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LEARNING WITH UNDERSTANDING

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This paper is concerned with meaningful learning. Psychologists have distinguished between meaningful and rote learning (e.g., Katona, 1940; Wertheimer, 1945/1959) largely by providing examples that contrast the two phenomena. The work reported in this paper is an attempt to develop a more explicit and detailed theoretical analysis of the nature of learning that occurs with understanding.

I will consider learning situations in which new procedures and concepts are acquired for solving problems. Systems for learning procedures that have been analyzed previously are of two general kinds that I will call (1) direct learning and (2) analogical learning. I will describe a third kind of learning system in this paper that I call schematic learning.

In direct learning, examples are presented that show performance of the procedures that the learner is to acquire. Anderson et al (in press), Neves (1981), and Vere (1978) have studied processes of acquiring procedures that match the actions shown in examples or written theorems that correspond to inferential procedures. Processes in which fragments of procedures become integrated, forming new procedural concepts, also have been studied (Anzai & Simon, 1979; Larkin, in press; Neves & Anderson, in press), as have processes in which existing procedures are corrected, extended, or refined (Brown & VanLehn, in press; Goldstein, 1974; Neches, 1981; Sussman, 1975).

In analogical learning, a new procedure is acquired by mapping components of a known procedure to a new domain (Rumelhart & Norman, in press). The procedures that are transfered constitute new concepts that can be used to represent situations in the new domain.

In schematic learning, new procedures and concepts are formed in the framework of a general conceptual structure. A schema can provide a framework either for learning from examples or for analogical learning. I will discuss two examples that have been worked out in the form of running computational models that simulate salient aspects of student subjects' learning and performance. The first example involves learning to solve proof problems in geometry. This illustrates the role of a schema in learning from examples. The second example, which illustrates the role of a schema in analogical learning, involves learning procedures for multidigit subtraction in arithmetic.

Learning from an Example Proof

My first example is learning from the solution of a simple proof problem that is given early in a high-school geometry course. The problem and its solution are in Figure 1. I will discuss learning that can occur on the basis of this example problem, but first consider the problem in Figure 2, a problem that Wertheimer (1945/1959) discussed. Note that the solutions of these two problems are very similar in form. Three steps in Figure 2 correspond to the third step in Figure 1, but otherwise there is a simple mapping between the two solution proofs.

It might be expected that anyone who has learned to solve the problem in Figure 1 would also be able to solve Figure 2. It turns out that there is considerable variation in the success different students have with Figure 2 when it is presented as a transfer problem. A set of protocols on the problem in Figure 2 was obtained from students who had completed study of proof problems about line segments, such as Figure 1, and had begun to study properties of angles. Some students had no idea how to proceed. Others solved Figure 2 easily, and one even complained about having to solve "the same problem" so many times.

Consider the question: What enables a student to apply the knowledge acquired for solving Figure 1 to find a solution to Figure 2 easily? One hypothesis is that the procedures learned for solving Figure 1 were associated with general concepts that can be applied when Figure 2 is encountered. A version of this hypothesis has been implemented in a simulation program (see Anderson et al, in press, for a more detailed description).

The general structure that I postulate as the basis of transfer is a schema called Overlap/Whole/Parts. In this schema there are two components called "wholes," each of which is divided into parts, and a part of one whole is identical to a part of the other. I assume that in meaningful learning based on Figure 1, the Overlap/Whole/Parts schema is formed. Overlap/Whole/Parts has two subschemata, the Whole/Part structures that are included in the pattern. It is reasonable to assume that ninth-grade students have understood relationships of parts that form whole quantities for several years, and that they have some procedures associated with that schema. For example, they can add numbers associated with subsets to find the number in a superset, or subtract one part from the whole to form the other part.

The Overlap/Whole/Parts schema is formed as a combination of two Whole/Parts schemata, constrained so that a part of each "whole" component is shared with the other. Procedures that are attached to Whole/Parts are available in situations where the more complex structure is applied. In addition, some new procedures are also acquired and associated with the Overlap/Whole/Parts schema. For example, when the whole-components of the two substructures are equal, this enables the inference that the sums of their parts are equal, and when these sums are equal, the unshared parts are equal. (Learning of these procedures is based on Steps 4 and 5 in Figure 1.)

Two characteristics of the acquired knowledge are significant. First, the procedures that are acquired are defined on the components of the problem representations, which are the schematized versions of problems. This makes the procedural knowledge transferrable to other situations where the same schemata can be applied--for example, to problems such as Figure 2, if the system can learn to represent adjacent angles with the Whole/Parts schema. The second significant feature is that new conceptual entities are acquired when the schema of Figure 3 is learned. The organized structures of the wholes-with-parts are arguments of the new procedures, and thus function as cognitive units as a result of the learning that occurs.

Learning Subtraction Analogically

My second example involves the role of a schema in learning that is based on an analogy. This research has been done in collaboration with Lauren Resnick, who presents a companion paper in these proceedings. In our research, the learner does not construct the analogical mapping, as in the system that Rumelhart and Norman (in press) studied. Rather, the mapping between domains is presented in detail by an instructor. Performance of students indicates that this instruction leads to understanding of the procedure, and we consider the questions of what knowledge is acquired that constitutes this understanding and of how the acquisition occurs.

The procedure that we have studied in this research is arithmetic subtraction. Children who were chosen to participate in the research had one of the subtraction "bugs" identified by Brown and Burton (1978). Examples of performance that involves bugs are in Figure 3. The first problem illustrates a "smaller-from-larger" bug, where the student subtracts the smaller digit from the larger one in each column, ignoring which is on top. The second and third problems illustrate a "don't-decrement-zero" bug, where borrowing from a zero does not include decrementing another number to its left or a change in the value of the zero digit.

The instruction that is given uses a procedure for subtracting with blocks. Different sizes of blocks represent different place values: small cubes for units, long (1 x 10) sticks for tens, flat (10 x 10) pieces for hundreds, and so on. Instruction occurs in three stages. First, a procedure is taught for subtracting with blocks. Second, there is a detailed mapping of that procedure to the procedure of subtracting with written numerals. Finally, the written procedure is made independent of the blocks.

The critical phase is in Step 2, where the correspondence between the procedures with the blocks and with the written numerals is made explicit. Each action in the blocks domain corresponds to an action in the written domain. For example, when a child removes a "tens"-block during a trade, the digit in that column is decremented by one, and when ten "ones"-blocks are added to the display, a small "one" is placed in the units column, indicating that ten has been added to that digit. This instruction can be considered as presentation of a component-by-component mapping between two procedures.

This instruction has been successful in changing children's performance, a form of debugging. Furthermore, children give us evidence that they have achieved significant understanding of arithmetic concepts and principles. One example was given by a student who had suffered from the smaller-from-larger bug. After instruction, this student was asked how the new procedure differed from the one the student used earlier. The student said, "I used to take the numbers apart; now I leave them together, ... and take them apart." We think that this shows that the student had achieved an understanding that the digits in one of the rows of the problem represent parts of a whole entity that is, that together they represent a number.

A second example was given by a student who had a don't-decrement-zero bug. After doing the problem: 403 - 275 correctly, including manipulations with blocks, the student was asked, "Do you know where the nine came from?" The student said, "It's nine tens, and the other ten is right here," and pointed to the small 1 that was written to the left of the 3 in the top number of the problem. We think that this shows that the student understood the principle of conservation involved in borrowing, that the numerals resulting from the borrowing procedure represent a quantity equal in value to reductions in another numeral.

Now consider the theoretical question: what knowledge is acquired in the instruction? Hypotheses about acquired knowledge should provide an explanation of the correct performance that results, as well as the evidence that students provide that they have achieved significant understanding. We will present two hypotheses. The simpler one uses an idea of schematic goals. The other hypothesis postulates that understanding of subtraction involves the Whole/Parts schema, the same structure to which we attribute understanding of the geometry problem considered above. The latter hypothesis has been implemented as a running program; the former is based on a suggestion by Robert Neches.

The hypothesis of schematic goals postulates that knowledge of the blocks procedure is organized in a way similar to Sacerdoti's (1977) system of hierarchical action knowledge, with higher-order actions providing a goal-based organization of lower-level actions in the procedure. Important goals for the blocks procedure include: (1) find an answer for each column; (2) if there are not enought blocks for a column, get some more; (3) if there are no blocks in a column where you need to get some more, get some blocks for that column. In the hypothesis of schematic goals, we assume that mapping instruction results in transferring the goals of the blocks procedure to the procedure with written numerals. We propose that these goals correspond to new cognitive units in the student's representation of subtraction with written numerals.

This organization can explain indications of understanding like those we presented earlier. The remark that the correct procedure "keeps the numbers together" is explained because the actions of Decrement-Top and Add-Ten are parts of the same general action. Similarly, the elementary actions Decrement, Makenine, and Addten are combined to form a larger structure, which could be the basis of the remark that "It's nine tens, and the other ten is right here."

The simulation that we have programmed is somewhat more complex than the hypothesis of schematic goals. Our reasons for implementing a more complex system were in protocols obtained as students learned about the procedures in he blocks domain. Instruction for this procedure involved a kind of discovery method, including questions such as, "Can you think of a way to get more blocks?" The student whose performance we tried to simulate showed several indications of understanding principles underlying the procedure without being shown the procedure. At one critical point, involving borrowing through zero, the student said "Ooh neat--Now I get it." We simulated the student's performance with a model in which adjacent digits are schematized as parts of a whole unit. Understanding of the part-and-whole relationship of adjacent digits enables the model to understanding borrowing through zero by co-ordinating a constraint of keeping a total quantity constant while adjusting the numbers of things in its parts. This is described in more detail in Resnick's companion paper.

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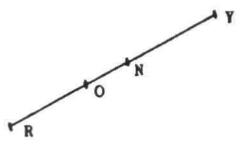
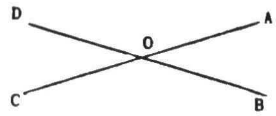


Given: RN = OY

Prove:: RO = NY

Statement	Reason
1. RN RO + ON	1. segment addition
2. OY = ON + NY	2. segment addition
3. RN OY	3. given
4. RO + ON = ON + NY	4. substitution
5. RO = NY	5. subtraction property

Figure 1



Given: $\overline{AOC}$, $\overline{BOD}$

Prove: ∠AOB = ∠COD

Statement	Reason
1. ∠AOC = ∠AOD + ∠COD	1. angle addition
2. ∠BOD = ∠AOB + ∠AOD	2. angle addition
3. ∠AOC = 180°	3. def. of straight ∠
4. ∠BOD 180°	4. def. of straight ∠
5. ∠AOC ∠BOD	5. substitution
6. ∠AOD + ∠COD = ∠AOB + ∠AOD	6. substitution
7. ∠COD = ∠AOB	7. subtraction

Figure 2

3 2 7	5 0 ¹2	⁶~~7~~ ¹0 ¹5
− 1 8 4	3 0 6	− 2 3 9
2 6 3	2 0 6	4 7 6

Figure 3