continuous and increasing from 0 to \( \infty \). Hence, the equation (3.13) has a unique solution in \( \lambda \). Furthermore, the solution \( \lambda = \lambda^* \) is the threshold in the optimal stopping rule, i.e., the throughput maximizing stopping rule is \( \{ n \geq 1 : R_n \geq \lambda^* \} \) and the maximum throughput is \( x^* = \lambda^* e^{-aT_s} \).

Proposition 3.1 implies that an optimal cooperative channel access strategy has the following form: any SU that successfully contends a channel will transmit whenever the transmission rate from the probing is bigger than or equal to a threshold \( \lambda^* \), and this threshold is common across all SUs and all channels.

### 3.3.3 Numerical computation of \( x^* \) and \( \lambda^* \)

We simplify the calculation of the maximum throughput and the corresponding threshold in the optimal stopping rule in the Proposition 3.1. We observe that under any threshold policy \( N = \min\{ n : R_n \geq \lambda \} \), the stopping time \( N \), i.e., the number of subrounds in a round, is distributed geometrically with parameter \( (1 - F(\lambda)) = \Pr(R \geq \lambda) \), the probability that the rate of the successfully contending SU is bigger than or equal to the threshold. Hence,
\[
\text{E}[N] = \frac{1}{1 - F(\lambda)}.
\]
(3.14)
Furthermore, under such a threshold policy,

$$E[R_N] = E[R|R \geq \lambda] = \frac{\int_{\lambda}^{\infty} r dF(r)}{1 - F(\lambda)}. \quad (3.15)$$

Along with equations (3.2), (3.3), (3.7), (3.8), we have

$$x = \frac{L \cdot \int_{\lambda}^{\infty} r dF(r) \cdot T_s e^{-aT_s}}{\frac{\tau}{M_{p_s}(1 - F(\lambda))} + \frac{M+L-1}{M} T_s} = \frac{L \cdot p_s \sum_{i=1}^{M} \int_{\lambda}^{\infty} r dF_i(r) \cdot T_s e^{-aT_s}}{\tau + \sum_{i=1}^{M} p_s (1 - F_i(\lambda)) \frac{M+L-1}{M} T_s}. \quad (3.16)$$

We therefore see from equations (3.16) and (3.12), or alternatively from Equation (5.3) directly, that the optimal throughput $\lambda^*$ is the unique solution for $\lambda$ in the fixed point equation

$$\lambda = \psi(\lambda) \triangleq \frac{\int_{\lambda}^{\infty} r dF(r) \cdot T_s}{\frac{\tau}{M_{p_s}(1 - F(\lambda))} + \frac{M+L-1}{M} T_s} = \frac{M+L-1}{M} p_s T_s \sum_{i=1}^{M} \int_{\lambda}^{\infty} r dF_i(r)}{\tau + \frac{M+L-1}{M} p_s T_s \sum_{i=1}^{M} (1 - F_i(\lambda))}. \quad (3.17)$$

Next proposition shows that $\lambda^*$ can be found by fixed-point iterations using the above equation. The relation $x^* = \lambda^* \cdot \frac{ML}{M+L-1} e^{-aT_s}$ in Equation (3.12) can be used to compute the throughput.

**Proposition 3.2.** The fixed-point iteration

$$\lambda^{(k+1)} = \psi(\lambda^{(k)}), \quad (3.18)$$

for $k \in \{0, 1, 2, \ldots\}$ and for any nonnegative $\lambda^{(0)}$ converges to the optimum threshold $\lambda^*$.

The proof is same as [7, Prop 3.4] and [49, Prop 4.2], and is omitted.

### 3.3.4 Fairness considerations via utility functions

It is clear that if there is a disparity between the rate distributions of the different SUs, in order to maximize the total throughput, the SUs with higher expected rates are more likely to transmit upon successful channel contention than those with lower rates. This is evident from the above discussion that the best cooperative strategy uses the same threshold across all SUs and each SU is equally likely to successfully contend the channel, and therefore SUs with worse channel
distributions have lesser transmission opportunities. A standard way of addressing these fairness issues are by considering concave utility functions. In this way, higher rates eventually lead to diminishing utility or value to the SUs, and thus ensuring that a scheme maximizing total utility provides more transmission opportunities to SUs with worse rates than a scheme maximizing total throughput. Motivated by generalized proportionality fairness \[55,56\], we can consider the utility functions

\[
U_\alpha(r) = \begin{cases} 
\log(r) & \text{if } \alpha = 1 \\
(1 - \alpha)^{-1} r^\alpha & \text{if } \alpha \in [0, 1).
\end{cases}
\]

(3.19)

Here, \(\alpha = 0\) leads to the same throughput maximization problem as before, and \(\alpha = 1\) leads to the commonly considered logarithmic utility function and proportional fairness. Thus, when the \(m\)-th user transmits at a rate \(R_m\), its utility is considered to be \(U(R_m)\) and the goal is to find a suitable maximize \(\frac{\mathbb{E}[U(R_N)\bigg| E[T_s]]}{E[T_s]}\). By similar derivation as earlier, the utility maximizing strategy is still a pure threshold policy, albeit the thresholds are on the utility function of the rate, instead of the rate itself. The common optimal threshold \(\lambda^*\) across all users is a solution to the fixed point equation

\[
\lambda = \frac{\sum_{i=1}^M \int_{1-F(\lambda)}^\infty U(r)dF_i(r)}{1-F(\lambda)} \cdot T_s = \frac{L_p s T_s \sum_{i=1}^M \int_{\lambda^*}^\infty U(r)dF_i(r)}{\tau + L_p s T_s \sum_{i=1}^M (1 - F_i(\lambda))}.
\]

(3.20)

We remark that the fairness comes at the cost of decreased total throughput.

In the next section, we consider a non-cooperative setup, and towards the end of the section, we observe that the best strategies in such a setup are at the other extreme, namely the greedy strategies of the SUs leads to lower total throughput compared to the cooperative case. There, we consider schemes that try to increase the total throughput and it may come at the cost of decreased fairness for SUs with worse channel conditions.

### 3.4 Non-cooperative channel access strategies

In the previous section, we considered channel access strategies in a setup where the different SUs cooperate with each other to maximize the total throughput. However, in many scenarios, the SUs do not cooperate and each SU tries to
maximize their own throughput at the cost of decreased channel access opportunity for other SUs. We analyze the performance and strategies in such a setup which are optimal in the sense that each SU’s throughput is maximal given the other SUs’s strategies.

3.4.1 Throughput analysis and optimality of threshold based policies

We analyze the throughput of the different SUs when they use channel access strategies $N \triangleq (N_1, N_2, \ldots, N_M)$ respectively. The channel access model in the non-cooperative setup is similar to that in the cooperative setup. Each SU explores the $L$ channels randomly and uniformly. If the $m$-th SU successfully accesses a channel, i.e., the PU on that channel is idle and no other SUs are exploring or transmitting on that channel, it uses strategy $N_m$ to decide whether to transmit or continue exploring to find a better channel. Similar to Equation (3.2), the contribution of $l$-th channel to the throughput of $m$-th SU is given by

$$x_{m,l} = \frac{E[R_{N_m}]E[T']}{E[T_{N,m}]}$$

the throughput of the $m$-th SU is

$$x_m = \sum_{l=1}^{L} x_{m,l} = \frac{L \cdot E[R_{N_m}]E[T']}{E[T_{N,m}]}.$$  (3.21)

Here, $T_{N,m}$ is the time taken by user $m$ for each round of successfully accessing and transmitting on a particular channel, $T'$ is the effective transmission time of the SU in a round, and $R_{N_m}$ is the transmission rate of the SU in a round. We have $E[R_{N_m}] = E[R|N_m]$, where $R$ is distributed according to $F_m(r)$ and $E[T'] = T_s e^{-\alpha T_s}$. Similar to Equation (3.7), we have $E[T_{N,m}] = (E[N_m]E[K_m]T + (L-1)T_s) + T_s = \frac{r}{p_{s,m}^'}E[N_m] + LT_s$, where the number of subrounds $N_m$, i.e., successful contentions until transmission is governed by the access strategy. The r.v. $K_m$ corresponds to the number of slots in a subround, distributed geometrically with parameter $p_{s,m}^'$, which is the probability that the $m$-th SU successfully contends a particular channel and depends on the strategies of other SUs along with other channel and system parameters. Note that $p_{s,m}^'$ is smaller if the other SUs access the channel more aggressively.
It is easy to see along the lines of Proposition 3.1 that a strategy $N_m$ that maximizes the throughput $x_m$ of the $m$-th SU, given the strategies $N_{-m} \triangleq (N_1, \ldots, N_{m-1}, N_{m+1}, \ldots, N_M)$ of other SUs, is a pure threshold policy $N_m = \min\{n \geq 1 : R_n \geq \lambda_n^m\}$ where the threshold $\lambda_n^m$ depends on $N_{-m}$. Hence, without loss of generality, we limit ourselves to strategies that are pure threshold policies and let the thresholds of the SUs be $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_M)$ respectively.

To simplify the derivation of throughputs $\{x_m\}$ under threshold based policies $\lambda$, we observe that the SUs divide the timeline of each channel into two types of events:

- Transmission event: an event of duration $\tau + T_s$, consisting of a time slot and a transmission period such which one of the SUs successfully contends the channel and transmits on it.

- Non-transmission event: an event of duration $\tau$, consisting of a time slot in which none of the SUs successfully contends the channel or an SU that successfully contends the channel decides not to transmit.

Consider a large number of such events $U'$ on the $l$-th channel. For ease of calculation, we first neglect the time spent by SUs for transmitting on other channels. Then for any time slot, the likelihood of the $i$-th SU successfully contending and deciding to transmit is $p_s(1 - F_i(\lambda_i))$. Thus, the expected number of transmission events is $U' \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i))$ and that of non-transmission events is $U'(1 - \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i)))$. The expected duration of $U'$ events is therefore $U' \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i))(\tau + T_s) + U'(1 - \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i)))\tau = U'(\tau + \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i))T_s)$. Next, we take into account the additional time SUs spend transmitting on other channels and therefore cannot contend or transmit on the $l$-th channel. Similar to the calculation of $T_N'$ for the cooperative case in Subsection 3.3.1, since the expected number of rounds across all users is $U' \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i))$, and the average additional time per round is $\frac{(L-1)T_s}{M}$, we see that the actual time taken for the $U'$ events is $U'(\tau + \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i))\frac{M+L-1}{M}T_s)$.

---

1While for ease of exposition, the number of events considered is $U'$ when the transmission time on other channels is neglected, we note that the actual number of events is $U' + U' \sum_{i=1}^{M} p_s(1 - F_i(\lambda_i))\frac{(L-1)T_s}{M\tau}$. 
Thus, the definition of Nash equilibrium aligns with our objective that for each user $i$, i.e., maximize its own utility by choosing $\lambda_i$. Here, the players are the set of SUs $S_u$. We can interpret the e\-ective transmission time corresponding to the $m$-th user on $l$-th channel as:

\[
x_m, l, U^* = \frac{U'(p_m(1-F_m(\lambda_m)))T_s e^{-aT_s}}{U'(\tau + \sum_{i=1}^{M} p_i(1-F_i(\lambda_i))(M+1-\frac{1}{M})T_s)}
\]

\[
x_m = \frac{L \cdot p_m \int_{\lambda_m}^{\infty} r dF_m(r) \cdot T_s e^{-aT_s}}{\tau + \sum_{i=1}^{M} p_i(1-F_i(\lambda_i))(M+1-\frac{1}{M})T_s}.
\]

We can interpret $x_m, l = \frac{\int_{\lambda_m}^{\infty} r dF_m(r)}{\tau + \sum_{i=1, \neq m} p_i(1-F_i(\lambda_i))(M+1-\frac{1}{M})T_s + L-1 \frac{1}{M} T_s}$, in the form of the expected time taken for the $m$-th SU to successfully access the $l$-th channel is $\frac{\tau + \sum_{i=1, \neq m} p_i(1-F_i(\lambda_i))(M+1-\frac{1}{M})T_s + L-1 \frac{1}{M} T_s}$, which is higher when other SUs access channel more aggressively, i.e., when $\{\lambda_i : i \neq m\}$ are small.

### 3.4.2 Game theoretic formulation and Nash equilibrium

Our objective is to find thresholds $\Lambda = (\lambda_1, \ldots, \lambda_M)$ so that for each SU $m$, the throughput $x_m(\Lambda)$ is maximal with respect to its choice of threshold $\lambda_m$, given the thresholds $\Lambda_{-m} \triangleq (\lambda_1, \ldots, \lambda_{m-1}, \lambda_{m+1}, \ldots, \lambda_M)$ of the other users. We closely follow the methodology in [7] and formulate this problem as a non-cooperative game. Here, the players are the set of SUs $\{1, 2, \ldots, M\}$. The strategy of the $m$-th user corresponds to its choice of threshold $\lambda_m \in [0, \infty)$. And the utility or payoff of the $m$-th user is its throughput $x_m(\Lambda)$. Each SU $m$ tries to achieve $\max_{\lambda_m} x_m(\Lambda)$, i.e., maximize its own utility by choosing $\lambda_m$.

A list of thresholds $\Lambda^* = (\lambda_1^*, \lambda_2^*, \ldots, \lambda_M^*)$ is a Nash equilibrium if for each SU $m$,

\[
x_m(\Lambda^*) \geq x_m(\lambda_m, \Lambda^*_{-m}) \quad \text{for all } \lambda_m \geq 0.
\]

Thus, the definition of Nash equilibrium aligns with our objective that for each SU $m$, $\lambda_m^*$ is the best choice of threshold given the threshold choices $\Lambda^*_{-m}$ of other
SUs. The following proposition shows the existence of Nash equilibrium and characterizes it. The result is similar to [7, Prop. 4.1].

**Proposition 3.3.** There exists a Nash equilibrium $\Lambda^*$ in the threshold selection game and each such equilibrium satisfies the equations in $\Lambda$

$$
\lambda_m = \psi_m(\Lambda) \triangleq \frac{M+L-1}{M}p_sT_s \int_{\lambda_m}^\infty rF_m(r) \frac{\tau + \frac{M+L-1}{M}p_sT_s \sum_{i=1}^M (1 - F_i(\lambda_i))}{L} \text{ for all } m. \tag{3.24}
$$

The corresponding maximal rate of each SU $m$ is given by $x_m(\Lambda^*) = \lambda_m^* \frac{ML}{M+L-1} e^{-aT_s}$.

**Proof:** The proof is similar to [7, Prop. 4.1]. We apply [57, Prop. 20.3] that guarantees the existence of Nash equilibrium if for each SU $m$, the set $A_m$ of strategies i.e., threshold choices, is a non-empty compact convex set, and the utility function $x_m(\Lambda)$ is quasi-concave in $A_m$. The first condition holds trivially, since $\lambda_m \in [0, \infty) = A_m$. To show that $x_m(\Lambda)$, or equivalently $\psi_m(\Lambda)$ is quasi-concave in $\lambda_m$, we use the definition [58] and show that all sublevel sets $S_c = \{\lambda_m : \psi_m(\lambda_m, \Lambda_{-m}) \geq c\}$ are convex for all $c$. Suppose $\lambda_m$ and $\lambda_m'$ are thresholds such that $\lambda_m < \lambda_m'$ and $\lambda_m, \lambda_m' \in S_c$, so that $\psi_m(\lambda_m, \Lambda_{-m}) > c$ and $\psi_m(\lambda_m', \Lambda_{-m}) > c$. Using the expression for $\psi_m(\Lambda)$ in Equation (3.24) and rearranging the terms, we have

$$
\int_{\lambda_m}^\infty (r - c) dF_m(r) \geq \frac{\tau + \frac{M+L-1}{M}p_sT_s \sum_{i \neq m} (1 - F_i(\lambda_i))}{L} \cdot \frac{c + \frac{M+L-1}{M}p_sT_s}{L} \text{ for all } m.
$$

and similarly for $\lambda_m'$. Consider any $\lambda_m'' \in [\lambda_m, \lambda_m']$. If $\lambda_m'' \geq c$, i.e., $\lambda_m' \geq \lambda_m'' \leq c$, then $\int_{\lambda_m}^{\lambda_m''} (r - c) dF_m(r) \geq \int_{\lambda_m}^{\lambda_m'} (r - c) dF_m(r)$. If $\lambda_m'' \leq c$, i.e., $\lambda_m \leq \lambda_m'' \leq c$, then $\int_{\lambda_m'}^{\lambda_m''} (r - c) dF_m(r) \geq \int_{\lambda_m'}^{\lambda_m} (r - c) dF_m(r)$. Thus, in both cases, $\int_{\lambda_m'}^{\lambda_m''} (r - c) dF_m(r) \geq \int_{\lambda_m}^{\lambda_m'} (r - c) dF_m(r)$. Therefore, $\psi_m(\Lambda)$ and $x_m(\Lambda)$ are quasi-concave in $\lambda_m$ guaranteeing the existence of Nash equilibrium.

Using similar arguments as Proposition 3.2, for each SU $m$, given the thresholds $\Lambda_{-m}$ of other users, the threshold $\lambda_m^*$ that maximizes throughput $x_m(\lambda_m, \Lambda_{-m})$ is unique and satisfies $\lambda_m = \psi_m(\lambda_m, \Lambda_{-m})$, and furthermore, this maximal rate is $x_m(\lambda_m^*) = \lambda_m^* \frac{ML}{M+L-1} e^{-aT_s}$. Thus, if $\Lambda^*$ is a Nash equilibrium, by definition, since each SU $m$ has a throughput maximizing unilateral strategy $\lambda_m^*$, it must satisfy $\lambda_m^* = \psi_m(\Lambda^*)$ and the corresponding throughput is $x_m(\Lambda^*) = \lambda_m^* \frac{ML}{M+L-1} e^{-aT_s}$. □
While Proposition 3.3 guarantees the existence of Nash equilibrium, it may not be unique in general for various rate distributions \((F_1(r), F_2(r), \ldots, F_M(r))\), not necessarily those induced by Rayleigh fading. In particular, an example in [7] shows that Nash equilibrium may not be unique even in the homogeneous case where the rate distributions are the same. However, they also show that there is a unique Nash equilibrium in the homogeneous case of Rayleigh fading rate distributions. It remains to resolve whether the Nash equilibrium is unique in general under non-homogeneous Rayleigh fading.

### 3.4.3 Best response strategies

We consider the three best response strategies considered in [7] that can be used by SUs to achieve the Nash equilibria, i.e., to compute the thresholds corresponding to the equilibria. Simulation of these strategies can be used as iterative algorithms for computing the Nash equilibria. We motivate and describe these strategies, and state their convergence guarantees.

#### A two-stage iterative algorithm

This algorithm is motivated by the properties of the Nash equilibrium from Proposition 3.3 that the threshold \(\lambda_m\) of the \(m\)-th SU is proportional to its maximum throughput given the thresholds \(\Lambda_{-m}\) of other SUs, and Equation (3.24), which suggests that the optimal \(\lambda_m\) can be obtained by solving \(\lambda_m = \psi_m(\Lambda)\) by fixed-point iterations. Accordingly, the algorithm initializes \(\Lambda^{(0)} = 0\), and in each outer iteration \(i\), it finds update \(\lambda^{(i)}_m\) as the solution of \(\lambda_m = \psi(\lambda_m, \Lambda_{-m}^{(i-1)})\). This is in turn done by fixed point iterations in an inner loop, similar to Equation (3.17) and Proposition 3.2. In other words, the threshold of each SU is updated to the best threshold corresponding to the thresholds of other SUs from the previous iteration. The following proposition provides a convergence guarantee for this algorithm.

**Proposition 3.4** ([7, Prop 4.5]). *If the threshold game has a unique Nash equilibrium \(\Lambda^*\), then the two-stage iterative algorithm converges to \(\Lambda^*\) as the number of iterations tends to infinity.*
A simplified one-stage iterative algorithm

This algorithm has similar motivations as the previous one. The outer and inner iteration loops of the two-stage algorithm are combined into a single iterative update equation. In each iteration $i$, each SU $m$ updates its threshold as $\lambda_m^{(i)} = \psi_m(\Lambda^{(i-1)})$ based on the thresholds $\Lambda^{(i-1)}$ in the previous iteration. The next Proposition shows that this response strategy has the additional advantage of convergence guarantees even the Nash equilibrium is not unique. However, the number of iterations is higher than the number of outer iterations of previous algorithm since we do not use the best threshold in each iteration.

**Proposition 3.5** ([7, Prop. 4.6]). The simplified one-stage algorithm converges to one of the Nash equilibria $\Lambda^*$ of the threshold selection game, as the number of iterations tend to infinity.

Online algorithm 1

The previous algorithms require that each SU has the knowledge of other SUs’ channel parameters and strategies. In most practical scenarios, such information is unavailable or expensive. Online algorithms enable the SUs to estimate each others’ channel statistics and strategies from the channel access times during the transmission process. These estimates are then used in place of their actual values in the threshold calculations. One such scheme is an adaptation of a scheme provided in [7] and comes with convergence guarantees.

To derive this scheme, we observe that Equation (3.24) for calculating the optimal threshold $\lambda_m$ of user $m$ depends on other SUs’ thresholds only through the quantity $\sum_{i=1}^{M} (1 - F_i)$. Secondly, we observe that for any particular channel, the expected number of time slots $K'$ between consecutive transmissions by SUs is

$$E[K'] = \frac{1}{p_s \sum_{i=1}^{M} (1 - F_i) + \frac{(L - 1)T_s}{M \tau}}, \tag{3.25}$$

similar to the calculation of $T_N'$ in equations (3.7) and (3.14). Hence, $K' - \frac{(L - 1)T_s}{M \tau}$ is an unbiased estimator of $\frac{1}{p_s \sum_{i=1}^{M} (1 - F_i)}$ and we use it appropriately as a replacement for $\sum_{i=1}^{M} (1 - F_i)$ in the threshold calculations. Accordingly, we rewrite
Equation (3.24) as
\[ g_m(\Lambda) \triangleq \frac{M + L - 1}{M} p_s T_s \int_{\Lambda_m}^{\infty} r dF_m(r) - \tau \lambda_m - \lambda_m = 0. \] (3.26)
Consider any particular channel, say channel 1, and let \( K'_{j} \) be the number of time slots taken between the \((j-1)\)-th and \( j \)-th transmission by the SUs on that channel. We compute \( \lambda \) based on inter-transmission times on this channel, and use the same thresholds across all channels. Let \( \Lambda^{(j)} \) be the thresholds of the SUs at the time of \( j \)-th transmission on this channel. Based on the above discussion, let
\[ \tilde{g}_m^{(j)} \triangleq \left( K'_{j} - \frac{(L - 1)T_s}{M} \right) \left( p_s \int_{\lambda_m^{(j)}}^{\infty} r dF_m(r) - \frac{M \tau}{(M + L - 1)T_s} \lambda_m^{(j)} \right) - \lambda_m^{(j)} \] (3.27)
be an approximation of \( g_m(\Lambda^{(j)}) \), so that \( \mathbb{E}[\tilde{g}_m^{(j)}] = g_m(\Lambda^{(j)}) \). Using stochastic approximation theory, in order to obtain solution \( \lambda \) to Equation (3.26), if the \( j \)-th transmission on the channel is by the \( m \)-th SU, we update only the threshold \( \lambda_m \) to
\[ \lambda_m^{(j+1)} = \left[ \lambda_m^{(j)} + \frac{1}{j_m} \tilde{g}_m^{(j)} \right]^{\beta_m} \] (3.28)
while keeping the other thresholds unchanged, i.e., \( \lambda_i^{(j+1)} = \lambda_i^{(j)} \) for \( i \neq m \). Here, \( j_m \) is the number of times \( m \)-th SU transmits among the first \( j \) transmissions. The quantity \( \beta_m \triangleq \frac{M + L - 1}{M} p_s T_s \int_{0}^{\infty} r dF_m(r) \) is an upper bound on a solution \( \lambda_m \) to the Equation (3.24). The truncation function \([z]^{\beta}_\gamma \triangleq \min\{\beta, \max\{\gamma, z\}\} \) limits the value of \( z \) to \( \gamma \) if \( z < \gamma \) and to \( \beta \) if \( z > \beta \). Note that we use the same thresholds \( \Lambda^{(j)} \) across all the channels between the times of \( j \)-th and \((j + 1)\)-th transmissions on channel 1. The following proposition provides a convergence guarantee for this algorithm.

**Theorem 3.1** ([7, Thm. 4.1]). If the threshold selection game has a unique Nash equilibrium \( \Lambda^* \), then the online algorithm 1 converges to \( \Lambda^* \) as the number of iterations tend to infinity.

Implementation of this algorithm requires that each of the SUs to track the state of channel 1, i.e., the inter-transmission times \( K'\tau \) on it. Furthermore, the convergence of the algorithm is slow when the number of channels \( L \) is large compared to the number of SUs \( M \), since only the transmissions on this channel, which would occur infrequently, are used to update the thresholds.
Online algorithm 2

We consider a modified algorithm which requires that each SU keep track of only its own inter-transmission times and across all channels. To derive this algorithm, we rewrite Equation (3.24) as

$$h_m(\Lambda) \triangleq \frac{M + L - 1}{ML} \int_{\lambda_m}^{\infty} \frac{r dF_m(r)}{1 - F_m(\lambda_m)} = \frac{\lambda_m}{T_s} \left( \frac{\tau + \frac{M + L - 1}{ML} p_m T_s \sum_{i \neq m} (1 - F_i(\lambda_i))}{L p_s (1 - F_m(\lambda_m))} - \frac{(M - 1)(L - 1)}{ML} T_s \right) - \lambda_m = 0. \quad (3.29)$$

We also rewrite Equation (3.22) in the form

$$x_m = \frac{\int_{\lambda_m}^{\infty} r dF_m(r)}{1 - F_m(\lambda_m)} \cdot T_s e^{-a T_s} \left( \frac{\tau + \frac{M + L - 1}{ML} p_m T_s \sum_{i \neq m} (1 - F_i(\lambda_i))}{L p_s (1 - F_m(\lambda_m))} - \frac{(M - 1)(L - 1)}{ML} T_s \right) + T_s,$$

so that the numerator can be interpreted as the expected number of bits transmitted by $m$-th SU in one round of transmission across all channels and the denominator is the total time of such a round of transmission. This implies that the inter-transmission time $K_m'' \tau$ of $m$-th SU across all channels is such that

$$E[K_m'' \tau] = \frac{\tau + \frac{M + L - 1}{ML} p_m T_s \sum_{i \neq m} (1 - F_i(\lambda_i))}{L p_s (1 - F_m(\lambda_m))} - \frac{(M - 1)(L - 1)}{ML} T_s. \quad (3.30)$$

Letting $K_{m,j}$ being the number of time slots between the $(j - 1)$-th and $j$-th transmissions of $m$-th SU across all channels, we accordingly consider the approximation

$$\tilde{h}_m^{(j)} \triangleq \frac{M + L - 1}{ML} \int_{\lambda_m^{(j)}}^{\infty} \frac{r dF_m(r)}{1 - F_m(\lambda_m^{(j)})} = \frac{\lambda_m^{(j)}}{T_s} K_{m,j}'' - \lambda_m^{(j)} \quad (3.31)$$

of $h_m(\Lambda^{(j)})$, so that $E[\tilde{h}_m^{(j)}] = h_m(\Lambda^{(j)})$. Starting with arbitrary thresholds $\Lambda^{(1)}$, for each SU $m$, we use the stochastic approximation updates

$$\lambda_m^{(j+1)} = \left[ \lambda_m^{(j)} + \frac{1}{j} \tilde{h}_m^{(j)} \right]^{\beta_m}, \quad (3.32)$$

at the end of its $j$-th transmission. Note that since the transmissions of SUs occur concurrently, the threshold updates of the SUs affect the channel access probabilities of each other reflected via the inter-transmission times $K_{m,j}'' \tau$. Clearly, this scheme requires each SU to only track its own inter-transmission times and does not require the SU to track any particular channel at all times and also utilizes the access dynamics of all channels to update its thresholds. While we do not provide theoretical convergence guarantees, numerical results for this scheme are provided in Section 3.7.
As noted in [7], it is easy to see that the total throughput $x(\Lambda^*) \triangleq \sum_{i=1}^{M} x_i(\Lambda^*)$ corresponding to any Nash equilibrium $\Lambda^*$ in the non-cooperative channel access model is at most the best total throughput $x(\Lambda^+) \triangleq \sum_{i=1}^{m} x_i(\Lambda^+)$ in the cooperative case, i.e., $x(\Lambda^*) \leq x(\Lambda^+)$, referred to as the price of anarchy. Here, $\Lambda^+ \triangleq (\lambda^*, \lambda^*, \ldots, \lambda^*)$ and $\lambda^*$ is the optimal threshold in the cooperative setup, as given by Proposition 3.1 and is the solution to Equation (3.17). This is because Proposition 3.1 implies that $\Lambda^+ = \arg \max_{\Lambda} \sum_{i=1}^{M} x_i(\Lambda)$. The next Proposition states that the throughput in the non-cooperative case is always strictly worse than that in the cooperative case.

**Proposition 3.6** ([7, Prop. 5.1]). If $M \geq 2$ and $f_m(r) > 0$ for all $m, r$ (as in the case of rate distributions induced by Rayleigh fading), $x(\Lambda^*) < x(\Lambda^+)$ for any Nash equilibrium $\Lambda^*$.

The efficiency of the non-cooperative case can be improved by using the well known mechanism of pricing, which essentially enforces the SUs to be partially cooperative. Using Equation (3.12) or otherwise, we observe that more aggressive channel strategies by one SU negatively impacts the channel access and throughput of other SUs. Hence, each SU is charged by channel access or usage, and the price is proportional to

$$\pi_m(\Lambda) \triangleq \frac{\frac{M+L-1}{M} T_s p_s (1 - F_m(\lambda_m))}{\tau + \frac{M+L-1}{M} T_s p_s (1 - F_m(\lambda_m))},$$

(3.33)

the effective fraction of time for which the $m$-th SU transmits, as seen in Equation (3.12). We then consider the utility function or the effective throughput as

$$x'_m(\Lambda) \triangleq x_m(\Lambda) - c \cdot \pi_m(\Lambda),$$

(3.34)

for some positive constant $c$. The next result states that the throughput corresponding to the Nash equilibrium thresholds in a game where each SU tries to
maximize its utility function is at least as good as that for the thresholds in the original game.

**Proposition 3.7** ([7, Prop. 5.2]). *For some positive value of $c$, there exists a Nash equilibrium $\tilde{\Lambda}$ for the new game such that $x(\tilde{\Lambda}) \geq x(\Lambda)$ for all Nash equilibrium $\Lambda^*$ of the original game.*

Note that the above proposition does not guarantee a strict improvement. In Section 3.7, we show numerical results that compare the maximum total throughputs in the cooperative case, the original non-cooperative game and using the pricing based scheme.

### 3.5 Interference to PUs

The transmission by SUs causes interference to the PUs – if an SU successfully contends a channel and starts transmitting when the PU is idle and the PU returns during the transmission, it experiences interference for the remaining duration of SU’s transmission. We calculate the fraction of PU’s transmission time for which it experiences interference in the long term when the SUs use pure threshold polices $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_M)$ for accessing the channel. As earlier, we assume for simplicity that the $T_s$ is a multiple of $\tau$ and the sensing and transmission boundaries of SUs are synchronized to multiples of $\tau$. We also assume that PU returns only at multiples of time $\tau$, which is reasonable when $\tau$ is much smaller than expected idle time $\frac{1}{a}$. After a PU returns, it experiences a collision for time $(T_s - j\tau)$ if a SU successfully contends and starts transmitting on the channel in the $(j+1)$-th time slot before the PU returns, for $j = 0, 1, \ldots, \left\lfloor \frac{T_s}{\tau} \right\rfloor - 1$. The probability of such an event is $e^{-a(j+1)\tau} \sum_{i=1}^{M-1} (1 - F_i(\lambda_i))$. Hence, the expected value of the collision time $T_c$ is

$$E[T_c] = \frac{1}{L} \left( 1 - \frac{1}{L} \right)^{M-1} \sum_{i=1}^{M} (1 - F_i(\lambda_i)) \sum_{j=1}^{\left\lfloor \frac{T_s}{\tau} \right\rfloor - 1} e^{-a(j+1)\tau} (T_s - j\tau)$$

$$= \frac{1}{L} \left( 1 - \frac{1}{L} \right)^{M-1} \sum_{i=1}^{M} (1 - F_i(\lambda_i)) \cdot \left( \frac{T_s e^{-a\tau}}{1 - e^{-a\tau}} - \frac{\tau e^{-2a\tau} (1 - e^{-aT_s})}{(1 - e^{-a\tau})^2} \right). \quad (3.35)$$
Note that $E[T_c] \approx \frac{1}{L}(1 - \frac{1}{L})^{M-1} \sum_{i=1}^{M}(1 - F_i(\lambda_i)) \cdot (T_s - \frac{1-e^{-aT_s}}{a} + \tau e^{-2aT_s}(1 - e^{-aT_s})) \cdot b \cdot T_s$ when $\tau \ll \frac{1}{a}$. 

Recall that the expected transmission time of a PU is $\frac{1}{L}$. Hence, the fraction of time for which a PU experiences collision or interference is

$$\eta_c \triangleq bE[T_c] = \frac{1}{L}(1 - \frac{1}{L})^{M-1} \sum_{i=1}^{M}(1 - F_i(\lambda_i)) \cdot \left(\frac{T_s e^{-aT_s} - \tau e^{-2aT_s}(1 - e^{-aT_s})}{1 - e^{-aT_s}}\right) \cdot b.$$ 

(3.36)

It is easy to see by taking derivatives with respect to $T_s$ or otherwise, that $\eta_c$ is an increasing function of $T_s$. Hence, if the interference is high, $T_s$ may be decreased. Alternatively, the thresholds $\Lambda$ may also be raised to reduce SUs’ transmissions and thereby decrease interference.

### 3.6 Accurate evaluation of throughputs using Markov chains

The expressions for throughput $x_m(\Lambda)$ of the $m$-th SU in Equation (3.22) is approximate due to the assumption that between any two consecutive transmissions on a particular channel, it spends time $(L-1)T_s$ transmitting on the remaining channels. To verify the accuracy of this approximation, we consider a more accurate evaluation of the throughputs using a Markov chain model along the lines of [8]. We have already assumed that the sensing and transmissions of SUs are synchronized and occur at multiples of $\tau$. However, the idle-busy states of the SUs in different time slots do not form a (one-step) Markov chain and in fact has infinite memory. Hence, similar to [8], we assume that the transmission times of the SUs are not exactly $T_s$, but instead the number of time slots over which it transmits is Geometrically distributed with parameter $\frac{\tau}{T_s}$, so the expected time is $T_s$. The advantage of this assumption is that due to the memoryless property of Geometric (and Exponential) distributions, the probability that a SU transmits for another time slot is simply $\frac{\tau}{T_s}$, independent of time for which it has transmitted so far. Thus, we are in a position to model the transmissions of SUs using a Markov chain. For any given time slot, we track the idle-busy states of the $M$ SUs using a $M$-length 0-1 vector $(b_1, b_2, \ldots, b_M)$ where $b_m = 1$ indicates that the $m$-th SU is transmitting
in that time slot and \( b_m = 0 \) indicates that it is contending for a channel in that slot. We calculate the probability of transition from a state \((b_1, \ldots, b_M)\) in one time slot to a state \((b'_1, \ldots, b'_m)\) in the next time slot. For this, we use the notation \( M_1^{(m)} = \sum_{i \neq m} b_i \) to denote the number of transmitting SUs in the current time slot and \( M_1^{(m)} = (M - 1) - M_1^{(m)} \) to denote the number of competing SUs, not counting the \( m \)-th user. We have \( \Pr((b'_1, \ldots, b'_M)|(b_1, \ldots, b_M)) = \prod_{i=1}^{M} \Pr(b'_i|(b_1, \ldots, b_M)). \)

It is easy to calculate the various transition probabilities. For instance, for the SU \( m = 1 \), \( \Pr(1|(0, b_2, \ldots, b_M)) = (1 - \frac{M_1^{(1)}}{L})(1 - \frac{1}{L})M_6^{(1)}(\frac{b}{a+\tau})e^{-a\tau}(1 - P_{fa})(1 - F_1(\lambda_1)), \)

\( \Pr(0|(0, b_2, \ldots, b_M)) = 1 - \Pr(0|(0, b_2, \ldots, b_M)), \)

\( \Pr(0|(1, b_2, \ldots, b_M)) = \frac{\tau}{T_s}, \) and \( \Pr(1|(1, b_2, \ldots, b_M)) = 1 - \frac{\tau}{T_s}. \) Under the stationary distribution of the Markov chain, the total throughput of all users can then be evaluated as \( x_{mc}(\lambda) = \sum_{b_1 \in \{0,1\}} \cdots \sum_{b_M \in \{0,1\}} \Pr(b_1, \ldots, b_M) \sum_{i=1}^{M} b_i e^{-aT_s} \int_{\lambda_i}^{\infty} r \, dF_i(r). \) Calculation of throughput by simulating this Markov chain is advantageous since we do not have to simulate all the \( L \) channels and track the activities of all \( M \) SUs at all times.

Apart from validating the throughput approximations, we also use the Markov chain model for efficiently simulating the performance of the online algorithm 2.

### 3.7 Numerical results

We present numerical results to evaluate the performance of our schemes. Unless otherwise stated, the values of the various parameters used are \( \rho = 10 \) dB, \( \tau_s = 20 \) ms, \( \tau_p = 20 \) ms, \( T_s = 400 \) ms, \( \frac{1}{a} = 500 \) ms, \( \frac{1}{b} = 250 \) ms and \( P_{fa} = 0.1, M = 5 \) and \( L = 10. \)

#### 3.7.1 Cooperative Setup

In the cooperative setting, we first examine the convergence of the iterative algorithm in Prop. 3.2 for determining the common threshold of the SUs in Table 3.1. We consider a homogeneous case where all SUs have the same average SNR, \( \rho. \) Table 3.1 shows that the convergence of the iterative algorithm is fast and takes a small number of iterations (\(< 5\) in our case) to achieve the convergence. We also
see that when the average SNR ($\rho$) of each SU increases, since the rate $x^*$ increases and $\lambda^* \propto x^*$ by Proposition 3.1, the threshold $\lambda^*$ increases as well.

**Table 3.1:** Convergence of the iterative algorithm in Prop. 3.2 for homogeneous case

<table>
<thead>
<tr>
<th>Average SNR, $\rho$ (dB)</th>
<th>$\lambda^{(0)}$</th>
<th>$\lambda^{(1)}$</th>
<th>$\lambda^{(2)}$</th>
<th>$\lambda^{(3)}$</th>
<th>$\lambda^{(4)}$</th>
<th>$\lambda^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>1.24803</td>
<td>1.51394</td>
<td>1.53620</td>
<td>1.53637</td>
<td>1.53637</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>1.68370</td>
<td>1.96753</td>
<td>1.98729</td>
<td>1.98740</td>
<td>1.98740</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>1.96215</td>
<td>2.24915</td>
<td>2.26697</td>
<td>2.26705</td>
<td>2.26704</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>2.16824</td>
<td>2.45464</td>
<td>2.47099</td>
<td>2.47104</td>
<td>2.47104</td>
</tr>
</tbody>
</table>

We examine the performance of the network as a function of various key operational parameters. We examine the maximal throughput $x^*$ and optimal threshold $\lambda^*$, as a function of the number of channels $L$ (Fig. 3.2) and the number of SUs $M$ (Fig. 3.3) in a homogeneous scenario where $\rho = 10$ dB and other parameters are fixed to the values stated earlier.

In Fig. 3.2, the increase in throughput is shown as a function of increase in the number of channels. When the number of channels is large compared to the number of SUs, the collisions between the SUs is small, and the throughput of each SU saturates to the scenario where there is only one SU. Thus, the total throughput saturates as well. The optimal threshold increases when the number of channels increases. With abundant transmission opportunities, the SUs can be more selective and transmit only when the channel quality is very good.

In Fig. 3.3, when the number of SUs is small relative to the number of channels, the network throughput increases as the number of SUs increases. However, with further increase in the number of SUs, the collisions among the SUs increases and becomes a dominant factor due to which the throughput decreases. We see that the threshold decreases monotonically with increase in the number of SUs. When the number of SUs is large, the SUs readily use the limited transmission opportunities by lowering the threshold.
3.7.2 Non-cooperative Setup

In the non-cooperative setting, we examine the convergence of the best response strategies discussed in Subsection 3.4.3 to the Nash equilibrium. We consider a non-homogeneous scenario where $L = 50$, $M = 10$, and the SUs have
Table 3.2: Convergence Behavior of two-stage iterative algorithm

<table>
<thead>
<tr>
<th>SU index</th>
<th>$\lambda_0$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU 1 ($\rho =2$ dB)</td>
<td>0</td>
<td>0.07811</td>
<td>0.07921</td>
<td>0.07923</td>
<td>0.07923</td>
</tr>
<tr>
<td>SU 3 ($\rho =6$ dB)</td>
<td>0</td>
<td>0.13734</td>
<td>0.13946</td>
<td>0.13949</td>
<td>0.13949</td>
</tr>
<tr>
<td>SU 5 ($\rho =10$ dB)</td>
<td>0</td>
<td>0.17035</td>
<td>0.17307</td>
<td>0.17311</td>
<td>0.17311</td>
</tr>
<tr>
<td>SU 7 ($\rho =14$ dB)</td>
<td>0</td>
<td>0.19360</td>
<td>0.19674</td>
<td>0.19679</td>
<td>0.19679</td>
</tr>
<tr>
<td>SU 9 ($\rho =18$ dB)</td>
<td>0</td>
<td>0.21162</td>
<td>0.21509</td>
<td>0.21515</td>
<td>0.21515</td>
</tr>
</tbody>
</table>

different and equally spaced SNRs, \{2, 4, 6, \ldots, 20\} dB. Tables 3.2 and 3.3 show the convergence behavior of the two-stage algorithm and the simplified one-stage algorithm. For both algorithms, the SUs are aware of each others’ strategies at all times and the Nash equilibrium is achieved within a few iterations, with the one-stage algorithm taking slightly longer. Figures 3.4 and 3.5 illustrate the convergence of the thresholds to the Nash equilibrium using the online algorithms 1 and 2. Since each SU calculates its threshold based on implicit estimates of other SUs’ thresholds and does not know them directly, the convergence is slower compared to the first two best response strategies and takes a few thousands of iterations to converge to the Nash equilibrium. The second online algorithm converges faster than the first one by efficiently tracking other SUs’ strategies, while having lesser and practical sensing requirements.

For simulating the second online algorithm, we use the Markov chain model that is computationally efficient to simulate, albeit for a different model of exponential transmission times. Hence, there is a discrepancy of about 0.03 nats/s/Hz in the converged thresholds. As an additional verification of the accuracy of the Markov simulations compared to the exact, but computationally difficult theoretical formula, we have observed a discrepancy of less than 8% in the special case of a homogeneous model with $L = 100$ and for various values of $M$. 
Table 3.3: Convergence behavior of simplified one-stage iterative algorithm

<table>
<thead>
<tr>
<th>SU index</th>
<th>( \lambda_0 )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU 1 (( \rho = 2 ) dB)</td>
<td>0.07798</td>
<td>0.07921</td>
<td>0.07923</td>
<td>0.07923</td>
<td>0.07923</td>
</tr>
<tr>
<td>SU 3 (( \rho = 6 ) dB)</td>
<td>0.13720</td>
<td>0.13946</td>
<td>0.13949</td>
<td>0.13949</td>
<td>0.13949</td>
</tr>
<tr>
<td>SU 5 (( \rho = 10 ) dB)</td>
<td>0.17022</td>
<td>0.17307</td>
<td>0.17311</td>
<td>0.17311</td>
<td>0.17311</td>
</tr>
<tr>
<td>SU 7 (( \rho = 14 ) dB)</td>
<td>0.19348</td>
<td>0.19674</td>
<td>0.19679</td>
<td>0.19679</td>
<td>0.19679</td>
</tr>
<tr>
<td>SU 9 (( \rho = 28 ) dB)</td>
<td>0.21151</td>
<td>0.21508</td>
<td>0.21515</td>
<td>0.21515</td>
<td>0.21515</td>
</tr>
</tbody>
</table>

Figure 3.4: Convergence of online algorithm 1 to Nash equilibrium

3.7.3 Comparison of total throughput and fairness across different settings and schemes

In Fig. 3.6, we compare the cooperative and non-cooperative settings and schemes for respective settings based on utility functions and pricing mechanisms, in terms of their total throughput and fairness. The comparisons are performed for a non-homogeneous case where \( L = 10 \), \( M = 5 \) and the average SNRs \( \rho \) of the SUs are \{1, 3, 5, 20, 25\} dB. The total throughput is maximal in the cooperative
setting with the optimal stopping policy described in Proposition 3.2. However, SUs with weaker average SNR get lesser transmission opportunities to make way for SUs with high SNR, as evident from the fact that the threshold is same across all SUs and illustrated in Fig. 3.6 (c) by the distribution of fractional transmission times. Maximization of the total utility $\sum_m U(x_m)$ for a concave utility function, say $U(x) = 2x^{0.5}$ corresponding to $\alpha = 0.5$ in Equation (3.19) leads to a fairer transmission opportunities to SUs with lower SNRs, reflected in the lower, albeit same threshold across the SUs in Fig. 3.6 (b) and fractional transmission times in Fig. 3.6 (c). On the other extreme, the Nash equilibrium in the non-cooperative setting without pricing yields low total throughput, but nearly uniform distribution of transmission times, shown in Fig. 3.6 (a) and 3.6 (c). The pricing-based best response strategy that charges SUs by the fraction of transmission time yields nearly the same total throughput as cooperative setting for a suitable pricing parameter, but uneven distribution of fractional times.

Since throughput and fairness are both important, for designing channel access strategy for SUs in practice, one uses the best threshold policy based on a concave utility function like $U(x) = \log(x)$ or $U(x) = 2x^{0.5}$ in a cooperative setting.
In a non-cooperative setting, one uses a best response strategy based on a pricing mechanism with a suitable pricing parameter.

**Figure 3.6:** Comparison of cooperative and non-cooperative settings with schemes based on utility functions and pricing mechanisms: (a) total throughput of SUs, $x^*$ (b) thresholds, $\lambda_m^*$ (c) fraction of transmission time, $\pi_m$ (d) throughput of each SU, $x_m^*$

### 3.8 Conclusions

In this chapter, we developed optimal channel-aware opportunistic spectrum access strategies for cognitive radio networks with multiple secondary users. We extended similar works on ad-hoc networks and cognitive radio networks by taking into account the complex interactions between PUs and SUs. We showed
that even with these interactions, the optimal channel access strategies are pure threshold policies in both cooperative and non-cooperative settings, with similar relationships between the optimal thresholds, maximum throughputs and various parameters. Furthermore, we established the existence of Nash equilibrium in the non-cooperative setting and developed best response strategies for SUs that rely only on their local observations and still converge to the equilibria. We studied the tradeoff between total throughput and fairness across cooperative and non-cooperative settings, and how schemes based on utility functions and pricing mechanisms can be used to bridge this gap. We also characterized the interference caused by SUs to PUs. Lastly, we extended a previous Markov model for simulating the network dynamics in the non-cooperative case, as well as validating the approximations. We are hopeful that the techniques and results in this chapter will be useful for deriving channel access strategies for more accurate and complex network models and schemes in future.

This chapter is adapted from Sheu-Sheu Tan, James Zeidler and Bhaskar Rao “Opportunistic Channel-Aware Spectrum Access for Cognitive Radio Networks with Multiple Secondary Users”, Submitted to *IEEE Transactions on Wireless Communications*. 
Chapter 4

Distributed Opportunistic Scheduling for Ad-Hoc Communications Under Delay Constraints

4.1 Introduction

Channel-aware scheduling for achieving the rich diversity inherent in wireless communications has recently emerged as a promising technique for improving spectrum efficiency in wireless networks. Most existing studies along this line require centralized scheduling [59–62], and little work has been done on developing distributed algorithms to provide diversity gains for ad hoc communications, partially due to the challenge of distributed learning of time-varying channel information.

Recent work [7] has taken some initial steps to develop distributed opportunistic scheduling (DOS) to reap multiuser diversity and time diversity in wireless ad hoc networks. The basic idea of DOS is that once a successful channel probing has been made (through a successful channel contention), the successful link may decide to continue data transmission if the observed channel condition is “good”;
otherwise, it may skip the transmission, and let all the links re-contend for the channel, in the hope that some links with better channel conditions can transmit after the re-contention. Intuitively speaking, for time varying channel conditions, different links at different time slots experience different channel conditions. It is likely that after further probing, the channel can be taken by a link with a better channel condition, resulting in higher network throughput. Hence, the multiuser diversity across links and the time diversity across slots can be exploited in a joint manner. On the other hand, each channel probing comes with a cost in terms of time. Clearly, there is a tradeoff between the throughput gain from better channel conditions and the cost for further channel probing. The desired tradeoff boils down to judiciously choosing the optimal stopping time for channel probing and the transmission rate, in the sense of maximizing the overall network throughput. Using the optimal stopping theory [9], it has been shown in [7] that the optimal scheme turns out to be a pure threshold policy, and the threshold is the optimal network throughput.

While significant progress has been made on opportunistic scheduling algorithms, scheduling with delay guarantees is still not well understood. Many wireless applications, such as multimedia traffic, have stringent delay requirements. For example, a VOIP application typically requires an average delay less than 200ms to maintain a normal conversation. For other applications, such as live video streaming or monitoring traffic, the tolerance to delay is even smaller, and there may be a limited lifetime during which a packet’s information remains valid. Then, the delivery of such packets has to be before the deadline, because otherwise the information would become outdated and useless.

Unfortunately, little work has been done on developing distributed opportunistic scheduling while taking delay into consideration. Without delay constraints, it is possible that the system may spend an arbitrarily long period of time on channel probing, looking for better channel conditions. This may significantly degrade the QoS performance of delay-sensitive applications, for which the delay performance is of critical importance. Therefore, distributed opportunistic scheduling must strive not only to maximize the throughput, but also to meet the
delay constraints. Needless to say, delay-driven scheduling is challenging, considering the distributed nature of ad hoc communications. A main objective of this study is to obtain a rigorous understanding of DOS under delay constraints, which is known to be important but hard.

In this chapter, we study DOS under the average delay constraint from two different perspectives, namely, network-wide average delay constraint and user-specific average delay constraint. Specifically, average delay constraint here refers to an ensemble constraint after taking average over many packet transmissions.

First, we consider the average delay constraint from a network-centric point of view, where links cooperate to maximize the overall network throughput, subject to the constraint that the network-wide average probing and transmission time is no greater than a given time constant $\alpha$. We note that the network-wide average delay constraint is applicable to applications such as event monitoring by sensor networks where a group of sensor nodes observe the same phenomenon and try to deliver the same messages to the sink node. Thus, a network-wide constraint is to ensure that every message reaches the sink node by a given deadline. Optimal scheduling under this network-wide average delay constraint is equivalent to a constrained optimal stopping problem. However, the standard techniques for constrained optimal stopping problems [63] cannot be used to solve our problem here. This is because those standard techniques are based on sample-path arguments, but the problem with average constraints involves averaging over many sample paths. Instead, we take a stochastic Lagrangian approach to transform the constrained problem into an unconstrained one. For the case where the rate follows a continuous distribution, we are able to show that the duality gap does not exist. Intuitively speaking, the continuity of channel fading distribution ensures that the channel exhibits sufficient randomization to close the gap between the constrained primal problem and the unconstrained dual problem [64]. We then characterize the corresponding threshold-based optimal scheduling algorithm and its throughput. Somewhat surprisingly, we find there exists a sharp transition for the optimal threshold tied to a critical time constant, $\alpha^*$, in the sense that if $\alpha$ is less than $\alpha^*$, the optimal threshold is upper-bounded by a function of $\alpha$; otherwise,
the imposed delay constraint has no impact on the optimal scheduling, and the optimal threshold is the same as that in the unconstrained case.

Next, we explore distributed scheduling under the average delay constraint from a user centric perspective, where each link seeks to maximize its own throughput subject to the constraint that the expected user delay should be no greater than its own delay constraint $\alpha_m, m = 1, \ldots, M$. We treat the threshold selection problem under the individual delay constraint as a non-cooperative game. We show that the Nash equilibrium exists for this constrained non-cooperative game. Our results reveal that there exists a vector of critical time constants $\{\alpha^*_m\}$, as the counterpart to that in the network-centric case, such that only when all the delay parameters $\alpha_m \geq \alpha^*_m$ can the Nash equilibrium become the one for the unconstrained case. We further provide an iterative algorithm to find the Nash equilibrium and show the convergence.

To the best of our knowledge, this is the first channel-aware distributed opportunistic scheduling under the delay constraints. We believe that these initial steps are useful for developing distributed scheduling for delay sensitive applications.

The rest of the chapter is organized as follows. We present the system model, and provide the background on the distributed opportunistic scheduling without delay constraints in Section 4.2. In Section 4.3, we study DOS under average delay constraints from the network-centric perspective and the user-centric perspective. The numerical results are presented in Section 4.4. Finally, we draw our conclusions and discuss the future work in Section 4.5.

4.2 System Model and Background

We consider a single-hop collocated random access network with $M$ links [7], where link $m$ contends for the channel with probability $p_m, m = 1, \ldots, M$. A collision model is assumed for the random access, where a successful channel contention of a link means that no other links transmit at the same time. Accordingly, the overall successful contention probability, $p_s$, is then given by $\sum_{m=1}^{M} (p_m \prod_{i \neq m} (1 - p_i))$. 
It is clear that the number of slots (denoted as $K$) for a successful channel contention is a Geometric random variable, i.e., $K \sim \text{Geometric}(p_s)$. Let $\tau$ denote the duration of a mini-slot for channel contention. It follows that the random duration corresponding to one round of successful channel contention is $K \tau$, with expectation of $\tau/p_s$. For convenience, we call the random duration of achieving a successful channel contention as one round of channel probing.

To get a more concrete sense of the dynamics of joint channel probing and distributed scheduling, we depict in Fig. 4.1 a sample realization of $N$ rounds of channel probing and one single data transmission. Let $s(n)$ denote the successful link at the $n$-th round of channel probing (successful channel contention) and $R_{n,s(n)}$ denote the corresponding transmission rate. Specifically, suppose after the first round of channel probing with a duration of $K_1$ slots, $R_{1,s(1)}$ is small due to poor channel condition, $s(1)$ will then give up its transmission opportunity and let all the links re-contend. This probing process continues for $N$ rounds until link $s(N)$ transmit as link $s(N)$ has a high transmission rate with good channel condition.

![Figure 4.1: Realization of channel probing and data transmission](image)

In a wireless network, $R_{n,s(n)}$ is random since it depends on the time varying channel conditions. We assume that $R_{n,s(n)}$, $n = 1, 2, \ldots$ are statistically independent, which in general holds in many practical scenarios of interest. For convenience, let $R_n$ denote the transmission rate corresponding to the $n$-th round successful channel probing, i.e., $R_n = R_{n,s(n)}$. Accordingly, $R_n$ has the following
compound distribution [7]:

\[ P(R_n \leq r) = \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} F_m(r), \quad (4.1) \]

where \( p_{s,m} \triangleq p_m \prod_{i \neq m} (1 - p_i) \) is the successful probing probability of user \( m \), and \( F_m(\cdot) \) denotes the distribution for each link \( m \in \{1, 2, \ldots, M\} \). In this study, we assume that \( F_m(r) \) is continuous in \( r \). Later, we will see that this continuous assumption on the channel statistics will ensure that there is no duality gap when the constrained problem is relaxed to an unconstrained dual problem, and hence is of critical importance to the existence of optimal solutions for the constrained problem.

### 4.2.1 Background: DOS Without Delay Constraint

In [7], we have studied distributed opportunistic scheduling (DOS) without delay constraint. Specifically, we have shown that the throughput maximization problem can be cast as a maximal rate of return problem in the optimal stopping theory [9,45], where the rate of return is the average network throughput, \( x \), given by

\[ x = \frac{E[R_N T]}{E[T_N]} \text{ where } T_n \triangleq \sum_{j=1}^{n} K_j \tau + T. \quad (4.2) \]

Note that \( N \) is a stopping time if \( \{N = n\} \) and is \( \mathcal{F}_n \)-measurable, where \( \mathcal{F}_n \) is the \( \sigma \)-field generated by \( \{(\rho|h_j|^2, K_j), j = 1, 2, \ldots, n\} \).

The distributed opportunistic scheduling algorithm that maximize the network throughput is given by the optimal stopping rule, \( N^* \), that solves the maximal rate of return problem in (4.2), i.e.,

\[ N^* \triangleq \arg \max_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \quad (4.3) \]

where

\[ Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}. \quad (4.4) \]

We have shown that the optimal stopping rule for the above problem can be found if the channel coefficients \( \{h_n, n = 1, 2, \ldots\} \) are independent, and \( \rho \) is finite. For completeness, we restate the following result from [7].
### Lemma 4.1

The optimal stopping rule $N^*$ exists, and is given by
\[ N^* = \min\{n \geq 1 : R_n \geq x^*\}. \]

The threshold in (4.5) is the maximum throughput $x^*$, which is the unique solution to
\[ E(R - x)^+ = \frac{x\tau}{p_s T}, \]
where $R$ is a random variable and has the same distribution as $R_n$.

### 4.3 DOS Under Average Delay Constraints

In this section, we generalize the above study to the case with average delay constraints. Specifically, we study the average time constraint from two different perspectives: a network-wide average delay constraint, or individual average delay constraints.

#### 4.3.1 DOS under the Network-Centric Average Delay Constraint

In this section, we treat distributed opportunistic scheduling under average delay constraint from a network-centric perspective. To this end, we formulate the DOS as a cooperative game in which all the links collaborate to maximize the overall network throughput under the network-wide average delay constraint, which equivalent to impose an additional constraint on $E[T_N]$.

We have the following problem:
\[ \mathbf{P}_c : \max_{N \in Q_\alpha} \frac{E[R_N T]}{E[T_N]}, \]
where
\[ Q_\alpha \triangleq \{N : N \geq 1, E[T_N] \leq \alpha\}. \]
Comparing $Q_\alpha$ with $Q$ in (5.19), it can be seen that the newly imposed delay constraint dictates that the average duration of channel probing and transmission must be no greater than $\alpha$. 

---

**Note:** The text is a continuation of the previous page, focusing on Lemma 4.1 and the section on DOS Under Average Delay Constraints. The mathematical expressions are formatted as per the guidelines, ensuring clarity and readability.
For convenience, define

\[ N_\alpha^* = \arg \max_{N \in Q_\alpha} \frac{E[R_NT]}{E[T_N]}, \quad x_\alpha^* \triangleq \sup_{N \in Q_\alpha} \frac{E[R_NT]}{E[T_N]}. \]  

(4.9)

Clearly, \( \alpha \) has to be greater than \( \frac{\tau}{p_s} + T \), because even if the data transmission follows immediately after a successful channel contention regardless of the channel condition, it takes a duration of \( \frac{\tau}{p_s} + T \) on average for the network to transmit a data packet. In other words, \( \frac{\tau}{p_s} + T \) is the minimum achievable average delay.

Let \( F_R(r) \) denote the continuous distribution of \( \{R_n\} \) and \( \alpha^* \) denote the unique solution to the following equation (in \( y \)):

\[
E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(y-T)} \right) \right)^+ \right] = \frac{\tau}{p_s T} F_R^{-1} \left( 1 - \frac{\tau}{p_s(y-T)} \right) .
\]

(4.10)

We have the following result regarding the optimal scheduling policy \( N_\alpha^* \) and the optimal throughput \( x_\alpha^* \) for the constrained optimization problem \( P_c \).

**Proposition 4.1.** I) When \( \alpha < \alpha^* \), the optimal scheduling policy \( N_\alpha^* \) is given by

\[
N_\alpha^* = \min \left\{ n \geq 1 : R_n \geq F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha-T)} \right) \right\} ,
\]

(4.11)

and the optimal throughput, \( x_\alpha^* \), is given by

\[
x_\alpha^* = \frac{T}{\alpha} F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha-T)} \right) + \frac{p_s T}{\alpha} \left( 1 - \frac{T}{\alpha} \right) E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha-T)} \right) \right)^+ \right].
\]

(4.12)

II) When \( \alpha \geq \alpha^* \), we have that \( N_\alpha^* = N^* \) and \( x_\alpha^* = x^* \), where \( N^* \) and \( x^* \) are shown in (4.5) and (5.3), respectively.

**Proof:** The proof of Proposition 4.1 hinges heavily on the tools in optimal stopping theory [9] and Lagrange duality theory [66]. We derive the optimal solution in the following four steps.
Step 1: Based on Theorem 1 in [9, Chapter 6], the main problem (4.7) is first transformed into the following problem:

$$\max E[R_N T] - xE[T_N], \text{ subject to } N \in Q_\alpha.$$  \hspace{1cm} (4.13)

Let $\phi(\alpha)$ denote the optimal value for the primal problem (4.13).

Step 2: Next, consider the problem (4.13) with Lagrangian relaxation:

$$\max E[R_N T] - (x + \lambda)E[T_N] + \lambda \alpha, \text{ subject to } N \in Q.$$  \hspace{1cm} (4.14)

Define

$$N^*(x, \lambda) \triangleq \arg \max_{N \in Q} E[R_N T] - (x + \lambda)E[T_N],$$  \hspace{1cm} (4.15)

and

$$V^*(x, \lambda) \triangleq E[R(N^*(x, \lambda))T] - (x + \lambda)E[T_{N^*(x, \lambda)}].$$  \hspace{1cm} (4.16)

Following the same procedure as in Lemma (4.1), it can be shown that

$$N^*(x, \lambda) = \min \{n \geq 1 : R_N \geq x + \lambda + \frac{V^*(x, \lambda)}{T} \},$$  \hspace{1cm} (4.17)

and

$$E \left[ \left( R_N - (x + \lambda) - \frac{V^*(x, \lambda)}{T} \right)^+ \right] = \frac{(x + \lambda)\tau}{T p_s}.$$  \hspace{1cm} (4.18)

Consequently, from the corollary 3.1 of [7], we have that

$$E[T_{N^*(x, \lambda)}] = \frac{\tau}{p_s \left[ 1 - F_R \left( x + \lambda + \frac{V^*(x, \lambda)}{T} \right) \right]} + T.$$  \hspace{1cm} (4.19)

Step 3: Solve the dual problem:

$$\psi(\alpha) \triangleq \min_{\lambda \geq 0} L_x(\lambda), \text{ subject to } \lambda \geq 0,$$  \hspace{1cm} (4.20)

where the dual objective is given by

$$L_x(\lambda) = V^*(x, \lambda) + \lambda \alpha.$$  \hspace{1cm} (4.21)

Let $\lambda^*(x) \triangleq \arg \min_{\lambda \geq 0} L_x(\lambda)$. By the complementary slackness condition in Theorem 4 in [64] and (4.19), we have that

$$\lambda^*(x) \left[ \frac{\tau}{p_s \left[ 1 - F_R \left( x + \lambda^*(x) + \frac{V^*(x, \lambda^*(x))}{T} \right) \right]} \right] + \lambda^*(x)T - \lambda^*(x)\alpha = 0.$$  \hspace{1cm} (4.22)
Step 4: By Theorem 1 in [9, Chapter 6], the optimal throughput \( x^* \) can be achieved if the the optimal value for the primal problem is 0. Accordingly, if there is no duality gap, the optimal value for the dual problem should be 0 too. Therefore, we characterize \( x^*_\alpha \) by solving the following equation:

\[
L_x(\lambda^*(x)) = 0. \tag{4.23}
\]

To this end, we consider the following two cases.

Case 1: If \( \lambda^*(x^*_\alpha) > 0 \), then it follows from (4.22) that

\[
x^*_\alpha + \lambda^*(x^*_\alpha) + \frac{V^*(x^*_\alpha, \lambda^*(x^*_\alpha)}{T} = F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right). \tag{4.24}
\]

Combining (4.24) and (4.18) yields that

\[
E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right)^+ \right] = \frac{(x^*_\alpha + \lambda^*(x^*_\alpha))\tau}{p_s T}. \tag{4.25}
\]

Using (4.25) in (4.24), we have that

\[
V^*(x^*_\alpha, \lambda^*(x^*_\alpha)) = TF_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right)
- \frac{p_s T^2}{\tau} E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right)^+ \right]. \tag{4.26}
\]

It then follows from (4.25) and (4.26),

\[
L_x(\lambda^*(x^*_\alpha)) = V^*(x^*_\alpha, \lambda^*(x^*_\alpha)) + \lambda^*(x^*_\alpha)\alpha
= TF_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right)
- \frac{p_s T^2}{\tau} E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right)^+ \right]
+ \alpha \left[ \frac{p_s T}{\tau} E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right)^+ \right] - x^*_\alpha \right]
\]

Since \( L_x(\lambda^*(x)) = 0 \), we have that

\[
x^*_\alpha = \frac{T}{\alpha} F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right)
+ \frac{p_s T}{\tau} \left( 1 - \frac{T}{\alpha} \right) E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right)^+ \right]. \tag{4.28}
\]
Note that
\[
\lambda^*(x^*_\alpha) = \frac{p_s T}{\tau} (4.29)
\]
\[
\times E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s (\alpha - T)} \right) \right)^+ \right] - x^*_\alpha > 0.
\]
It follows that (4.29) is equivalent to
\[
\frac{p_s T}{\tau} E \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s (\alpha - T)} \right) \right)^+ \right].
\] (4.30)

\textbf{Case 2:} Otherwise, if \( \lambda^*(x^*_\alpha) = 0 \), we have that
\[
L_x(\lambda(x^*_\alpha)) = V^*(x^*_\alpha, 0) = 0. \] (4.31)

It follows from (4.18) that
\[
E \left[ (R_N T - x^*_\alpha T)^+ \right] = \frac{x^*_\alpha T}{p_s}, \] (4.32)
which is exactly (5.3). Therefore, \( x^*_\alpha = x^* \).

Observing that the left side of (4.31) is monotonically strictly increasing (by the continuous assumption of \( F_R(r) \)) in \( \alpha \) from 0 to \( \infty \) as \( \alpha \) grows from \( \frac{\tau}{p_s} + T \) to \( \infty \), and the right side is monotonically strictly decreasing \( \frac{p_s T}{\tau} E \left[ R_{(\alpha)} T \right] \) to 0, therefore, there exists a unique \( \alpha^* \) that is the solution of (4.10). And when \( \alpha < \alpha^* \), we have Case 1, and when \( \alpha \geq \alpha^* \), we have Case 2. Using the above results, it can also be verified that the conditions in Theorem 4 in [64] are satisfied due to the continuity of \( F_R(r) \), and therefore, there is no duality gap. The proof is concluded. \( \square \)

\textbf{Remarks:}

1. The above result reveals that the optimal stopping rule under the network-wide average time constraint scenario, \( N_{\alpha^*} \), is a pure threshold policy and the threshold hinges heavily on the time constraint \( \alpha \). Interestingly, there exists a sharp transition associated with the critical time constant, \( \alpha^* \), in the sense that if \( \alpha \) is less than \( \alpha^* \), the optimal threshold depends on \( \alpha \); otherwise, the imposed constraint has no impact on the optimal scheduling, and the optimal policy remains the same as if the time constraint were removed.
2. We observe from the proof that under the continuous assumption on the channel statistics, the strong duality holds. In general, \( \psi(\alpha) \) is the concave hull of \( \phi(\alpha) \) that is not necessarily concave \([64]\) (see Fig. 4.2 for a pictorial illustration). Accordingly, there may exist duality gaps. Somewhat surprisingly, it turns out when the rate distribution is continuous, \( \psi(\alpha) \) coincides with \( \phi(\alpha) \). Our intuition is as follows: when the channel exhibits sufficient randomization, then there is no duality gap, and solving the relaxed problem is equivalent to solving the primal problem. In contrast, when the channel distribution is discrete, the duality gap is zero only at countably many points \([67]\). We note that the underlying rationale key idea behind the above result is kin to the hidden convexity property established in \([68,69]\).

\[ \psi(\alpha) = \phi^{**}(\alpha) \]

\[ \phi(\alpha) \]

\[ L_\lambda(\lambda) \]

3. Observe that to compute the optimal threshold offline, network-wide channel statistical information is required. We note that an online algorithm can be developed, based on local information only, to find estimate the optimal
threshold similar to that in [7].

### 4.3.2 An Iterative Algorithm for Computing $\alpha^*$

Comparing (5.3) and (4.10), it is easy to see that

$$x^* = F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha^* - T)} \right),$$  \hspace{1cm} (4.33)

or equivalently,

$$\alpha^* = T + \frac{\tau}{p_s(1 - F_R(x^*))}.$$  \hspace{1cm} (4.34)

Based on (4.34), we can use the iterative algorithm developed in [7] to find $\alpha^*$. More specifically, we first use the following iterative algorithm to compute $x^*$:

$$x_{k+1} = \Phi(x_k) \text{ for } k = 0, 1, 2, ..., \hspace{1cm} (4.35)$$

where $\Phi(x) = \int_{\frac{x}{p_s}}^{\infty} r dF_R(r)$ and $x_0$ is a positive initial value. Then, $\alpha^*$ can be obtained from (4.34).

Since $F_R^{-1} \left( 1 - \frac{\tau}{p_s(x-T)} \right)$ is monotonically increasing in $x$, based on the above relationship of $x^*$ and $\alpha^*$, we can further simplify the optimal threshold policy presented in Prop. 4.1 as follows.

**Corollary 4.1.** The optimal scheduling policy $N^*_\alpha$ is given by

$$N^*_\alpha = \min \left\{ n \geq 1 : R_n \geq \min \left( x^*, F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right) \right\}, \hspace{1cm} (4.36)$$

and the corresponding optimal throughput $x^*_\alpha$ is given by

$$x^*_\alpha = x^* \mathbf{I}(\alpha \geq \alpha^*) + \frac{T}{\alpha} F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \mathbf{I}(\alpha < \alpha^*) + \frac{p_s T}{\tau} \left( 1 - \frac{T}{\alpha} \right) \mathbf{E} \left[ \left( R_N - F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \right)^+ \right] \mathbf{I}(\alpha < \alpha^*), \hspace{1cm} (4.37)$$

where $\mathbf{I}(\cdot)$ is an indicator function, and $x^*$ and $\alpha^*$ are given in (5.3) and (4.10), respectively.
4.3.3 DOS under Individual Average Delay Constraints

In the above section, the average delay constraint is with respect to the whole network. In some cases, different users may have different individual delay requirements. Therefore, it is of great interest to study the impact of the individual average time constraint. More specifically, we focus on DOS from a user-centric perspective, where each link seeks to maximize its own throughput under its individual average delay constraint $E[T_m] \leq \alpha_m$, for $m = 1, ..., M$. To this end, we treat joint channel probing and distributed scheduling as a non-cooperative game, where each user chooses its threshold in a selfish manner to maximize its own throughput, subject to its own individual average time constraint.

For a given set of thresholds across links, $\{\phi_m, m = 1, 2, ..., M\}$, it is easy to see that the expected channel probing and transmission time for link $m$, $E[T_m]$, is given by [7]

$$E[T_m] = \tau + \sum_{i \neq m}^{M} p_{s,i} (1 - F_i(\phi_i)) T_p + T. \quad (4.38)$$

Here, the effective channel probing time is $\tau + \sum_{i \neq m}^{M} p_{s,i} (1 - F_i(\phi_i)) T_p$ and the data transmission time is $T$. The average throughput of link $m$ can be expressed as (see Lemma 4.1 in [7])

$$\vartheta_m(\Theta) = \frac{p_{s,m} \int_{\phi_m}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(\phi_i))}, \quad (4.39)$$

where $\Theta = \{\phi_1, \phi_2, ..., \phi_M\}$ is the threshold vector.

Following [7], we cast the threshold selection problem across different links as a non-cooperative game, in which each individual link chooses its threshold $\phi_m$ to maximize its own throughput, $\vartheta_m$, given its own average probing and transmission time constraint, $\alpha_m$. Let $G = \{\{1, 2, ..., M\}, \times_{m \in \{1, 2, ..., M\}} A_m, \{\vartheta_m, m \in \{1, 2, ..., M\}\}$ denote the non-cooperative threshold selection game. The links in $\{1, 2, ..., M\}$ are the players of the game, $A_m = \{\phi_m | 0 \leq \phi_m < \infty\}$ is the action set of the player $m$, and $\vartheta_m$ is the utility function for the player $m$. The non-cooperative game can be formulated as follows:

$$(G) \max_{\phi_m \in A_m} \vartheta_m(\Theta), \quad \text{subject to } E[T_m] \leq \alpha_m, \quad (4.40)$$

$\forall m = 1, 2, ..., M.$$
4.3.4 Nash Equilibrium under Individual Average Delay Constraints

Next, we investigate the corresponding Nash equilibrium under individual average delay constraints.

**Definition 3.1.** A threshold vector $\mathbf{\phi}^* = \{\phi_1^*, \phi_2^*, ..., \phi_M^*\}$ is said to be a Nash equilibrium of game $G$ if

$$
\theta_m(\phi_m^*, \mathbf{\phi}^*) \geq \theta_m(\phi_m, \mathbf{\phi}^*), \forall \phi_m \in A_m,
$$

where $\mathbf{\phi}^* \overset{\Delta}{=} [\phi_1, ..., \phi_{m-1}, \phi_{m+1}, ..., \phi_M]^T$.

Definition 3.1 reveals that when the Nash equilibrium is achieved, no link can increase its throughput by changing its threshold from the equilibrium, given the thresholds of other links.

Along the same line as in network-centric case, we need the following conditions on each individual average delay constraint $\alpha_m$:

$$
\alpha_m \geq \frac{\tau + \sum_{i \neq m}^{M} p_{s,i} T}{p_{s,m}} + T.
$$

(4.42)

To this end, we examine the existence of Nash equilibrium. Using Prop. (4.1), we have the following result on the existence of the Nash equilibrium for the game $G$.

**Proposition 4.2.** Under the condition in (4.42), there exists a Nash equilibrium for the game $G$, and

$$
\phi^*_m = \min \left( x^*_m, \frac{F_R^{-1} \left( 1 - \frac{\tau + \sum_{i \neq m}^{M} p_{s,i} (1 - F_i(\phi^*_i) T)}{p_s(\alpha_m - T)} \right) }{p_{s,m}} \right),
$$

(4.43)

where $x^*_m$ satisfies the following equation:

$$
x^*_m = \frac{p_{s,m} \int_{x_m'}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(\phi^*_i))}.
$$

(4.44)

The proof follows directly from [7].
It is not surprising to see, Prop. 4.2 reveals that the optimal threshold in
the Nash equilibrium is upper-bounded by a function of the time constraint \( \alpha_m \),
and other users’ thresholds. When \( \alpha_m \to \infty, \forall m \), we have that
\[
\phi_m^* \to x_m^* \to x_m^*,
\]
where \( x_m^* \) can be computed by
\[
x_m^* = \frac{p_{s,m} \int_{x_m^*}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(x_i^*))}, \forall m,
\]
which boils down to the unconstrained case in [7].

Define for \( m = 1, \ldots, M \),
\[
\alpha_m^* \triangleq T + \frac{\tau + \sum_{i \neq m} p_{s,i} (1 - F_i(x_i^*))T}{p_s (1 - F_m(x_m^*))}.
\]
As the counterpart to the critical time constant \( \alpha^* \) in the network-centric case,
the vector \( \{\alpha_m^*\} \) defined above serves as the critical time constant vector for the
individual average delay constraint case. Particularly, we have the following result:

Proposition 4.3.
\[
\phi_m^* = x_m^* \text{ if and only if } \alpha_m \geq \alpha_m^*, \forall m.
\]

Proof: If \( \alpha_m \geq \alpha_m^*, \forall m \), it is straightforward to examine that \( \phi_m^* = x_m^* \) satisfies
(4.43) and (4.44). On the other hand, if there exists any \( m \) such that \( \alpha_m < \alpha_m^* \),
then it can be seen from (4.43) that \( \phi_m^* < x_m^* \), and the proof is concluded.

Prop. 4.3 reveals that only when all the time constants are larger than the
 corresponding critical time constants can the Nash equilibrium points under the
 individual average delay constraint belong to that for the unconstrained case.

4.3.5 Iterative Algorithm for Finding Nash Equilibrium

Based on (4.43), we have the following best response strategy to compute
the Nash equilibrium:
\[
\phi_m(k + 1) = \min(x_m^*(k), F^{-1}_R(1 - \frac{\tau + \sum_{i \neq m} p_{s,i} (1 - F_i(\phi_i(k)))T}{p_s (\alpha_m - T)})),
\]
(4.47)
and \( x^*_m(k) = \frac{p_{s,m} \int_{x^*_m(k)}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(\phi_i(k)))} \),
for \( k = 0, 1, 2, \ldots \) \( \forall m = 1, 2, \ldots, M \). \hfill (4.48)

The following proposition establishes the convergence of the above best response strategy.

**Proposition 4.4.** Suppose that the Nash equilibrium is unique. Then, for any non-negative initial value \( \Theta(0) \), the sequence \( \{\Theta(k)\} \), generated by the iterative algorithm in (4.47), converge to the Nash equilibrium \( \Theta^\ast \), as \( k \to \infty \). The proof follows directly from [7].

### 4.4 Numerical Results

In this section, we study, by numerical examples, the performance of the DOS under delay constraints. Unless otherwise specified, we assume that \( \tau, T, p_m \) and \( M \) are chosen such that \( \delta = \tau/T = 0.1, p_s = \exp(-1) \). We consider the continuous rate case only, assuming that the instantaneous rate is given by the Shannon channel capacity, i.e.,

\[
R_n = \log(1 + \rho|h_n|^2) \text{ nats/s/Hz},
\hfill (4.49)
\]

where \( \rho \) is the normalized average SNR, and \( h_n \) is the random channel coefficient with a complex Gaussian distribution \( \mathcal{CN}(0, 1) \). Accordingly, the distribution of the transmission rate is given by

\[
F_R(r) = 1 - \exp\left(-\frac{\exp(r) - 1}{\rho}\right). \hfill (4.50)
\]

#### 4.4.1 The Case with Network-Wide Average Delay Constraint

We first provide numerical examples for DOS under the network-wide average delay constraint. It follows from (4.37) and (4.50) that the maximal throughput, \( x^*_\alpha \), can be expressed as in (4.51),

\[
x^*_\alpha = x^* \mathbb{I}(\alpha \geq \alpha^\ast) + \frac{\gamma^T}{\alpha} \left[ \frac{1}{\rho} \right] \gamma \exp\left(-\frac{\exp(\gamma)}{\rho}\right) + E_1\left(\frac{\exp(\gamma)}{\rho}\right) \right] \left[ -CT\gamma \left(\exp\left(-\frac{\exp(\gamma) - 1}{\rho}\right)\right) \right] \mathbb{I}(\alpha < \alpha^\ast), \hfill (4.51)
\]
where \( \gamma = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right), C = \frac{p_s}{\tau}(1 - \frac{T}{\alpha}) \), \( x^* \) satisfies the following fixed point equation:

\[
x^* = \frac{p_s}{\delta} \exp\left(\frac{1}{\rho}\right) E_1\left(\frac{\exp(x^*)}{\rho}\right),
\]

and \( E_1(x) \) is the exponential integral function defined as

\[
E_1(x) \triangleq \int_x^{\infty} \frac{\exp(-t)}{t} dt.
\]

**Figure 4.3:** Maximal throughput for different values of \( \alpha \) at different \( \rho \)

In Fig. 4.3, we compare the optimal throughput \( x^*_\alpha \) under delay constraints, against that corresponding to two other schemes: 1) the throughput, denoted as \( x_\alpha \), corresponding to the naive threshold policy \( \phi = F_R^{-1}\left(1 - \frac{\tau}{p_s(\alpha - T)}\right) \) that always enforces that the average delay equal the delay constraint, and 2) the throughput, denoted as \( x^* \), corresponding to the optimal threshold policy \( \phi = x^* \) with no delay constraint. We plot them for different \( \alpha \) at different average SNR \( \rho = 5, 10, 15, 20, 25 \). It is clear that \( x^*_\alpha \) is an increasing function of the average SNR \( \rho \) for a given \( \alpha \) as expected. Another important observation from Fig. 4.3 is that \( x_\alpha \) intersects with \( x^* \) at its peak point, which is exactly the critical time constant, \( \alpha^* \). Note that \( x_\alpha = x^*_\alpha \) when \( \tau/p_s + T \leq \alpha \leq \alpha^* \), and \( x^*_\alpha = x^* \) when \( \alpha > \alpha^* \). That is to say, \( \alpha^* \) is the transition point beyond which, the optimal scheduling policy would be the same as that without the delay constraint. Intuitively speaking, the optimal
scheduling policy under delay constraints can be viewed as a marriage between the naive threshold scheme and the optimal threshold scheme without considering the constraint.

Figure 4.4: $E[T_N]$ of network versus average probing and transmission time constraint of $\alpha$

To gain a deeper understanding of the critical time point, $\alpha^*$, we plot in Fig. 4.4 the average probing and transmission time,

$$E[T_N] = \frac{\tau}{p_s(1 - F_R(\phi))} + T,$$

under the above mentioned three schemes. It can be seen from Fig. 4.4 that the average probing and transmission time would always be the same value of $\alpha$, under the threshold policy with $\phi = F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right)$. For the threshold policy with $\phi = x^*$, the average probing and transmission time is a constant regardless of $\alpha$. The critical constraint point, $\alpha^*$ is shown in this figure as the crossing point between these two threshold methods. This also explains the physical meaning of this critical point: when $\alpha \leq \alpha^*$, the threshold policy with $\phi = F_R^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right)$ is optimal since it “pushes” the average time towards the allowed limit to achieve
a higher throughput; when \( \alpha > \alpha^* \), since the threshold policy with \( \phi = x^* \) already satisfies the delay constraint while achieving the maximum throughput, it is also optimal in the case under the delay constraint. This phenomena can be also observed from Fig. 4.3: after the critical point, the throughput is decreasing if the scheduling policy still follows the original time constraint policy with \( \phi = F_{R}^{-1} \left( 1 - \frac{\tau}{p_s(\alpha - T)} \right) \). Therefore, the optimal scheduling policy (shown in circle) under the delay constraint should be the same as the case without the delay constraint, when \( \alpha > \alpha^* \).

Figure 4.5: \( \alpha^* \) for different \( \rho \) at different \( p_s \)

It is clear from the above discussions that the network throughput suffers a loss due to the delay constraint when \( \alpha \) falls into the range of \( [\tau/p_s + T, \alpha^*] \). Thus, \( \alpha^* \) could be used to measure the criticality of the delay constraint: the smaller \( \alpha^* \), the less the criticality of the delay constraint. To get a better understanding of the characteristics of the critical time, we plot in Fig. 4.5 \( \alpha^* \) as a function of \( \rho \) and the contention successful probability \( p_s \). It can be seen from the figure that \( \alpha^* \) is a decreasing function of \( \rho \) for a given \( p_s \). That means the effect of delay constraint is less critical at high SNR. We also observe that \( \alpha^* \) is a decreasing function of \( p_s \) for a given \( \rho \). This is because a larger \( p_s \) indicates that the users can access and probe the channel with smaller cost (in terms of contention time), and hence, can meet
more stringent delay requirements. Overall, Fig. 4.5 suggests that the importance of our proposed approach becomes increasingly prominent in low SNR and/or low successful contention probability regions.

### 4.4.2 The case with Individual Average Time Constraints

In this section, we examine the DOS performance with individual user’s average delay constraint.

**Table 4.1**: Critical time for each individual link

<table>
<thead>
<tr>
<th>Link</th>
<th>$\alpha_m^* (p_m = 0.125)$</th>
<th>$\alpha_m^* (p_m = 6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 1</td>
<td>66.12 ($\rho_1 = 2$)</td>
<td>82.38 ($p_1 = 0.075$)</td>
</tr>
<tr>
<td>Link 2</td>
<td>64.81 ($\rho_2 = 4$)</td>
<td>69.51 ($p_2 = 0.100$)</td>
</tr>
<tr>
<td>Link 3</td>
<td>64.14 ($\rho_3 = 6$)</td>
<td>62.62 ($p_3 = 0.125$)</td>
</tr>
<tr>
<td>Link 4</td>
<td>63.70 ($\rho_4 = 8$)</td>
<td>58.84 ($p_4 = 0.150$)</td>
</tr>
<tr>
<td>Link 5</td>
<td>63.39 ($\rho_5 = 10$)</td>
<td>56.96 ($p_5 = 0.175$)</td>
</tr>
</tbody>
</table>

We first examine in Table 4.1 the behavior of $\alpha_m^*$ with fixed contention probability of $p_m = 0.125$ but with different average SNR ($\rho_m$) across links. The result shows that the link with higher average SNR has a lower $\alpha_m^*$ for the same reasoning explained in Fig. 4.5. We next examine the behavior of critical time with a fixed average SNR, but with the different contention probability ($p_m$) across links. It can be observed that the link with higher contention probability has a lower critical time constant. Similar to the reason for the network centric case, the link with higher contention probability has the lower unconstrained expected probing and transmission time, and therefore the time constraint is less stringent. The results imply that the effects of imposing a constraint on the individual link becomes less significant when the average SNR and/or the contention probability of the link increases.

Needless to say, for a non-cooperative game, a key aspect of study is the characteristic of Nash equilibrium point. Table 4.2 illustrates the convergence
Table 4.2: Convergence Behavior of the best response strategy

<table>
<thead>
<tr>
<th>Link</th>
<th>$x_0$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Link 1 ($\rho = 2$, $\alpha_m = 64.0$)</td>
<td>1.000</td>
<td>0.171</td>
<td>0.097</td>
<td>0.082</td>
<td>0.077</td>
<td>0.077</td>
</tr>
<tr>
<td>Link 2 ($\rho = 4$, $\alpha_m = 64.5$)</td>
<td>1.000</td>
<td>0.260</td>
<td>0.220</td>
<td>0.183</td>
<td>0.176</td>
<td>0.175</td>
</tr>
<tr>
<td>Link 3 ($\rho = 6$, $\alpha_m = 65.0$)</td>
<td>1.000</td>
<td>0.320</td>
<td>0.267</td>
<td>0.264</td>
<td>0.263</td>
<td>0.263</td>
</tr>
<tr>
<td>Link 4 ($\rho = 8$, $\alpha_m = 65.5$)</td>
<td>1.000</td>
<td>0.367</td>
<td>0.302</td>
<td>0.299</td>
<td>0.298</td>
<td>0.298</td>
</tr>
<tr>
<td>Link 5 ($\rho = 10$, $\alpha_m = 66.0$)</td>
<td>1.000</td>
<td>0.405</td>
<td>0.331</td>
<td>0.327</td>
<td>0.326</td>
<td>0.326</td>
</tr>
</tbody>
</table>

behavior of the iterative best response strategy defined in section (4.3.5) with $M = 5$, $p_m = 0.125$ and different $\rho$ and $\alpha_m$ for each individual link. It is clear from Table 4.2 that the thresholds for all the links converge to the equilibrium point within a few ($\approx 5$) iterations. Note that the time constraint for each individual link, $\alpha_m$, in this example is less than its corresponding critical time constant (refer to the corresponding critical point in table (4.1)). Thus, the table shows that the algorithm achieved convergence even when the imposed constraint is less than the corresponding critical time constant.

Prop. 4.2 shows that there exists a Nash equilibrium which satisfy the (4.43), which is verified by Table 4.3. It is clear from the table that when the imposed constraint of all the links, $\alpha_m$, $m = 1, 2, ..., M$, are bigger than the corresponding critical time constraint of $\alpha_m^*$, the threshold for each link with delay constraint ($x_m^*$, shown in forth column) will be the same as the scenario without the constraint ($x_m^{*\star}$, shown in third column). When the imposed delay constraint is less than the $\alpha_m^*$, the threshold (shown in fifth column) will be different than the corresponding $x_m^{*\star}$. 
4.5 Conclusions

In this chapter, we considered an ad-hoc network where many links contend for the channel using random access, and studied distributed opportunistic scheduling (DOS) under average delay constraint from two different perspectives. First, we study DOS with delay constraints from a network-centric perspective, with the objective to maximize the overall throughput subject to the average constraint on the network-wide probing and transmission time. We showed that the optimal DOS strategy under such delay constraint is a pure threshold policy. Specifically, we found that there exist a critical time constant, \( \alpha^* \): if the imposed delay constraint is less than the \( \alpha^* \), the optimal threshold is a function of \( \alpha \); otherwise, the imposed delay constraint has no effect on the optimal scheduling and the optimal policy remains the same as if the delay constraint did not exist. We showed that the critical time \( \alpha^* \) is a decreasing function of average SNR and contention probability. Next, from the user-centric constraint perspective, each individual link has its own average time constraint. In this case, the threshold selection for different
links is treated as a non-cooperative game, in which every link strives to maximize its throughput subject to its own individual delay constraint. We explore the existence of the Nash equilibrium and showed that the Nash equilibrium can be achieved by iterative algorithms. Similar to the network-centric case, we show that there exists a critical time vector \( \{\alpha^*_m\} \) such that only when all delay constraints \( \alpha_m \geq \alpha^*_m, \forall m \), then the delay constraints have no impacts on the optimal threshold in the Nash equilibrium.

In summary, we took some initial steps towards studying channel-aware distribution scheduling in ad hoc networks under a delay constraint for real-time traffic. In particular, we characterized the fundamental tradeoff between the throughput gain from better channel conditions and the cost for channel probing that may cause the delay. As expected, the scheduling policy tends to be more conservative by setting a smaller threshold when the time constraint becomes tighter.

This chapter is adapted from Sheu-Shue Tan, Dong Zheng, Junshan Zhang, and James Zeidler, “Distributed Opportunistic Scheduling for Ad-Hoc Communications Under Time Constraints”, *IEEE Intl. Conference on Computer Communications (INFOCOM)*, pp. 1–9, Mar. 2010.
Chapter 5

Channel Distribution Information Based Cross-Layer Design of an Opportunistic Scheduler for MIMO Networks

5.1 Introduction

One of the key technological advances proposed for next generation communication systems involves the use of multiple antennas at both the transmitting and receiving nodes of the network. Multiple antennas have been shown to be effective for providing spatial filtering, spatial multiplexing and diversity gain [70]. One of the fundamental challenges in exploiting the additional spatial processing gain is the significant difference in the time scale associated with routing and scheduling transmissions between nodes and the time scale with which the channel state information (CSI) must be updated. Hence, it is of interest to develop scheduling protocols that would address the challenge of different time scales.

Scheduling protocols that employ multiple antennas have often been based on the assumption of perfect knowledge of the instantaneous CSI to achieve maximum network throughput [10–14]. CSI represents the ground truth of the commu-
Scheduling protocols that can exploit accurate CSI potentially achieve significant performance gains due to enhanced interference mitigation and due to the temporal diversity gains provided. The use of CSI in PHY layer links generally involves only a small number of nodes, i.e., the transmitting node and its corresponding receiving node. On the other hand, the use of CSI in the MAC layer involves a large number of nodes, i.e., the entire network. Due to this, maintaining accurate CSI in the MAC layer is relatively less feasible compared to that in the PHY layer. This is especially important in decentralized tactical ad hoc networks with high mobility and hostile interference where the channel conditions are time-varying [15, 16]. In addition, CSI acquisition in a multi-user network requires significant amount and frequency of feedback between the users and can thus introduce significant complexity and delay in the design of appropriate scheduling protocols. Although the fast changing CSI can provide time diversity, the difficulties in obtaining accurate CSI makes it infeasible to design a scheduler that uses CSI for the whole network. It is thus imperative to use some other channel information at the MAC layer that would provide stability on a larger time scale.

Researchers have explored the usage of channel distribution information (CDI) at the PHY layer for Multiple-Input-Multiple-Output (MIMO) systems [15–21]. In particular, the work in [16] highlights the importance of using CDI in a mobile system, as the changes in channel spatial correlation are minimal or small in the interval of about 100ms even when the user is moving at 629mph [15]. Results in [16–18] develop CDI-based beamforming as an alternate approach to CSI beamforming [71] in Multi-User Multiple-Input-Multiple-Output (MU-MIMO) systems. The temporal stability of CDI allows for reduced frequency of feedback in time-varying channels. It also allows the MAC and PHY layers to share common information about link characteristics, thereby enabling truly cross-layer optimization of the link schedule. However, CDI has the drawback that it is incapable of capturing the instantaneous channel conditions and harnessing the temporal diversity as CSI does. The MAC layer therefore has the option of a throughput-stability tradeoff depending on the type of information used for scheduling [22, 23].

In this chapter, we present both CSI and CDI designs for new cross-layer op-
portunistic channel access scheduling frameworks in MU-MIMO networks. Specifically, CDI in the form of a full spatial correlation matrix is used in the MAC layer and involves a large number of nodes over a longer time scale that is governed by channel statistics. On the other hand, CSI is used in the individual links in the PHY layer for capturing instantaneous channel conditions. The channel-aware distributed scheduler is formulated as a maximum rate-of-return problem using optimal stopping theory [7, 12] and consists of two main phases. Since MIMO networks have interference suppression ability, multiple links can be scheduled to access/transmit simultaneously. Hence, in the first phase, we divide the links in the network into multiple disjoint groups where each group consists of several links that are considered for simultaneous data transmissions. Ideally, CSI is a good candidate for optimally grouping the simultaneous links. However, since CSI fluctuates rapidly, it is intractable and impractical to use it for grouping the simultaneous links. Thus, CDI is used to group the simultaneous access/transmission links in the first phase. In the second phase, the users of the same group will contend for the channel together and will access the channel together if their sum-rate is above a predefined threshold derived from the optimal stopping theory. CSI is used to decide which group of users should transmit and when.

By further taking into account the tradeoff between feedback requirements in the link context and the system throughput, CSI and/or CDI will be used in beamforming depending on the network conditions and available information. We consider various combinations of CDI and CSI at the transmitters and the receivers. In a practical network, accurate interference information from other users is generally more difficult to obtain than the CSI information for the desired signal at any particular receiver. Hence, we consider a combination where CDI is used for the interfering channel and CSI is used for the channel associated with the desired signal. The concept of a beamforming vector based on CDI at the interfering channel and CSI for the desired signal is new and developed in this chapter.

Our simulation results demonstrate that CDI can be used as an alternative low level temporal stability quantity to increase the network performance in the
MAC and the PHY layer. Our proposed joint CSI-CDI scheduler which is based on CDI-grouping can obtain about 90% network throughput of that achieved by CSI-grouping. Depending on the CSI/CDI information used at the transmitter/receiver in the link context, simulation results show that the network throughput can maintain roughly 68 – 85% of that optimal CSI-based beamformer throughput, but with better stability and less amount of feedback. The results also suggest that the benefit of using CSI in the MAC layer is not significant since CSI can cancel the interference very efficiently at the PHY layer with beamforming. However, CDI with its multi-user diversity and temporal stability can benefit the MAC layer better than CSI. To summarize, by using CDI in the network context and using CSI/CDI on a link basis, we can reap the benefits of both the temporal stability of CDI and time diversity of CSI. We also bridge the gap between the PHY layer’s coherence time when using CSI and the MAC layer’s larger network time scale.

The rest of the chapter is organized as follows. We present the system model in Section 5.2 and provide details of beamforming with CSI/CDI in Section 5.3. We then present the scheduling protocol in Section 5.4. The simulation results are presented in Section 5.5. Finally, we draw our conclusions in Section 6.6.

For notational purposes, throughout this chapter, scalars are written in lower-case, while vectors and matrices are written as bold-face in lower or upper-case, respectively. Standard matrix operations of transpose, conjugate, and conjugate transpose are defined as \{·\}^T, \{·\}^*, and \{·\}^H, respectively. The function \text{vec}(·) represents the matrix column stacking operator, and \text{mat}(·) represents its inverse, i.e., \text{mat}(\text{vec}(A)) = A. Finally, \otimes denotes the matrix Kronecker product.

\section{5.2 System Model}

The single-hop multi-user MIMO \textit{ad hoc} network consists of \(N_s\) active links (users), i.e., \(N_s\) pairs of source-destination (S-D) nodes, \{\(S_k, D_k\)\}, for \(k = 1, 2, 3, ..., N_s\). Specifically, each source (transmitter) and destination (receiver) node is equipped with \(M \geq 2\) antennas. For the sake of clarity, we present a system with \(M = 2\) but the proposed scheme can be generalized to \(M > 2\). The \(N_s\) users in the
network exploit MIMO opportunistic scheduling with contention-based medium access to contend/probe and opportunistically transmit over the MIMO channel. Given this MIMO model we first present the signal model at the PHY layer. Next, we detail the channel distribution information and then describe probing and data transmission at the MAC layer.

5.2.1 Signal Model

For the MIMO signaling at the PHY layer, we focus on the interference channel as shown in Fig. 5.1. The interference channel is modeled as multiple point-to-point connections which cause interference with each other if present.

![Interfering channel](image)

Figure 5.1: Interfering channel.

Let $\mathcal{L}$ denote the set of simultaneous links. For example, the links shown in Fig. 5.1 can be written as the set of $\mathcal{L} = \{\{S_i, D_i\}, \{S_j, D_j\}\}$. For the clarity of presentation, we represent duples in the set of $\mathcal{L}$ as $\{m, n\}$, where $m$ represents the transmit source node, and $n$ represents the receive node. The cardinality of $\mathcal{L}$ represents the total number of simultaneous links.

Consider the link represented by the duple $\{m, n\}$, the received signal can be written as

$$y_{\{m,n\}}(t) = \sqrt{P_{\{m,n\}}}H_{\{m,n\}}(t)b_{\{m,n\}}(t)x_{\{m,n\}}(t)$$

$$+ \sum_{\{i,j\} \neq \{m,n\}} \sqrt{P_{\{i,j\}}}H_{\{i,j\}}(t)b_{\{i,j\}}(t)x_{\{i,j\}}(t) + \eta_{\{m,n\}}(t).$$
In this expression, $P$ is the per-stream fixed transmit power. The matrix $H_{\{m,n\}}(t)$ is the $N_r \times N_t$ channel transfer matrix from the source node $m$ to the destination node $n$, where $N_r$ is the number of receive antennas and $N_t$ is the number of transmit antennas. The vector $b_{\{m,n\}}(t)$ is the $N_t \times 1$ transmit beamforming vector for the $\{m,n\}$ link. The link attempts to transfer a single stream of information represented by the symbol $x_{\{m,n\}}(t)$ with $E[x_{\{m,n\}}(t)]^2 = 1$. The noise $\eta_{\{m,n\}}(t)$ is additive white Gaussian noise (AWGN) with variance of $\sigma^2$. All other links where $\{i,j\} \neq \{m,n\}$ are considered interference to the desired stream. In schemes that use perfect CSI for beamforming constructions, for every change in $H_{\{m,n\}}(t)$, all the beamformers must be updated. This work assumes that the channel remains constant over each transmission duration. Therefore, for notational convenience, the time variable $t$ will be omitted for the channels and beamformers.

In addition to the transmit beamforming vector $b_{\{m,n\}}$, there exists an associated receive beamforming $1 \times N_r$ vector, $w_{\{m,n\}}$ for each receive vector in (5.1), which is used for detection purposes. Without lost of generality, we assume that the beamforming vector is normalized to have unit power, i.e., $\|b_{\{m,n\}}\|^2 = \|w_{\{m,n\}}\|^2 = 1$. Given this received vector and under the assumption that all links will interfere with each other, the signal to interference plus noise ratio (SINR), denoted by $\rho_{\{m,n\}}$, can be calculated as

$$\rho_{\{m,n\}} = \frac{P_{\{m,n\}} | w_{\{m,n\}}^H H_{\{m,n\}} b_{\{m,n\}} |^2}{\sigma^2 + \sum_{\{i,j\} \neq \{m,n\}}^L P_{\{i,j\}} | w_{\{m,n\}}^H H_{\{i,n\}} b_{\{i,j\}} |^2}. \quad (5.2)$$

The sum-rate of the simultaneous transmission links in the interference channel can then be expressed as

$$C = \sum_{\{m,n\}}^L \log \left(1 + \rho_{\{m,n\}}\right). \quad (5.3)$$

### 5.2.2 Channel Distribution Information (CDI)

The channel distribution information (CDI) is comprised of the spatial correlation matrices of the channel. The spatial correlation of the transfer matrix is created by the angular properties of the multipath propagation as well as the
antenna configuration. Specifically, the full spatial correlation matrix for the link \( \{m, n\} \) can be written as

\[
\mathcal{R}_{\{m,n\}} = E[\text{vec}(\mathbf{H})_{\{m,n\}} \text{vec}(\mathbf{H})_{\{m,n\}}^H].
\]  

(5.4)

When only CDI is known, the exact channel \( H \)'s are assumed to be a random variable drawn from a complex normal distribution:

\[
\text{vec}(\mathbf{H})_{\{m,n\}} \sim CN(0, \mathcal{R}_{\{m,n\}}).
\]  

(5.5)

As the multi-antenna precoding strategy and scheduling design utilizes the spatial structure in the channel, it is important that the model closely reflects this spatial information. For the purpose of simulation, we benefit from the measurements taken by Brigham Young University [72] to model the spatial correlation in the multi-user MIMO channel with realistic correlation values.

### 5.2.3 Channel Probing and Data Transmission

For the channel access at the MAC layer, MIMO opportunistic scheduling with contention-based random access is used. Recall that we deploy beamforming and each link transmits a single stream. With \( M \) antennas at each node, \( M \) simultaneous transmission links can be supported. Hence, \( M \) links can be grouped together in order to contend for the channel and transmit the data simultaneously. Based on the CDI in the form of a full spatial correlation matrix, the simultaneous transmission links in the network are grouped. As CDI is used in the scheduler design, a hybrid network is adopted in which the network operates with end-to-end goals in a distributed fashion but uses a central repository to track and store network status updates and coordinations.

The grouped links will contend, probe and access for the channel together. We formulate the probing/access process as a maximum rate-of-return problem in the optimal stopping theory framework [7]. An optimal stopping rule is a strategy for deciding when to take a given action based on the past events in order to maximize the average return. The average return is the net gain between the reward and the cost. In our work, reward refers to the data transmission and
the cost is the duration spent to look for a good channel plus the time for actual transmission. The optimal stopping rule governs the group that succeeds in the contention to either earn the reward by carrying out the data transmissions or give up the transmission opportunity for the sake of getting a possibly better data rate at the expense of spending longer time in looking.

To illustrate the dynamics of the channel probing/access, a sample realization of $N$ rounds of channel probing with one single successful data transmission is depicted in Fig. 5.2. In this figure, the state of a slot being ‘1’ denotes the successful channel contention, ‘0’ denotes either unsuccessful channel contention or lack of active links (idle), $\tau$ is the duration of a slot for channel contention and $N$ is the stopping time. The process of channel contention/probing and transmission is as follows.

**Figure 5.2:** A sample realization of channel contention/probing and data transmission.

1. **Channel contention/probing:**
   After grouping, the grouped source nodes will contend for the channel together with probability $p_v$. A collision model is assumed, where a channel contention of grouped source nodes is said to be successful if no other source groups are transmitting at the same time. The random duration of achieving a successful channel contention is defined as one round of channel probing. After each probing round, the destination nodes of the successful group return information about the link conditions in a form of achievable sum-rate to their corresponding source nodes. Let $s(n)$ denote the links of the successful group at the $n^{th}$ round of channel probing and $R_{n,s(n)}$ denote the corresponding achievable sum-rate.
2. Compare the rate with a threshold for transmission decision:
With the feedback information, the successful group will compare the achievable sum-rate $R_{n,s(n)}$ to a threshold ($x^*$) pre-designed using optimal stopping theory:

- Give up the channel and re-contend if $R_{n,s(n)} < x^*$.
- Transmit if $R_{n,s(n)} \geq x^*$.

Specifically, suppose after the first round of channel probing with a duration of $K_1$ slots, $R_{1,s(1)}$ is small due to poor channel conditions, then $s(1)$ will forgo its transmission opportunity and let all the source groups re-contend. This probing process continues for $N$ rounds until grouped links of $s(N)$ transmit when $s(N)$ has a high transmission rate that exceeds the threshold.

### 5.3 Beamforming with CSI/CDI

Before embarking on a detailed discussion of proposed scheduling method, this section provides the construction of beamforming vectors with CSI or CDI that will be used in the scheduling protocol. The paper in [17] derived the beamforming vector based on CSI ($b_{\{m,n\}}, w_{\{m,n\}}$) or CDI ($\bar{b}_{\{m,n\}}, \bar{w}_{\{m,n\}}$) at the transmitter and the receiver. Note that the same type of channel information (CSI or CDI) is used for both the desired signal and the interference signal in [17]. In this chapter, we include a new beamforming vector that is based on CSI for the desired signal and CDI from the interfering channel. To construct this beamformer, the following optimization metric will be used:

$$\hat{C} = \sum_{\{m,n\} \in \mathcal{L}} \log \left( 1 + \frac{\mathcal{N}_{\{m,n\}}}{\mathcal{D}_{\{m,n\}}} \right),$$

where $\mathcal{N}_{\{m,n\}}$ is the numerator of the SINR values ($\rho_{\{m,n\}}$) in (5.2) whereas $\mathcal{D}_{\{m,n\}}$ is the expected value of the denominator of the SINR values. Intuitively, (5.6) represents the “sum-rate” computed from the signal to the average interference plus noise ratio (SAINR).
Given (5.6) as the objective function for optimization purposes, the transmit beamforming vector, $\hat{b}_{\{m,n\}}$, and receive beamforming vector, $\hat{w}_{\{m,n\}}$, of this scenario can be found by following a similar procedure as in [17], i.e., taking the partial derivative of sum-rate in (5.6) and equating each derivative to zero. The transmit beamforming vector ($\hat{b}_{\{m,n\}}$) can then be expressed as:

$$
\hat{b}_{\{m,n\}} = \left( \frac{\sigma^2}{P} \frac{\mathcal{N}_{\{m,n\}}}{D_{\{m,n\}}(\mathcal{N}_{\{m,n\}} + D_{\{m,n\}})} \right)^{-1} \hat{\Lambda}_{\{m,n\}},
$$

where

$$
d_{\{m,n\}} = \frac{\mathcal{N}_{\{m,n\}}}{D_{\{m,n\}}(\mathcal{N}_{\{m,n\}} + D_{\{m,n\}})}, \quad \hat{\Lambda}_{\{m,n\}} = \frac{\hat{A}^H_{\{m,m\}} \hat{b}_{\{m,n\}}}{D_{\{m,n\}}},
$$

$$
\hat{S}_{\{i,j\}} = E[H^T_{\{i,j\}} \otimes H^H_{\{i,j\}}],
$$

$$
\hat{A}_{\{l,i,j\}} = \text{mat}(\tilde{S}_{i,l} \text{vec}(w_{\{i,j\}}w^H_{\{i,j\}})), \quad \{i, j\} \neq \{m, n\},
$$

$$
\hat{A}_{\{l,m,n\}} = H^H_{\{l,n\}} w_{\{m,n\}} w^H_{\{m,n\}} H_{\{l,n\}}.
$$

The receive beamforming vector ($\hat{w}_{\{m,n\}}$) can be written as:

$$
\hat{w}_{\{m,n\}} = \left( \frac{\sigma^2}{P} d_{\{m,n\}} I + \sum_{\{i,j\}} d_{\{i,j\}} \hat{A}_{\{n,i,j\}} \right)^{-1} \tilde{\Lambda}_{\{m,n\}},
$$

where

$$
\tilde{\Lambda}_{\{m,n\}} = \frac{\tilde{\hat{A}}^H_{\{m,m\}} \hat{w}_{\{m,n\}}}{\tilde{D}_{\{m,n\}}}, \quad \tilde{\hat{S}}_{\{i,j\}} = E[H^*_{\{i,j\}} \otimes H_{\{i,j\}}],
$$

$$
\tilde{\hat{A}}_{\{l,i,j\}} = \text{mat}(\tilde{\hat{S}}_{l,t} \text{vec}(b_{\{i,j\}}b^H_{\{i,j\}})), \quad \{i, j\} \neq \{m, n\},
$$

$$
\tilde{\hat{A}}_{\{l,m,n\}} = H^H_{\{m,l\}} b_{\{m,n\}} b^H_{\{m,n\}} H_{\{m,l\}}.
$$

Note that in (5.8) and (5.10), the instantaneous channel information is used for the terms associated with the desired channel and the statistical information contained in the channel correlation matrices ($\hat{S}_{\{m,n\}}$ and $\tilde{\hat{S}}_{\{m,n\}}$) is used for the terms associated with the interfering channel. The beamforming vectors that are constructed based on CSI ($b_{\{m,n\}}$, $w_{\{m,n\}}$), or based on CDI ($\bar{b}_{\{m,n\}}$, $\bar{w}_{\{m,n\}}$) in [17], have the same form as in (5.7) and (5.9), but with instantaneous CSI or CDI replace the associated terms accordingly.

Since both transmit and receive beamforming vectors are implicitly functions of themselves, an iterative algorithm suggested in [17] is used for optimization purposes. In this approach, each node alternates between updating transmit and receive beamforming vector.
5.4 Scheduling protocol

To harvest the benefits provided by CSI and CDI, the proposed joint CSI-CDI MIMO opportunistic scheduler with optimal stopping rule is divided into two phases. In the first phase, links in the network are divided into groups based on CDI such that multiple links can contend, access and transmit simultaneously. In the second phase, based on the channel knowledge available at the transmitter and receiver, CSI and/or CDI is used to construct transmit/receive beamforming vectors and decide when/which link pair should transmit. In the following two subsections, we explain these two phases in more detail.

5.4.1 Phase I: CDI-based grouping

A system with multiple antennas due to the spatial degrees of freedom enables the system to support multiple simultaneous transmissions. Thus, multiple links in the network can be divided into disjoint groups depending on the number of parallel spatial channels formed by the multiple transmit/receive antennas. In our MIMO system with $M = 2$, two links are paired and contend for the channel together to transmit the data simultaneously. The criterion we use for pairing the simultaneous contention/transmission links is based on the low level quantity of CDI.

Let $v$ denote a particular pair of simultaneous links and $V_p$ denote the set of linked-pairs for the $p^{th}$ network pairing pattern. A network pairing pattern is the method used to groups links in the network. For example, the possible network pairing patterns for a network with four source nodes are shown in Fig. 5.3. In this figure, for clarification, let each source node refer to one user. Given four users in the network shown in the figure, the possible network pairing patterns would be: $\{[1, 2][3, 4]\}, \{[1, 3][2, 4]\}$ and $\{[1, 4][2, 3]\}$, where $\{[A, B][C, D]\}$ indicates the pairing pattern of the user $A$ paired with the user $B$, and the user $C$ paired with the user $D$ to access the channel together. For a system with $N_s$ number of source
nodes, the possible different network pairing patterns is given as
\[ N_p = \prod_{t=1}^{N_s} N_s^{2t+2} C_2 \left( \frac{N_s}{2} \right)! \]  
(5.11)

We can see that based on (5.11), the total number of possible different network pairing patterns for four source nodes \( N_s = 4 \) shown in Fig. 5.3 is three.

![Network pairings](image)

**Figure 5.3:** Different network pairing patterns

Given the number of possible different pairing patterns \( N_p \), the optimization metric of the network that we use for pattern selection is
\[ \bar{C}_p = \sum_{v \in V_p} \bar{R}_v. \]  
(5.12)

Here, \( \bar{R}_v \) is the CDI-based sum-rate of the \( v \)-th pair which consists of two simultaneous transmission links with CDI-based transmit (\( \tilde{\mathbf{b}} \)) and receive (\( \tilde{\mathbf{w}} \)) beamforming vectors, and \( \bar{C}_p \) is the total CDI-based sum-rate of the network. The \( \bar{R}_v \) can be written as
\[ R_v = \sum_{\{m,n\}} \log \left( 1 + \frac{P_{\{m,n\}} \tilde{\mathbf{w}}_{\{m,n\}}^H \mathbf{mat} \left( \tilde{\mathbf{S}}_{\{m,n\}} \mathbf{vec} \left( \tilde{\mathbf{b}}_{\{m,n\}} \tilde{\mathbf{b}}_{\{m,n\}}^H \right) \right) \tilde{\mathbf{w}}_{\{m,n\}}}{\sigma^2 + \sum_{\{i,j\} \neq \{m,n\}} P_{\{i,j\}} \tilde{\mathbf{w}}_{\{i,j\}}^H \mathbf{mat} \left( \tilde{\mathbf{S}}_{\{i,n\}} \mathbf{vec}(\tilde{\mathbf{b}}_{\{i,j\}} \tilde{\mathbf{b}}_{\{i,j\}}^H) \right) \tilde{\mathbf{w}}_{\{m,n\}}} \right). \]  
(5.13)

Note that \( \tilde{\mathbf{S}}_{\{i,j\}} \) in (5.13) is defined in (5.10), and only CDI is used in (5.13).

Our objective function is to find a network pairing pattern with optimum paired-links, \( p_{opt} \), that maximizes the total CDI-based sum-rate of the network:
\[ p_{opt} = \arg \max_{p \in \{1,2,\ldots,N_p\}} \bar{C}_p, \]  
(5.14)
where $C_p$ is given by (5.12).

### 5.4.2 Phase II: Optimal stopping rule and beamformer construction with different channel knowledge scenarios

After the pairing process, the paired-links start the contention process that leads to the second phase. In this phase, the paired-links contend for the channel and are opportunistically selected for simultaneous data transmissions. CSI is used to decide when and which paired-links should be selected for transmissions based on the pre-designed threshold. Depending on the state of the network and the available channel information, the beamforming weights that were embedded in the scheduling decision can be formed using CSI and/or CDI. We first formulate the contention/access process as a maximal-rate-of-return problem and then lay out the threshold and beamformer design for different channel knowledge scenarios.

Considering a single hop *ad hoc* network with $\frac{N}{2}$ pairs of source nodes, the $v^{th}$ paired-links contends for the channel with probability $p_v, v = 1, 2, ..., \frac{N}{2}$ using random access. The successful contending/probing probability of the $v^{th}$ paired-links is given as

$$p_{s,v} = p_v \prod_{i \neq v} (1 - p_i). \quad (5.15)$$

Accordingly, the overall successful channel contention probability of each slot, $p_s$, is given by

$$p_s = \sum_{v \in V_{opt}} p_v \prod_{i \neq v} (1 - p_i) = \sum_{v \in V_{opt}} p_{s,v}. \quad (5.16)$$

It is clear that the number of slots (denoted as $K$) for a successful channel contention is a geometric random variable, i.e., $K \sim \text{Geometric}(p_s)$. It follows that the random duration corresponding to one round of successful channel contention is $K \tau$, and its expectation is $E[K] \cdot \tau = \tau / p_s$, where $\tau$ is the duration of a contention’s slot.

We treat the contention-based distributed opportunistic scheduling as a team game in which all links in the network collaborate to maximize the overall network throughput. Using the optimal stopping theory, the throughput optimization problem can be formulated as a maximal-rate-of-return problem, in which the
rate-of-return is referring to the average throughput [7]. By invoking the renewal theorem, the rate-of-return after \( N \) probing rounds is given as [7]

\[
x = \frac{E[R_N T]}{E[T_N]}, \quad \text{where} \quad T_n = \sum_{j=1}^{n} K_j \tau + T.
\]  

(5.17)

Here, \( R \) is the transmission rate, \( T \) is the data transmission duration, \( T_N \) is the total time duration including both the transmission time \( T \), and the time elapsed over the \( N \) probing rounds, and \( K_j \) is the number of slots for a successful channel contention/probing during the \( j^{th} \) probing round. The data transmission is carried out at the \( N^{th} \) probing round. Clearly, \( R_N \) and \( T_N \) are the stopped random random variables since \( N \) is a stopping time.

According to [9], for the problem of maximizing the long-term average throughput, which is cast as a maximal-rate-of-return problem, a key step is to characterize the optimal stopping rule \( N^* \) and the optimal throughput \( x^* \), as

\[
N^* \triangleq \arg \max_{N \in Q} \frac{E[R_N T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_N T]}{E[T_N]},
\]  

(5.18)

where

\[
Q \triangleq \{ N : N \geq 1, E[T_N] < \infty \}.
\]  

(5.19)

As shown in [7], the optimal stopping rule \( N^* \) for distributed opportunistic scheduling that maximizes the average return exists, and is a threshold policy. It is also shown in [7] that maximum network throughput \( x^* \) is indeed an optimal threshold.

We next present the optimal stopping rule for different channel knowledge scenarios at the transmitter, the receiver, as well as at the desired and interfering channels. Specifically, we consider four different scenarios shown in Table 5.1 and explained below.

**CSI at transmitter and CSI at receiver (CSIT-CSIR)**

In this CSIT-CSIR scenario, both the transmitter and the receiver have the instantaneous knowledge of CSI. Thus, the beamforming vectors at the transmitter \( (b) \) and the receiver \( (w) \) can be constructed based on the CSI [17]. The data rate of the \( v^{th} \) pair (link \( \{m, n\} \) and \( \{i, j\} \)) for this scenario \( (R_v^{(Pair-CSIT-CSIR)}) \) can
Table 5.1: Different Channel Knowledge Scenarios

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Channel Knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSIT-CSIR</td>
<td>Tx</td>
</tr>
<tr>
<td>CDIT-CSIR</td>
<td>All CDI</td>
</tr>
<tr>
<td>CDIT-CDIR</td>
<td>All CDI</td>
</tr>
<tr>
<td>CSIS-CDII</td>
<td>Desired Signal CSI; Int. CDI</td>
</tr>
</tbody>
</table>

be expressed as

\[
R_v^{(\text{Pair-CSIT-CSIR})} = \log \left(1 + \frac{P_{\{m,n\}} \left| w_{\{m,n\}} H_{\{m,n\}} b_{\{m,n\}} \right|^2}{\sigma^2 + P_{\{i,j\}} \left| w_{\{i,j\}} H_{\{i,j\}} b_{\{i,j\}} \right|^2}\right) + \log \left(1 + \frac{P_{\{i,j\}} \left| w_{\{i,j\}} H_{\{i,j\}} b_{\{i,j\}} \right|^2}{\sigma^2 + P_{\{m,n\}} \left| w_{\{i,j\}} H_{\{i,j\}} b_{\{m,n\}} \right|^2}\right).
\]

\[5.20\]

From the characterization of the optimal stopping rule in section 5.4.2, the optimal stopping rule \(N^*\) for distributed opportunistic scheduling of this scheme is given by

\[N^*(\text{Pair-CSIT-CSIR}) = \min\{n \geq 1 : R^{(\text{Pair-CSIT-CSIR})}_{(n)v} \geq x^*(\text{Pair-CSIT-CSIR})\}\]

\[5.21\]

The optimal stopping rule in (5.21) reveals that the optimal scheduling policy for this scheme is a threshold policy as follows: After one round of channel probing, if the successful link pair discovers that the current rate \(R^{(\text{Pair-CSIT-CSIR})}_{(n)v}\) is higher than the optimal threshold \(x^*(\text{Pair-CSIT-CSIR})\), the successful contended link pair will carry out the data transmission; else, it simply skips the transmission opportunity and lets all the pairs re-contend. The proof of the optimal threshold follows directly from [7]. Note that the stopping rule for all the remaining scenarios will be the same except that the rate \(R_v\) and the threshold \(x^*\) will be different for different scenarios.

The optimum threshold \(x^*(\text{Pair-CSIT-CSIR})\) in (5.21) is the maximum through-
put, and is the unique solution to the equation:

$$E(R^{(Pair-CSIT-CSIR)} - x^{(Pair-CSIT-CSIR)})^+ = \frac{x^{(Pair-CSIT-CSIR)}_T}{p_s T}. \quad (5.22)$$

By rearranging, we have the optimal threshold (maximum throughput) as the unique solution to the equation:

$$\frac{E[R_N T]}{E[T_N]} = x^{s(Pair-CSIT-CSIR)}$$

$$= \frac{\sum_{v \in V_{popt}} p_{s,v} \int_{x^{s(Pair-CSIT-CSIR)}}^\infty r d[F_{R_v}^{(Pair-CSIT-CSIR)}(r)]}{\delta + \sum_{v \in V_{popt}} p_{s,v} (1 - F_{R_v}^{(Pair-CSIT-CSIR)}(x))},$$

where $\delta = \frac{\pi}{T}$. The notation $F_{R_v}^{(Pair-CSIT-CSIR)}(r)$ is used for the CDF of the sum-rate of the $v$-th pair. As different links have different spatial correlation matrices, the rate distribution for each link will be different. Note that the optimal threshold is the same for all the links even if the rate distribution for each link is different. The threshold is obtained by solving the fixed point equation in (5.23), which in general does not admit a closed-form solution. In what follows, an iterative algorithm shown in [7] is used to compute $x^{s(Pair-CSIT-CSIR)}$.

It is of interest to compare the amount of feedback needed for each scenario. Fig. 5.4(a) gives a schematic diagram of feedback for this scenario. In this diagram, $Q$ is a single bit feedback that lets the transmitter know if it should transmit or just skip this opportunity. If $Q = 1$, indicating that the transmitter should transmit, the receiver also needs to feed back channel matrix $H$ so that the transmitter can construct the transmit beamforming vector based on the instantaneous $H$. This scenario is the ideal scenario where CSI is available at both the transmitter and receiver. However, the overhead cost of estimation and feedback amount incurred for this scenario is high.

**CDI at transmitter and CSI at receiver (CDIT-CSIR)**

For this CDIT-CSIR scenario, the receiver is assumed to have the information of the instantaneous CSI, and constructs the receive beamforming vector based on CSI. However, the transmitter does not have the information of CSI and hence the transmit beamforming vector is constructed based on the CDI. The data
rate of the $v^{th}$ pair (link $\{m, n\}$ and $\{i, j\}$) is given by

$$R_{v}^{(Pair\text{-}CDIT\text{-}CSIR)} = \log \left( 1 + \frac{P_{\{m,n\}} | w_{\{m,n\}}^H H_{\{m,n\}} b_{\{m,n\}} |^2}{\sigma^2 + P_{\{i,j\}} | w_{\{i,j\}}^H H_{\{i,j\}} b_{\{i,j\}} |^2} \right)$$

$$+ \log \left( 1 + \frac{P_{\{i,j\}} | w_{\{i,j\}}^H H_{\{i,j\}} b_{\{i,j\}} |^2}{\sigma^2 + P_{\{m,n\}} | w_{\{m,n\}}^H H_{\{m,n\}} b_{\{m,n\}} |^2} \right).$$

Here, $b_{\{m,n\}}$ is the CDI-based transmit beamforming vector and $w_{\{m,n\}}$ is the CSI-based receive beamforming vector.

The optimal threshold (maximum throughput) for this scenario, $x^*(Pair\text{-}CDIT\text{-}CSIR)$, is the unique solution to:

$$x^*(Pair\text{-}CDIT\text{-}CSIR) = \frac{\sum_{v \in \mathcal{V}_{\text{opt}}} p_{s,v} \int_{x^*(Pair\text{-}CDIT\text{-}CSIR)}^{\infty} r \frac{d F_{R_v}^{(Pair\text{-}CDIT\text{-}CSIR)}(r)}{dx} (x)}{\delta + \sum_{v \in \mathcal{V}_{\text{opt}}} p_{s,v} (1 - F_{R_v}^{(Pair\text{-}CDIT\text{-}CSIR)}(x))}$$
where $F_{R_v(Pair-CDIT-CSIR)}(r)$ is the CDF of the sum-rate of the $v^{th}$ pair for this scenario.

Figure 5.4(b) shows a schematic diagram of feedback for this scenario. By using CDI at the transmitter, the receiver does not need to feedback the instantaneous CSI to the transmitter for the transmit beamforming design. Hence, the receiver only needs to feed back the single bit, $Q$, and the corresponding data rate for transmission if $Q = 1$. As the channel matrix is not needed for feedback, the information to feedback is smaller compared to that of CSIT-CSIR scenario.

### 5.4.3 CDI at transmitter and CDI at receiver (CDIT-CDIR)

For this scenario, both the transmitter and receiver use CDI to construct the transmit and receive beamforming vectors. The data rate of the $v^{th}$ pair for this scenario is given by

$$R_{v(Pair-CDIT-CDIR)} = \log \left( 1 + \frac{P_{\{m,n\}} | \bar{w}^H_{\{m,n\}} \bar{H}_{\{m,n\}} \bar{b}_{\{m,n\}} |^2}{\sigma^2 + P_{\{i,j\}} | \bar{w}^H_{\{i,j\}} \bar{H}_{\{i,j\}} \bar{b}_{\{i,j\}} |^2} \right)$$

$$+ \log \left( 1 + \frac{P_{\{i,j\}} | \bar{w}^H_{\{i,j\}} \bar{H}_{\{i,j\}} \bar{b}_{\{i,j\}} |^2}{\sigma^2 + P_{\{m,n\}} | \bar{w}^H_{\{m,n\}} \bar{H}_{\{m,n\}} \bar{b}_{\{m,n\}} |^2} \right),$$

where $\bar{b}_{\{m,n\}}$ and $\bar{w}_{\{m,n\}}$ is the CDI-based transmit and receive beamforming vector, respectively.

The schematic diagram of feedback for this scenario is shown in Fig. 5.4(c). As we notice from this figure, the amount of feedback for this scenario (CDIT-CDIR) is the same as CDIT-CSIR since both of them use CDI to construct transmit beamforming vectors at the transmitter.

The equation of the optimal threshold for this scenario is the same as (5.25) except that the rate CDF for CDIT-CSIR $F_{R_v(Pair-CDIT-CSIR)}(r)$ is replaced by the rate CDF for CDIT-CDIR, $F_{R_v(Pair-CDIT-CDIR)}(r)$:

$$x^{\star(Pair-CDIT-CDIR)} = \frac{\sum_{v \in V_{\text{opt}}} p_{s,v} \int_{x^{\star(Pair-CDIT-CDIR)}}^{\infty} r d[F_{R_v(Pair-CDIT-CDIR)}(r)]}{\delta + \sum_{v \in V_{\text{opt}}} p_{s,v}(1 - F_{R_v(Pair-CDIT-CDIR)}(x))}.$$
5.4.4 CSI at desired signal and CDI at interfering channel (CSIS-CDII)

As the interfering channel is more susceptible to errors, in this scenario, the transmitter and the receiver construct the beamforming vectors based on the CDI for the interfering channel and CSI for the channel associated with the desired signal. The data rate of the $v^{th}$ pair (link $\{m,n\}$ and $\{i,j\}$) can be expressed as

$$R_v^{(Pair-CSIS-CDII)} = \log \left( 1 + \frac{P_{\{m,n\}} |\hat{\mathbf{w}}_{\{m,n\}}^H \mathbf{H}_{\{m,n\}} \hat{\mathbf{b}}_{\{m,n\}}|^2}{\sigma^2 + P_{\{i,j\}} |\hat{\mathbf{w}}_{\{i,j\}}^H \mathbf{H}_{\{i,j\}} \hat{\mathbf{b}}_{\{i,j\}}|^2} \right)$$

$$+ \log \left( 1 + \frac{P_{\{i,j\}} |\hat{\mathbf{w}}_{\{i,j\}}^H \mathbf{H}_{\{i,j\}} \hat{\mathbf{b}}_{\{i,j\}}|^2}{\sigma^2 + P_{\{m,n\}} |\hat{\mathbf{w}}_{\{i,j\}}^H \mathbf{H}_{\{m,n\}} \hat{\mathbf{b}}_{\{m,n\}}|^2} \right),$$

where $\hat{\mathbf{b}}_{\{m,n\}}$ and $\hat{\mathbf{w}}_{\{m,n\}}$ are the transmit and receive beamforming vectors, respectively, for this CSIS-CDII scheme, which are constructed based on the CSI for the desired signal and the CDI for the interfering channel (explained in Section 5.3).

The maximum throughput (optimal threshold) is the unique solution to the following equation:

$$x^{\ast\ast}(Pair-CSIS-CDII) = \frac{\sum_{v \in V_{\text{opt}}} p_{s,v} \int_{x^{\ast\ast}(Pair-CSIS-CDII)}^\infty r \cdot d [F_{R_v^{(Pair-CSIS-CDII)}}(r)]}{\delta + \sum_{v \in V_{\text{opt}}} p_{s,v} (1 - F_{R_v^{(Pair-CSIS-CDII)}}(x))}.$$

(5.28)

A schematic diagram of feedback for this scenario is shown in Fig. 5.4(d). Notice that the source node does not need any feedback information from the interference node to construct the beamforming vector.

5.5 Simulation Results

In this section, we study the performance of the joint CSI-CDI opportunistic channel-aware scheduler by simulation results. We consider a MIMO mobile ad hoc network which has $N_s = 8$ active links. That is, there are 8 source nodes in the network and each source node has its own designated receive node. Although we limit the network to $N_s = 8$ for the sake of simplicity, the work detailed here is
applicable for all possible values of $N_s$ with any number of links. Each of the receivers as well as the transmitters in the network have $M = 2$ antennas, i.e., $N_t = N_r = 2$. Unless otherwise specified, we assume that $\tau$, $T$ and $p_v$ are chosen such that $\delta = \tau/T = 0.1$ and $p_s = 1/e$.

We employ the measurements taken by Brigham Young University [72] to realistically estimate the spatial correlation in the multi-user MIMO channel. Specifically, the full spatial correlation matrix for the $\{m, n\}$ link can be estimated by performing post-processing on the measured dataset:

$$\mathbf{R}_{\{m,n\}} = \frac{1}{W} \sum_{i=1}^{W} \text{vec} \left( \tilde{\mathbf{H}}_{\{m,n\}}(i) \right) \text{vec} \left( \tilde{\mathbf{H}}_{\{m,n\}}(i) \right)^H,$$

where $\tilde{\mathbf{H}}_{\{m,n\}}$ is the measured channel, index $i$ denotes samples of the measured channel, and $W$ is the size of the estimation window.

Once the spatial correlation matrix has been estimated for each link, link channels can be realized from the measured correlation matrix using

$$\mathbf{H}_{\{m,n\}} = \text{mat} \left\{ \sqrt{\mathbf{R}_{\{m,n\}}} \text{vec}(\mathbf{H}_w) \right\},$$

where $\mathbf{H}_w$ is an $N_r \times N_t$ matrix with i.i.d. zero mean unit variance, complex Gaussian entries, and $\sqrt{\cdot}$ is the matrix square root operator. The full correlation model in (5.30) along with statistically measured samples from (5.29) provides a method for obtaining channel realizations with realistic correlation values for simulations.

We are interested in comparing the different pairing patterns in terms of the total CDI-based sum-rate for each pattern. Figure 5.5 shows an example of the total CDI-based sum-rate of a network with all possible network pairing patterns. The 8 source nodes can be divided into four disjoint source pairs. The total possible different network pairing patterns with eight source nodes can be calculated from (5.11) as 104. Hence, the index of network pairing pattern on the $x$-axis has value up to 104. The total CDI-based sum-rate of a network for a specific network pairing pattern can be obtained from (5.12). As shown in this figure, different network pairing patterns give a different sum-rate of the network. The best pair and the worst pair provide a difference of 13 bits/sec/Hz. Thus, by using CDI, we
can select a network pairing pattern (best pair), which gives us the highest total CDI-based sum-rate that will last for a longer duration.

![Graph showing total CDI-based sum-rate for different network pairing patterns.]

**Figure 5.5**: Total CDI-based sum-rate of the network for different network pairing patterns.

We next perform simulations to study the performance of the joint CSI-CDI scheduler in the following ways:

- We compare our proposed CDI grouping scheme with other schemes.
- We evaluate the performance of the joint CSI-CDI scheduler for different channel information scenarios.
- We compare the network throughput of the joint CSI-CDI scheduler with the optimum group of node pairings and alternate group of node pairings.

### 5.5.1 Comparison of our proposed CDI grouping scheme with other schemes

Figure 5.6 compares the network throughput of the proposed joint CSI-CDI scheduler, based on CDI-grouping, with other alternative schedulers based
on: CSI-grouping, Pessimum-grouping, TG(Two-Group)-MIMO-grouping and No-grouping. All use the CSIT-CSIR scenario in which the transmitter and the receiver construct the transmit/receive beamforming vectors with CSI in the PHY layer. For CSI-, CDI- and Pessimum-grouping, two links are grouped together for simultaneous transmission and thus each transmission contains two links. In the scheme of CSI-grouping, CSI is used to group the simultaneous transmission links for every channel access. The best CSI-grouping pattern which has the highest total instantaneous sum-rate of the network will be used for channel contention/access. Although the scheduler based on CSI-grouping is not practical, it serves as an upper bound for our comparison. For CDI-grouping scheduler, the best CDI-based network pairing pattern shown in Fig. 5.5 is used. For Pessimum-grouping scheme, the worst CSI-grouping pattern which has the lowest total instantaneous sum-rate of the network is used for every channel access. Note that Pessimum-grouping scheme is the worst case scenario for transmission containing two links and serves as the lower bound for our comparison. The scheme of No-grouping is the scheduler which has no grouping and each link is contending/accessing the channel by itself. Hence, each transmission has only one link. The TG-MIMO-grouping scheme is proposed in [12]. In this scheme, the channel time is divided into meta slots composed of two mini-slots. Each user first randomly categorizes itself into one of the two groups. A source node will contend in the mini-slot belonging to its group. Each probing round is completed with one of three possible states \(\{0, 1\}, \{1, 0\}, \{1, 1\}\) where ‘0’ is unsuccessful channel contention and ‘1’ is successful channel contention. The source node will choose the transmission strategy that maximizes the transmission rate: only link \(i\) transmits, only link \(j\) transmits or both link \(i\) and link \(j\) transmit together. Thus, for TG-MIMO-grouping, each transmission can have one link or two links depending on the contention outcomes as well as the achievable transmission rate. By comparing schedulers based on CSI-, CDI-, Pessimum-, TG-MIMO- and No-grouping in the network context, all with CSIT-CSIR in the link context, we observe that CSI-grouping performs the best, followed by CDI-grouping, Pessimum-grouping, TG-MIMO-grouping and lastly No-grouping. We notice that CDI-grouping sacrifices some performance gain com-
pared to the optimal CSI-grouping as expected, but CDI-grouping still manages to achieve impressive performance by maintaining 90% of the optimal CSI-grouping.

![Graph showing network throughput comparison](image)

**Figure 5.6:** Comparison of network throughput of scheduler based on CSI-grouping, CDI-grouping, Pessimum-grouping, TG-MIMO-grouping and No-grouping for the CSIT-CSIR scenario.

### 5.5.2 Evaluation of the Performance of joint CSI-CDI scheduler for different channel information scenarios

Under CDI-grouping scheme, we compare the average network throughput under different channel information scenarios at the link level: CSIT-CSIR, CDIT-CSIR, CSIS-CDII and CDIT-CDIR in Fig. 5.7. The transmitter and the receiver will construct the beamforming weights based on the available channel information at the transmitter and the receiver, as well as at the desired channel and the interfering channel. It can be seen from Fig. 5.7 that CSIT-CSIR performs the best, followed by CDIT-CSIR, CSIS-CDII and lastly CDIT-CDIR. The network throughput of CDIT-CSIR, CSIS-CDII, CDIT-CDIR can maintain roughly 85%, 79%, and 68% of the CSIT-CSIR network throughput, respectively. Obviously, the more CSI knowledge the link obtains, the better the network performs, but with the
expense of higher amount of feedback (refer to Fig. 5.4). It is worth noting that CDIT-CSIR performs slightly better than CSIS-CDII. This result suggests that CSI information (including desired signal and interference signal) at the receiver is more effective in combating the interference compared to those which have CSI for the desired signal but not the interference signal at both the transmitter and the receiver.

Figure 5.7: Comparison of network throughput with CDI-grouping for different combinations of CSI and CDI at the transmitter and the receiver.

5.5.3 Comparison of the network throughput of the joint CSI-CDI scheduler with the optimum group of node pairings and alternate group of node pairings

It is of interest to compare the network throughput of the joint CSI-CDI scheduler with the optimum group of node pairings and alternate group of node pairings with degraded performance. Figure 5.8 depicts the performance of the joint CSI-CDI scheduler based on the optimum CDI-grouping pattern, the worst CDI-grouping pattern, and the scheduler based on random grouping for the scenar-
ios of CSIT-CSIR (5.8(a)), CDIT-CSIR (5.8(b)), CSIS-CDII (5.8(c)), and CDIT-
CDIR (5.8(d)). (A specific group of node pairing is referred to as a “pattern” in
this figure.) As we can see from Fig. 5.8, the network throughput of the proposed
CSI-CDI scheduler with the optimum CDI-grouping pattern is always superior to
that of the random grouping pattern and the worst pattern. Particularly, the gain
of the proposed joint CSI-CDI scheduler is larger when more CDI information
is used in the system. The result suggests that the impact of finding the best
grouping is smaller when more CSI information is available. This is because when
perfect CSI is available, the interference cancellation is very effective no matter
what combination of pattern is chosen. CDI on the other hand, proportionally
benefits from multi-user diversity at the MAC much more than CSI. When more
CDI information is used, different pairing patterns will give a larger difference in
the throughput as the same CDI-based beamforming weight will be used regardless
of the channel gains. Therefore, a wise grouping of source nodes becomes more
vital when more CDI information is used. In other words, a good CDI-grouping
pattern from the MAC layer provides the PHY layer a good basis to construct the
CDI-based beamforming vectors that are more effective, and thus enables a truly
cross-layer optimization across the MAC and PHY layer. We also observe in Fig.
5.8 that the gain of using optimum patterns becomes larger when average SNR
is higher. This observation is more obvious when more CDI information is used.
The reason is that when the channel condition is bad, the system performs poorly
regardless of which pairing pattern is used. However, when the channel condition
is relatively better, a good pairing in an average sense will have better interference
mitigation ability.

5.6 Conclusions

This chapter shows that CDI can be used as an additional scheduling tool
for the MAC layer in optimizing the overall network performance. We developed a
distributed cross-layer MAC protocol based jointly on CSI and CDI for MIMO ad
hoc networks that bridges the gap between the PHY layer’s coherence time and the
MAC layer’s larger network time scale. Particularly, building on optimal stopping theory, we proposed a two-phase joint CSI-CDI opportunistic scheduler with reduced frequency and amount of feedback required between nodes. Based on CDI, in the form of a spatial correlation matrix, the simultaneous transmission links are grouped to contend for the channel together using random access. Depending on the available information at the transmitter/receiver and desired/interfering channel, beamforming weights can be constructed using CSI or CDI. The successfully contended group will access the channel and transmit together if their sum-rate is above a predefined threshold.

Simulation results show that CDI can be used to improve the network performance when deploying perfect CSI in the network context is intractable. Based on CDI-grouping in the network context, our proposed joint CSI-CDI scheduler obtained impressive performance by achieving 90% of the network throughput attained by CSI-grouping. Given this form of joint scheduler, depending on the available information in the link context, the network throughput can maintain roughly 68 – 85% of that optimal throughput with CSI beamforming. Since beamforming especially with CSI at the PHY layer is very effective in cancelling the interference, deploying CDI at the MAC layer for the network is more plausible.

This chapter is adapted from Sheu-Sheu Tan, James Zeidler and Bhaskar Rao “Channel Distribution Information Based Cross-Layer Design of an Opportunistic Scheduler for MIMO Networks”, In preparation for submission to IEEE Transactions on Vehicular Technology. This chapter also contains materials from Sheu-Sheu Tan, Adam Anderson and James Zeidler, “The role of Channel Distribution Information in the Cross-Layer Design of Opportunistic Scheduler for MIMO Networks”, IEEE 44th ASILOMAR Conference on Signals, Systems and Computers, Nov. 2010.
Figure 5.8: Comparison of network throughput of the joint CSI-CDI scheduler based on the optimum CDI-grouping pattern, the worst CDI-grouping pattern, and the scheduler based on random grouping for (a) CSIT-CSIR (b) CDIT-CSIR (c) CSIS-CDII (d) CDIT-CDIR.
Chapter 6

Adaptive Modulation for OFDM-based Multiple Description Progressive Image Transmission

6.1 Introduction

The growing demand for wireless multimedia services requires reliable and high-rate data communications over a wireless channel. Ideally, wireless multimedia systems have to be adaptive based on the channel conditions.

In this chapter, we include the role of the application layer and investigate the use of adaptive modulation in an OFDM system used for transmitting progressively-coded images with multiple description coding. Specifically, each description is mapped into one of the subchannels of the OFDM waveforms [24]. In most of the literature, such as [25–27], temporal coding is used. In this chapter, we use a cyclic redundancy check (CRC) to check the validity of each description, and erase all descriptions that do not pass the CRC. Then, Reed Solomon (RS) erasure decoding is used across the descriptions.

To achieve minimal image distortion, we need to optimize the constellation
size and code rates jointly. However, due to the complexity of jointly optimizing adaptive modulation and channel coding, the problem is decomposed into two sub-problems. First, we decide the constellation sizes to maximize the system throughput prior to RS decoding, then we decide the code rates to minimize distortion.

In much of the literature, the same constellation size is used for all the subchannels when applying adaptive modulation for image transmission [25, 26]. However, we propose adopting different constellations for different subchannels to avoid the problem of overwhelming some of the subchannels by imposing a higher order modulation size than the quality of their channels can sustain. Specifically, two schemes of M-QAM adaptive modulation are considered. The first is a variable rate, fixed power scheme; for each subchannel, a constellation size is assigned which maximizes the system throughput prior to RS decoding, with equal power allocation for all subchannels. The second is a variable rate, variable power scheme, which maximizes the system throughput prior to RS decoding by changing the constellation size and the allocated power at each subchannel.

The remainder of this chapter is organized as follows: Section 6.2 outlines the system and channel models. Section 6.3 describes the adaptive modulation techniques, and Section 6.4 presents the RS error protection framework. Section 6.5 demonstrates the simulation results and Section 6.6 presents the conclusions.

6.2 System and Channel Models

6.2.1 System Model

For the transmission of progressively-coded images over an OFDM system with $L$ subchannels [24], an embedded bitstream is first converted into $L$ descriptions using an FEC-based multiple description coder. Then, Reed-Solomon (RS) encoding is used to code across the descriptions and provide unequal error protection for the multiple descriptions, where the rates of the codes are a non-decreasing function of the level of importance of the data. Lastly, a cyclic redundancy check (CRC) is appended to each description for error detection. Note that the terms
description and packet are used interchangeably in this chapter.

Coding across the subchannels normally requires a consistent code alphabet. However, this chapter proposes to have variable modulation alphabet sizes across the subchannels using adaptive modulation. Hence, a mapping from the modulated symbols to the RS code symbols is needed. For adaptive modulation, the constellation size \( M \) is restricted to \( 2^n \), where \( n \) is an even number varying from 2 to \( N_b \). For \( N_b = 6 \), the resulting constellation choices are 4-QAM, 16-QAM and 64-QAM. With adaptive modulation, the number of symbols modulating the subchannels is the same, but the number of bits may vary from subchannel to subchannel. As a \( GF(2^{10}) \) RS code is adopted, each RS code symbol contains 10 bits. We mapped the modulated symbols to the RS code symbols as shown in Fig. 6.1. Five 4-QAM, 16-QAM, and 64-QAM modulated symbols are grouped as one, two, and three RS code symbols, respectively.

### 6.2.2 Channel Model

In a frequency-selective OFDM channel, the entire frequency band of \( B_T \) Hz containing \( L \) subcarriers is assumed to be divided into \( N_c \) independent channels (blocks). Each of the \( N_c \) independent channels has bandwidth approximately equal to the coherence bandwidth of the channel and consists of \( M_c \) identically correlated subcarriers. Furthermore, slow Rayleigh fading is assumed at each subchannel, such that the fading coefficient remains constant over a packet. We assume neither inter-symbol interference nor inter-carrier interference exists.

### 6.3 Adaptive Modulation

This section illustrates our adaptive modulation schemes for the transmission of progressive images via multiple descriptions. We first consider the scenario where the transmitter has the instantaneous channel state information. We next consider the scenario where the transmitter only has the average value of the channel state information.
6.3.1 Adaptive Modulation based on Instantaneous Channel State Information

For this scenario, the constellation size is based upon the instantaneous channel state. A pilot-symbol-assisted estimation technique can be used to determine the corresponding channel state. However, in this chapter, perfect channel estimation and an error-free feedback channel are assumed such that both the transmitter and the receiver have the perfect knowledge of the channel state information.
Variable Rate, Fixed Power Scheme

This scheme responds to the channel fluctuations by varying the constellation size at each subchannel to maximize the system throughput prior to RS decoding, (average number of received bits from all the subchannels) with equal power allocation for all the subchannels. The system throughput prior to RS decoding, $T_{uc}$, is given as

$$T_{uc} = \sum_{l=1}^{L} \Gamma_{l,M}(\gamma) \quad (6.1)$$

where $\Gamma_{l,M}(\gamma)$ is the throughput for the $l^{th}$ subchannel prior to RS decoding with a constellation size of $M$, $L$ is the number of subchannels and $\gamma$ is the received instantaneous SNR.

The SNR range is divided into $|\cdot|$ regions, where $|\eta|$ denotes the cardinality of the region set. When the received SNR, i.e., $\gamma$, is in the $\eta^{th}$ region, a constellation of size $M = 2^n$ is transmitted. The region boundaries $\{\gamma_b\}, b = 0, 2, 4, \ldots, N_b$ are determined from the function $\Gamma_{l,M}(\gamma)$, which is given by

$$\Gamma_{l,M}(\gamma) = (1 - PER_{l,M}(\gamma)) \left( z \log_2 M \right) \quad (6.2)$$

where $z$ is the total number of modulated symbols in one packet, $\log_2 M$ is the number of bits in one symbol, and $PER_{l,M}(\gamma)$ is the conditional packet error rate of subchannel $l$ for a constellation size of $M$, conditioned on the channel state, given by

$$PER_{l,M}(\gamma) = 1 - \left( 1 - SER_{l,M}(\gamma) \right)^z. \quad (6.3)$$

In (6.3), $SER_{l,M}(\gamma)$ is the conditional symbol error probability of subchannel $l$, conditioned on the channel state. For a Gray-encoded MQAM square constellation it is given by [73] as

$$SER_{l,M}(\gamma) = 1 - \left( 1 - \frac{4(\sqrt{M} - 1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M - 1}}\right) + \frac{2(\sqrt{M} - 1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M - 1}}\right)^2 \right). \quad (6.4)$$
By substituting (6.3) and (6.4) into (6.2), $\Gamma_{l,M}(\gamma)$ can be rewritten as

$$\Gamma_{l,M}(\gamma) = \left(1 - \frac{4(\sqrt{M} - 1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\gamma}{M - 1}}\right) + \left[2(\sqrt{M} - 1) Q\left(\sqrt{\frac{3\gamma}{M - 1}}\right)\right]^2\right) \cdot (z \log_2 M).$$

(6.5)

Figure 6.2: Throughput of one subchannel prior to RS decoding, $\Gamma_{l,M}(\gamma)$, for $M=4, 16, 64$.

Fig. 6.2 depicts the throughput of one subchannel prior to RS decoding, $\Gamma_{l,M}(\gamma)$, for $M = 2, 4, 6$. As is well known, the throughput for each constellation tends to flatten out for large SNR. Hence, switching to a higher constellation at the appropriate threshold is desirable to achieve higher throughput. The boundaries of the switching thresholds $\{\gamma_b\}, b = 0, 2, 4, .., N_b$ are the points where the throughput of the two adjacent constellations cross. If the SNR of a particular subchannel is in the region bounded by $\gamma_b$ and $\gamma_{b+2}$, then the subchannel will be assigned a constellation of size $M = 2^{b+2}$. 
Variable Rate, Variable Power Scheme

For the variable rate, variable power scheme, both the constellation size and
the allocated power for each subchannel are changed in order to maximize the sys-
tem throughput prior to RS decoding, subject to a constraint of total transmission
power, from all the subchannels. Mathematically, the problem can be formulated
as
\[
\text{Max: } \sum_{l=1}^{L} g_l(\gamma), \text{ subject to } \sum_{l=1}^{L} P_l = P_{\text{total}} \quad (6.6)
\]
where \(P_l\) is the allocated power at the \(l\text{th}\) subchannel, \(P_{\text{total}}\) is the total power
constraint, and \(g_l(\gamma)\) is the composite throughput function at the \(l\text{th}\) subchannel,
which is given in (6.7), and is illustrated in Fig. 6.2. As \(N_b = 6\) is assumed in this
chapter, \(g_l(\gamma)\) is thus a composite throughput function of 4-QAM, 16-QAM and
64-QAM:

\[
g_l(\gamma) = \Gamma_{l, M=4}(\gamma), \quad 0 \leq \gamma < \gamma_2 \\
g_l(\gamma) = \Gamma_{l, M=16}(\gamma), \quad \gamma_2 \leq \gamma < \gamma_4 \\
g_l(\gamma) = \Gamma_{l, M=64}(\gamma), \quad \gamma_4 \leq \gamma \quad (6.7)
\]

An algorithm has been developed that is summarized in the flowchart shown
in Fig. 6.3. It is based upon a greedy approach to allocate power and choose
constellation size.

We employ a utility-cost function as in [74]. The utility measures the
amount of benefit that the receiver is likely to achieve from receiving the packet,
and the cost measures how much one has to pay to achieve a certain utility. The
throughput prior to RS decoding is taken to be the utility value, and the required
power is the cost. Overall, the function \(g_l(\gamma)\) depicted in Fig. 6.2 (given in (6.7))
contains both the utility and the cost. Based on the function \(g_l(\gamma)\), the algorithm
assigns the power successively to maximize the argument of the utility function per
unit power on each step. The constellation size of each subchannel is determined
by its final power assignment.

Before we describe the algorithm in detail, we consider a set of integer values
Determine: Constellation, $M$ for $l = 1, 2, ..., L$

Initialize: For $l = 1, ..., L$, $P_l \leftarrow 0$, $\sum_{l=1}^{L} P_l = 0$

Calculate: For each subchannel,

\[
R(\lambda_l) = \frac{g_l(P_l + \lambda_l \Delta P) - g_l(P_l)}{\lambda_l \Delta P}, \quad l = 1, 2, ..., L
\]

where $\lambda_l = 1, ..., L$

Select: Choose $l$, and $\lambda$ with maximum slope, $R(\lambda)$ (Select the smallest index if a tie)

\[
\hat{l}, \hat{\lambda} \leftarrow \arg \max_{l, \lambda} \frac{g_l(P_l + \lambda \Delta P) - g_l(P_l)}{\lambda \Delta P}
\]

Update: $P_j \leftarrow P_j + \hat{\lambda} \Delta P$, $\sum_{l=1}^{L} P_l \leftarrow \sum_{l=1}^{L} P_l + \hat{\lambda} \Delta P$

Figure 6.3: Flow chart of variable rate, variable power algorithm

for

\[
\lambda \in \left\{1, \ldots, \left[\frac{P_{\text{total}} - \sum_{l=1}^{L} P_l}{\Delta P}\right]\right\}
\]

(6.8)

where $\sum_{l=1}^{L} P_l$ is the current total power allocated for all the subchannels, hence, $P_{\text{total}} - \sum_{l=1}^{L} P_l$ is the current remaining power budget. The floor $\lfloor x \rfloor$ denotes the greatest integer $\leq x$, and $\Delta P$ is a fixed, small increment of power. Several values for $\Delta P$ have been tested and 0.1 appeared to be an appropriate value.

The algorithm is described as follows:
1. **Initialization:** Set the power of all subchannels to zero.

2. **Calculate the slope:** For each subchannel, calculate the slope of the throughput when an increment $\lambda \Delta P$ of power is applied. The slope $R_l(\lambda)$ is defined as

$$R_l(\lambda) = \frac{g_l(P_l + \lambda \Delta P) - g_l(P_l)}{\lambda \Delta P}$$

(6.9)

where $0 \leq \lambda \Delta P \leq P_{total} - \sum_{i=1}^{L} P_i$, $l \in \{1,2,...,L\}$,

$\lambda \in \left\{1,\ldots,\left\lfloor \left( P_{total} - \sum_{i=1}^{L} P_i \right)/\Delta P \right\rfloor \right\}$, and $P_l$ is the current power at the $l^{th}$ subcarrier. The incremental throughput $g_l(P_l + \lambda \Delta P) - g_l(P_l)$ represents the utility achieved at a cost of $\lambda \Delta P$ units of power applied to the $l^{th}$ subchannel.

3. **Select and update:** Selection is made by choosing subchannel $\hat{l}$, with corresponding value $\hat{\lambda}$, that corresponds to the steepest slope $R_l(\lambda)$ (i.e., the largest throughput increment per unit power). If there is a tie, select the subchannel with the smaller index. The corresponding increment of power, $\hat{\lambda} \Delta P$, is then assigned to the $\hat{l}^{th}$ selected subchannel, and the total power budget is reduced by the allocated amount. This iterative power allocation process terminates either when the total transmission power constraint is reached, or when the remaining power is less than $\Delta P$.

4. **Determine the constellation size:** The final step of the algorithm is to decide the constellation size for each subchannel based on its final allocated power. The assigned power determines the value of $\gamma$, and thus the constellation size, for $N_b = 6$, is indicated below:

$$\gamma = 0, \quad \text{No Transmission}$$

$$0 < \gamma < \gamma_2, \quad M = 4$$

$$\gamma_2 \leq \gamma < \gamma_4, \quad M = 16$$

$$\gamma_4 \leq \gamma, \quad M = 64$$

(6.10)
6.3.2 Adaptive Modulation Based on Average Throughput Maximization

In order to maximize the system throughput prior to RS decoding, this scheme varies the constellation size at each subcarrier according to the channel conditions (i.e., maximizing the average number of successfully received bits from all subcarriers). The system throughput prior to RS decoding, $T_{uc}$, can be written as

$$T_{uc} = \sum_{l=1}^{L} \Gamma_{l,M} (\bar{\gamma}) \tag{6.11}$$

where $\Gamma_{l,M} (\bar{\gamma})$ is the average throughput for the $l^{th}$ subcarrier prior to RS decoding, where $\bar{\gamma}$ is the average SNR.

Let $\mu$ denote the number of unique constellation choices. The average SNR ($\bar{\gamma}$) range is divided into $\mu$ regions. When the average SNR falls into a particular region, the constellation size associated with that region is transmitted. The region boundaries $\{ \bar{\gamma}_b \}, b = 0, 2, 4, ...N_b$ are determined from the function $\Gamma_{l,M} (\bar{\gamma})$, which is given by

$$\Gamma_{l,M} (\bar{\gamma}) = \int (1 - PER_{l,M} (\gamma)) \left( (\log_2 M) z \right) f_{\gamma} (\gamma) d\gamma \tag{6.12}$$

where $\log_2 M$ is the number of bits in one symbol, $z$ is the total number of modulated symbols in one packet, and $f_{\gamma} (\gamma)$ is the probability density function of SNR, $\gamma$. $PER_{l,M} (\gamma)$ in (6.12) is the conditional packet error rate of subcarrier $l$, conditioned on $\gamma$ for a constellation size of $M$. Note that packet error rate is synonymous with the description loss probability, and is given by

$$PER_{l,M} (\gamma) = 1 - \left( 1 - SER_{l,M} (\gamma) \right)^z. \tag{6.13}$$

In (6.13), $SER_{l,M} (\gamma)$ is the conditional symbol error probability of subcarrier $l$, conditioned on $\gamma$. For a Gray-encoded M-QAM square constellation [73], this conditional symbol error probability is given by (6.4) as earlier.

We also consider the scenario where the transmitter only knows the average channel state information of each subcarrier. The probability density function of
the SNR is modeled as a Chi Square distribution with two degrees of freedom [11]:

\[ f_{\gamma}(\gamma) = \frac{1}{\pi} e^{-\gamma}. \]  

(6.14)

Fig. 6.4 depicts the average throughput of one subcarrier prior to RS decoding, \( \Gamma_{t,M}(\overline{\gamma}) \), for \( M = 4, 16, 64 \). The boundaries of the switching thresholds \( \{\overline{\gamma}_b\}, \ b = 0, 2, 4, \ldots, N_b \) are the points where the throughput of the two adjacent constellations cross from constellation size of \( 2^{2n} \) to constellation size of \( 2^{2(n+1)} \). If the average SNR \( \overline{\gamma} \) of a particular subcarrier is in the region bounded by \( \overline{\gamma}_b \) and \( \overline{\gamma}_{b+2} \), then the subcarrier will be assigned a constellation size of \( M = 2^{b+2} \).

![Figure 6.4: Average throughput of one subcarrier prior to RS decoding, \( \Gamma_{t,M}(\overline{\gamma}) \), for \( M=4, 16, 64 \).](image)

**Adaptive Modulation Based on Proposed Variance Aware Method**

If throughput and distortion were related linearly, then maximizing average throughput would be the same as minimizing average distortion. Since in general their relationship is not linear, maximizing average distortion can be very different from minimizing average distortion if the variance of the throughput is large caused by high channel quality variability. We would therefore like to modify the objective
function used in the previous subsection as it considers only the average throughput in determining the switching thresholds.

We first describe the motivation that we used to design a new cross-layer objective function for switching-threshold determination. The rate distortion function for an $N (0, \sigma^2)$ Gaussian source with a squared-error distortion is given by [75]

$$ R(D) = \begin{cases} \frac{1}{2} \log \frac{\sigma^2}{D} & 0 \leq D \leq \sigma^2 \\ 0 & D > \sigma^2 \end{cases} $$

(6.15)

where $D$ is the distortion and $R(D)$ is the rate. We can rewrite the above equation to express the distortion in terms of the rate as

$$ D(R) = \sigma^2 2^{-2R} $$

(6.16)

Without loss of generality, $\sigma^2$ can be set to unity. Note that $R$ in (6.16) is the source rate, i.e., the number of bits used to represent one pixel. If we define the horizontal and vertical dimensions of an image to be $Y$ and $Z$ pixels, respectively, then (6.16) can be rewritten as

$$ D(T) = 2^{- \frac{YZ}{2\pi} T} $$

(6.17)

where $T$ is the number of successfully decoded source bits sequentially from the beginning of the data stream. Since $T$ is random, assume, for simplicity, that its pdf is that of a truncated Gaussian. That is, let

$$ f_T(T) = \begin{cases} \frac{1}{1 - \phi\left(-\frac{\mu_T}{\sigma_T}\right)} \frac{1}{\sqrt{2\pi}\sigma_T} e^{-\frac{(T-\mu_T)^2}{2\sigma_T^2}} & \\ 0 & \end{cases} $$

(6.18)

where

$$ \phi(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy. $$

(6.19)
Then, the average distortion, $D_{ave}$, is given by

$$D_{ave} = \int_{0}^{\infty} 2^{-\frac{2}{2\pi}x} e^{-\frac{2}{\sigma^2} (\frac{\mu_T}{\sigma})^2} d\Gamma$$

$$= \int_{0}^{\infty} e^{-\frac{2\ln 2}{Y \times Z} T} \frac{1}{1 - \phi \left( -\frac{\mu_T}{\sigma} \right)} e^{\frac{(T-\mu_T)^2}{2\sigma^2}} d\Gamma$$

$$= \frac{1}{1 - \phi \left( -\frac{\mu_T}{\sigma} \right)} e^{\frac{2\ln 2}{Y \times Z} (\mu_T - \ln 2 \sigma^2)} \left( 1 - \phi \left( -\frac{\mu_T - \frac{2\sigma^2 \ln 2}{Y \times Z}}{\sigma^2} \right) \right) .$$

If $\frac{\mu_T}{\sigma}$ is large, then $\phi \left( -\frac{\mu_T}{\sigma} \right) \ll 1$, and the above average distortion can be approximated as

$$D_{ave} \approx e^{\frac{2\ln 2}{Y \times Z} (\mu_T - \ln 2 \sigma^2)}$$

(6.21)

To minimize (6.21), we need to maximize

$$\mu_T - \left( \frac{\ln 2}{Y \times Z} \right) \sigma^2 .$$

(6.22)

By analogy with (6.22), to determine the switching threshold, we propose to use the objective function of

$$\text{MAX} \left\{ \Gamma_{l,M} (\overline{\sigma}) - K\overline{\sigma}_{l,M} \right\}$$

where $\Gamma_{l,M} (\overline{\sigma})$ is the average throughput of subcarrier $l$ prior to RS decoding defined in (6.12), and $\overline{\sigma}_{l,M}^2$ can be shown to be given by

$$\overline{\sigma}_{l,M}^2 = \frac{\sigma_{l,M}^2}{N_e} .$$

(6.24)

In (6.24), $\sigma_{l,M}^2$ denotes the variance of the throughput of subcarrier $l$. Note that we are not actually assuming that the throughput has a truncated Gaussian distribution. We are simply using the result of (6.22) to motivate the functional form of combining the means and the variance of throughput. When all the channels are correlated, $N_e = 1$ and $\overline{\sigma}_{l,M}^2$ is the largest, whereas when all the channels are independent, $N_e = L$, which makes the $\overline{\sigma}_{l,M}^2$ $L$ times smaller. In (6.23), $K$ is a scaling factor in units of subcarriers per bit. Note that when $K = 0$, the objective function becomes the average throughput maximization.
6.4 RS Error Protection Framework

After determining the constellation sizes, the channel code rates have to be determined. This section presents the error protection framework by considering the RS rate assignment in order to minimize the distortion of the image.

For the transmission of a progressively-coded image, an embedded bitstream is first converted into $L$ descriptions using multiple description coding. Descriptions for an embedded bitstream are then protected by $J$ RS codewords, where each codeword contains a segment $(S_j, j \in [1, J])$ of the information data (see Fig. 6.1), and each segment consists of $m_j \in \{1, 2, ..., N\}$ source symbols. Without loss of generality, we let one source symbol correspond to one RS code symbol. Let $f_j = N - m_j$ denote the number of RS parity symbols that protects the segment $S_j$. Then, by adding a set of constraints $f_1 \geq f_2 \geq \ldots \geq f_J$ and $f_j \in \{1, 2, ..., N - 1\}$, the receiver can recover at least the first $j$ segments, given that no more than $f_j$ RS code symbols are erased. Fig. 6.1 illustrates a mechanism for mapping an embedded bitstream from a source encoder into multiple descriptions. For a progressively-coded source, prefixes of the bitstream can be used to reconstruct the source with a certain fidelity. However, occurrence of an error along the bitstream will result in the loss of the subsequent bits. For the example in Fig. 6.1, if the number of erased code symbols is no more than 3, 5 and 6, source segments up to 1, 2 and 3 can be reconstructed, respectively.

The analysis of RS error protection framework is similar for both the cases of adaptive modulation based on instantaneous and average SNR. Without lost of generality, we only present the case for instantaneous SNR. For a given $E = (f_1, ..., f_J)$, the conditional distortion, $D(E, \gamma)$, conditioned on the received SNR, $\gamma = [\gamma_1, \gamma_2, ..., \gamma_L]$, can be written as [76]

$$D(E, \gamma) = \sum_{j=0}^{J} P_j(E, \gamma) \phi(T_j(E))$$

(6.25)
where

\[ P_j(F, \gamma) = \begin{cases} 
\text{Prob}(X > f_1 | \gamma) & j = 0 \\
\text{Prob}(f_j + 1 < X \leq f_j | \gamma) & j = 1, \ldots, J - 1 \\
\text{Prob}(X \leq f_J | \gamma) & j = J. 
\end{cases} \]  

(6.26)

In above equations, \( \phi \) represents the operational rate-distortion function of the source encoder, \( X \) is the number of erased RS code symbols and \( T_j(F) = \sum_{k=1}^{j} m_k \) is the number of source symbols in the first \( j \) segments. Note that \( T_0(F) = 0 \), and thus \( \phi(T_0(F)) = \phi(0) \) corresponds to the distortion when no transmitted source symbol is correctly decoded, and so the decoder must reconstruct the source without using any of the transmitted information.

The FEC optimization goal is to determine the set of RS parity assignments that minimizes the conditional average distortion expressed in (6.25). Given a set of received SNRs, \( \gamma \), and given the operational distortion-throughput function \( \phi(T_j(F)) \), the problem is as follows:

\[ D^* (\gamma) = \min_{F \in \mathbb{F}} \{ D(F, \gamma) \} = \min_{F \in \mathbb{F}} \left\{ \sum_{j=0}^{J} P_j(F, \gamma) \phi(T_j(F)) \right\} \]  

(6.27)

where \( \mathbb{F} \) denotes the set of \( J \)-tuples \((f_1, f_2, \ldots, f_J)\) such that \( f_1 \geq f_2 \geq \ldots \geq f_J \).

The hill climbing approach proposed in [77] is adopted to find the optimal FEC assignment for the RS codewords. At each iteration, the algorithm examines \( 2QJ \) possible assignments; \( Q \) is the maximum number of parity symbols that can be added or subtracted to a codeword in one iteration, and \( J \) is the number of codewords. The conditional distortion is evaluated using (6.25) after adding or subtracting one parity symbol to each codeword. Lastly, the FEC allocation, \( F \), with the lowest distortion is chosen.

### 6.5 Simulation Results and Discussions

In this section, simulations are performed on a 128 x 128 gray-scale Lena image, which is encoded using the Set Partitioning in Hierarchical Trees (SPIHT) [78] algorithm to produce an embedded bitstream. To analyze the image quality,
the peak-signal-to-noise ratio (PSNR) performance metric is used, which defined as

\[ PSNR = 10 \log \frac{255^2}{E\left[D^*\left(\gamma\right)\right]} \text{dB} \quad (6.28) \]

where \( E\left[D^*\left(\gamma\right)\right] \) is the expectation of the distortion in (6.27).

### 6.5.1 Adaptive Modulation based on Instantaneous SNR

We compare the two proposed schemes against a fixed rate, fixed power baseline scheme. For the latter scheme, adaptive modulation is not used, but rather a 4-QAM constellation is used for all the subchannels. The parameters for the simulations are as follows: \( L = N_c = 16, \ M_c = 1, \ z = 255 \text{ symbols}, \ J = 51 \) and \( P_{\text{total}} = 16 \).

Fig. 6.5 compares the PSNR of the received image for the three systems, and the variable rate, variable power performs the best. The PSNR for the non-adaptive scheme saturates at high SNR, as the low constellation size limits the maximum source data that can be transmitted. For the adaptive modulation systems, the amount of transmitted source data varies according to the channel conditions, which, in turn, results in a higher PSNR. In particular, at low SNR, the variable rate, variable power scheme gives a significant gain. When the channel conditions are bad, system performance can be improved by transmitting fewer descriptions (descriptions with a zero power assignment are, by definition, not transmitted), each with a higher probability of being received. At high SNR, when the channels are all relatively good, changing the constellation (variable rate, fixed power) alone is sufficient to enhance the system performance.

To obtain a better insight for the performance of the three schemes, Fig. 6.6 illustrates the packet error rate of the three systems. It can be observed that the packet error rate of the variable rate, fixed power scheme is similar to the fixed rate, fixed power scheme at low SNR. This is because the variable rate, fixed power scheme has a high probability of choosing the lowest order constellation (i.e., 4-QAM) at low SNR, namely the same constellation that the fixed rate, fixed power scheme uses. When the SNR increases, the variable rate, fixed power scheme will
Figure 6.5: Comparison of PSNR for three schemes: Variable rate variable power, variable rate fixed power and fixed rate fixed power.

switch to a constellation size higher than 4-QAM, which results in a higher packet error rate. The packet error rate for the variable rate, variable power system is the lowest among the three systems. This is because if one particular subchannel is bad, this scheme will not assign power to it. Instead, the power will be allocated to the relatively good channels, and thus increase the probability of correct detection.

Fig. 6.7 compares the performance between a system with a uniform constellation for all the subchannels and a system with variable constellation for each subchannel (variable rate, fixed power). For the system with a uniform constellation, the system throughput prior to RS decoding for three different constellation sizes, 4-QAM, 16-QAM and 64-QAM, are evaluated, and the constellation size with the highest throughput is chosen for all subchannels. That is, for both of the schemes, the constellation size is changed based on the channel conditions. However, for the uniform constellation scheme, all subchannels have to adopt the same constellation, whereas for the variable constellation scheme, each subchannel can adapt its constellation size to its channel conditions. It can be observed
that when SNR is low, the two systems perform similarly, for the same reason as discussed relative to Fig. 6.6. In addition, similar to Fig. 6.6, when SNR is high, each subchannel in the variable constellation scheme will choose a constellation size that fits its current channel condition, thus yielding better performance than the uniform scheme.

6.5.2 Adaptive Modulation based on Average SNR

We first concentrate on the case where all the subcarriers are correlated, i.e., $M_c = L = 16$, in which case the throughput variance is the largest. We compare the PSNR of the transmitted image using the average throughput maximization and using the variance-aware method. Simulations for different test images, i.e., ‘Shuttle’, ‘Goldhill’, ‘Lena’ and ‘Tiffany’, have been carried out. As can be seen from Fig. 6.8, these images have different degrees of complexity. However, we observed similar trends for all of these images. Hence, we only present the result for ‘Shuttle’ (the most complex image among these four images) in Fig. 6.9, and for ‘Lena’ in Fig. 6.10. With the same number of successfully received bits, ‘Shuttle’ has the most distortion and the lowest PSNR among the four test images,
**Figure 6.7**: Comparison between variable constellation size at different subcarriers and same constellation size for all subcarriers.

approximately 8dB below the least complex image (‘Tiffany’).

As observed in Fig. 6.9 and Fig. 6.10, the variance-aware method outperforms the average throughput maximization method by as much as 4dB. Note that a dramatic drop in PSNR for average throughput maximization occurs when switching from 4-QAM to 16-QAM, and from 16-QAM to 64-QAM. This performance decline occurs when changing to a higher constellation size at a SNR value lower than the optimal threshold. The result suggests that when only the average channel state information is available, the threshold for switching to a higher constellation should be more conservative.

Consider now how the optimal $K$ value changes with different types of images. We observed that the optimum $K$ values for these four images are $K_{opt} = 0.005$ for ‘Shuttle’, $K_{opt} = 0.006$ for ‘Goldhill’, $K_{opt} = 0.007$ for ‘Lena’ and $K_{opt} = 0.007$ for ‘Tiffany’. Fig. 6.9 and 6.10 present the PSNR for an optimum K with the range of SNR from 10 to 42dB. To better understand the sensitivity of $K$, Fig. 6.11 is plotted. In Fig. 6.11, we take the average difference between the PSNR with the optimum K (as shown in Fig. 6.9 for ‘Shuttle’ and Fig. 6.10 for ‘Lena’).
and the PSNR with a $K$ ranging from 0 to 0.016. The range of $K$ was chosen to be approximately twice the optimum $K$ value. For each value of $K$, the average difference in PSNR is defined as

$$
\Delta = \frac{\sum_{i=1}^{N} |f_i(K_{\text{opt}}) - f_i(K)|}{N} \quad (6.29)
$$

where $N$ is the total number of SNR points in the PSNR curve used to get the difference. Specifically, we use $N = 17$ for SNR ranging from 10dB to 42dB with 2dB intervals. The plots suggest that the test images are not very sensitive to the optimal value of $K$. Results from slight variations ($\pm 0.003$) in $K_{\text{opt}}$ are still close to the optimal performance.

The previous plots (Fig. 6.9 through Fig. 6.11) are for cases where all the 16 subcarriers are identically correlated; the subsequent three figures examine the performance of the ‘Lena’ image with eight correlated subcarriers (Fig. 6.12), four

**Figure 6.8:** Test images (a) Shuttle, (b) Goldhill, (c) Lena, and (d) Tiffany.
correlated subcarriers (Fig. 6.13) and all independent subcarriers (Fig. 6.14). The optimum $K$ for ‘Lena’ obtained from the case of fully correlated subcarriers, $K_{opt} = 0.007$, is used for the simulations of the cases of eight correlated subcarriers, four correlated subcarriers and all independent subcarriers. Again, the performance of the variance-aware method is superior to the average throughput maximization method. However, we observe that the gain decreases with reduced number of correlated channels. This is because the throughput variance decreases with fewer correlated channels.

6.6 Conclusions

In this chapter, we proposed an adaptive modulation technique for transmitting progressive images with multiple description coding. The constellation size and power allocation for each description were chosen to maximize throughput. Compared with a system of fixed power and fixed constellation assignment, our proposed adaptive scheme resulted in superior performance. In particular, in

Figure 6.9: PSNR plot of ‘Shuttle’ image for average throughput maximization method and variance-aware method, $N_c = 1, \ M_c = 16$. 
Figure 6.10: PSNR plot of ‘Lena’ image for average throughput maximization method and variance-aware method, $N_c = 1$, $M_c = 16$.

a low SNR regime, changing both the power and constellation size will give us significant gain. In a high SNR regime, the extra gain achieved from allocating different power to each subchannel in addition to changing the constellation size is relatively small compared to the gain achieved from changing the constellation size alone. This suggests that rate adaptation is the key to achieve high performance at high SNR.

For the adaptive modulation based on average SNR, this chapter identifies the throughput variance as an important factor for determining the constellation size switching threshold. Large throughput variance caused the average throughput maximization results to diverge from average distortion minimization ones. This chapter provides an objective function which considers both the average and the variance of the throughput in determining the switching threshold. For adaptive modulation in progressive image transmission with multiple description coding, simulation results suggest that our variance-aware method increases the system performance significantly.

This chapter contains materials from “Adaptive Modulation for OFDM-
Figure 6.11: Average difference in PSNR (dB), $\Delta$ versus different value of $K$.

Based Multiple Description Progressive Image Transmission” presented at IEEE Global Telecommunications Conference (Globecom) in December of 2008 and “Variance Aware Adaptive Modulation for OFDM-based Multiple Description Progressive Image Transmission” presented at IEEE International Conference on Communications (ICC) in May of 2010. Both these papers were co-authored with Minjoung Rim, Pamela Cosman, and Lawrence Milstein.
Figure 6.12: PSNR plot of ‘Lena’ image for average throughput maximization method and variance-aware method with 8 subcarriers correlated, $N_c = 2, \ M_c = 8$.

Figure 6.13: PSNR plot of ‘Lena’ image for average throughput maximization method and variance-aware method with 4 subcarriers correlated, $N_c = 4, \ M_c = 4$. 
Figure 6.14: PSNR plot of 'Lena' image for average throughput maximization method and variance-aware method with all subcarriers independent, $N_c = 16$, $M_c = 1$. 
Chapter 7

Conclusions

In this dissertation, we studied physical layer aware cross-layer optimization for cognitive radio, MIMO and OFDM systems. In Chapter 2, we developed opportunistic channel access schemes where the transmissions are interleaved with periodic sensing in cognitive networks. For the proposed scheme, we obtained the optimal threshold and the optimal transmission period that jointly maximize the average throughput. Thereafter, in Chapter 3, we took into account the complex interactions between PUs and SUs and studied the tradeoff between total throughput and fairness across cooperative and non-cooperative settings, and how schemes based on utility functions and pricing mechanisms can be used to bridge this gap. In Chapter 4, we took some steps towards studying channel-aware distribution scheduling in ad-hoc networks under a delay constraint for real-time traffic. In particular, we characterized the fundamental tradeoff between the throughput gain from better channel conditions and the cost for channel probing that may cause the delay. In Chapter 5, we developed a distributed cross-layer MAC protocol based jointly on CSI and CDI for MIMO ad-hoc networks that bridges the gap between the physical layer’s coherence time and the MAC layer’s larger network time scale. Finally, in Chapter 6, we proposed an adaptive modulation technique for transmitting progressive images with multiple description coding.
7.1 Future Work

There are several avenues for continued research along the works in this dissertation. We are hopeful that the techniques and results in chapters 2 and 3 for developing channel access schemes for general cognitive radio networks will be useful for deriving channel access strategies for more accurate and complex network models and schemes as well as for addressing other concerns like energy-efficient transmission. In the multiuser setting, we consider a fixed transmission time when studying the complex interactions between PUs and SUs. In the future work, the transmission time can be optimized to improve the overall performance of the network.

In chapter 4, we considered average delay constraint. It would also be of interest to consider individual packet lifetime constraint. Different from average delay constraint, the packet life time constraint is imposed on scheduling of individual packets and is essentially a constraint on sample-path realizations. Besides considering delay constraints, imposing an outage constraint for each user would be useful for video streaming applications.

Chapter 5 presented preliminary simulation results for simple networks and in future, the schemes may be analyzed in the context of more realistic, random and large networks. There are still many issues regarding implementing CDI and CSI in the scheduling design. In particular, the collection and tracking of the CDI can be challenging.

Chapter 6 is a novel and special application of cross-layer optimization between application and physical layers and is one among many of such existing and future tightly-coupled system designs. We had only considered image transmission, it would be useful to consider adaptive modulation with unequal error protection in the video transmission applications.
Bibliography


