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TYPE-II SUPERCONDUCTORS

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ABSTRACT

The longitudinal critical current of a bulk type-II superconductor near the transition field H_{c2} has been calculated from the Ginzburg-Landau equations via a simple extension of Abrikosov's treatment of the mixed state, subject to the approximation that the self-field of the current can be ignored. The critical current is given by

$$(4\pi J_c/c) = (2/3)^{3/2} (H_{cb}/\lambda) (1-H/H_{c2})^{3/2} / \beta(1-1/2\kappa^2).$$

It is suggested that this expression may be approached for real wires in the limit of small radius.

1. INTRODUCTION AND DISCUSSION

We have calculated the longitudinal critical current of a bulk type-II superconductor near the transition field H_{c2} , subject to the approximation that the self-field of the current can be ignored. The calculation is a simple extension of the Abrikosov¹ treatment of the mixed state (based on the Ginzburg-Landau² equations). The result is

$$(4\pi J_c/c) = (2/3)^{3/2} (H_{cb}/\lambda) (1-H/H_{c2})^{3/2} / \beta (1-1/2\kappa^2).$$

This differs from the result obtained by a simple free energy argument³ only by a numerical factor of order unity. It is of the same form and order of magnitude as the critical surface current density,^{4,5} except for the substitution of H_{c2} for H_{c3} . Although the result is derived subject to the approximation $H_{c2} - H \ll H_{c2}$, it is interesting to note that for zero field the result is identical to the Ginzburg-Landau critical current of a thin film in zero field⁶ except for the numerical factor $\beta(1-1/2\kappa^2)$ which is of order unity.

Unfortunately the current densities permitted by our equation are so large that in most circumstances it is a rather poor approximation to ignore their self-field. For instance, if we consider a material with $H_{cb}/\lambda \approx 2.5 \times 10^7$ Oe/cm and $H_{c2} \approx 80$ kOe, then at 60 kOe we have $J_c \approx 10^6$ A/cm². The self-field of the current at the surface of a standard 10 mil wire would be $H \approx 5$ kOe. If the current flows parallel to the axis of the wire, then the product $\alpha = |J \times H| \approx 5 \times 10^6$ kOe A/cm² at the surface. "Pinning strengths" of this order of magnitude can be achieved in some materials, to which--unfortunately-- the theory does not strictly apply, since such materials tend to be rather inhomogeneous. Such scanty data as are available

on longitudinal critical currents indicate that our predicted values are approached for materials with strong pinning^{3,7} but not for homogeneous well-annealed alloys.⁸

The self-field of the current can be reduced considerably by making the radius of the wire smaller and going to higher longitudinal fields (lower current densities). In this way we can to some extent replace bulk pinning with surface pinning, and so should be able to see the predicted behavior in more homogeneous materials. However, we then run into the problem of separating the contribution of surface currents⁹ from that of the bulk. At H_{c2} itself, for instance, the current is carried entirely by the surface.

Despite the difficulties we have mentioned, our expression for the bulk critical current should be correct in the limit of small radius and $H_{c2} - H \ll H_{c2}$.

2. DERIVATION OF LONGITUDINAL CRITICAL CURRENT

We shall follow Abrikosov¹ closely. We assume an infinite superconductor. The external field H_0 and vector potential A are assumed to be directed along the z and y axes respectively. The units of field and length are the thermodynamic critical field $\sqrt{2} H_{cb}$ and penetration depth λ respectively. In these units the Ginzburg-Landau² equations can be written

$$\left(\frac{i\nabla}{\kappa} + A\right)^2 \Psi = \Psi (1 - |\Psi|^2) \quad (1)$$

$$-\nabla \times (\nabla \times A) = A |\Psi|^2 + \frac{i}{2\kappa} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) \quad (2)$$

Near the transition field $|\Psi|^2 \ll 1$, so that we may approximate the field with

$$A = H_0 x. \quad (3)$$

We include a phase factor $\exp(ijz)$ in Ψ to insure the presence of a longitudinal current, but neglect throughout the magnetic field due to this current, which amounts to neglecting A_z . We assume that Ψ is otherwise a function of x only:

$$\Psi = \exp(ijz) f(x). \quad (4)$$

Then, neglecting the term $\Psi|\Psi|^2$, we obtain for $f(x)$ the harmonic oscillator equation

$$d^2f/dx^2 + (\kappa^2 - j^2)[1 - \kappa^2 H_0^2 x^2 / (\kappa^2 - j^2)] f = 0 \quad (5)$$

which has solutions when $H_0 = (\kappa^2 - j^2)/\kappa(2n + 1)$. The nucleation field is given by the largest eigenvalue

$$H_0 = (\kappa^2 - j^2)/\kappa \quad (6)$$

and corresponds to the solution

$$\Psi = \exp(ijz) \exp[-(\kappa^2 - j^2)x^2/2]. \quad (7)$$

Equation (1) is also satisfied by the functions

$$\Psi = \exp \left\{ ijz + iky - \frac{(\kappa^2 - j^2)}{2} \left[x - \frac{k}{(\kappa^2 - j^2)} \right]^2 \right\}. \quad (8)$$

We choose a general solution of the form

$$\Psi = e^{ijz} \sum_n C_n e^{ikny} \psi_n(x),$$

$$\psi_n = \exp \left\{ - \frac{(\kappa^2 - j^2)}{2} \left[x - \frac{kn}{(\kappa^2 - j^2)} \right]^2 \right\}, \quad (9)$$

where k, C_n are arbitrary constants.

Let A_0 be the vector potential at the nucleation field,

$$A_0 = (\kappa^2 - j^2)x/\kappa. \quad (10)$$

We note the useful identity

$$\partial \Psi / \partial x = -\kappa [(i/\kappa) \partial / \partial y + A_0] \Psi, \quad (11)$$

which may be easily verified by differentiation of Eq. (9).

We substitute Ψ and A_0 into the equation for the current, Eq. (2), to obtain the first order correction to A . On making use of the identity Eq. (11), we get

$$\partial^2 A / \partial x^2 = - (1/2\kappa) (\partial / \partial x) |\Psi|^2 \quad (12)$$

$$H = \partial A / \partial x = H_0 - (1/2\kappa) |\Psi|^2 \quad (13)$$

$$A = H_0 x - (1/2\kappa) \int^x |\Psi|^2 dx' \quad (14)$$

which are identical to Abrikosov's results, since the phase j is contained only in $|\Psi|^2$. The constant H_0 was shown by Abrikosov to be the external field strength.

We now add to the factors $C_n \psi_n$ in Eq. (9) small terms $\psi_n^{(1)}$:

$$\Psi = \Psi^{(0)} + \Psi^{(1)} = \sum e^{ikny} (C_n \psi_n + \psi_n^{(1)}). \quad (15)$$

We substitute Ψ together with A given by Eq. (14) into Eq. (1). Keeping terms of first order in small quantities, and subtracting the linear equation satisfied by $\Psi^{(0)}$, we obtain

$$\left[\left(\frac{i\nabla}{\kappa} + A_0 \right)^2 - 1 \right] \Psi^{(1)} = \left[\left(\frac{i\nabla}{\kappa} + A_0 \right)^2 - \left(\frac{i\nabla}{\kappa} + A \right)^2 - |\Psi^{(0)}|^2 \right] \Psi^{(0)} \quad (16)$$

On multiplication by $\exp(-ikny)$ and integration over y , we get an inhomogeneous equation for $\psi_n^{(1)}$. For a solution to exist, the inhomogeneous part must be orthogonal to the solution of the corresponding homogeneous equation. But this is simply ψ_n . We multiply the inhomogeneous

geneous part by ψ_n , integrate over x , and set the result equal to zero:

$$\iint dx dy e^{-ikny} \psi_n \left[\left(\frac{i\nabla}{\kappa} + A_0 \right)^2 - \left(\frac{i\nabla}{\kappa} + A \right)^2 - |\Psi|^2 \right] \Psi = 0 \quad (17)$$

We discard the superscript on Ψ hereafter.

Setting $A_0 + A \approx 2A_0$ and discarding $\nabla \cdot A$, we get from Eq. (17) after applying Eq. (11)

$$\iint e^{-ikny} \psi_n \left[-\frac{2}{\kappa} (A_0 - A) \frac{d}{dx} - |\Psi|^2 \right] = 0. \quad (18)$$

At this point some caution is necessary. We wish to obtain a polynomial in $|\Psi|^2$ under the integral after we multiply by C_n^* and sum over n . For this purpose we must transfer the derivative to the term $(A_0 - A)$ by partial integration, which eliminates x as a coefficient. However, we must do this before performing the sum if the evaluated term is to be zero. The order of operations is as follows: we partially integrate half the derivative term, then multiply by C_n^* and sum over n . After adding the complex conjugate equation, we finally get

$$\iint |\Psi|^2 \left[\frac{1}{\kappa} \frac{d}{dx} (A_0 - A) - |\Psi|^2 \right] = 0. \quad (19)$$

The phase j appears explicitly only in the derivative term. On evaluation we obtain Abrikosov's relation

$$\frac{\kappa^2 - j^2 - \kappa H_0}{\kappa^2} \overline{|\Psi|^2} + \left(\frac{1}{2\kappa^2} - 1 \right) \overline{|\Psi|^4} = 0, \quad (20)$$

in which the quantity $(\kappa^2 - j^2)/\kappa$ has taken the place of κ as the bulk nucleation field. From this we obtain

$$\overline{|\Psi|^2} / 2\kappa = [(\kappa^2 - j^2)/\kappa - H_0] / (2\kappa^2 - 1) \beta \quad (21)$$

where

$$\beta = \overline{|\Psi|^4} / (\overline{|\Psi|^2})^2 \quad (22)$$

and β is independent of H_0 .

The natural unit of current is

$$(c/4\pi) (\sqrt{2} H_{cb}/\lambda),$$

which ensures that $J = \nabla \times H$ in this system of units. Then the mean longitudinal current is given by

$$J = (j/\kappa) \overline{|\Psi|^2} = 2j [(\kappa^2 - j^2)/\kappa - H_0] / (2\kappa^2 - 1) \beta. \quad (23)$$

We differentiate J with respect to j and set the derivative equal to zero to find the value of j for which J is a maximum:

$$\partial J / \partial j \propto \kappa^2 - \kappa H_0 - 3j^2 = 0 \quad (24)$$

or

$$j^2 = \frac{1}{3} (\kappa^2 - \kappa H_0).$$

The maximum occurs at a value of j only $1/\sqrt{3}$ of the maximum possible value of j .

In ordinary cgs units, the critical current is then given by

$$\frac{4\pi J_c}{c} = \left(\frac{2}{3}\right)^{3/2} \frac{H_{cb}}{\lambda} \frac{(1 - H/H_{c2})^{3/2}}{\beta(1 - 1/2\kappa^2)}. \quad (25)$$

The magnetization at the transition falls to two-thirds of its equilibrium value in the absence of current. The smallness of the change in the magnetization accounts for the fact that a simple free energy argument gives very nearly the same result for the critical current.

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